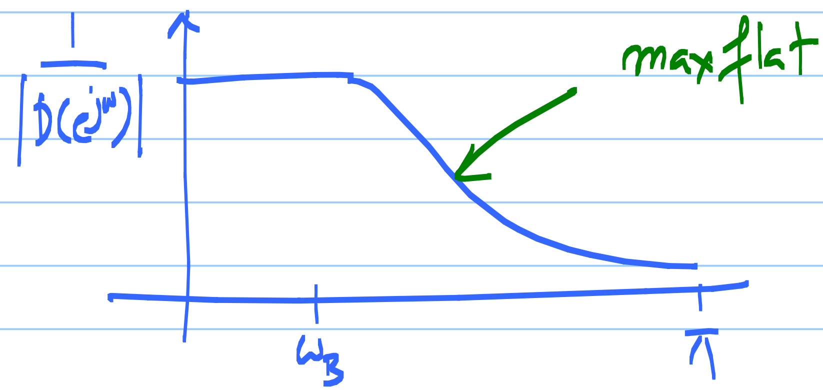
ECE 615 – Lecture 12

Note Title 10/15/2013

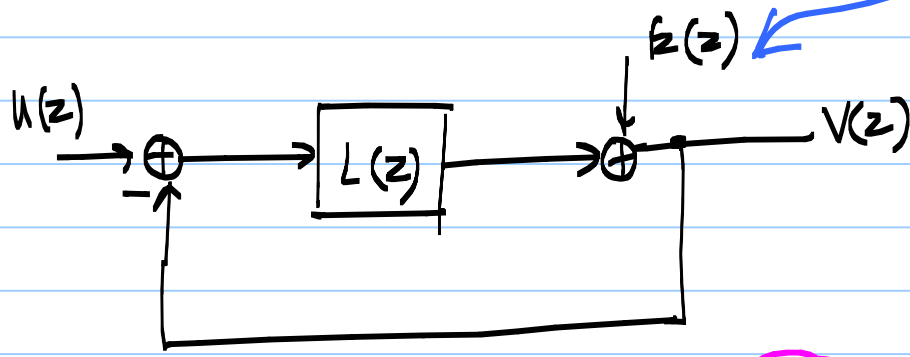
Systematic Design of NTF ⟶ HW 4

$$NTF(z) = \frac{\prod_i (1 - z^{-1} z_i)}{D(z)}$$



Sensitivity of a feedback loop

disturbance injected



$$V(z) = U(z)\cdot\frac{L(z)}{1+L(z)} + E(z)\,\frac{1}{1+L(z)}$$

$|L| \to \infty$ in the signal band

$$\left|\frac{1}{1+L}\right|$$

Sensitivity of the loop $\quad \dfrac{1}{1+L(e^{j\omega})}$

↳ how effectively the loop suppresses the disturbance

↳ loop is insensitive to the disturbance at low frequencies

$\Rightarrow$ $|L(e^{j\omega})|$ is high at low-frequencies

Sensitivity for $\Delta\Sigma$ loop $\rightarrow$ NTF

* $h(0) = \underline{NTF(\infty) = 1}$ ← Realizability Condition

zero at DC

Complex zero pairs

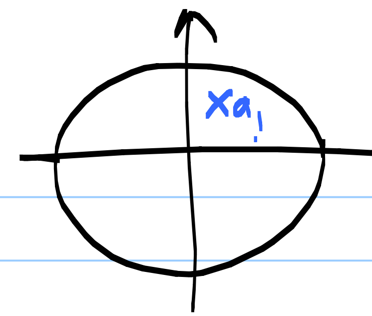
$$NTF(z) = \frac{(1+a_1 z^{-1})(1+a_2 z^{-1}+a_3 z^{-2})(\cdots)}{(1+b_1 z^{-1})(1+b_2 z^{-1}+b_3 z^{-3})(\cdots)}$$

poles must be within the unit circle

& zeros are on the unit circle.

