ECE 615 - Lecture 8

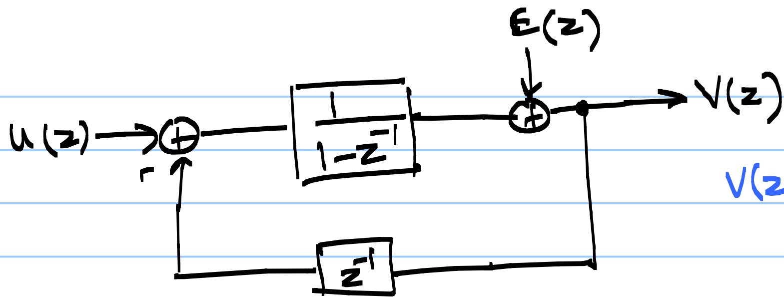
Note Title 9/26/2013

Decimation filter

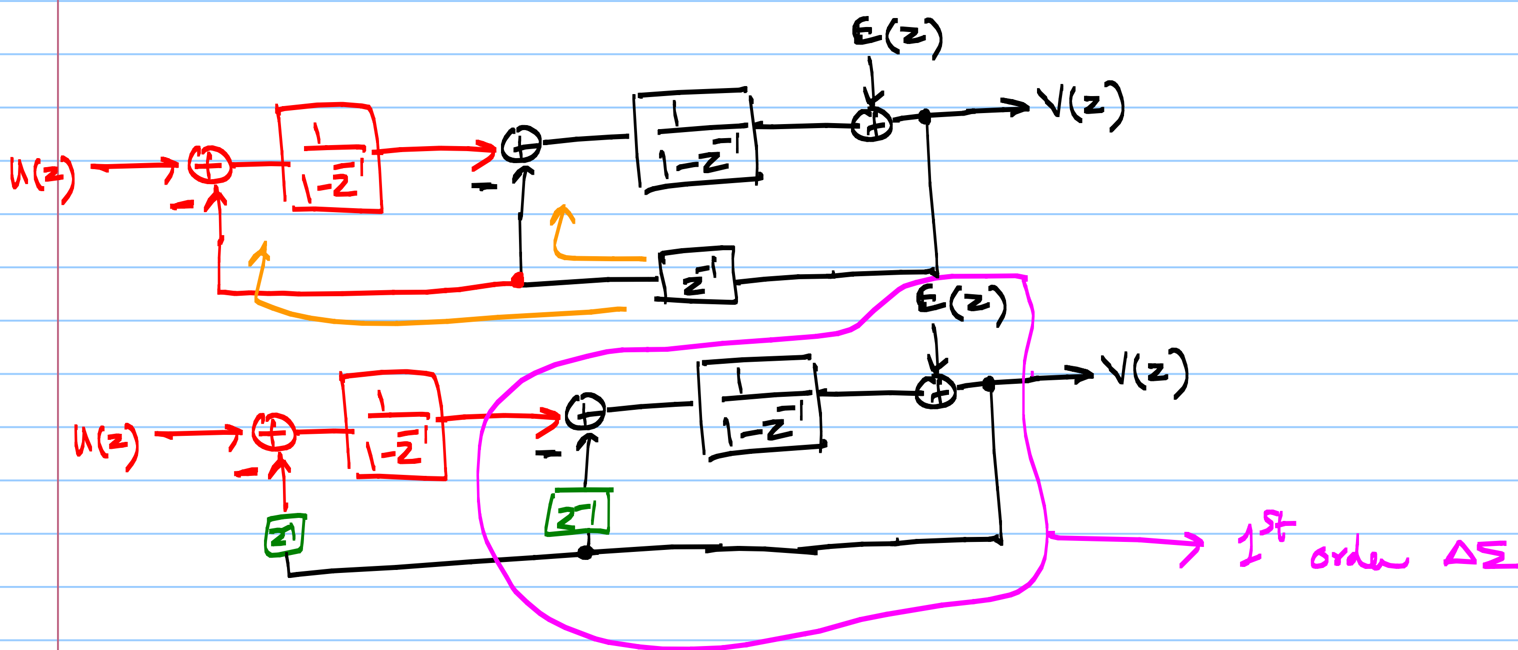
First order $\Delta\Sigma \Rightarrow$ sinc^2

