Assignment 3

ECE 615 – Mixed Signal IC Design

Problem 1 Consider the first- and second-order delta-sigma modulators shown in Figure 1. In both the modulators a 4-bit quantizer (i.e. 16 levels) is used. Use the *ds_quantize* function from the toolbox which uses a fixed LSB size of $\triangle = 2$. The input sinewave has an amplitude of 70% of the full-scale amplitude (i.e. $A = 0.7 \cdot FS$) and a frequency of $f_{in} = 1\,KHz$. The sampling frequency employed in the modulator is $f_s = 256\,KHz$. Do not use the *synthesizeNTF* function in the toolbox for this assignment as we haven't learned its algorithm yet.

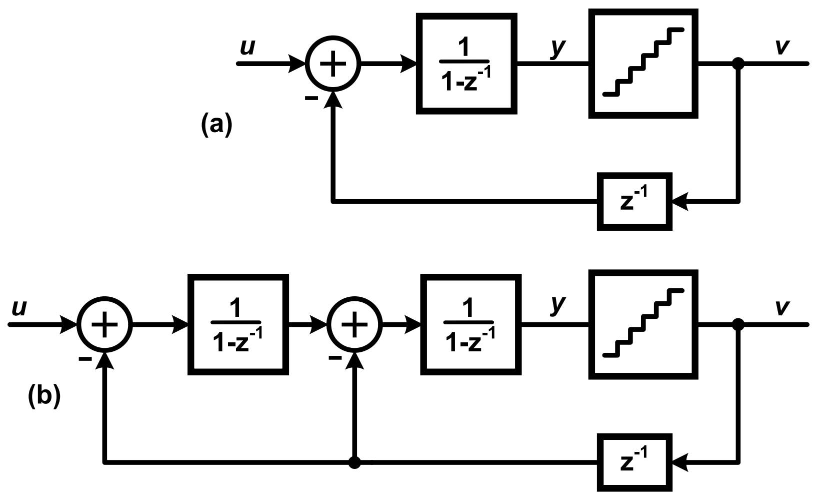

Figure 1: The First- and second-order $\triangle\Sigma$ modulators for problem 1.

1. Explore the *simulateDSM* function in the toolbox and find out the various ways in which the modulator's loop-filter can be defined. Simulate both the modulators using MATLAB for 2^{16} output samples. Attach the simulation code in your submission.

2. Overlay the PSD of the outputs (v) of both the modulators in the same plot with logarithmic frequency scale. Use a 4096-point Hann window for the PSD computations. What are the NTF slopes in the signal band of interest for both of the modulators?

3. For the first-order modulator, replace the quantizer with an additive uniformly distributed white noise-source given by $e \sim U[-1, 1]$. Plot the PSD of the output v and compare with the PSD obtained in part (2) with the quantizer. What do you observe ? Can you explain the anomalies observed in the result from part (2) ?

4. Repeat part (3) for the simple case of a single-bit quantizer. Can you explain your observations ?

5. To avoid the problem observed in part (3), we can *dither* the quantizer input. Dither involves adding a small amount of random noise source at the quantizer input. Repeat part (2) above,

by adding a Gaussian distributed white dither noise given by $d \sim n(0, 0.1)$ (i.e. zero mean and with a standard deviation of $\sigma = 0.1$) at the quantizer input. What do you observe in the PSD now compared to what you saw for part (2) ? How does this PSD plot compare with the one seen in part (3) ?

For further reading, details on dithering in delta-sigma modulators can be found in Chapter 3 of reference [2].

Problem 2 Decimation Filters: In this problem we will analyze and simulate the degradation in the ideal performance of the delta-sigma modulator due to a poor decimation filter. Consider the setup shown in Figure 2. Here, a delta-sigma modulator employing an oversampling ratio of OSR is excited by a signal with maximum frequency content at a frequency f_B. The modulator is followed by a digital decimation filter. The output rate of the modulator is decimated by a factor equal to OSR. The modulator internally employs a quantizer introducing a uniformly-distributed white quantization noise $e[n]$ with its variance given by σ_e^2.

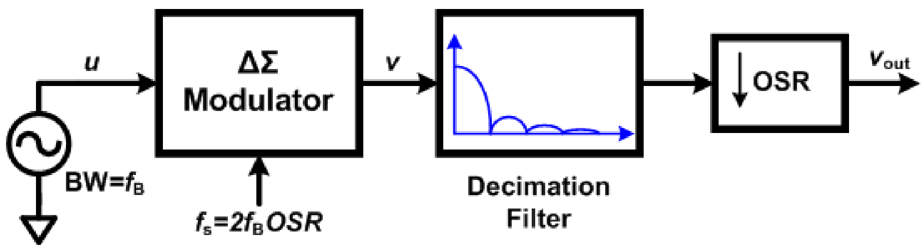

Figure 2: A $\triangle\Sigma$ modulator followed by the decimation filter.

