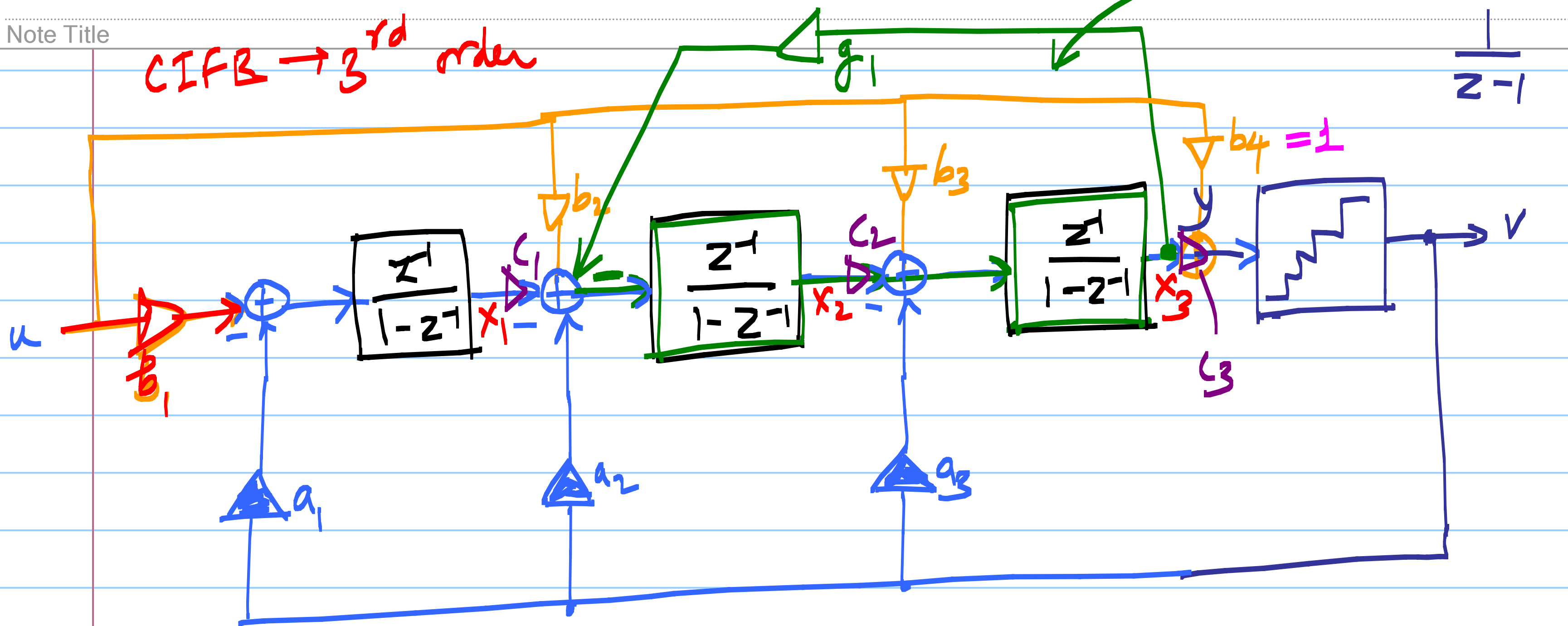
ECE 615 - Lecture 16

Note Title 10/29/2013

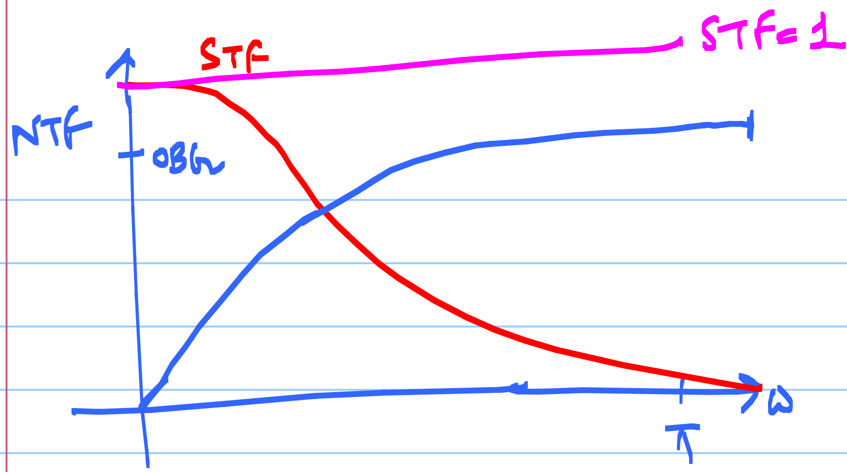
CIFB → 3rd order

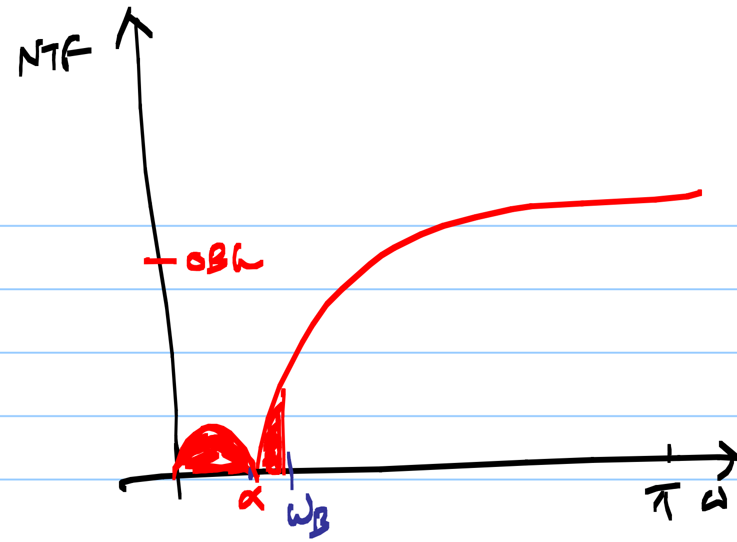


$$NTF(z) = \frac{(1-z^{-1})^3}{D(z)}$$

"c_1, c_2, c_3"
↓
coeff for "range scaling"

