ECE 615 – Lecture 10

Note Title 10/3/2013

"OBG" ↑ → more wiggling

↓

quantizer clips more often

↓

k_1 drops

↓

poles in NTF move closer to the unit circle

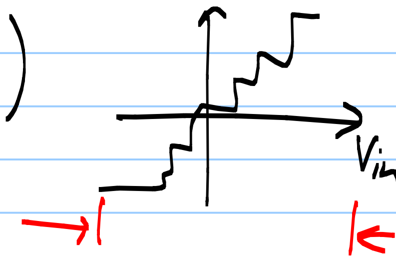
↓

high-Q response (peaking NTF response)

runaway loop

Multibit $\Delta\Sigma$ Modulator (n-bit quantizer)

* If we increase the # of quantizer levels (2^n)



$\Rightarrow$ LSB size decreases $\left(\Delta = \frac{FS}{2^n}\right)$

$\Rightarrow$ variance of quantization noise at the quantizer input

$NTF = (1-z^{-1})^N$

$$\sigma_y^2 = \frac{\Delta^2}{12}\left(\sum_n h^2[n] - 1\right) = \begin{cases} 5\,\frac{\Delta^2}{12} & N=2 \\ 19\,\frac{\Delta^2}{12} & N=3 \\ 69\,\frac{\Delta^2}{12} & N=4 \end{cases}$$

$h[n] \overset{\mathcal{Z}}{\Longleftrightarrow} NTF(z)$

order of the modulator

$\Delta \downarrow 2 \Rightarrow \sigma_y^2 \downarrow 4$

* Maximum Stable Amplitude (MSA) $\longrightarrow$ u_{max}

MSA < FS of the quantizer

$(1-z^{-1})^3$

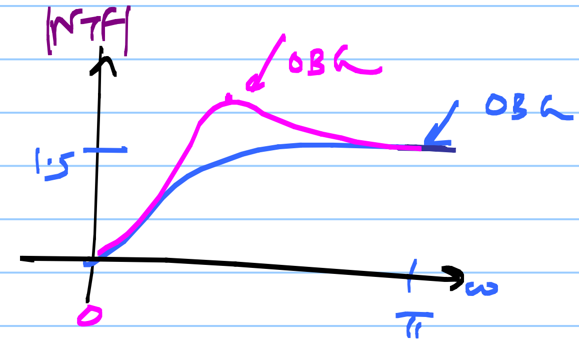
$\Rightarrow$ NTF order, $N \uparrow$ $\Rightarrow$ MSA $\downarrow$

$n \uparrow \Rightarrow \Delta \downarrow \Rightarrow$ MSA $\uparrow$

A good value for max stable amplitude: MSA $\simeq 0.8$

* for a single-bit $\Delta\Sigma$ modulator.

$$OBG \leq 1.5 \leftarrow \text{Lee's Criterion}$$



$$OBG = \max_{\omega} |NTF(e^{j\omega})|$$

$$= \text{infinity norm} \Rightarrow \|NTF\|_{\infty}$$

$$NTF(z) = (1-z^{-1}) \Rightarrow OBG = 2$$

$$(1-z^{-1})^2 \Rightarrow OBG = 4$$

Single-bit $\Delta\Sigma$ $\Rightarrow$ easy circuit-level design (low-power)

