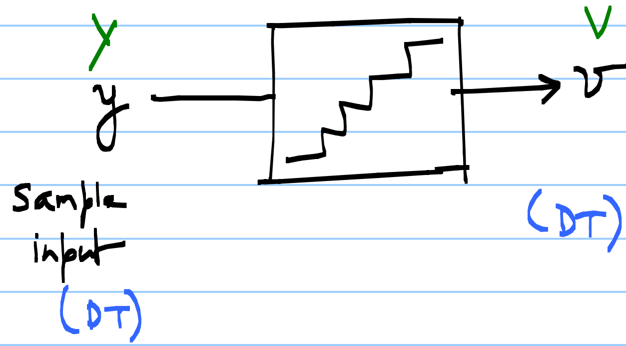
ECE 615 - Lecture 4

Quantizer



→ Amplitude level is quantized

Y → V

$v = Q(y)$ → non-linear operation

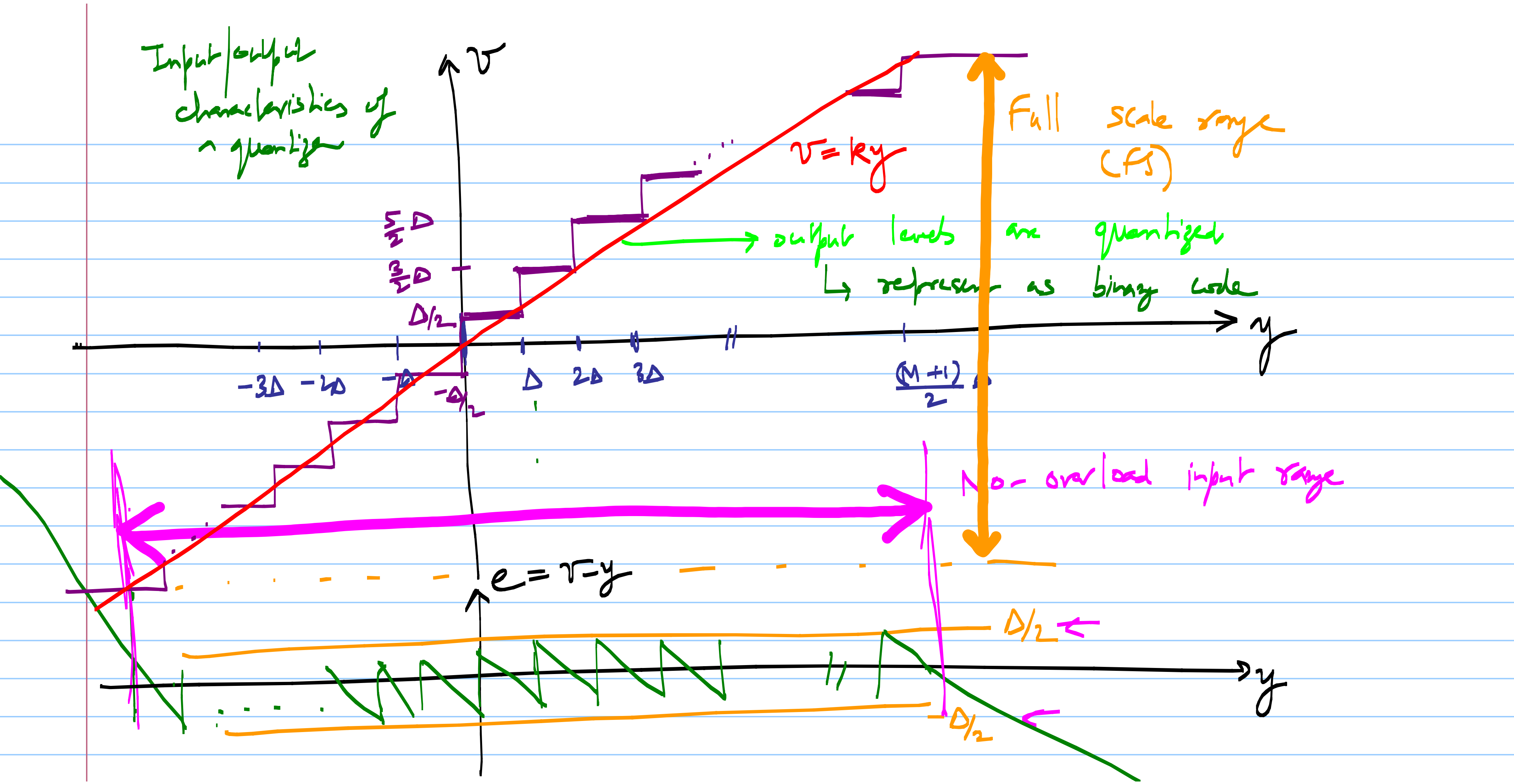
* The amplitude of the analog DT signal (y) is quantized so that it assumes one of the finite number of allowable values (or levels)
  ↳ These levels are represented by a binary code

