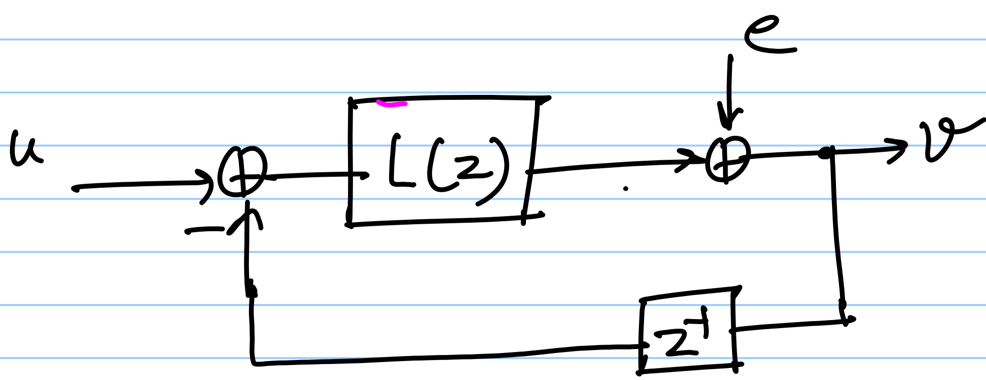
ECE 615 - Lecture 10

Note Title 2/11/2016



$|L(z)| \to \infty$ at $z = 1$

$\Rightarrow \omega = 0$

$\Rightarrow$ DC

$|u - v| \to 0$ in the signal band

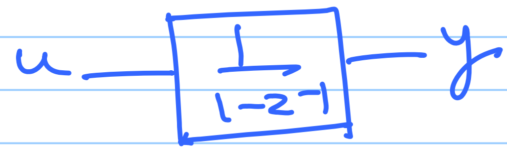
Uncorrelated with $U(z)$

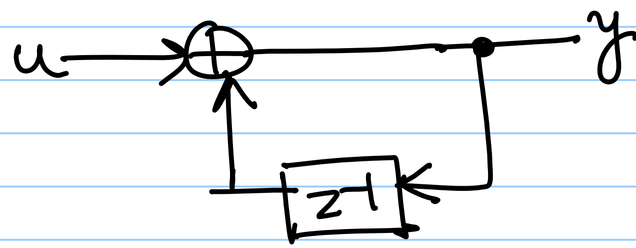
$$V(z) = \boxed{\frac{L(z)}{1 + z^{-1} L(z)}} \cdot U(z) + \boxed{\frac{1}{1 + z^{-1} L(z)}} \cdot E(z)$$