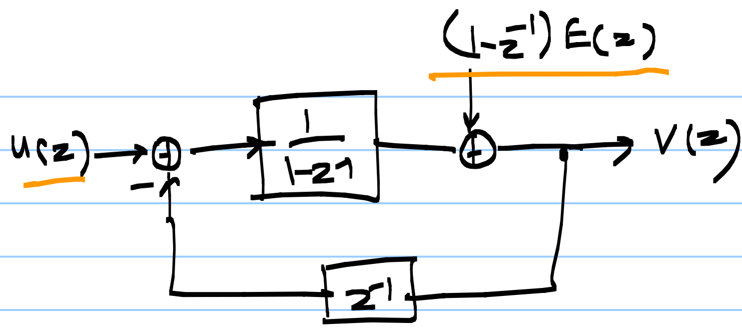
2^{nd} - order $\Delta\Sigma \Rightarrow$ sinc^3

2nd order ΔΣ Modulator



$$V(z) = U(z) + (1 - z^{-1})E(z)$$

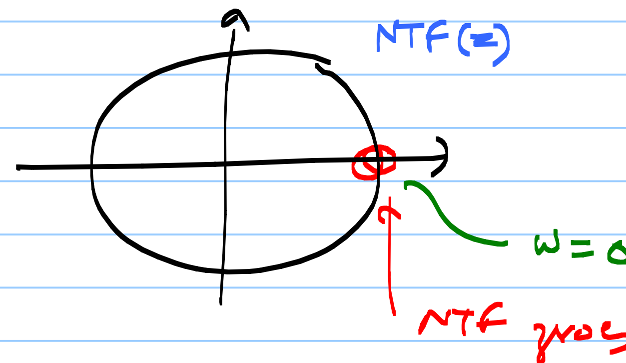


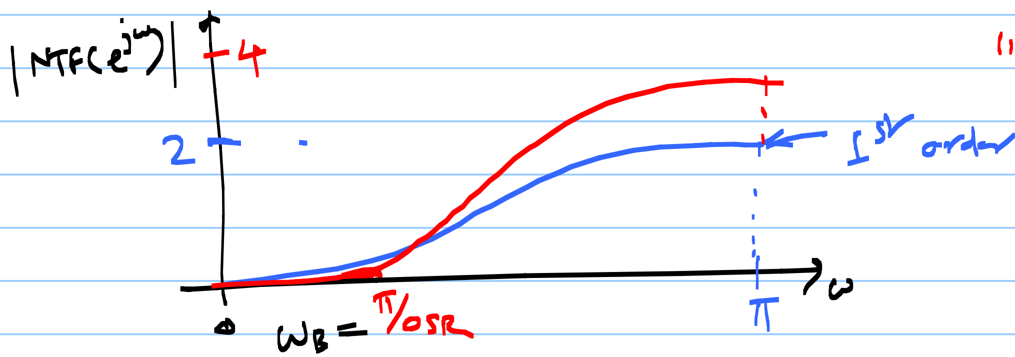


"Double differentiation of the quantization noise"

$$NTF(z) = (1-z^{-1})^2$$

$$STF(z) = 1$$





"two NTF zeros at DC!"

$(1-z^{-1})^2\Big|_{z=e^{j\omega}}$

for $\omega \ll \pi$

$|1-e^{-j\omega}| \simeq \omega$

$$IBN = \frac{\Delta^2}{12\pi}\int_0^{\pi/OSR} |(1-e^{-j\omega})^2|^2 \, d\omega$$

$$\simeq \frac{\Delta^2}{12\pi}\int_0^{\pi/OSR} \omega^4 \, d\omega$$

$$= \frac{\Delta^2}{12\pi} \frac{\omega^5}{5}\Big|_0^{\pi/OSR} = \frac{\Delta^2}{12\pi}\left(\frac{\pi}{OSR}\right)^5 \cdot \frac{1}{5}$$

$$= \boxed{\frac{\Delta^2}{60}\pi^4 \; OSR^{-5}}$$

$\frac{15}{6}$

2X OSR $\Rightarrow$ IBN $\downarrow$ 15 dB $\Rightarrow$ SQNR $\uparrow$ 15 dB $\Rightarrow$ 2.5 bits extra resolution