* Uniform Quantizer: fixed level spacing (Δ)

* Quantizer is a memoryless non linear block, defined by its input-output characteristics

* Unipolar quantizer: Quantizes only +ve inputs
  Bipolar ": Quantizes +ve as well as −ve inputs

Quantization Error : $e = v - y$

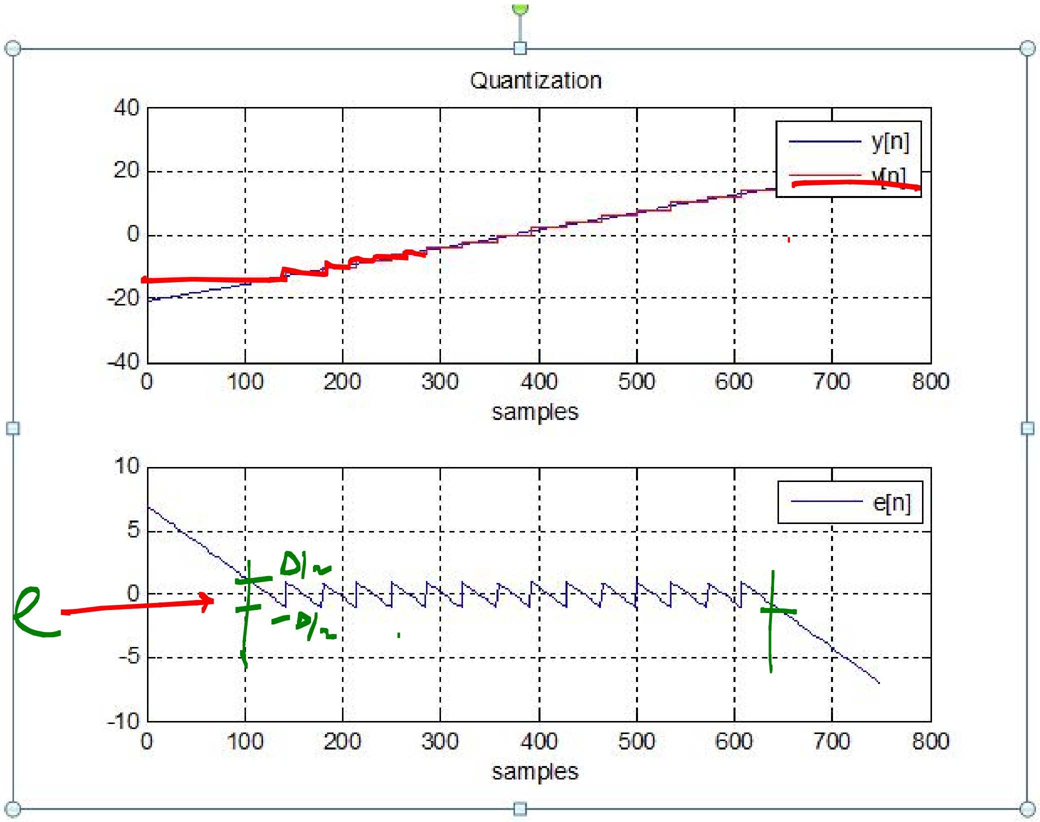


Input/output characteristics of a quantizer
v
v = ky
Full scale range (FS)
output levels are quantized
↳ represent as binary code
5/2 Δ
3/2 Δ
Δ/2
−3Δ −2Δ −Δ
−Δ/2
Δ 2Δ 3Δ
(M+1)/2 Δ
y
No-overload input range
e = v−y
Δ/2
Δ/2
y

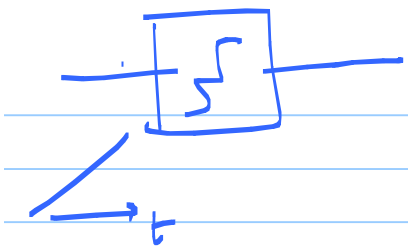
least significant bit

Definitions:

- input step size = Δ = LSB = δ in $\Delta\Sigma$ Toolbox
- number of steps = M ⟶ odd or even
- number of levels = $(M+1)$ = nLev
- number of bits ⇒ $\lceil \log_2(M+1) \rceil$ ⟵ MATLAB ceil(log2(nLev))
- no-overload input range ⟶ range for which $-\frac{\Delta}{2} \le e \le \frac{\Delta}{2}$
  $= y \in [-(M+1), M+1]$
- full scale range (FS) = difference between the lowest & highest levels in the quantizer



Quantization
y[n]
v[n]
samples
e[n]
samples



* Nature of the quantization error? → Mathematically difficult to analyze!