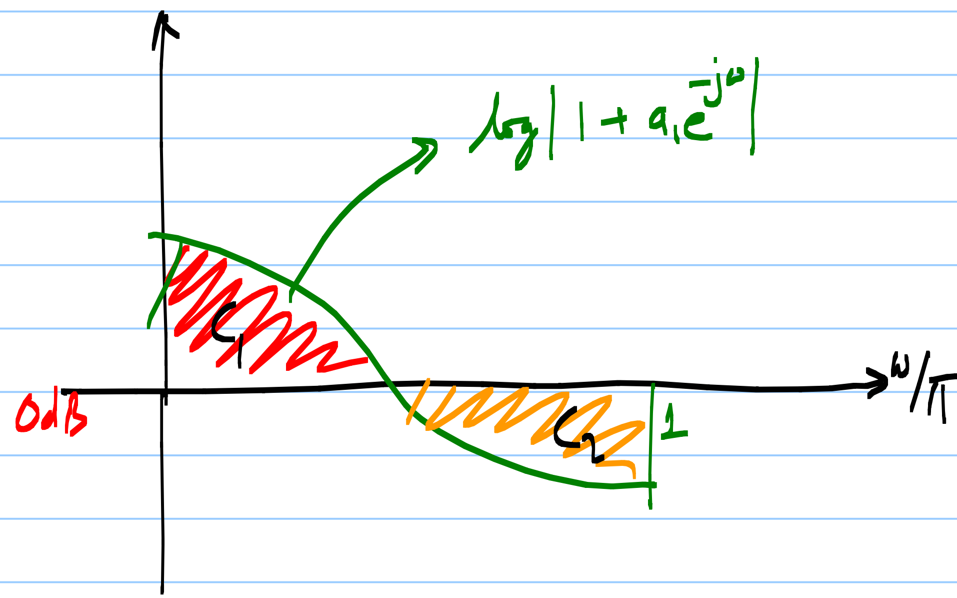
$$\frac{1}{1+az^{-1}}$$

pole at a_1



it can be shown that

$$\int_0^{\pi} \log\left|1+a_1 e^{-j\omega}\right| d\omega = 0 \quad \text{if } |a_1| < 1$$



$C_1 = C_2$

Area above 0dB line = Area below 0dB line

Similarly.
$$\int_0^{\pi} \log\left|1 + a_2 e^{-j\omega} + a_3 e^{-j2\omega}\right| d\omega = 0.$$

$$\int_0^{\pi} \log\left|NTF(e^{j\omega})\right| \cdot d\omega = \int_0^{\pi} \log\left|\frac{(1+a_1 e^{-j\omega})(1+a_2 e^{-j\omega}+a_3 e^{-j2\omega})}{(1+b_1 e^{-j\omega})(1+b_2 e^{-j\omega}+b_3 e^{-j2\omega})}\right| d\omega$$

$$= \int_0^{\pi} \log\left|1+a_1 e^{-j\omega}\right| + \int - \int \log\left|1+b_1 e^{-j\omega}\right| + \ldots$$