1. Consider a first-order delta-sigma modulator (with $NTF(z) = 1 - z^{-1}$) be followed by a Sinc decimation filter. The impulse response of the Sinc filter is given as

$$h_1[n] = \begin{cases} \frac{1}{N}, & 0 \leq n \leq N-1 \\ 0, & otherwise \end{cases} \tag{1}$$

For this problem setup, we have $N = OSR$.

(a) Find the transfer function $H_1(z)$ of the Sinc filter and sketch its magnitude response $\left|H_1(e^{j\omega})\right|$.

(b) Sketch the DTFT spectra at all the block outputs. Use ω as the variable on the frequency axis.

(c) Show that the resulting quantization noise power (i.e the variance) σ_{q1}^2 at the output v_{out} is given by

$$\sigma_{q1}^2 = \frac{2\sigma_e^2}{OSR^2} \tag{2}$$

How does this compare with the resulting noise power when an ideal brick-wall filter is used with a cut-off frequency of $\frac{\pi}{OSR}$?

(d) How will you implement this decimation filter using discrete-time accumulator and comb-filters ? What is the droop in the signal band at $f = f_B$?

(e) Plot the resultant spectrum for the noise at the block outputs by using/modifying the following MATLAB script :

```
% Creating NTF(z) = 1-z^-1
b = [1 -1]; a = 1;
NTF = dfilt.df2t(b,a);
% Sinc filter of length 8, H1
N = 8; b1 = [1 0 0 0 0 0 0 -1]/N;
a1 = [1 -1];
H1=dfilt.df2t(b1,a1);
% Cascade of NTF followed by H1
Hcas=dfilt.cascade(NTF,H1);
% Plots
fvtool(NTF, H1, Hcas);
```

2. Now, in order to increase the effectiveness of the decimation filter and reduce the quantization noise aliased into the signal band, a $Sinc^2$ digital filter is employed.The transfer function of the $Sinc^2$ filter is given by

$$H_2(z) = \left[\frac{1}{N}\frac{\left(1 - z^{-N}\right)}{\left(1 - z^{-1}\right)}\right]^2 \tag{3}$$

(a) Sketch its magnitude response $\left|H_2(e^{j\omega})\right|$. Can you find the impulse response $h_2[n]$ of the $Sinc^2$ filter ?

(b) Sketch the DTFT spectra at all the block outputs and compare with your MATLAB plots. Use ω as the variable on the frequency axis.

(c) Show that the resulting quantization noise power σ_{q2}^2 at the output v_{out} is given by

$$\sigma_{q2}^2 = \frac{2\sigma_e^2}{OSR^3} \tag{4}$$

How does this compare with the resulting noise power when an ideal brick-wall filter is used with a cut-off frequency of $\frac{\pi}{OSR}$?

(d) How will you implement this decimation filter using discrete-time accumulator and comb-filters ? What is the droop in the signal band at $f = f_B$?

3. Now, consider a second-order delta-sigma modulator (with $NTF(z) = \left(1 - z^{-1}\right)^2$) be followed by the Sinc decimation filter.

(a) Sketch the DTFT spectra at all the block outputs and compare it with MATLAB generated plots.

(b) Find an expression for the resulting quantization noise power σ_{q1}^2 at the output v_{out} ? How does this compare with the resulting noise power when an ideal brick-wall filter is used with a cut-off frequency of $\frac{\pi}{OSR}$?

(c) Repeat parts (a) and (b) when a $Sinc^2$ digital filter is used.

(d) Repeat parts (a) and (b) when a $Sinc^3$ digital filter is employed. The transfer function of the $Sinc^3$ function is given by

$$H_3(z) = \left[\frac{1}{N} \frac{\left(1 - z^{-N}\right)}{\left(1 - z^{-1}\right)} \right]^3 \tag{5}$$

(e) How much is the signal droop at $f = f_B$? From the above exercise what can you intuit about the cut-off rate of the decimation filter when compared to the modulator's response at frequencies near $f = f_B$?

In practice, the decimation filtering scheme required for high-resolution higher-order modulators is more complicated than a simple cascade of *Sinc* filters. FIR filters are usually employed in the decimation chain to compensate for the droop due to the $Sinc^k$ filter. For details refer to Section 3.5 of the textbook. For a practical example of decimation filtering scheme refer to page 21 of the reference [3].

Hint: You may find the following definite integrals useful

$$\int_0^{\pi} sin^4 \left(\frac{Nx}{2} \right) dx = \frac{3\pi}{8} \tag{6}$$

$$\int_0^{\pi} \left(sin \left(\frac{x}{2} \right) sin \left(\frac{Nx}{2} \right) \right)^2 dx = \frac{\pi}{4} \tag{7}$$

$$\int_0^{\frac{\pi}{2}} \left(\frac{sin^{M+1}(Nx)}{sin(x)} \right)^2 dx = \frac{\pi N}{2} \left(\prod_{m=1}^{M} \frac{2m-1}{2m} \right) \tag{8}$$