. Conditions for simplifying assumption:

① y stays within the no-overload input range (Shall not overload the quantizer)

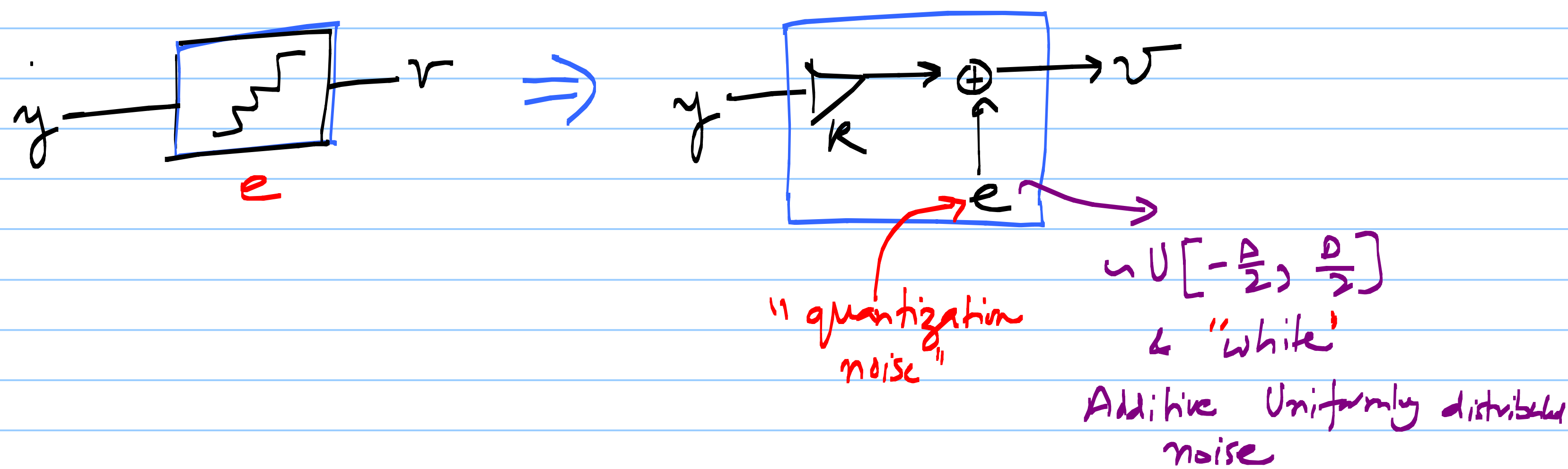
② $e[n]$ is uncorrelated with the input $y[n]$

③ spectrum of $e[n]$ is "white" ⇒ $E\{e[n]\,e[n+m]\} = \delta(m)\,\sigma^2$

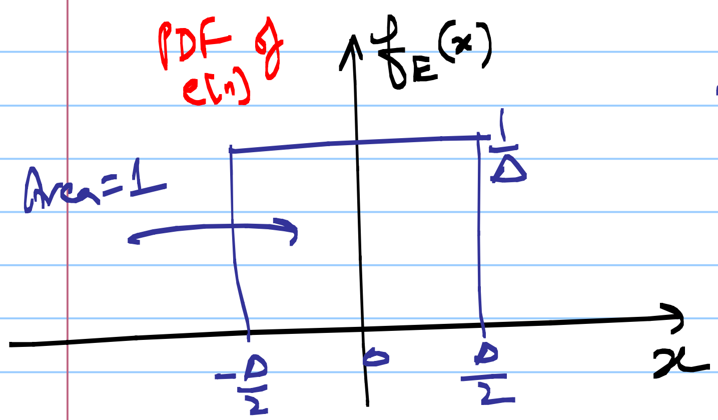
④ quantization error is uniformly distributed ⇒ $e \sim U\left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right]$

↳ makes the calculations easy.

"Linearized Quantizer Model"



* Probability Distribution Function (PDF) of the quantization noise (e)



mean : $\mu = E(e) = 0$ by definition

$$\sigma_e^2 = E\left[(e-\mu)^2\right] = E(e^2)$$

$$= \int_{-\infty}^{\infty} x^2 f_E(x)\, dx$$

$$= \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} x^2 dx = \frac{2}{\Delta} \left.\frac{x^3}{3}\right|_0^{\Delta/2}$$

Quantization Noise Power

$$\boxed{\sigma_e^2 = \frac{\Delta^2}{12}} = \frac{(LSB)^2}{12}$$

* When does this linearized model break down?