↳ but can't use aggressive NTF

OBG ≤ 1.5

Derivation Skipped

\# of levels

$$\max u(n) \le M + 2 - \|h\|_1$$

1-norm of the NTF

$$\sum_n |h[n]|$$

Ex $M = 16$ levels, $NTF(z) = (1-z^{-1})^3 \Rightarrow \|h\|_1 = 8$

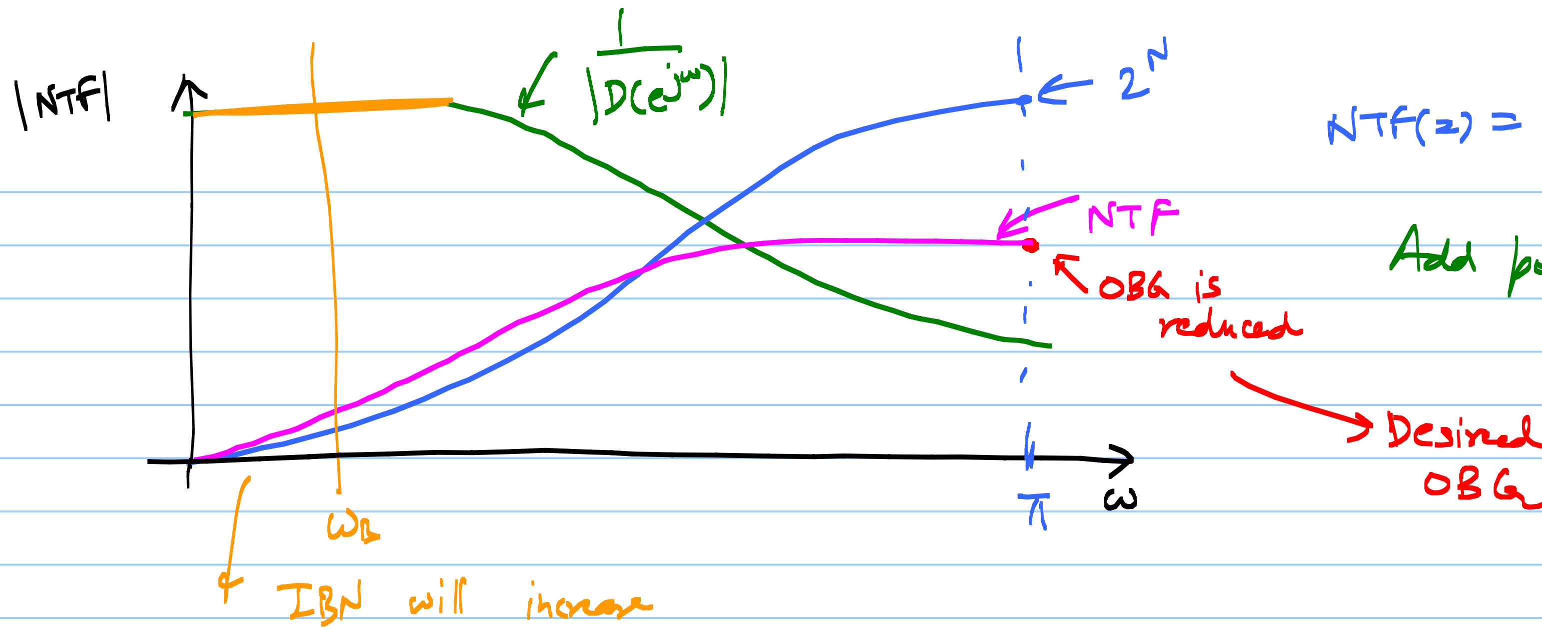
$$\Rightarrow \frac{\max u(n)}{FS} = 62.5\,\%$$

$\boxed{OBG = 8}$

Systematic NTF Design Procedure

* for higher-order NTF $= \cancel{(1-z^{-1})^N}$, MSA reduces drastically as OBG $= 2^N$

* We want the MSA to be 75% – 80% of the FS



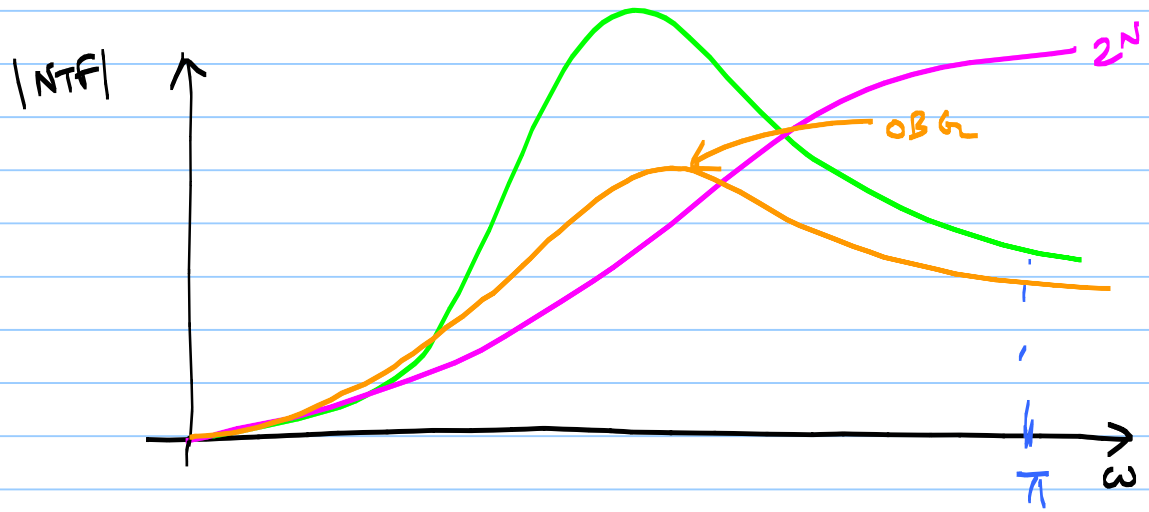
$$NTF(z) = \frac{(1 - z^{-1})^N}{D(z)}$$