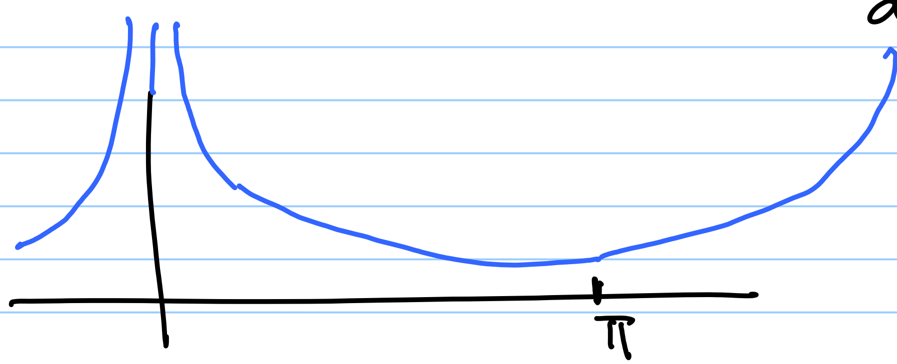
STF(z) signal transfer function

NTF(z) : noise transfer function

Let $L(z) = \frac{1}{1-z^{-1}}$







$$\frac{Y(z)}{U(z)} = \frac{1}{1-z^{-1}}$$

$$\Rightarrow Y(z) - z^{-1} Y(z) = U(z)$$

$$\Rightarrow y(n) - y(n-1) = u(n)$$

$$\Rightarrow y(n) = y(n-1) + u(n)$$

accumulator

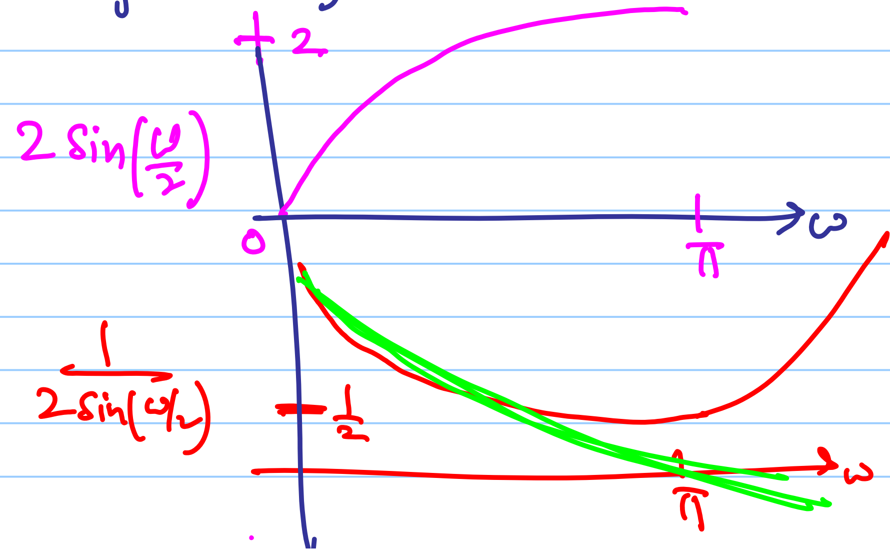
discrete integrator

$$X(z) = \frac{1}{1-z^{-1}}$$

$$X(e^{j\omega}) = \frac{1}{1-e^{-j\omega}} = \frac{1}{e^{-j\frac{\omega}{2}}\left(e^{j\frac{\omega}{2}} - e^{-j\omega/2}\right)}$$

$$= e^{j\frac{\omega}{2}} \cdot \frac{1}{2j\sin(\omega/2)}$$

$$|X(e^{j\omega})| = \frac{1}{2\sin(\omega/2)}$$



$$STF(z) = \frac{L(z)}{1+z^{-1}L(z)} = \frac{\frac{1}{1-z^{-1}}}{1+\frac{z^{-1}}{1-z^{-1}}} = \frac{\cancel{\frac{1}{1-z^{-1}}}}{\frac{1-\cancel{z^{-1}}-\cancel{z^{-1}}}{\cancel{1-z^{-1}}}} = 1$$

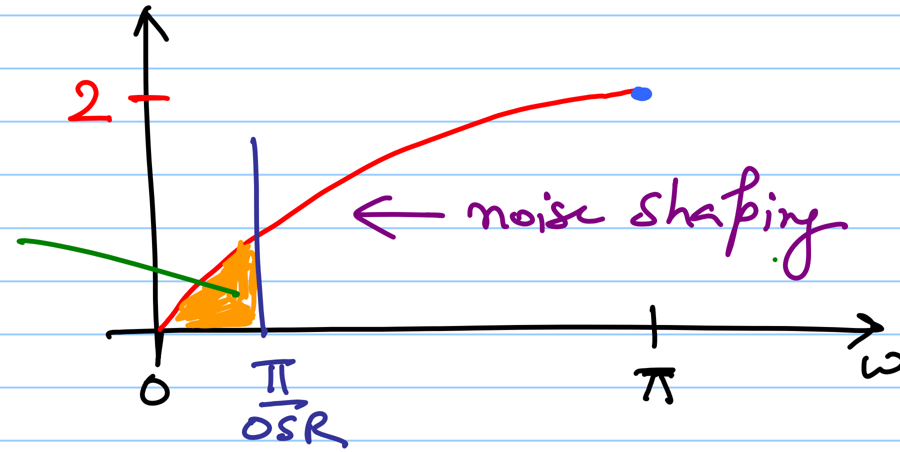
$$NTF(z) = \frac{1}{1+z^{-1}L(z)} = \frac{1}{1+\frac{z^{-1}}{1-z^{-1}}} = 1-z^{-1}$$