↳ y is not varying fast ⇒ e is correlated with y
input (y) is not busy

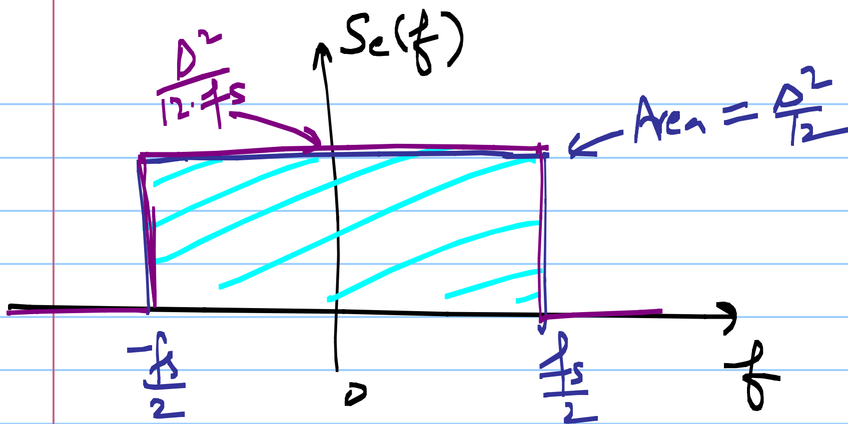
↳ y is periodic with a frequency harmonically related to f_s.

↳ quantizer overload

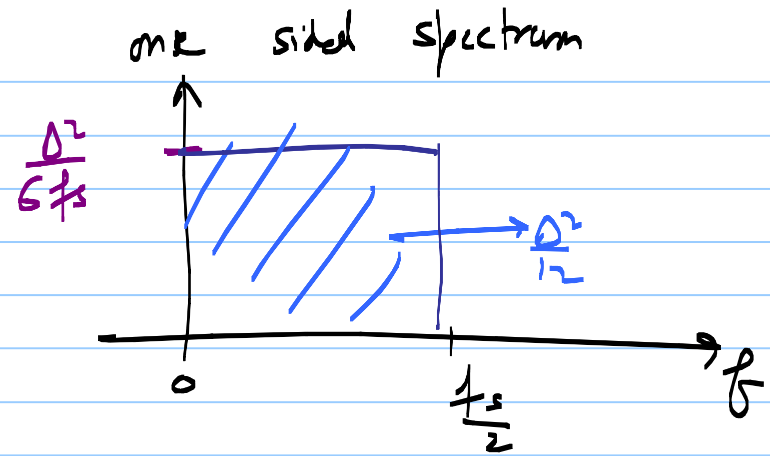
* $\frac{f_{in}}{f_s} = \begin{cases} \frac{m}{N}, \; m \ll N \Rightarrow \text{closely spaced tones in FFT of } e(n) \Rightarrow \text{quantization noise} \\ \text{irrational} \Rightarrow \text{continuous spectrum} \end{cases}$

↳ FFT leakage

PSD (power spectral Density of the quantization noise)



$\Rightarrow$



Let $y = A\sin(\omega_{in}t)$ is quantized with a quantizer with $LSB = \Delta$.

Signal to quantization noise ratio

⇒ $SQNR = \dfrac{\text{Signal power}}{\text{quantization Noise power}} = \dfrac{P_s}{\Delta^2/12}$

⇒ for a N-bit ADC ⟶ 2^N ← no of levels in the quantizer output

full scale range $= 2^N \cdot \Delta$

maximum amplitude of the input sine wave $= A_{max} = \dfrac{2^N\Delta}{2} = 2^{N-1}\Delta$

maximum signal power (P_s) $= \dfrac{(2^{N-1}\Delta)^2}{2}$ $\quad \because \dfrac{A_{max}^2}{2}$ for sine

$$\text{Peak SQNR} = \frac{P_s}{\Delta^2/12} = \frac{(2^{N-1}\Delta)^2}{2\cdot\frac{\Delta^2}{12}}$$