⇒ poles set the OBG

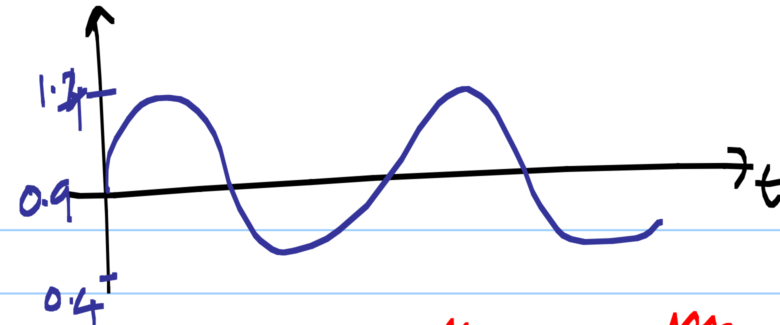




$$NTF(z) = \frac{(1-z^{-1})(1-e^{\pm j\alpha}z^{-1})}{D(z)}$$

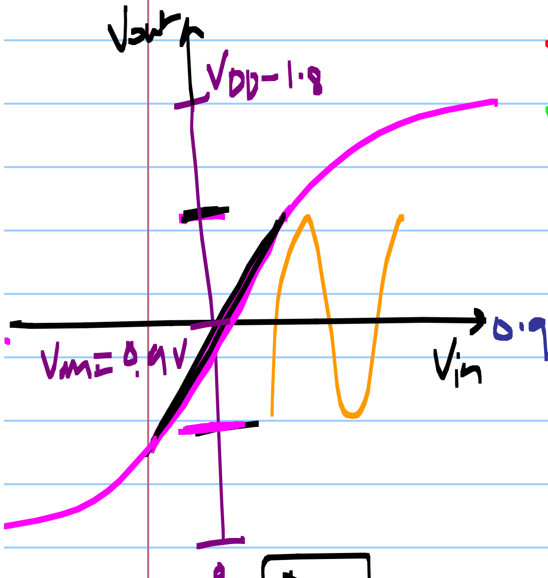
$f_S = 1V$