$\boxed{NTF(z) = 1-z^{-1}}$ & $STF = 1$

$$NTF(e^{j\omega}) = 1-e^{-j\omega} = e^{-j\frac{\omega}{2}} \times 2j \sin\left(\frac{\omega}{2}\right)$$

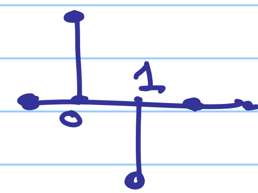
$$|NTF(e^{j\omega})| = |2\sin(\omega/2)| \leftarrow \text{High-pass response}$$

(in-band noise) IBN



$$NTF(z) = 1 - z^{-1}$$

$$h[n] = [1, -1]$$



$$\sum_n h[n] = 0 \rightarrow H(e^{j0})$$

$$\underbrace{\sum_n (-1)^n h[n] = 2}_{\text{intuitively HPF}} \rightarrow H(e^{j\pi})$$

output quantization noise

$E(z) \cdot NTF(z)$

$e[n] \circledast h[n]$

PSD of o/p quantization noise

$$= S_{n_o}(\omega) = S_e(\omega) \cdot \left|NTF(e^{j\omega})\right|^2$$

$$= \frac{\Delta^2}{12\pi} \cdot \left|2 \sin(\omega/2)\right|^2$$

for $\omega < \frac{\pi}{OSR}$

$\Rightarrow \omega \ll \pi$

$$\simeq \boxed{\frac{\Delta^2}{12\pi} \cdot \omega^2}$$ at low frequencies

random process $\qquad\qquad\qquad\qquad$ random process

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

noise $\longrightarrow$ $S_y(\omega) = S_x(\omega)\,|H(e^{j\omega})|^2$

power spectral density

(PSD)

Inband Noise

$$IBN = \int_0^{\pi/OSR} \frac{\Delta^2}{12\pi} \omega^2 \, d\omega$$

$$= \frac{\Delta^2}{12\pi} \cdot \left. \frac{\omega^3}{3} \right|_0^{\pi/OSR} = \frac{\Delta^2}{12\pi} \left(\frac{\pi}{OSR}\right)^3 \cdot \frac{1}{3}$$

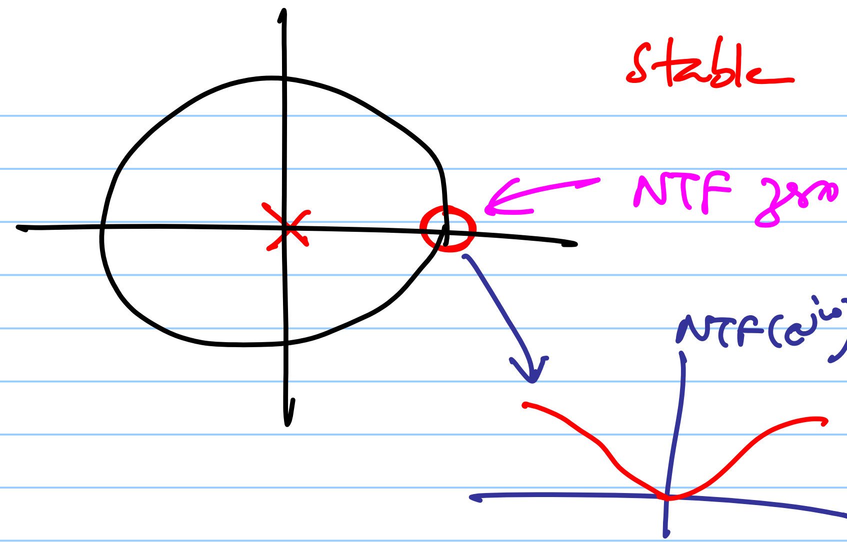
$$= \frac{\Delta^2 \pi^2}{36 \cdot OSR^3}$$

