ECE 615 - Lecture 7

Note Title 9/24/2013

First-order noise shaping

$$IBN = \frac{\Delta^2 \pi^2}{36} \cdot OSR^{-3}$$

⇒ for 2x OSR ↑

⇒ Noise ↓ 9dB

⇒ ENOB ↑ 1.5bit

$$\text{Total quantization noise} = \frac{\Delta^2}{12\pi}\int_0^{\pi} |NTF(e^{j\omega})|^2 \, d\omega$$

$$= \frac{\Delta^2}{12\pi}\left(\sum_n |h(n)|^2\right) \cdot \pi$$

$$= \frac{\Delta^2}{12} \cdot 2$$

Oversampling w/o noise shaping

$$IBN = \frac{\Delta^2}{12} \cdot OSR^{-1}$$

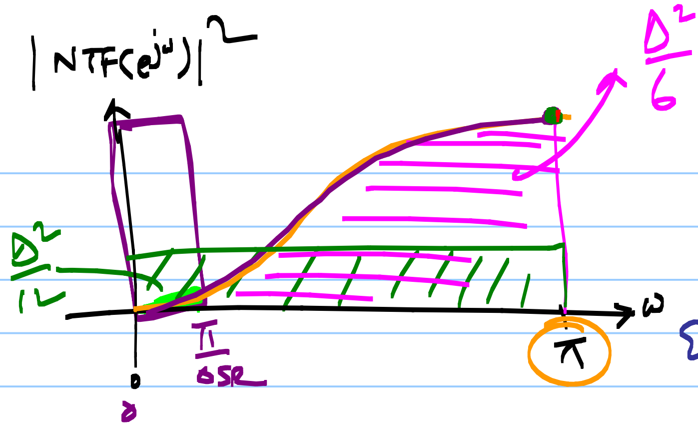
ENOB ↑ 0.5 Bit

$$\text{Total quantization noise} = \frac{\Delta^2}{12}$$

$$NTF(z) = 1 - z^{-1}$$

$$h[n] = \{1, -1\}$$

$$\sum (h[n])^2 = 2$$



Parseval's Theorem:

for DTFT:

$$\text{Energy} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Frequency — Time Domain

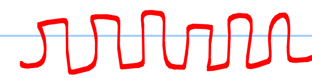
$$NTF(z) = 1 - z^{-1}$$

* what is $NTF(e^{j\omega})\Big|_{\omega=\pi} = 1 - e^{-j\omega}\Big|_{\omega=\pi} = 1 - (e^{-j\pi}) = 2$

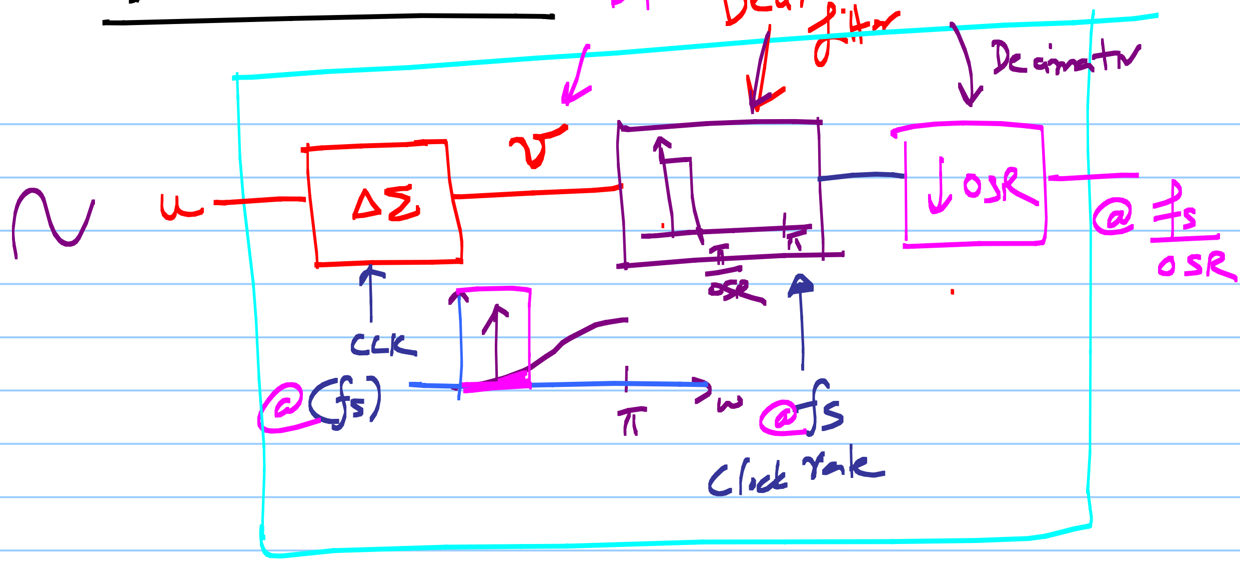
Out of band gain (OBG)