$$= \frac{2^{2N-2}}{2}\cdot 12 = \frac{3}{2}\cdot 2^{2N}$$

$$SQNR_{dB} = 10\log_{10}\left(\frac{3}{2}\cdot 2^{2N}\right)$$

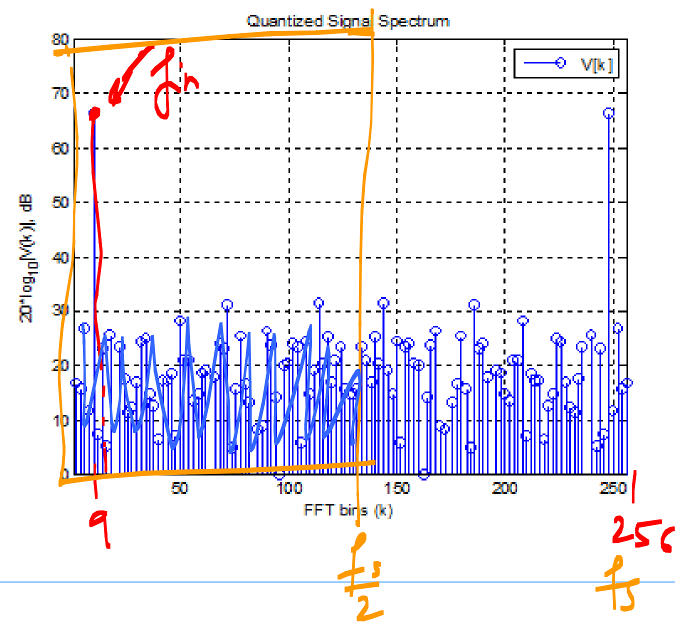
$$SQNR = 6.02N + 1.76\ dB$$

6.02N: N levels of quantizer

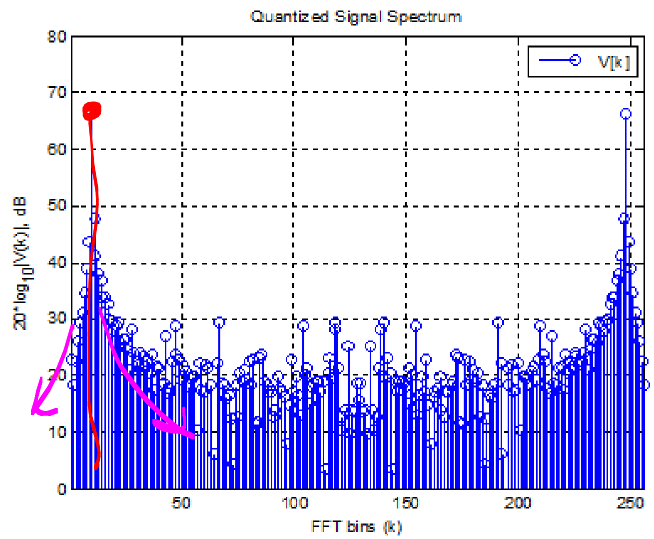
1.76 dB: sine wave input

Matlab Simulation

① $\frac{f_{in}}{f_s} = \frac{m}{N} = \frac{9}{256}$

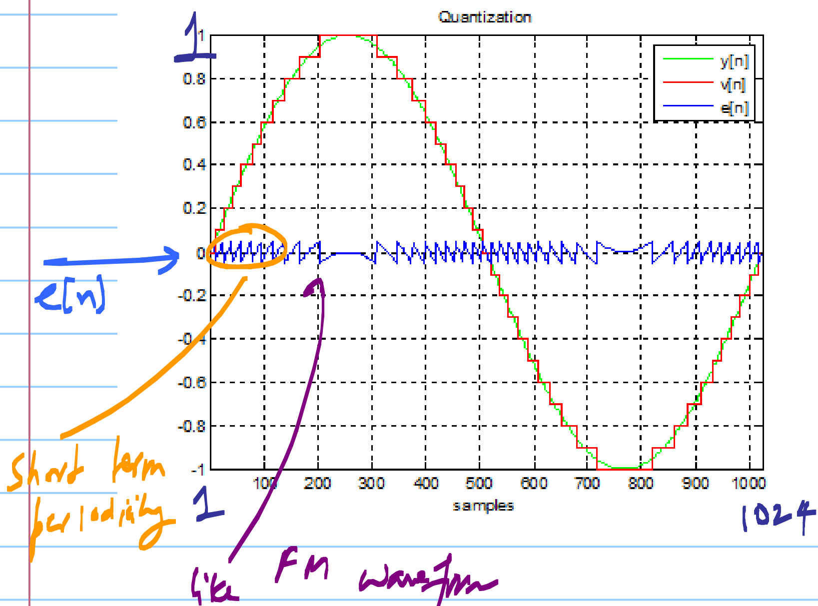


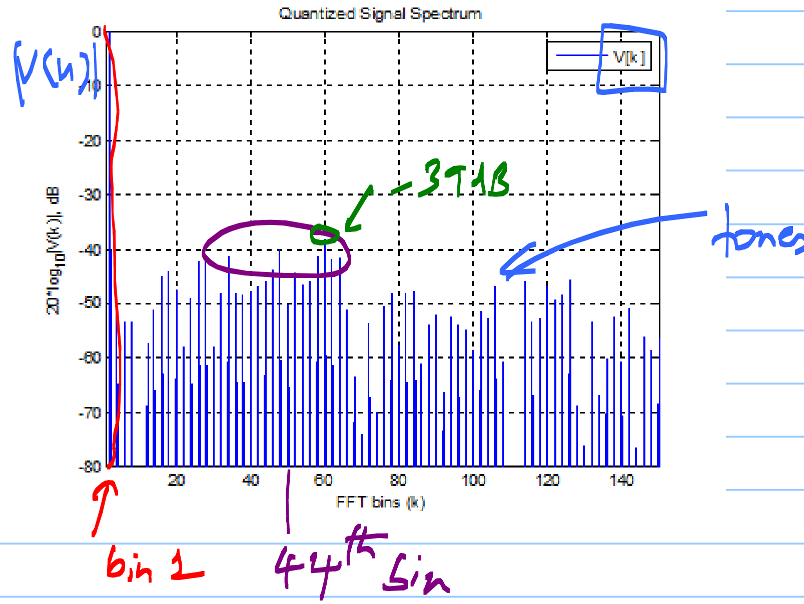
② $\frac{f_{in}}{f_s} = \frac{9.1}{256}$