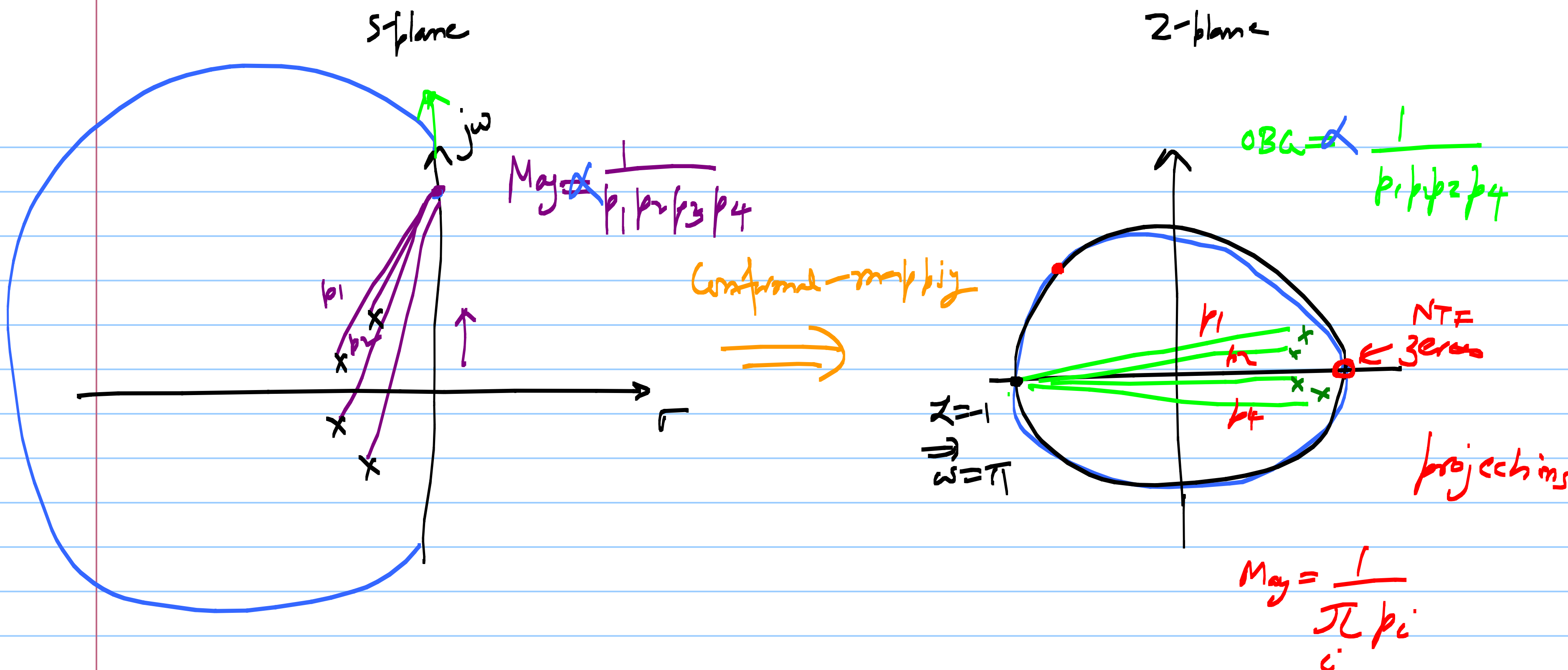
Add poles

for a given order ⇒ OBG ⟺ SQNR

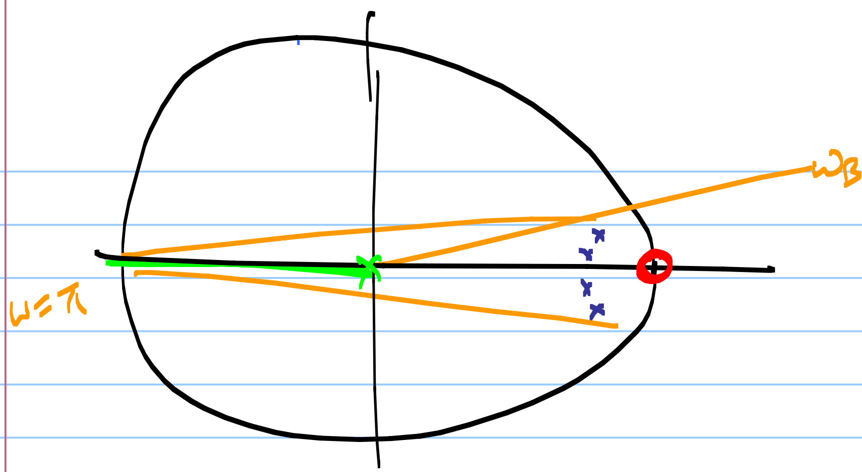
Stability ⟺ performance

$\frac{1}{D(z)} \leftarrow$ LPF

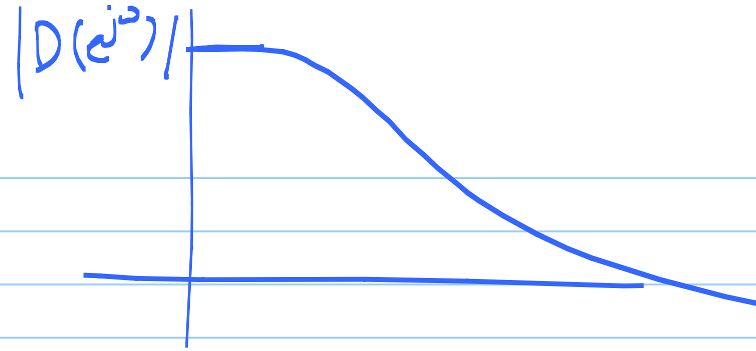




* Gain at $\omega=\pi$ is lower as the poles are forther away from $z=-1$



ω_B
$\omega=\pi$



$|D(e^{j\omega})|$

* Recall the $\boxed{\text{realizability condition}}$ (delay free loops)

$\underline{\underline{h[n]}}$ $\qquad h[0] = 1 \Rightarrow NTF(\infty) = 1$