Ex

$N_o = 4$ bit

$OSR = 64$

2nd order $\Delta\Sigma$ modulator

$N_{inc} = \log_2(64) \cdot 2.5$

$= 6 \times \frac{5}{2} = 15$ bits

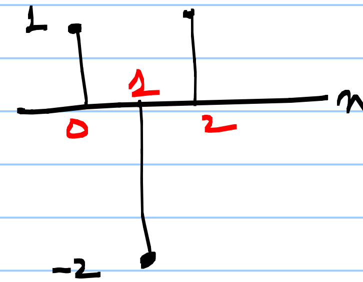
$N_{eff} = N_o + N_{inc} = 19$ bits

$\rightarrow 512 \times 10^3$ levels

$$NTF(z) = (1 - z^{-1})^2 = 1 - 2z^{-1} + z^{-2}$$

$$\updownarrow$$

$$h[n] = [1, -2, 1]$$

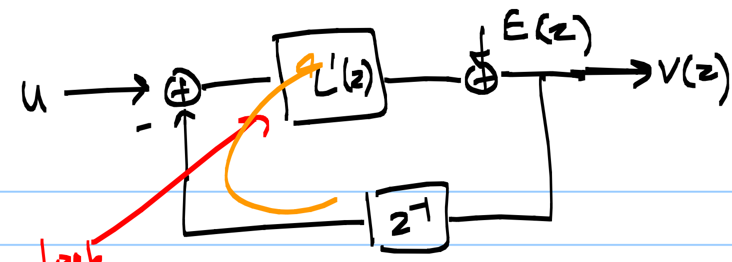


$$NTF(z) = \sum_n h(n)\, z^{-n}$$

$$NTF(e^{j\omega}) = \sum_n h(n)\, e^{-j\omega n}$$

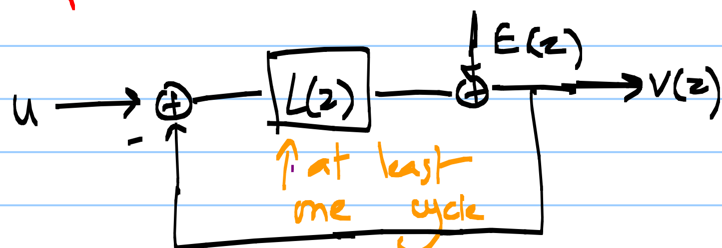
$$OBG = |NTF(e^{j\pi})| = \left|\sum_n h(n)\, e^{-j\pi n}\right| = \sum_n (-1)^n h(n) = 1 + 2 + 1 = 4$$

$$OBG = 4$$



loop filter

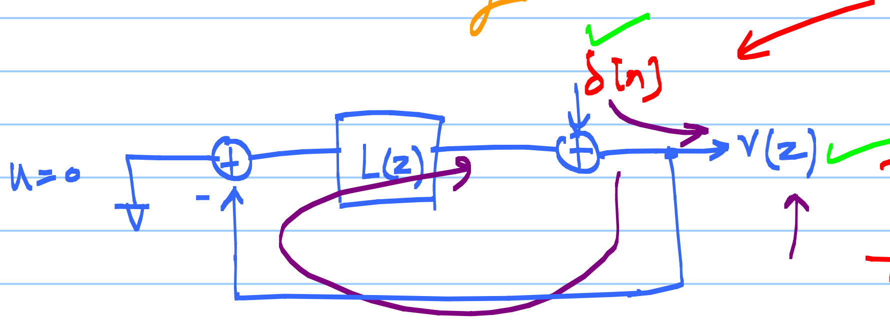
move the delay z^{-1} inside $L'(z)$ block



at least one cycle delay

"No DELAY FREE LOOPS"

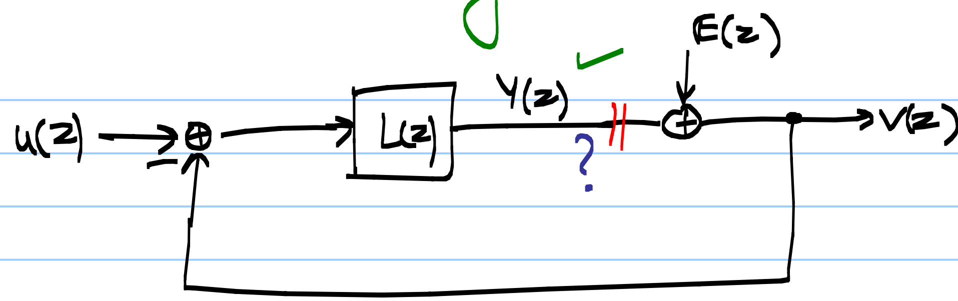
want to find the impulse response seen by the quantization noise