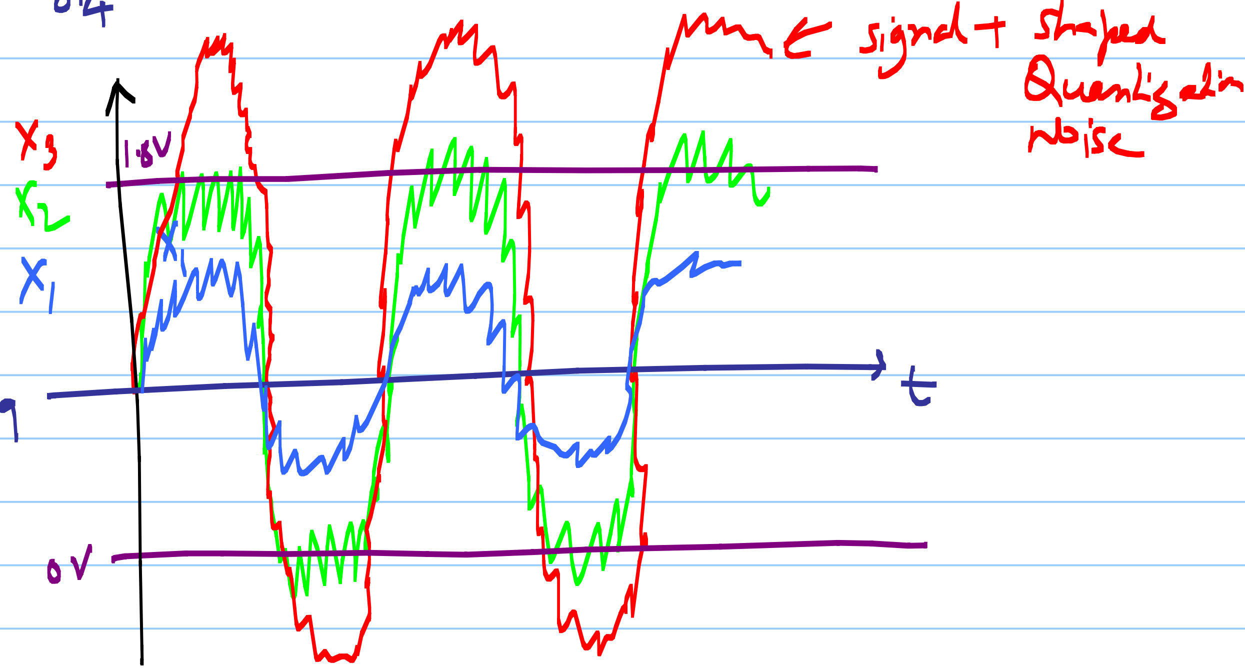
$u = 0.5\, f_S$
$= 0.5V$

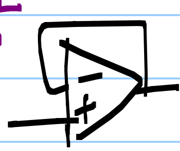


$V_{cm} = 0.9V$

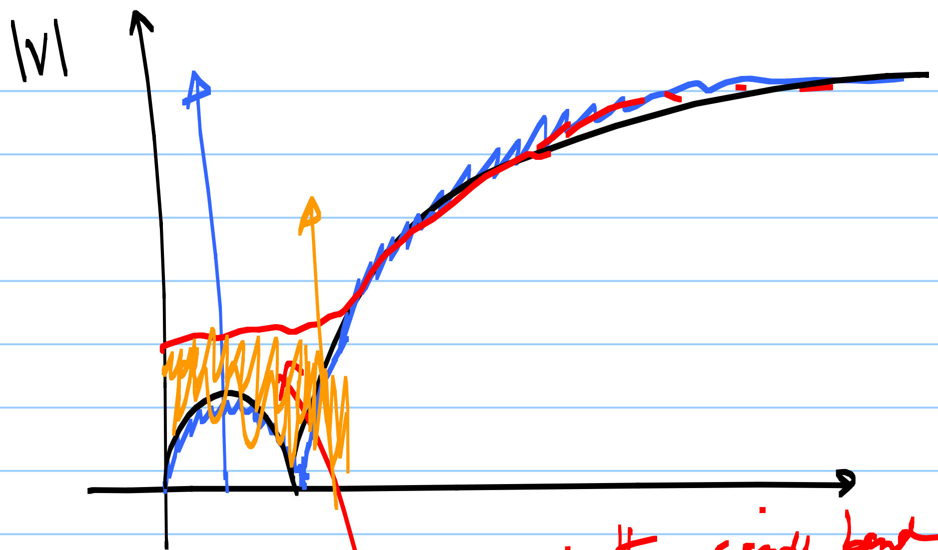
$$\frac{f_{in}}{f_S} = \frac{m}{N_{FFT}}$$