$$\boxed{IBN = \frac{\Delta^2 \pi^2}{36} \cdot OSR^{-3}}$$

$\frac{\Delta^2}{12} OSR^{-1}$ for plain oversampling

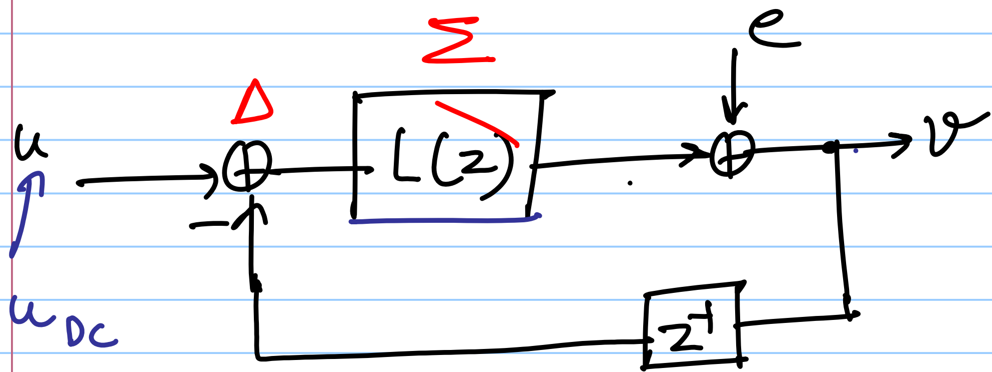
$$NTF(z) = 1 - z^{-1}$$

$$= \frac{z-1}{z}$$



$$L(z) = \frac{1}{1-z^{-1}} \Leftarrow 1^{st}\text{-order}$$

1^{st}-order modulator



$$\bar{u} = \lim_{N\to\infty} \frac{1}{N} \sum_{n} v[n]$$

Oversampling

$IBN = \frac{\Delta^2}{12} OSR^{-1}$

$\Rightarrow$ 2X . OSR Noise $\downarrow$ 3dB

$\Rightarrow$ SQNR $\uparrow$ by 3dB

$\Rightarrow$ ENOB $\uparrow$ 0.5 bits

Total noise $= \frac{\Delta^2}{12}$