$$\boxed{h[0] = 1} = NTF(\infty)$$

NTF realizability condition

Further understanding



$V(z) = STF \cdot U(z) + NTF(z) \cdot E(z)$

$Y(z) = V(z) - E(z)$

$= \underbrace{STF(z) \cdot U(z)}_{\text{input component}} + \underbrace{(NTF(z) - 1)E(z)}_{\text{Q-noise circulating inside the } \Delta\Sigma\text{-loop}}$

In time-domain

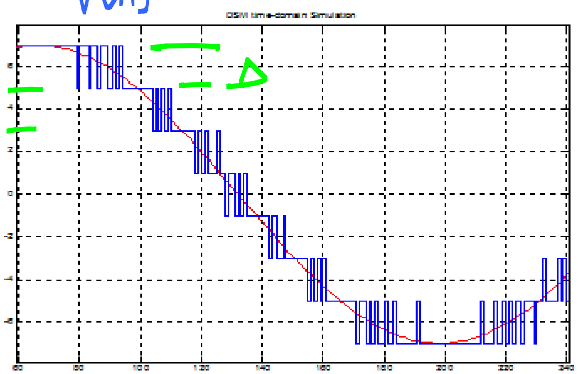
$STF \simeq 1$ at low frequencies

$NTF(z) - 1 \xleftrightarrow{\mathcal{Z}^{-1}} h[n] - \delta[n]$ at the quantizer input (y)

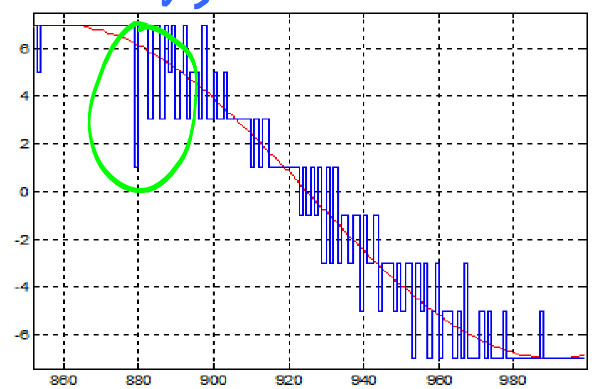
$N_0=8$

$\Delta\Sigma_1$ | $\Delta\Sigma_2$

$v[n]$



$v[n]$



OBG=2 | OBG=4

LSB jump=1 | worst case LSB jump=3

higher OBG ⇒ larger LSB jumps

↳ more wiggling in the output

$$||x[n]|| = \sum_n |x[n]|$$

1- norm of $h[n] - \delta[n]$

Maximum LSB jump $= \Delta \cdot ||h[n] - \delta[n]||_1$

for $NTF(z) = (1-z^{-1})^3$

$OBG = 8$

* How about 3^{rd} or higher order noise shaping!

$$NTF(z) = (1 - z^{-1})^N$$

$$IBN = \frac{\Delta^2}{12\pi} \int_0^{\pi/OSR} \omega^{2N} \cdot d\omega$$

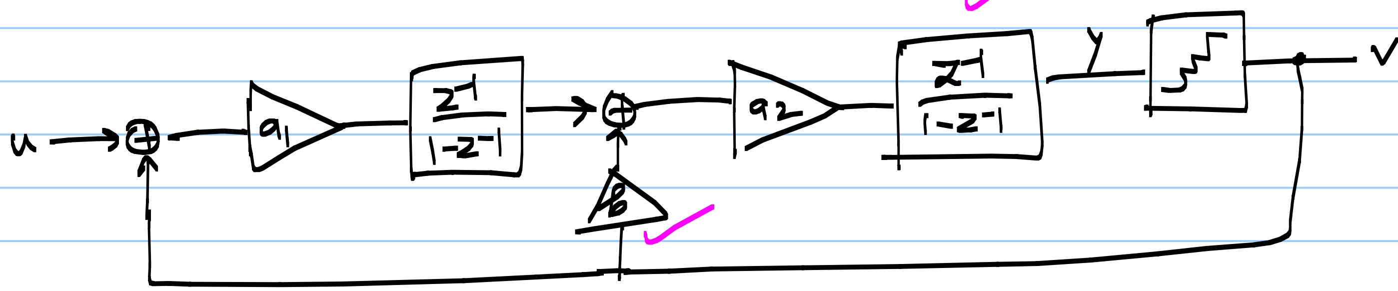
$$= \frac{\Delta^2}{12\pi} \cdot \frac{\pi^{2N+1}}{(2N+1)} \cdot OSR^{-(2N+1)}$$