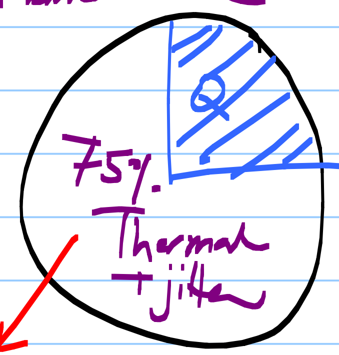
* We want to restrict the states within the linear range of the integrator.



Notch in the signal band gets filled up with Folded Q-noise

Sufficient dithering
↳ thermal noise of the circuit for dithering

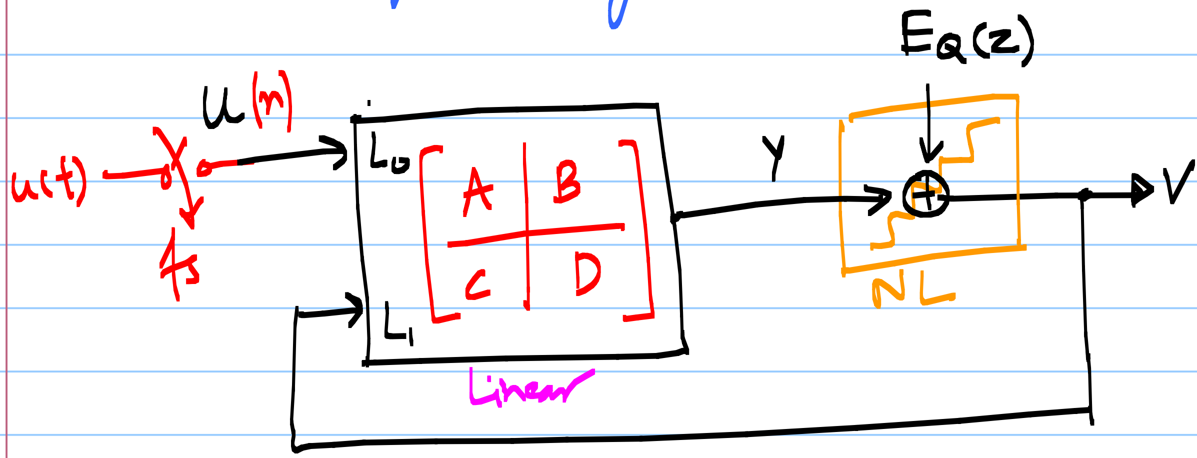
Input-referred in-band noise



$\propto \sqrt{\frac{kT}{C}}$

Dynamic Range Scaling (DRS)

"Linear System Theory"



* The loop filter can be described as a linear system

$$x[n+1] = A\,x[n] + B\begin{bmatrix} u[n] \\ v[n] \end{bmatrix}$$

next state — previous

$x \rightarrow$ states
integrator outputs

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

3×1 vector

$$y(n) = C\,x[n] + D\begin{bmatrix} u(n) \\ v(n) \end{bmatrix}$$

for M^{th}-order modulator,

$x(n) \in \mathbb{R}^{M\times 1} \rightarrow$ state vector

$A \in \mathbb{R}^{M\times M} \rightarrow$ interconnections within the loop filter

$B \in \mathbb{R}^{M \times L} \Rightarrow$ describes how $u(n)$ & $v(n)$ are applied to the loop filter