First-order noise-shaping

$IBN = \frac{\Delta^2 \pi^2}{36} OSR^{-3}$

$\Rightarrow$ Noise $\downarrow$ 9dB

SQNR $\uparrow$ 9dB

ENOB $\Rightarrow$ 1.5 bits per doubling with OSR

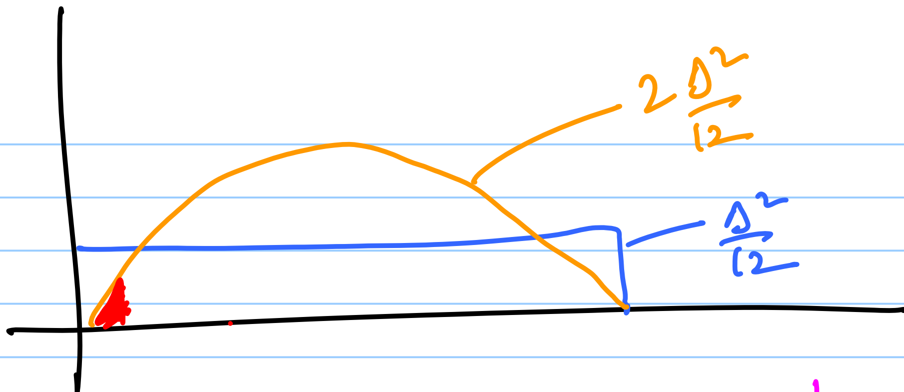
Total noise

$$= \frac{\Delta^2}{12\pi} \int_0^{\pi} |NTF(e^{j\omega})|^2 d\omega$$

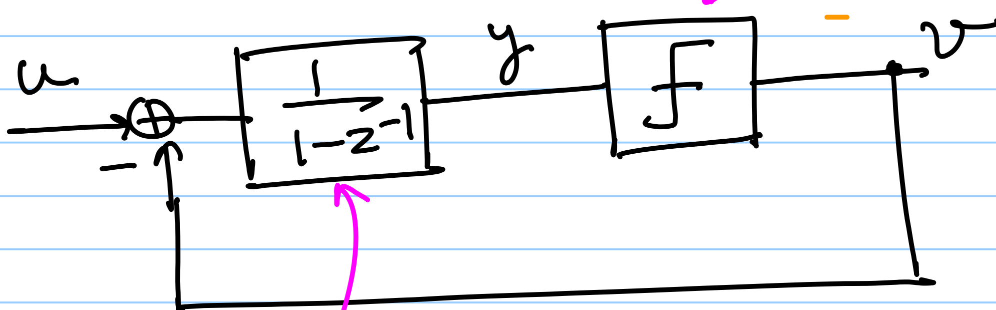
Parseval's Theorem

$$= \frac{\Delta^2}{12\pi} \left(\pi \sum_n |h(n)|^2\right)$$

$$= \frac{\Delta^2}{12\pi} \times 2\pi = 2\left(\frac{\Delta^2}{12}\right)$$

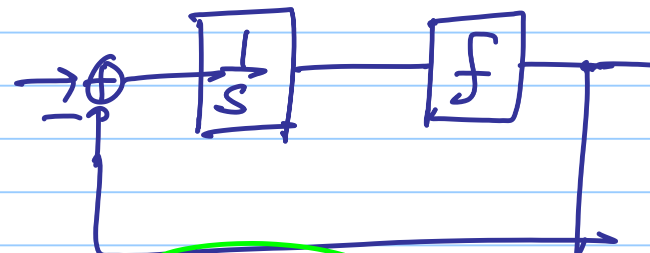


comparator

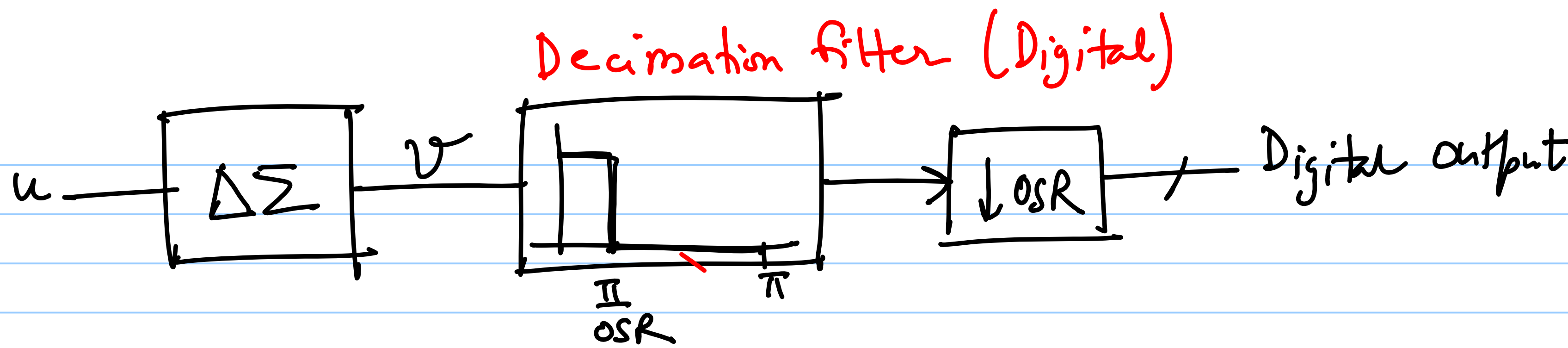


