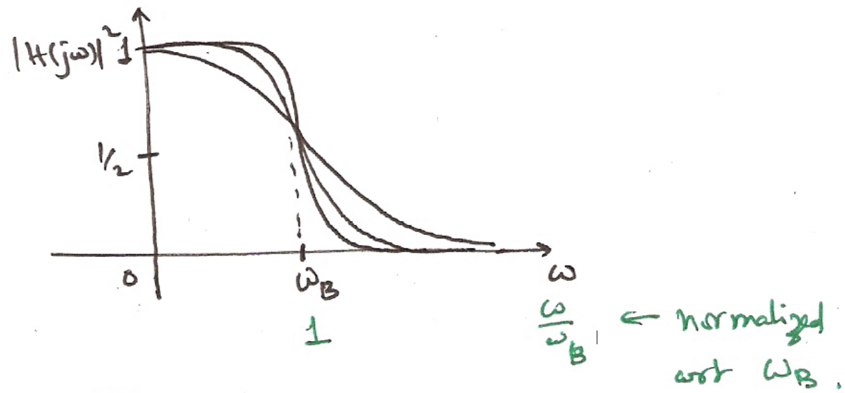
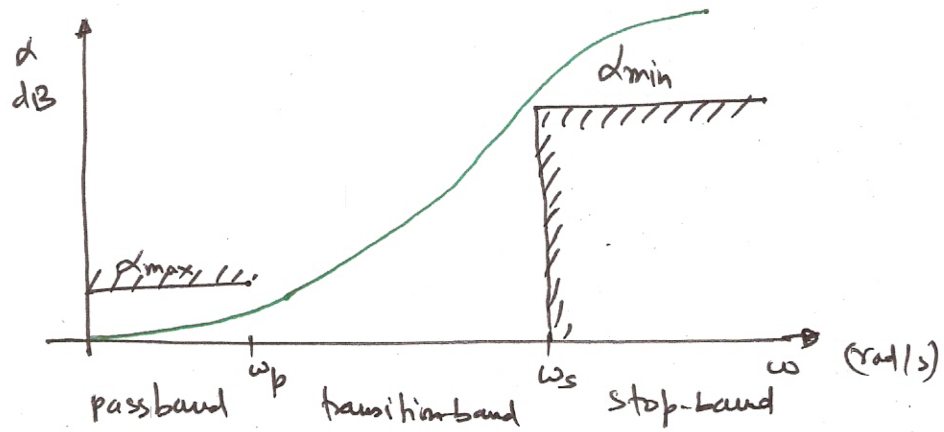
Butterworth LP filter Example.

$$H(s) = \frac{1}{1+\left(\frac{s}{\omega_B}\right)^n}$$



$$\Rightarrow \quad |H(j\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{\omega_B}\right)^{2n}}$$



Attenuation:

$$\alpha(\omega) = -20 \log |H(\omega)|.$$

$$|H(\omega)| = 10^{-\frac{\alpha(\omega)}{20}}$$

Define passband corner at $\omega = 1$ $\Rightarrow$ normalize ω as $\frac{\omega}{\omega_p}$

$\varepsilon \rightarrow$ "ripple width"

$$\Rightarrow \quad |H(j\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{\omega_B}\right)^{2n}} = \frac{1}{1+\varepsilon^2\left(\frac{\omega}{\omega_p}\right)^{2n}} \quad \leftarrow \text{renormalized}$$

at $\frac{\omega}{\omega_p} = 1 \Rightarrow$ $\qquad \Rightarrow \varepsilon = \left(\frac{\omega_p}{\omega_B}\right)^n$

$$|H(j1)|^2 = \frac{1}{1+\left(\frac{\omega_p}{\omega_B}\right)^{2n}} = \frac{1}{1+\varepsilon^2} \quad \Rightarrow \boxed{\omega_B = \varepsilon^{-1/n}\omega_s} \longrightarrow (1)$$

$\Rightarrow$ At the passband corner

$$\alpha_{max} = 10 \log (1+\varepsilon^2)$$

$$\Rightarrow \boxed{\varepsilon^2 = 10^{0.1\,\alpha_{max}} - 1} \longrightarrow (2)$$

At the passband:

$$\alpha_{min} = 10 \log_{10}\left(1 + \varepsilon^2 \omega_s^{2n}\right) \longrightarrow (3)$$

from (2) and (3)

$$\alpha_{min} = 10 \log\left[1 + \left(10^{0.1\alpha_{max}} - 1\right)\omega_s^{2n}\right]$$

$$\Rightarrow \boxed{\omega_s^{2n} = \frac{10^{0.1\alpha_{min}} - 1}{10^{0.1\alpha_{max}} - 1}}$$

$$\Rightarrow \boxed{n = \frac{\log\left[\dfrac{10^{0.1\alpha_{min}} - 1}{10^{0.1\alpha_{max}} - 1}\right]}{2 \log \omega_s}} \longrightarrow (4) \leftarrow \text{gives the order.}$$

Design procedure:

1. using Eq (2) determine parameter $\varepsilon \rightarrow$ sets α_{max}
2. using Eq (4) calculate the degree n and round it to the next largest integer
3. Calculate the normalizing frequency $\boxed{\omega_B = \varepsilon^{-1/n} \omega_p}$

$\hookrightarrow$ 3-dB Bandwidth

Matlab functions:

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