$C \in \mathbb{R}^{1 \times M}$
$D \in \mathbb{R}^{1 \times L}$ $\Big\} \Rightarrow$ compute $y(n)$ from the states and

$D \rightarrow$ direct path from inputs to $y(n)$

$\underline{\underline{M=3}}$

$$A = R^{3\times 3}$$

$$\dot{x} = Ax$$

$$x(n+1) = Ax(n)$$

$$\begin{array}{c|ccc} & \dot{x}_1 & \dot{x}_2 & \dot{x}_3 \\ \hline x_1 & & c_1 & \\ x_2 & & & c_2 \\ x_3 & & -g_1 & \end{array}$$

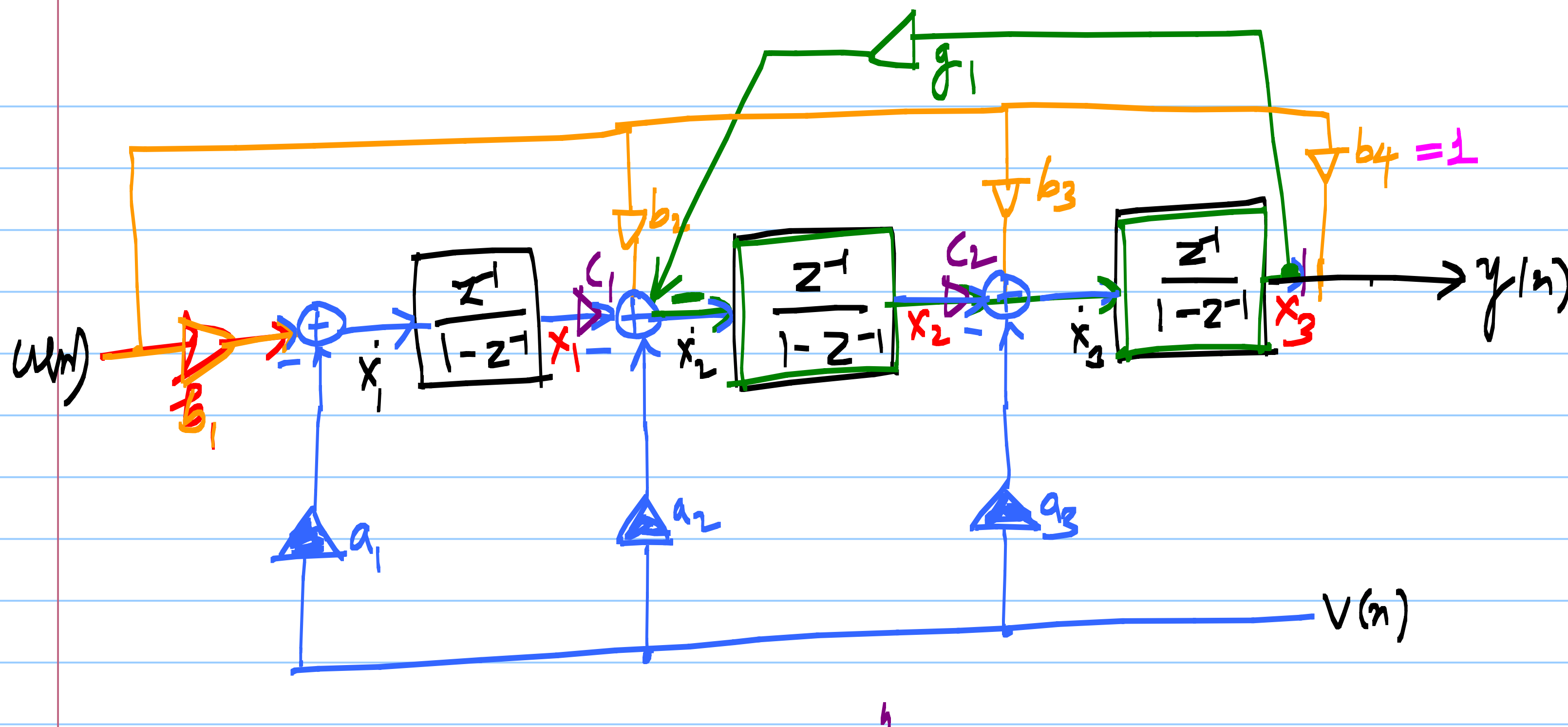
$$\dot{x}_2 = x_1 \cdot c_1$$

$$\dot{x}_3 = x_2 \cdot c_2$$

$$B = \left[\begin{array}{c|c} b_1 & -a_1 \\ b_2 & -a_2 \\ b_3 & -a_3 \end{array}\right]$$