$$= 0$$

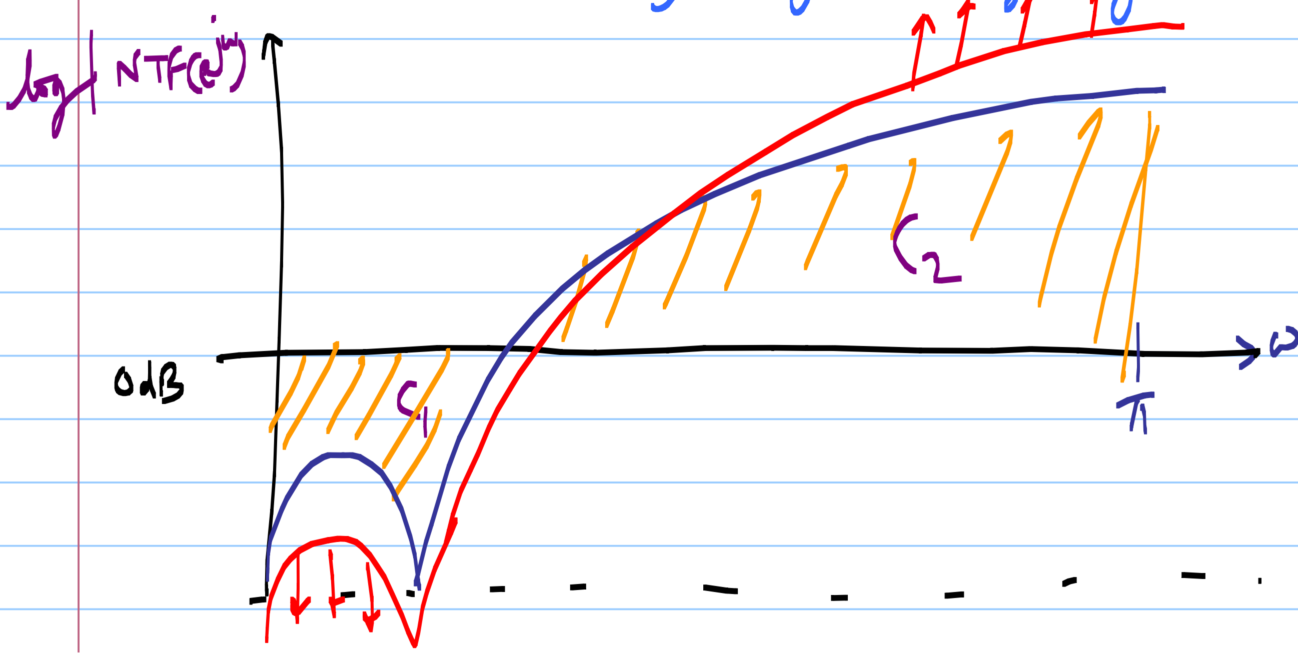
when all NTF poles are within unit circle

zeros on the unit circle

Bode's Sensitivity Theorem

$$\boxed{\int_0^{\pi} \log \left| NTF\left(e^{j\omega}\right) \right| d\omega = 0}$$

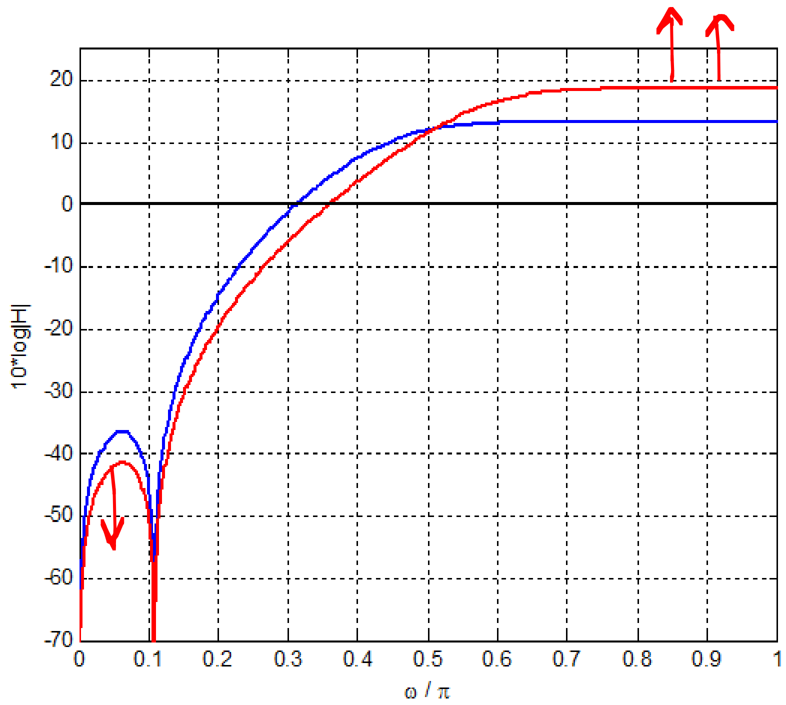
the integral of the log-magnitude of a 'stable' NTF = 0