③ $\frac{f_{in}}{f_s} = \frac{1}{1024}$ ← slow input to observe quantization noise periodicity

$\Delta = 0.1$





for $\Delta = 0.1$ $\Rightarrow$ number of local periods $= 44$

↳ most tones lie around 44^{th} bin

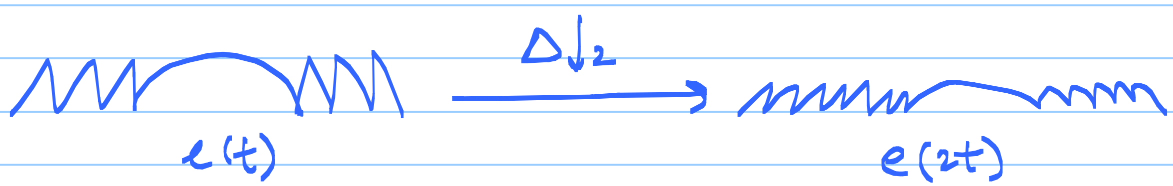
for $\Delta = 0.2$ $\Rightarrow$ number of local periods $= 20$

↳ most tones around 20^{th} bin

* Δ is halved

tone power $\propto \Delta^2 \Rightarrow \frac{1}{4}^{th} \Rightarrow -6\,dB$ lower

*



$\Rightarrow \frac{1}{2} E(f/2)$

the tones spread out in frequency by 2

⇒ another tone power reduction by 2dB

⇒ total reduction in the tone with max magnitude

= 9dB reduction

⇒ tone power reduction = 9dB with halving Δ

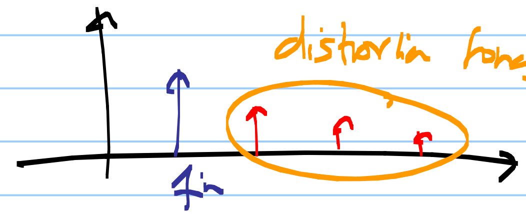
$\Delta \downarrow_2$ ⇒ SQNR ↑ by 6 dB

SFDR ↑ by 9dB

Frequency Domain Measures :

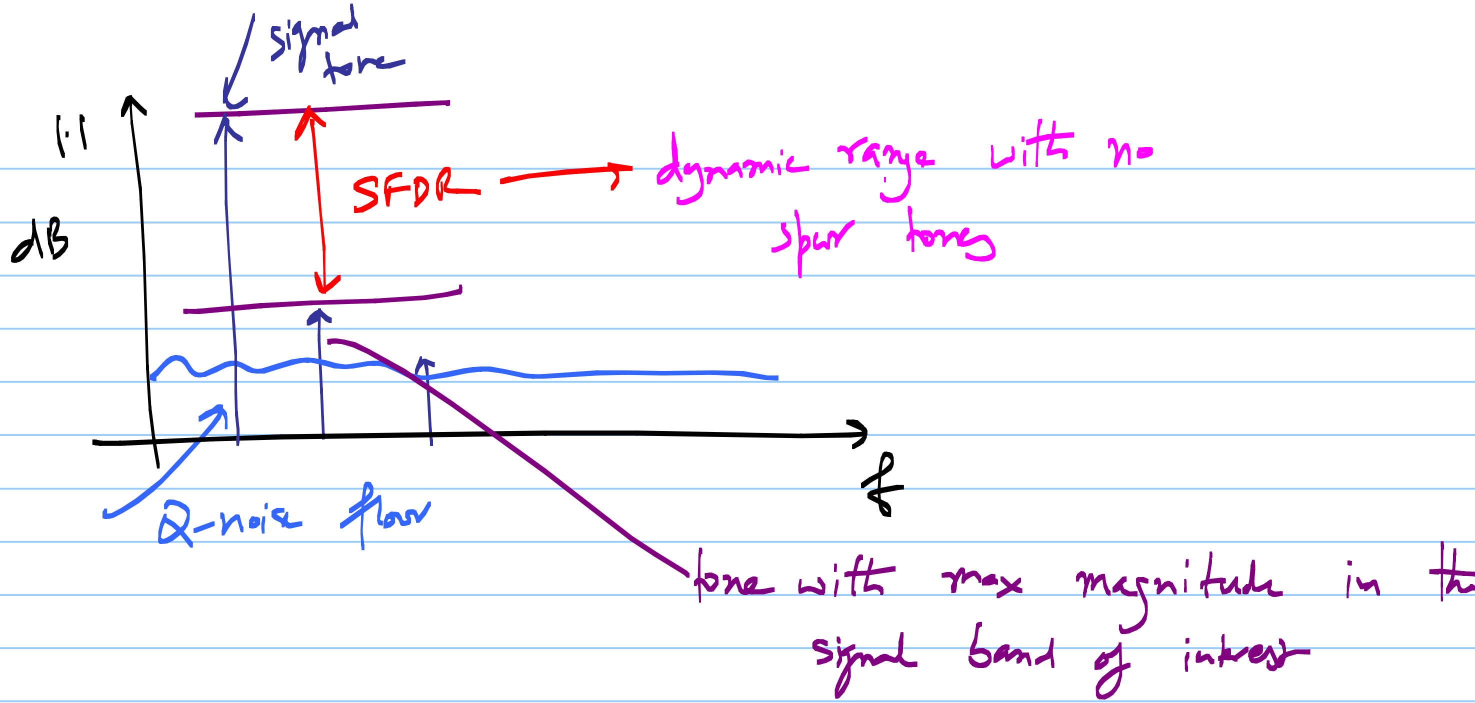
* SNDR = $10 \log_{10}\left(\frac{P_S}{P_N + P_{distortion}}\right)$