$$C = \begin{bmatrix} 0 & 0 & c_3 \end{bmatrix}$$

$$D = \begin{bmatrix} b_4 & 0 \end{bmatrix}$$



u(n)
b_1
$\dot{x}_1$
z^{-1}
$1-z^{-1}$
x_1
c_1
b_2
$\dot{x}_2$
x_2
c_2
b_3
$\dot{x}_3$
x_3
$b_4 = 1$
g_1
y(n)
a_1
a_2
a_3
V(n)

Find $\underbrace{L(z)}$ from $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$

$$L_0(z) = \left.\frac{Y(z)}{U(z)}\right|_{V(z)=0}$$

$$L_1(z) = \left.\frac{Y(z)}{V(z)}\right|_{U(z)=0}$$

CT in linear systems course

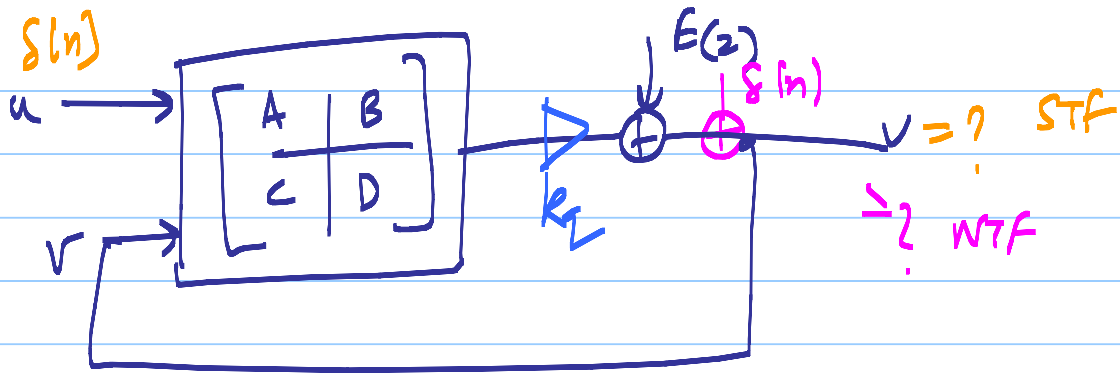
$$L(s) = C(sI-A)^{-1}B + D$$

DT

$$\begin{bmatrix} L_0(z) \\ L_1(z) \end{bmatrix} = C\left(z^{-1}I_{m\times m} - A\right)^{-1} B + D$$

$$NTF(z) = \frac{1}{1+L_1(z)}$$

$$STF(z) = \frac{L_0(z)}{1+L_1(z)}$$



δ(n)
u
v
A
B
C
D
E(z)
δ(n)
k_q
v = ? STF
= ? NTF

$$D_{cL} = \begin{bmatrix} k_q D_1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} STF(z) \\ NTF(z) \end{bmatrix} = C_{cL}\left(z^{-1} I - A_{cL}\right)^{-1} B_{cL} + D_{cL}$$

Synthesize NTF $\longrightarrow$ NTF

$$\text{NTF} \xrightarrow{\text{mapNTF("frm")}} \text{ABCD}$$

CIFB
CRFB
CIFF
CRFF

$X_{lim} = 0.5V$

$$x_{max} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

found by simulation

$$X_{new,max} = S \cdot X_{max}$$

$$\begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$S = \begin{bmatrix} 1/x_1 & & & 0 \\ & 1/x_2 & & \\ & & \ddots & \\ 0 & & & \frac{1}{x_m} \end{bmatrix} = \begin{bmatrix} 1/2 & & 0 \\ & 1/4 & \\ 0 & & 1/6 \end{bmatrix}$$

