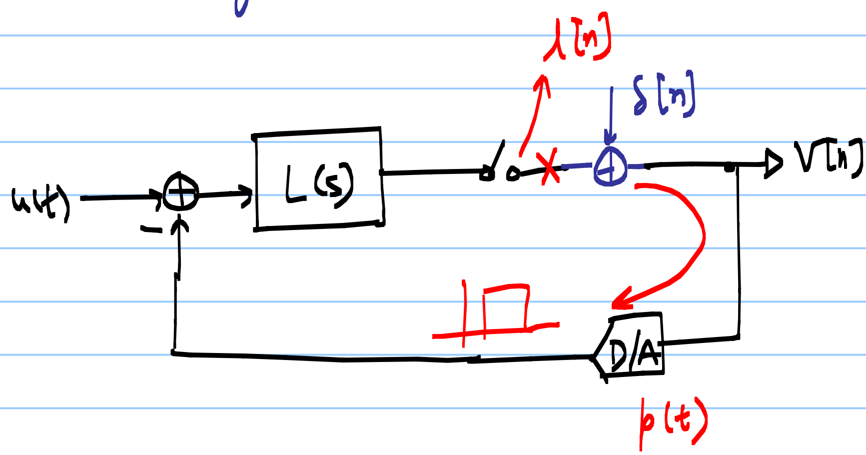
ECE 615 - Lecture 20

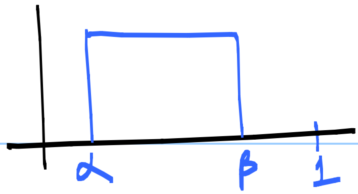
Note Title 11/12/2013

Impulse Invariant Transform based symbolic calculating

Numerical fitting



$l[n]$



Normalized to $T_s = 1\,s$
(1 Hz)

I.I.T.

$$l[n] = \mathcal{Z}^{-1}\{L(z)\} = \mathcal{L}^{-1}\{R_D(s)\cdot L(s)\}\Big|_{t=nT_s}$$

① Start with a DT ΔΣ modulator, with an open loop response

$$L(z) = \frac{1}{NTF(z)} - 1$$

② Map $L(z)$ to a CT ΔΣ modulator, with a DAC pulse shape

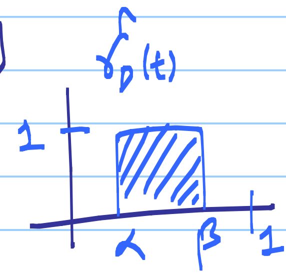
"$\hat{r}_D(t)$" $= p(t)$

$$\{L(z), \hat{r}_D(t)\} \xrightarrow{IIT} L(s)$$

Use Tables

(3) Write $L(z)$ as a partial fraction expansion

↳ rectangular pulse shape $t \in [\alpha, \beta]$



$$\hat{R}_{\alpha,\beta}(s) = \frac{1}{s}\left[e^{-\alpha s} - e^{-\beta s}\right]$$

$u(t-\alpha) - u(t-\beta)$

④ Use table to convert each of the partial fraction 'pole' from z-domain to s-domain

↳ recombine to obtain $L(s)$

J.A. Cherry

when using IIT,

a z-domain pole z_k with multiplicity of 'ℓ'

maps to a CT pole at $s=s_k$ with the same multiplicity

$$s_k = \ln(z_k)$$

'l' poles at z_k $\xRightarrow{\text{IIT}}$ 'l' poles at $s_k = \ln(z_k)$

$$\frac{1}{(z-1)^{l}} \rightarrow \frac{y_0}{s^{l} - 0} = \frac{y_0}{s^{l}}$$

scaling coefficient

$$\underbrace{\frac{z^{-1}}{1-z^{-1}}}_{\text{delaying integrator}} \rightarrow \frac{1}{s}$$

CT integrator

A poor intuition

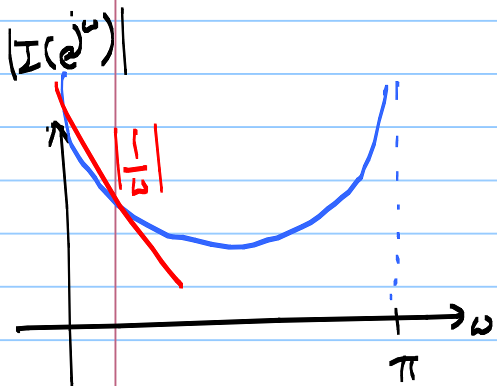
$$I(z) = \frac{z^{-1}}{1-z^{-1}} = \frac{1}{z-1} = \frac{1}{e^{sT_s}-1}$$

$$= \frac{1}{e^{s}-1} \qquad \text{for } T_s = 1s$$

$$= \frac{1}{\left(\cancel{1} + s + \frac{s^2}{2!} + \dots\right) - \cancel{1}}$$

$$= \frac{1}{s + \frac{s^2}{2!} + \frac{s^3}{3!} + \dots} \simeq \frac{1}{s} \qquad \text{for } |sT_s| \ll 1$$

$$\Rightarrow \omega \ll \pi$$



Ex 1 $\qquad$ 2nd-order CT-$\Delta\Sigma$M

NRZ DAC $\qquad (\alpha, \beta) = (0, 1)$

$$NTF(z) = (1 - z^{-1})^2$$

$$\Rightarrow \quad L(z) = \frac{NTF(z) - 1}{NTF(z)} = \frac{-2z^{-1} + z^{-2}}{(1 - z^{-1})^2}$$

$$L(z) = \frac{-2z + 1}{(z-1)^2}$$

$$= \frac{-2}{(z-1)} + \frac{-1}{(z-1)^2}$$ $\Leftarrow$ partial fraction expansion of $L(z)$