* NTF gain at $\omega = \pi$ $\Rightarrow$ $e^{-j\omega n}\Big|_{\omega=\pi}$

$= e^{-j\pi n} = (-1)^n$

high-freq component at $\omega = \pi$ $\longrightarrow$ 

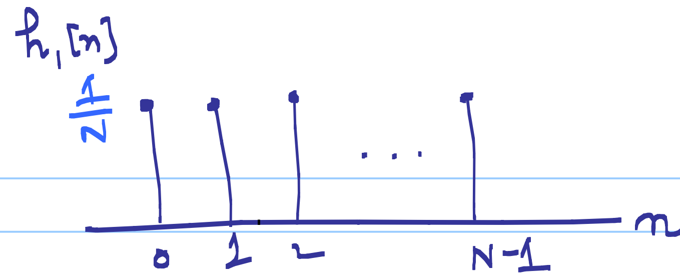
Decimation Filter :



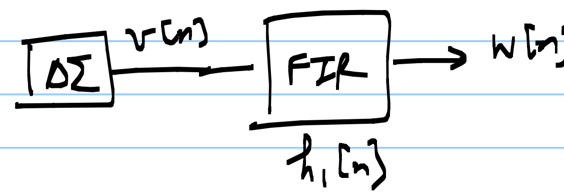
$f_s = (2 f_B) \cdot OSR$

↑ Sample rate

* FIR filter



↳ computes a running average over $v[n]$



$$w[n] = \frac{1}{N}\sum_{i=0}^{N-1} v[n-i]$$

$$h_1[n] = \begin{cases} 1/N, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

$$H_1(z) = \frac{1}{N}\sum_{n=0}^{N-1} z^{-n} = \frac{1}{N}\cdot\left(\frac{1-z^{-N}}{1-z^{-1}}\right) =$$

$N = 8$

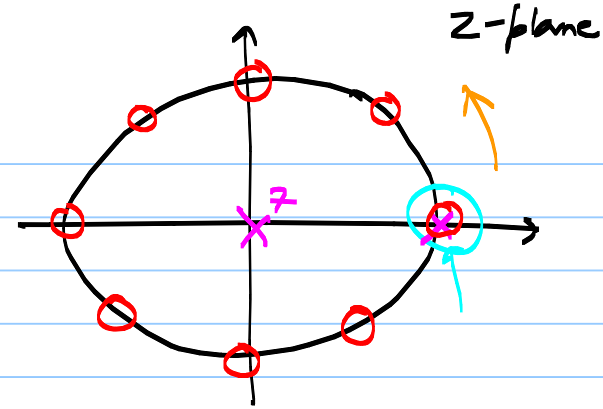
$$H_1(z) = \frac{1}{8} \cdot \frac{1 - z^{-8}}{1 - z^{-1}}$$