$C_1 = C_2$

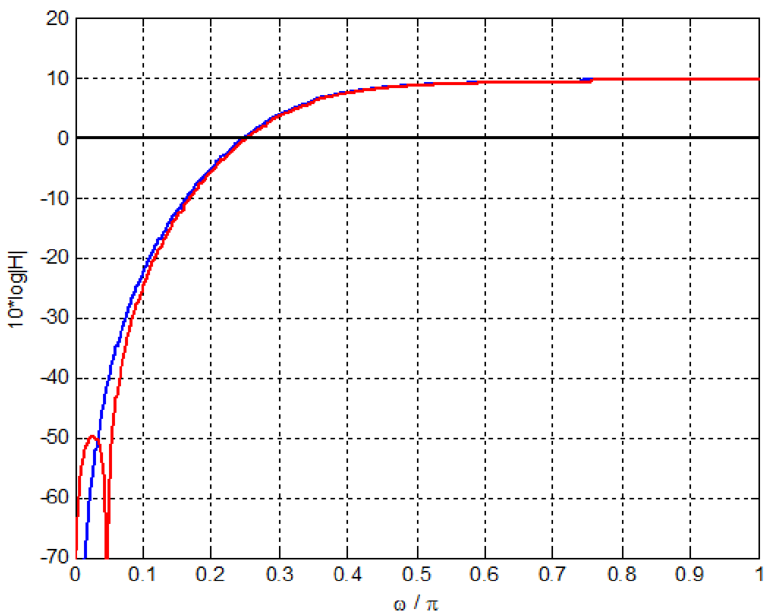
Trade-off between IBN & OBG

"Good in-band performance comes at the cost of poor OBG"



Same order

different OBG

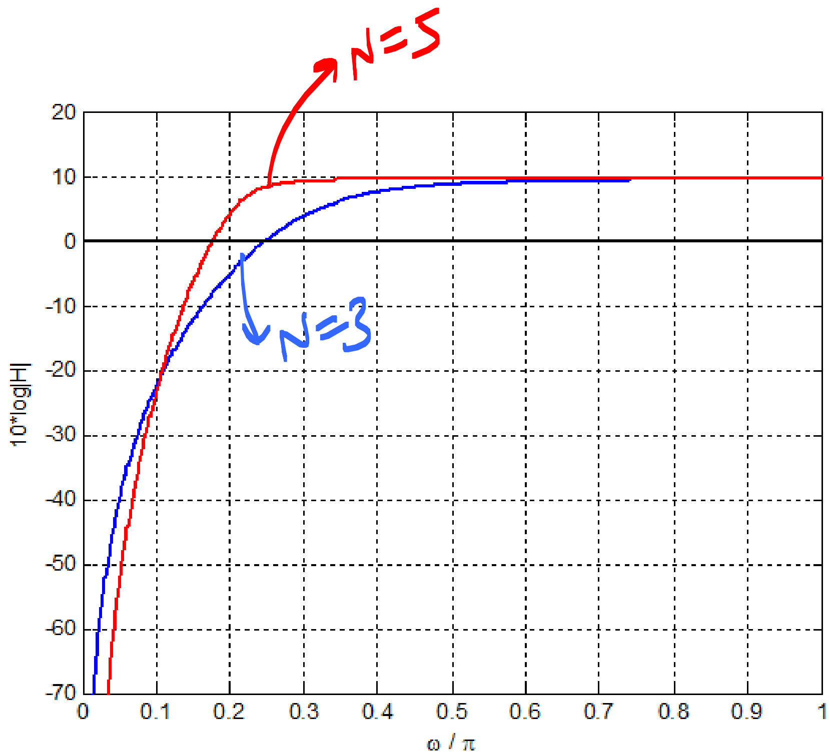


Same OBG

but complex zeros

result in better SQNR

File: BodeSensitivity3.m



Same OBG

higher-order

↳ better SQNR

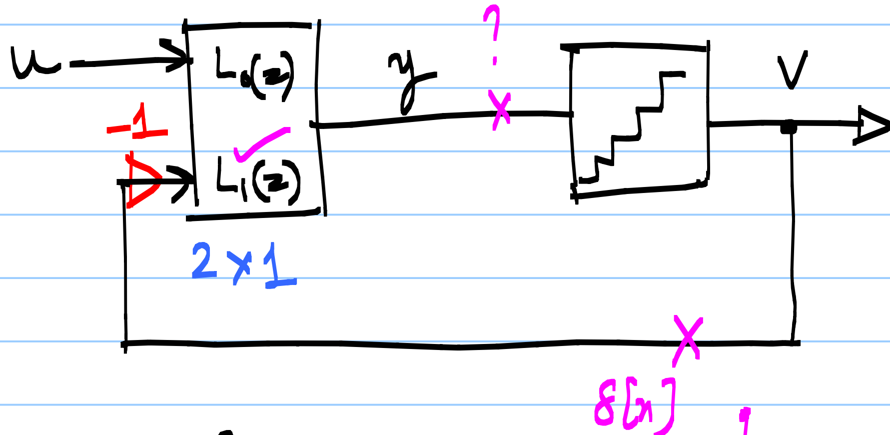
Higher-order Modulators

$$NTF(z) = \frac{N(z)}{D(z)}$$

Architectures?