Switched Capacitor Integrator

Discrete-time $\Delta\Sigma$

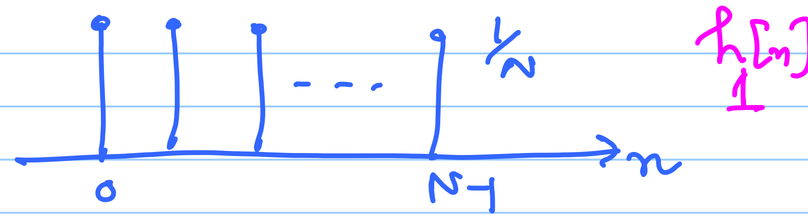


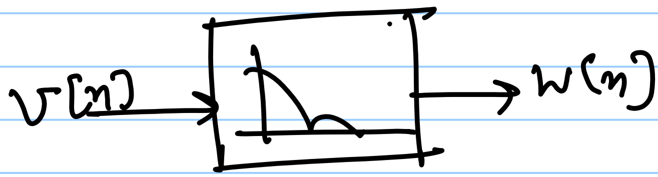
CT-$\Delta\Sigma$



Decimation filter (Digital)
u
ΔΣ
v
π/OSR
π
↓OSR
Digital output

FIR filter





$$w(n) = \sum_{i=-\infty}^{\infty} h[i]\, v[n-i]$$

$$= \frac{1}{N} \sum_{i=0}^{N-1} v[n-i]$$

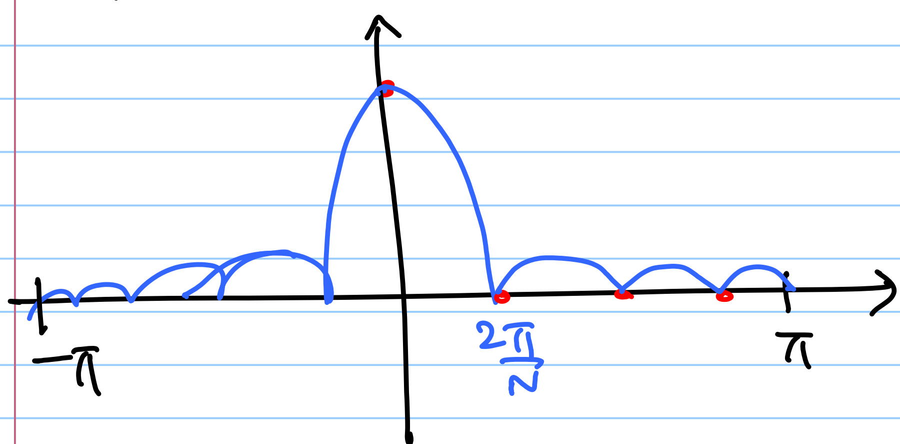
→ moving average filter over the modulator output.

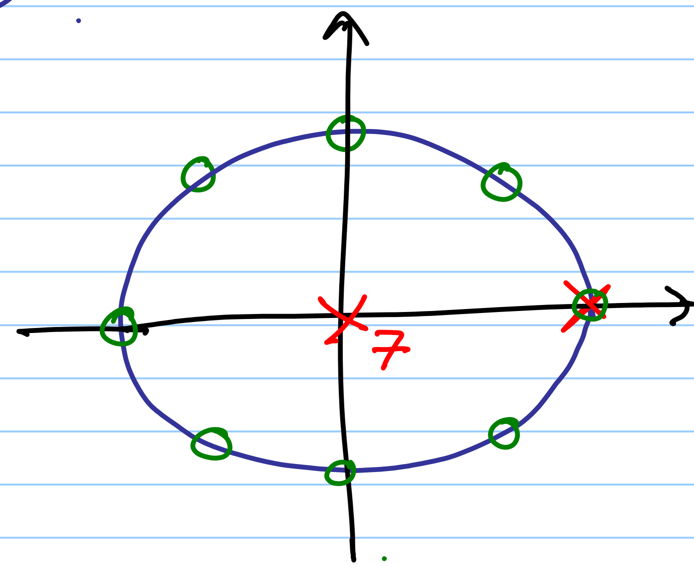
$$h_1[n] = \begin{cases} 1/N, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