Signal to Noise + Distortion ratio



* Effective number of bits, ENOB (Neff) $\triangleq \frac{SNDR - 1.76}{6.02}$

* SFDR → spur free dynamic range



$$SFDR = 10 \log_{10}\left(\frac{\text{signal power}}{\text{largest spurious tone power}}\right)$$

Harmonic Distortion :

* Distortion metrics (IIP_3, IIP_5, IM_3, IM_5) RF course

$HD_k \Rightarrow$ Harmonic distortion wrt the k^{th} harmonic

$$= 10 \log \left(\frac{X_k^2}{X_1^2} \right)$$

X_k^2 = rms of the k^{th} harmonic component

HD_3, HD_5

THD = Total HD

$$= 10 \log \left(\frac{\sum_{k=2} X_k^2}{X_1^2} \right)$$

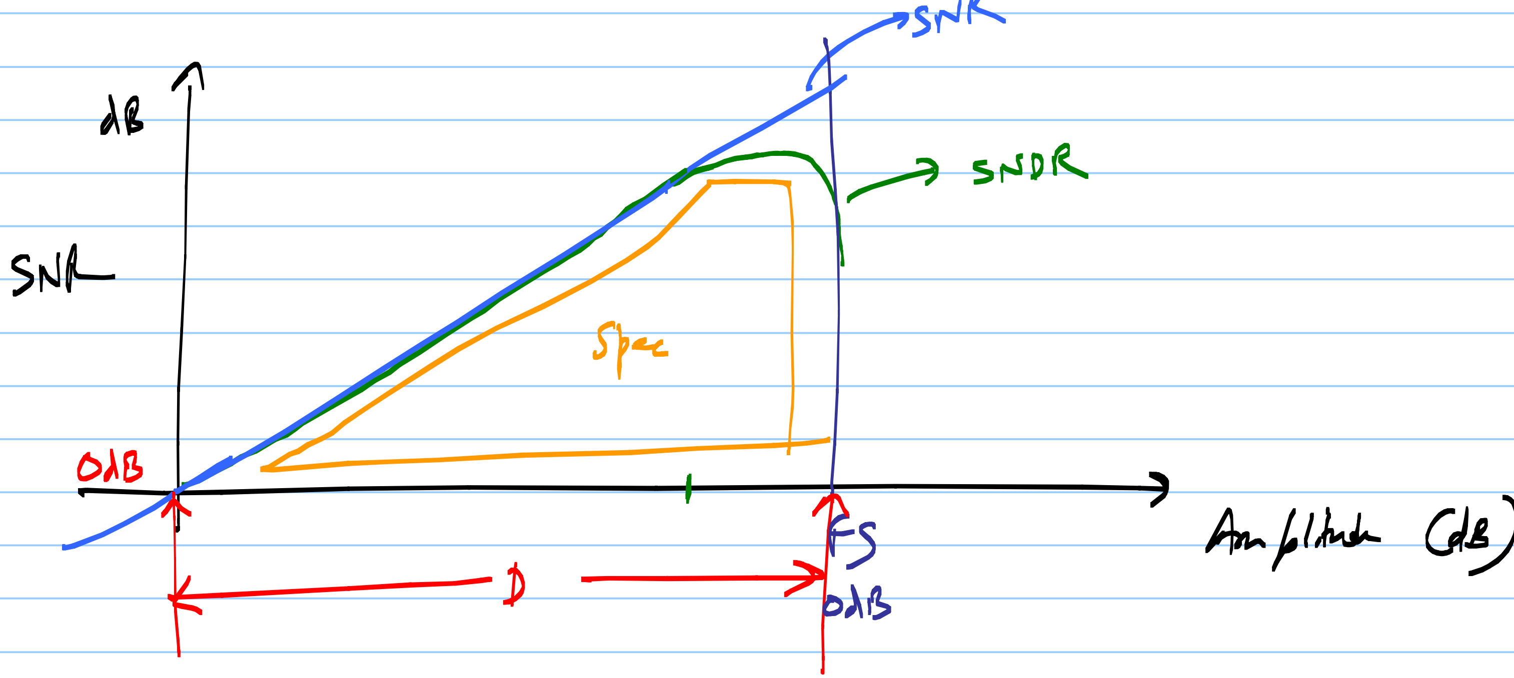
Dynamic Range (DR)

$$= 10 \log_{10}\left(\frac{\text{Maximum signal power detected}}{\text{Smallest signal power detected}}\right)$$