$$= \frac{1}{8} \frac{(z^8 - 1)}{z^7 (z-1)}$$

Zeros $\Rightarrow$ $z^8 - 1 = 0$

$z^8 = 1$

$\Rightarrow$ Eight roots of unity ← Complex Numbers

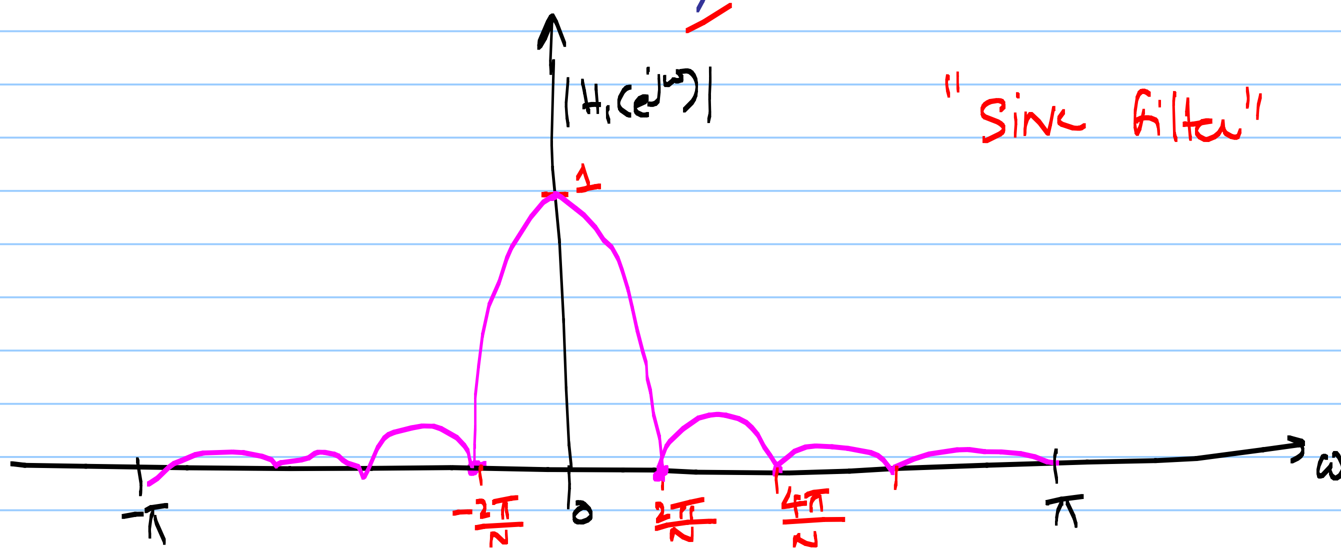


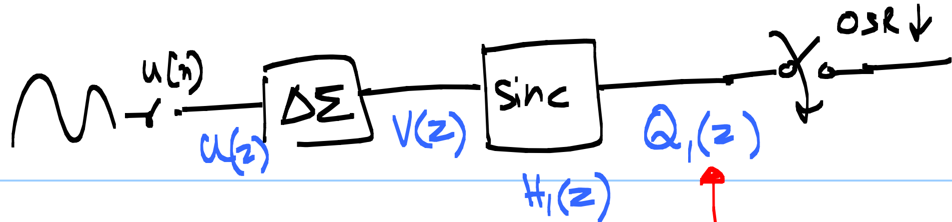
$z = e^{-j\omega}$

$$H_1(e^{j\omega}) = \frac{1}{N} \cdot \frac{1 - e^{-j\omega N}}{1 - e^{-j\omega}} = \frac{1}{N} \cdot \frac{|e^{-j\omega \frac{N}{2}}| \left(e^{j\omega\frac{N}{2}} - e^{-j\omega\frac{N}{2}}\right)}{|e^{-j\frac{\omega}{2}}| \left(e^{j\omega/2} - e^{-j\omega/2}\right)}$$

$$|H_1(e^{j\omega})| = \frac{1}{N} \cdot \frac{\sin\left(\frac{N\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$$, Draw for $N=8$

"Sinc filter"