$$H_1(z) = \frac{1}{N}\left(\frac{1-z^{-N}}{1-z^{-1}}\right)$$

$z^N - 1 = 0$

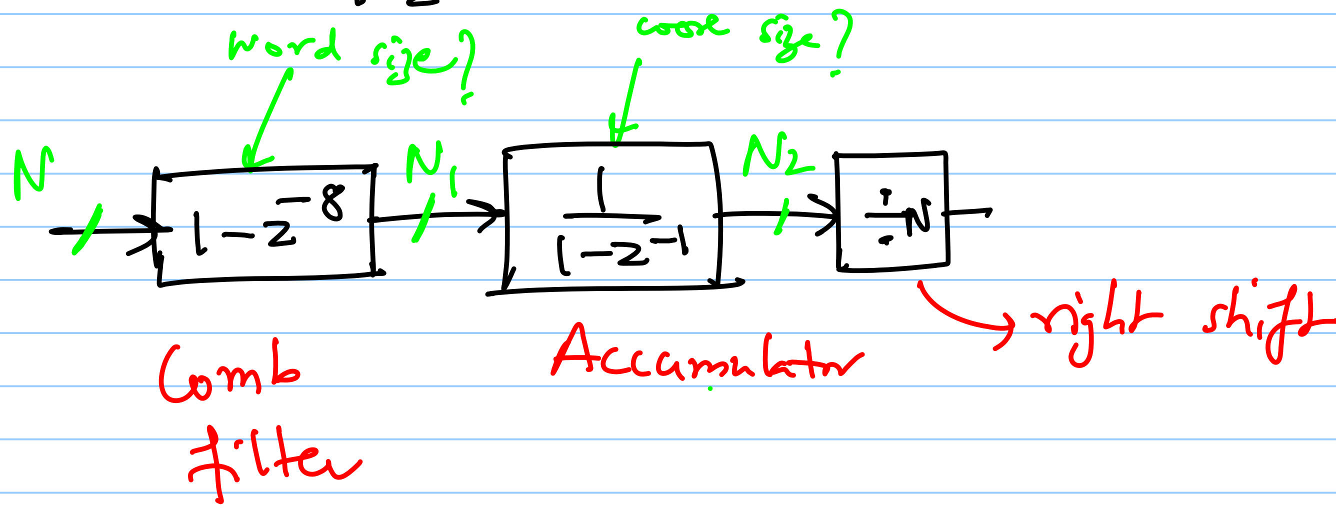
for $N=8$

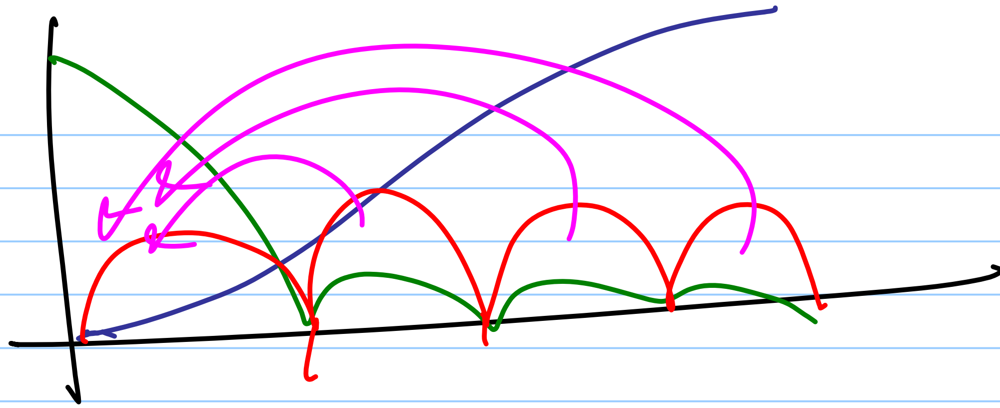




$$H(z) = \frac{1}{N} \cdot \frac{1 - z^{-N}}{1 - z^{-1}}$$

$N = 8$





"Sinc filters"