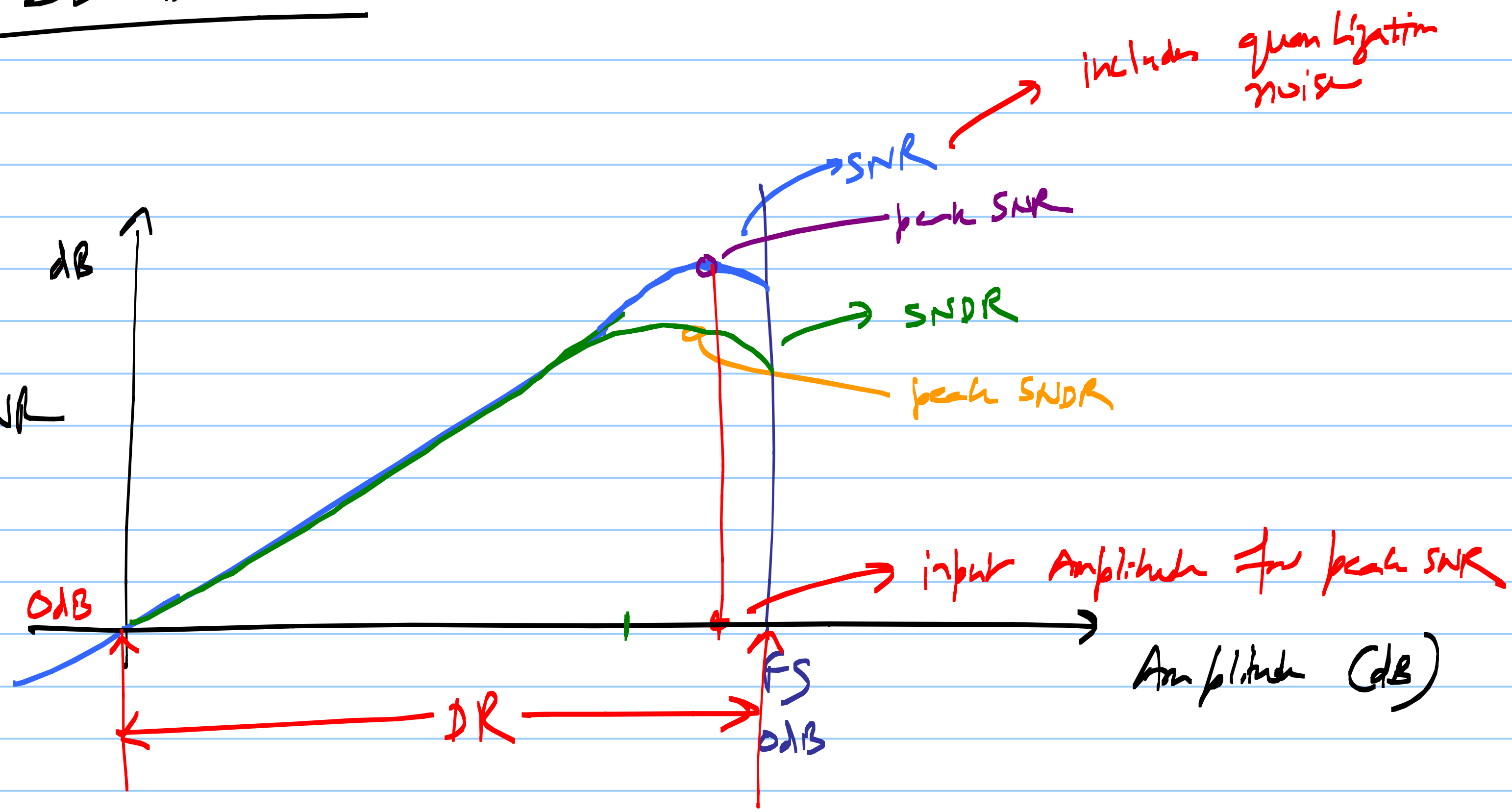
⇒ The range from the full scale (FS) to the smallest detectable signal is the dynamic range

↳ circuit noise

For a Nyquist Rate ADC



fw a ΔΣ ADC


dB
SNR
0dB
SNR
includes quantization noise
peak SNR
SNDR
peak SNDR
input Amplitude for peak SNR
FS
0dB
DR
Amplitude (dB)