$X(z) \leftarrow Z$

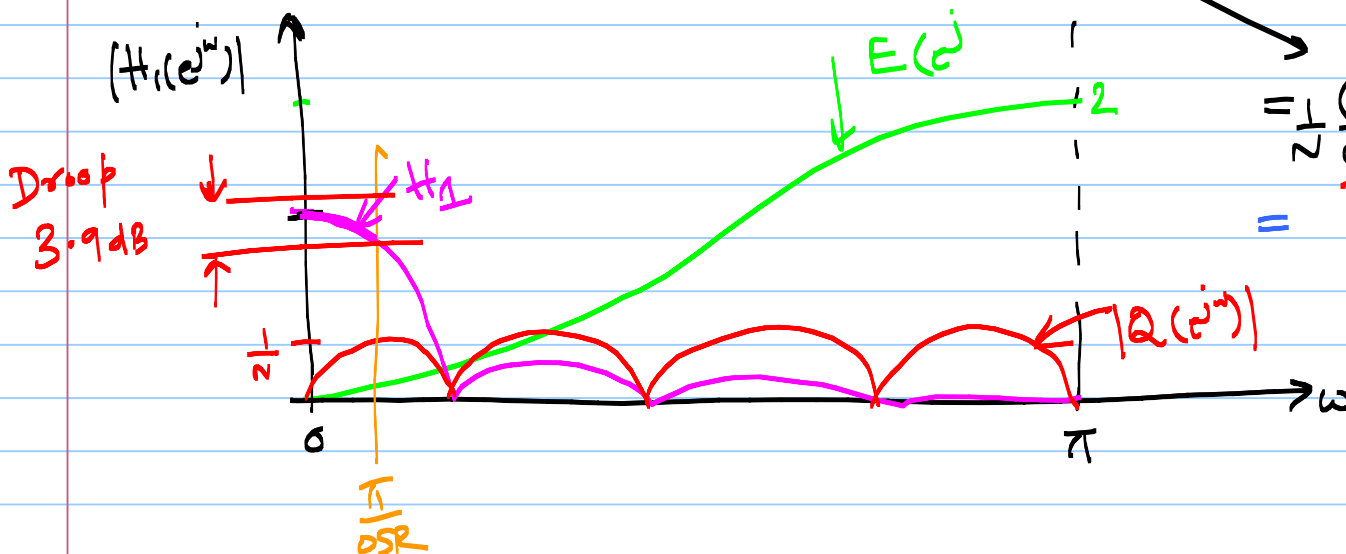
$X(e^{j\omega}) \leftarrow$ DTFT

$X(k) \leftarrow$ FFT

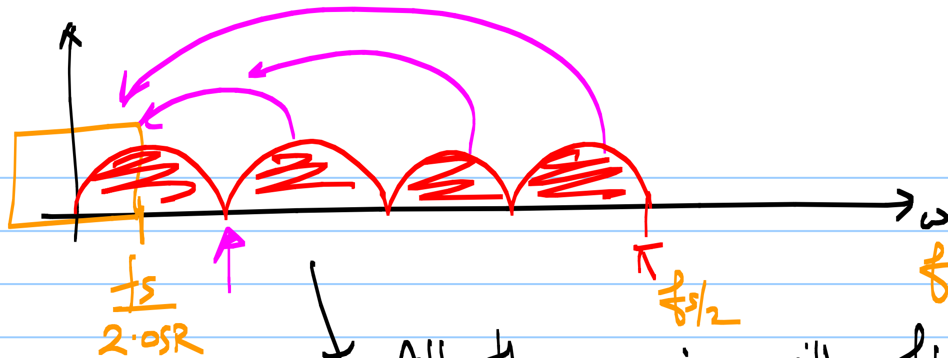
$$Q_1(z) = H_1(z) \cdot NTF(z) \cdot E(z)$$

$$= \frac{1}{N} \frac{(1-z^{-N})}{(1-z^{-1})} \times (1-z^{-1}) \cdot E(z)$$

$$= \frac{1}{N} \cdot (1-z^{-N}) \cdot E(z)$$



$N = OSR = 8$



All these noise will fold into the base band after decimation

total output quantization noise power

$$\sigma_{q_1}^2 = \int_0^{\pi} \frac{1}{N^2} \left|1 - e^{-j\omega N}\right|^2 \cdot \left(\frac{\Delta^2}{12\pi}\right) \cdot d\omega$$