"ΔΣ Toolbox"

In this notation



2×1

$$NTF(z) = \frac{1}{1+L_1(z)}$$

$$STF(z) = \frac{L_0(z)}{1+L_1(z)}$$

$L_1 \Leftarrow$ loop filter seen by the feedback path

$L_0 \Rightarrow$ loop-filter seen by the input

$$NTF = \frac{1}{1-L_1(z)}$$

Observations

* $|NTF| \simeq \frac{1}{|L_1|}$ in the signal band

↑ $|L_1|$ has large gain in the signal band

$\Rightarrow$ quantization noise is reduced

* $|STF| \simeq \frac{|L_0|}{|L_1|} \simeq 1$ in the signal band

$\Rightarrow$ signal is not distorted

loop filter $\qquad L_1(z) = \dfrac{N_1(z)}{D_1(z)}$

$$NTF(z) = \frac{1}{1 - L_1(z)} = \frac{1}{1 - \frac{N_1(z)}{D_1(z)}} = \frac{D_1(z)}{D_1(z) - N_1(z)}$$

* poles of loop filter $\Rightarrow$ zeros of the NTF

  $\hookrightarrow$ important

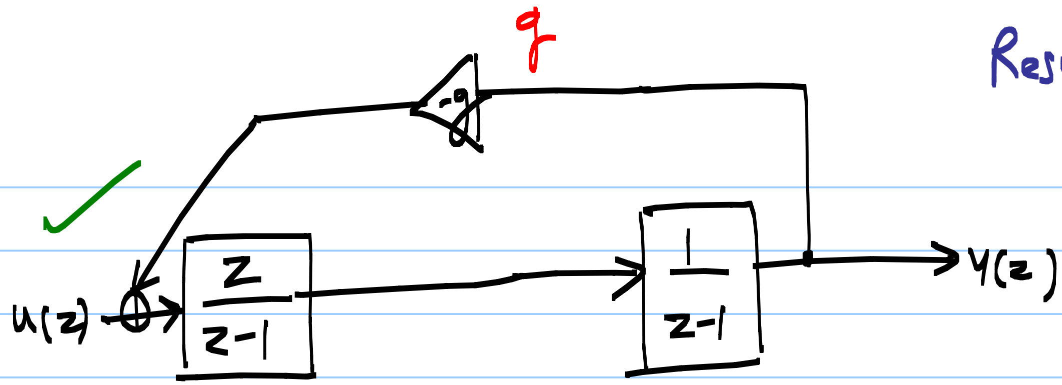
* NTF & STF share the same poles

  $\hookrightarrow$ roots of $1 - L(z) = 0$

$$NTF(z) \longrightarrow \begin{matrix} L_0(z) \\ L_1(z) \end{matrix}$$

Loop-filter architectures

Resonator



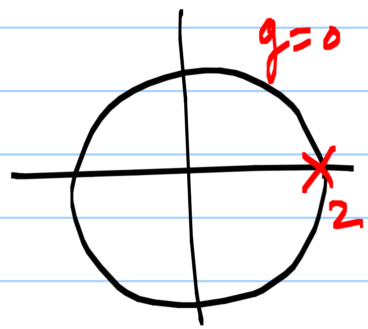
$\frac{1}{1-z^{-1}}$ $\frac{z^{-1}}{1-z^{-1}}$