"Scaling matrix"

linear transformation of the state-space

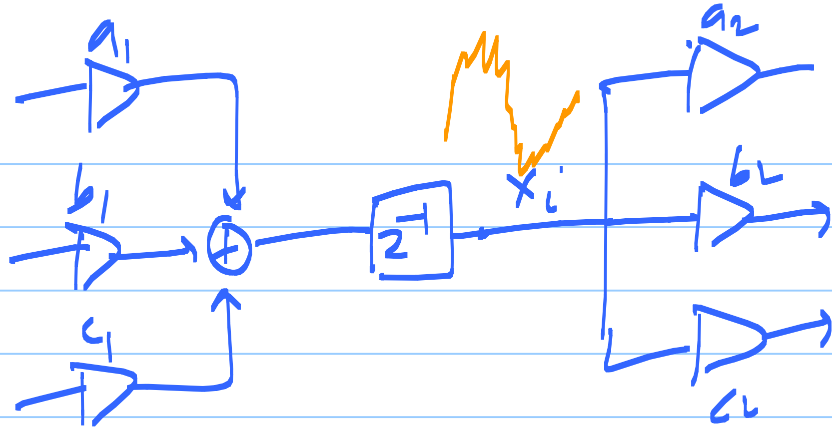
$$x_s = Sx$$

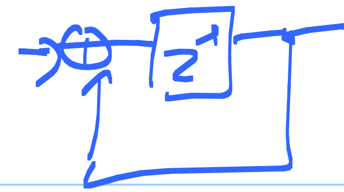
for $x_s = S \cdot x \Leftarrow$ Dynamic Range Scaling

$$ABCD_s = \left[\begin{array}{c|c} SAS^{-1} & SB \\ \hline CS^{-1} & D \end{array}\right] = \left[\begin{array}{c|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array}\right]$$

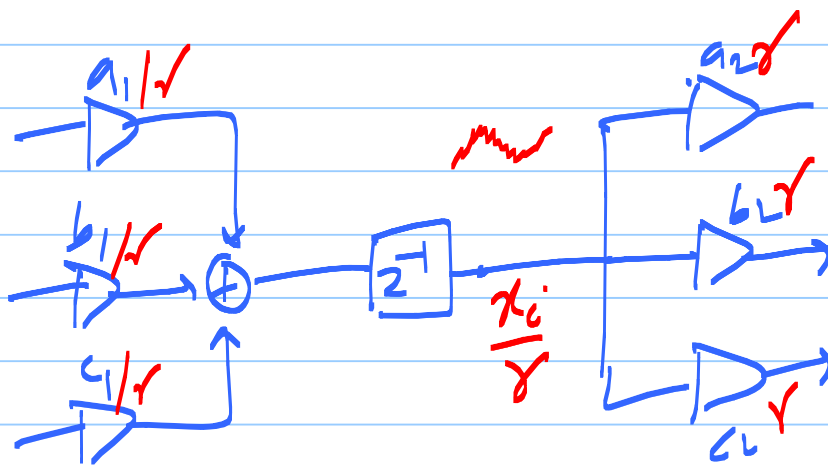
scaled ABCD matrix

scale ABCD





$$\frac{z^{-1}}{1-z^{-1}}$$

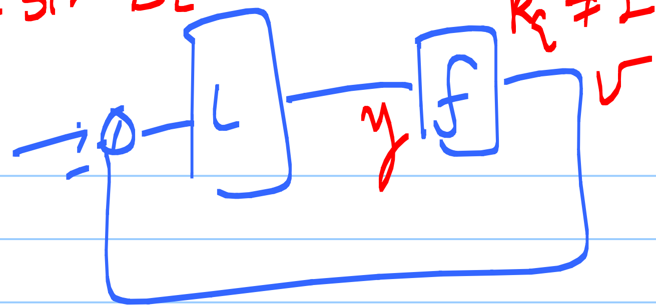


DRS

$C \neq 1$

For single-bit $\Delta\Sigma$

$k_q \neq 1$



$$k_q = \frac{E[|y|]}{E[y^2]}$$