

# Assignment 5

ECE 615 – Mixed Signal IC Design

## Problem 1 Continuous-time $\Delta\Sigma$ Modulator with Butterworth NTF:

1. Determine the NTF of a third-order  $\Delta\Sigma$  modulator with a Butterworth high-pass response and an out of band gain (OBG) equal to 3. For the Butterworth response, all the NTF zeros are at DC (i.e.  $z = 1$ ). Assume that the design is normalized to a clock frequency ( $f_s$ ) of 1 Hz.
2. The modulator is to be implemented using the **CIFB** architecture. Also, assume in this problem that the excess loop delay (ELD) is equal to zero. What is the loop-filter response  $L(z) \triangleq L_1(z)$  seen by the quantization noise ?
3. Plot the impulse responses,  $l(n)$  of the loop-filter seen by the quantization noise, and  $h(n)$  of the NTF( $z$ ). Also show the respective pole-zero plots.
4. Now, using the forward impulse-invariant transform table, Table II in [?], analytically find the transfer function of the continuous-time loop-filter  $L(s)$  for the **NRZ** DAC pulse-shape. Sketch the ideal block-level implementation of the resulting CT  $\Delta\Sigma$  modulator. Plot the frequency response of  $\hat{L}(s)$  using MATLAB.
5. Drive the designed loop-filter  $\hat{L}(s)$  with the DAC pulse response in time-domain and plot the continuous-time open-loop response  $l(t)$ . Show that the sampled loop-response (with  $T_s = 1$ ) is same as the impulse response  $l[n]$  seen in part (3).
  - (a) From this observation, what can you conclude about the NTF of the overall CT DSM?
  - (b) Is it overall a discrete-time or a continuous-time system? Can you ever write NTF and STF responses as  $NTF(s)$  and  $STF(s)$  respectively?
6. Determine the  $STF(e^{j\omega})$  for your design and sketch the spectrum. Can you plot the STF spectrum using MATLAB?
7. Consider the MATLAB  $\Delta\Sigma$  toolbox function **realizeNTF\_ct** used for synthesizing CT  $\Delta\Sigma$  modulators.
  - (a) Verify your answer in part (4) with the result using the toolbox. What algorithm does the toolbox function use for determining the ABCD matrix description of  $\begin{bmatrix} L_0(s) & L_1(s) \end{bmatrix}^T$ .
  - (b) Use **mapCtoD** toolbox function to ensure that  $L(s)$  from parts (4&6) results in the desired  $NTF(z)$ .
8. Repeat parts (4) to (7) for a **RZ** DAC pulse-shape given by  $r_{DAC}(t) = (0, 0.5)$ .

## Problem 2 Excess-Loop Delay in CT $\Delta\Sigma$ Modulators:

1. Determine the NTF of a third-order  $\Delta\Sigma$  modulator with a Butterworth high-pass response and an out of band gain (OBG) equal to 2.5. Assume that the design is normalized to a clock frequency ( $f_s$ ) of 1 Hz. Plot the NTF response and the pole-zero plot.
2. Assuming an **NRZ** DAC pulse-shape and a **CIFF** architecture, determine the loop-filter coefficients  $\{k_1, k_2, k_3, k_4\}$  for the continuous-time loop-filter  $\hat{L}(s)$ . Initially assume that the excess loop delay ( $\tau$ ) is zero. Sketch the loop-filter block diagram. You may use MATLAB to do this computation.
3. Plot the CT and sampled loop-responses  $l(t)$  and  $l[n]$  respectively for the CT-DSM.
4. Now, due to design considerations an excess loop delay of 40% of the clock-period is introduced (i.e.  $\tau = 0.4$ ).
  - (a) Find the equivalent  $NTF(z)$  and plots its response and the pole-zero plot. What do you observe?
  - (b) Plot  $l(t)$  and  $l[n]$  for the CT-DSM with the excess loop-delay.
  - (c) Show that the direct path in the loop-filter arising due to the ELD compensation, can be implemented as a direct path around the quantizer.
5. Find a method to compensate for the ELD so that the NTF is restored to the response seen in part (1).
  - (a) Use analytical calculations to determine the resulting loop-filter coefficients. You will realize that the usual method of computing the modified loop-filter coefficients using the Table III in [?] is unweildy for this design. You may use the algorithm introduced in [?].
  - (b) Verify your hand-calculations with the toolbox function **realizeNTF\_ct**.
  - (c) Plot  $l(t)$  and  $l[n]$  for the CT-DSM, and demonstrate that the ELD is fixed.
6. Repeat parts (3-5) for a **RZ** DAC pulse-shape given by  $r_{DAC}(t) = (0, 0.5)$  with  $\tau = 0.4$ . Do you need the direct path around the quantizer ?

## References