$$= \frac{\Delta^2}{12\pi} \cdot \frac{1}{N^2} \cdot \int_0^{\pi} \left|1 - e^{-j\omega N}\right|^2 d\omega$$

$$= \frac{\Delta^2}{12\pi} \cdot \frac{1}{N^2} \cdot 2$$

$$\sigma_{q_1}^2 = \sigma_e^2 \cdot \frac{2}{N^2} = \frac{\Delta^2}{12} \cdot 2 \cdot OSR^{-2}$$

Here $\because N = OSR$

→ 1 bit ↑ per 2x OSR

If we had an ideal LPF, $\sigma_q^2 = \sigma_e^2 \cdot \frac{\pi^2}{3} \cdot OSR^{-3}$ ← 1.5 bits

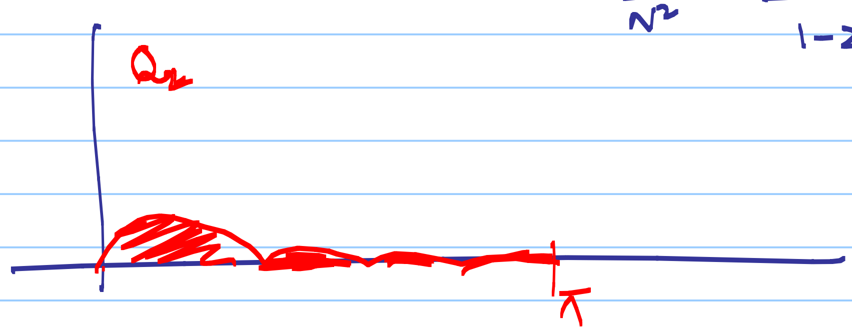
⇒ Sinc filter is nearly 'N' times less effective than the ideal LPF.

Sinc^2 Decimation Filter

$$H_2(z) = \left(\frac{1}{N} \cdot \frac{1-z^{-1}}{1-z^{-N}}\right)^2$$

$$Q_2(z) = \frac{1}{N^2} \cdot \left(\frac{1-z^{-N}}{1-z^{-1}}\right)^2 \cdot (1-z^{-1})\, E(z)$$

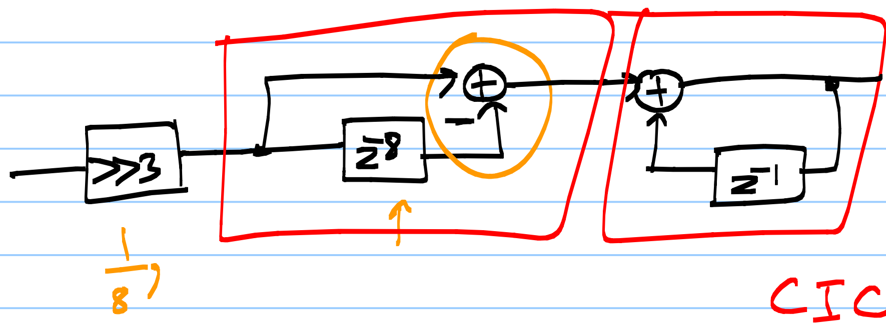
$$= \frac{1}{N^2} \cdot \frac{(1-z^{-N})^2}{1-z^{-1}} \cdot E(z)$$



$$\sigma_{q_2}^2 = \sigma_e^2 \cdot \; 2 \times OSR^{-3}$$

$$\frac{1}{z} \cdot \frac{(1-z^{-8})}{(1-z^{-1})} = \boxed{\frac{1}{z}} \; \boxed{(1-z^{-8})} \; \boxed{\frac{1}{(1-z^{-1})}}$$

Comb filter



CIC

Comb- Integrator Cascade