→ $(N+\frac{1}{2})$ bits increase in resolution for 2x OSR

Boser-Wooley Modulator

feedback topology

delaying integrator



$$NTF(z) = \frac{(1-z^{-1})^2}{D(z)} \qquad STF(z) = \frac{a_1 a_2 z^{-1}}{D(z)}$$

$$D(z) = (1-z^{-1})^2 + a_2 b\, z^{-1}(1-z^{-1}) + a_1 a_2 z^{-2}$$

$$NTF(z) = (1-z^{-1})^2 \quad \& \quad STF(z) = z^{-2}$$

satisfy conditions $a_1 a_2 = 1$ & $a_2 b = 2$

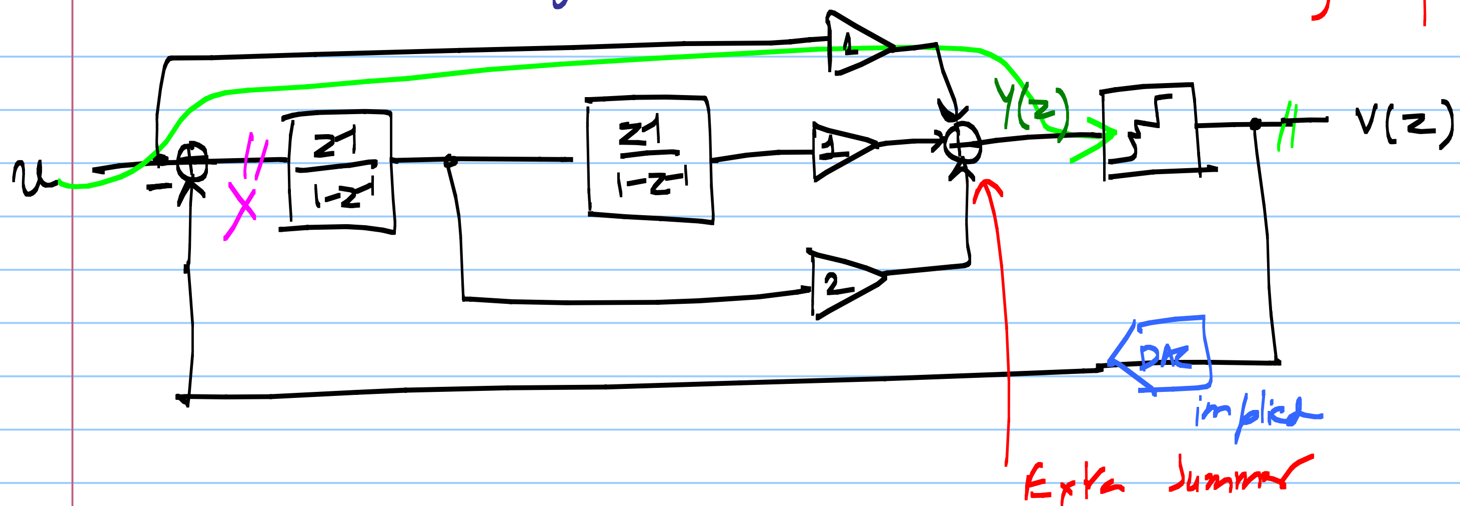
① $a_1 = a_2 = 1$ & $b = 2$

② $a_1 = 1/2,\ a_2 = 2$ & $b = 1$

We will later see that "Dynamic Range Scaling" eliminates the ambiguities in design.

Silva- Steensgaard Structure

feedforward Topology



$$V(z) = \underbrace{U(z)}_{STF=1} + \underbrace{(1-z^{-1})^2}_{NTF} E(z)$$

$$X(z) = U(z) - V(z)$$

$$= -(1-z^{-1})^2 E(z)$$ ← only noise & no signal content

+ loop filter requirements are relaxed
but need an extra summing block