$(\alpha, \beta) = (0, 1)$

& $s_k = h(z_k) = 0$

$$\frac{1}{(z-1)} \rightarrow 1/s$$

$$\frac{1}{(z-1)^2} \rightarrow \frac{1 - 0.5s}{s^2}$$

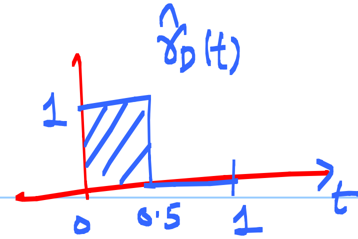
from the IIT Table

$$L(s) = -2\left(1/s\right) - \left(\frac{1 - 0.5s}{s^2}\right)$$

$$\boxed{L(s)_{NRZ} = -\frac{1 + 1.5s}{s^2}}$$

for an NRZ DAC

for an R2 DAC $\quad (\alpha, \beta) = (0, 0.5)$



$$L(s)_{R2} = -\left(\frac{\alpha + \alpha \cdot 5 s}{s^2}\right)$$

* Numerator coeff are higher for R2 pulse-shape

Numerical Determination of Loop filter Coefficients

$$k_1 \begin{bmatrix} 0 \\ 1 \\ 1 \\ \vdots \end{bmatrix} + k_2 \begin{bmatrix} 0 \\ 0.5 \\ 1.5 \\ 2.5 \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 3 \\ 4 \\ 5 \\ \vdots \end{bmatrix}$$

c_1 c_2 $l[n]$

$N \times 1$

N is our choice

$$\begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} l(0) \\ l(1) \\ l(2) \\ \vdots \end{bmatrix}$$

$N \times 2$ $\quad 2 \times 1$ $\quad N \times 1$

"optimization"

$$\mathbf{C}\, k = l$$

$$(C^T C)\, k = C^T l$$

$$\vec{k}^T = [k_1 \;\; k_2] \leftarrow \boxed{k^* = (C^T C)^{-1} (C^T l)}$$

pseudo-inverse

least mean square (LMS) data fitting

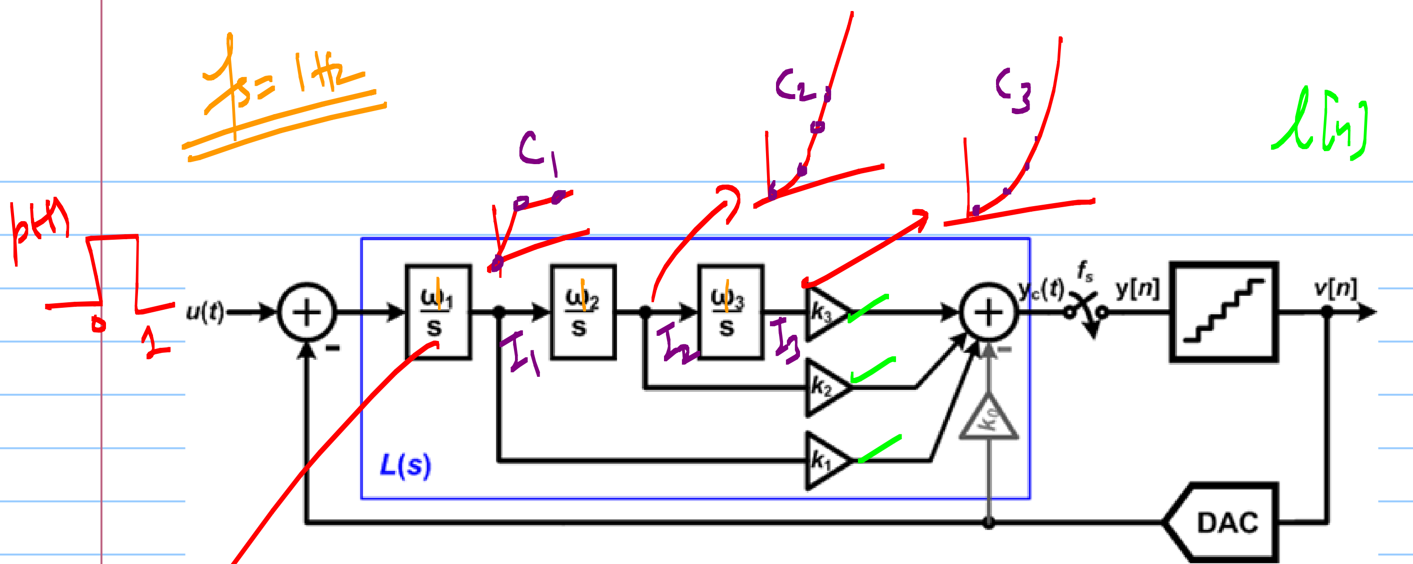
In $\Delta\Sigma$ Toolbox $\rightarrow$ realizeNTF_ct()

$\uparrow$ $L(z)$, (α, β)

$\downarrow$ $L(s)$

use numerical fitting

$f_s = 1\,\text{Hz}$



$$\begin{bmatrix} C_1 & C_2 & C_3 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} l(0) \\ l(1) \\ l(2) \\ \vdots \end{bmatrix}$$