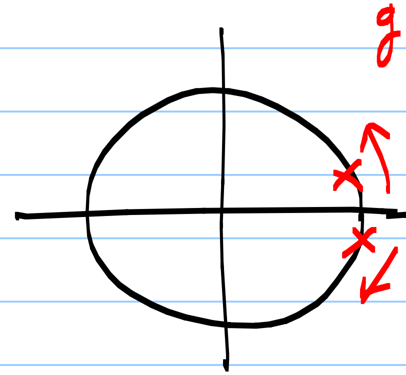
small amount of local feedback

↓

$g>0$

$g=0$





Complex poles in $L_1(z)$

↳ Complex NTF zeros

$$u \cdot \frac{z}{(z-1)^2} - \frac{gz}{(z-1)^2} \cdot y = y$$

$$\Rightarrow \quad R(z) = \frac{y}{u} = \frac{z}{(z-1)^2 + gz} = \frac{z}{z^2 - (2-g)z + 1}$$

Let $\cos\alpha = 1 - \frac{g}{2}$

$$= \frac{z}{z^2 - (2\cos\alpha)z + 1}$$

$$= \frac{z}{(z - e^{j\alpha})(z - e^{-j\alpha})}$$

Complex poles of L_1 at $e^{\pm j\alpha}$

where $\alpha = \cos^{-1}\left(1 - \frac{g}{2}\right)$

Complex roots at $\omega = \pm\alpha$

$= \pm\cos^{-1}\left(1 - g/2\right)$

$$\cos\alpha = 1 - g/2$$

$$1 - 2\sin^2\frac{\alpha}{2} = 1 - g/2$$

$$\Rightarrow \quad \sin\left(\frac{\alpha}{2}\right) = \pm\frac{\sqrt{g}}{2}$$

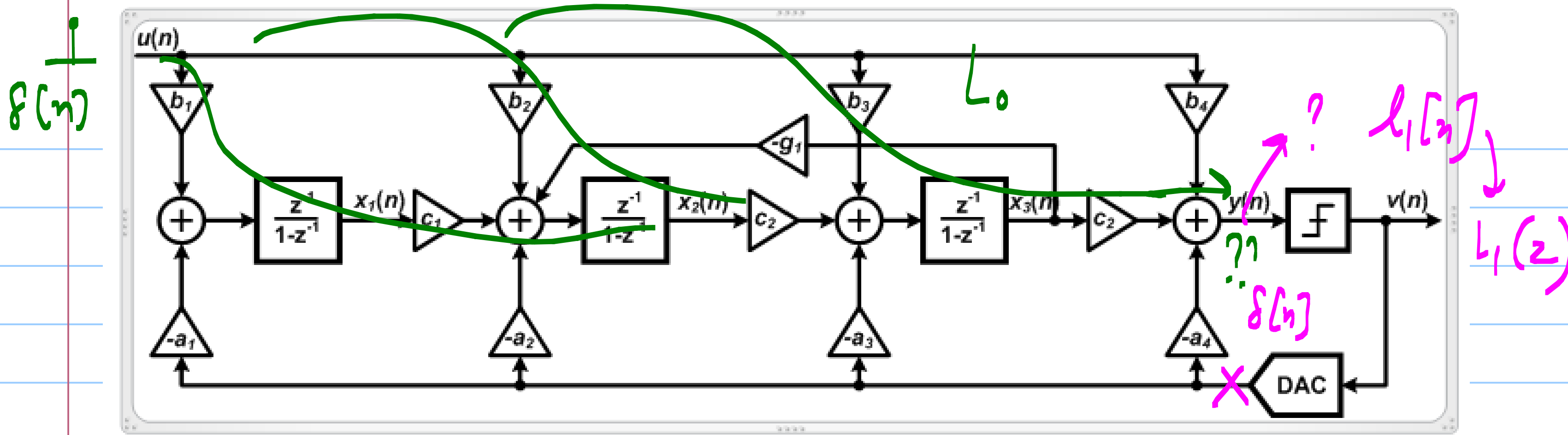
$$\Rightarrow \quad \alpha \simeq \pm\sqrt{g} \quad \text{for} \quad \alpha \ll \pi \quad \because \sin\alpha \simeq \alpha$$

poles at $z = e^{\pm j\sqrt{g}}$ → Complex NTF zeros

NTF

realize NTF( ) in the DΣ Toolbox

[a, b, g] values



By inspection:

$$L_1(z) = -\frac{a_1 + a_2(z-1) + \cdots + a_N(z-1)^{N-1}}{(z-1)^N} \Leftarrow NTF(z)$$