If $$NTF(z) = \frac{(1-z^{-1})^4}{a_0 + a_1 z^{-1} + \cdots + a_4 z^{-4}} = \frac{N(z)}{D(z)}$$

HPF

LPF

$NTF(z=\infty) = 1$

$NTF(z^{-1}=0) = 1 \Rightarrow \boxed{a_0 = 1}$

* Which pole configuration to use?

Approximation problem

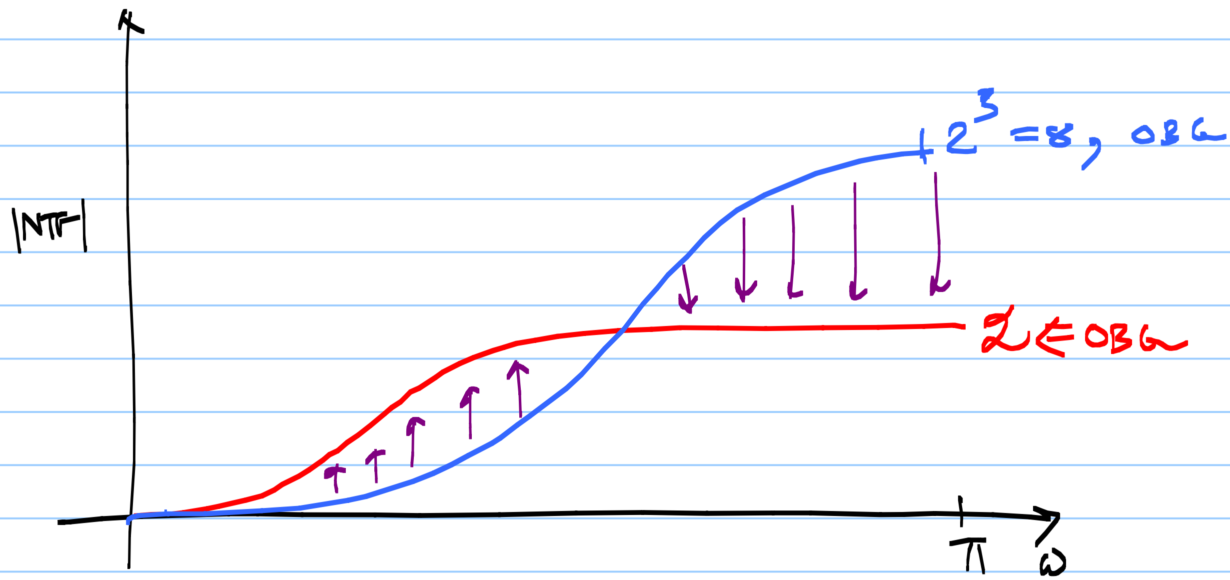
* Approximate the ideal brickwall HPF response with a polynomial NTF(z).

1. Butterworth
2. Inverse Chebyshev
3. Maximally flat all-pole transfer function, etc.

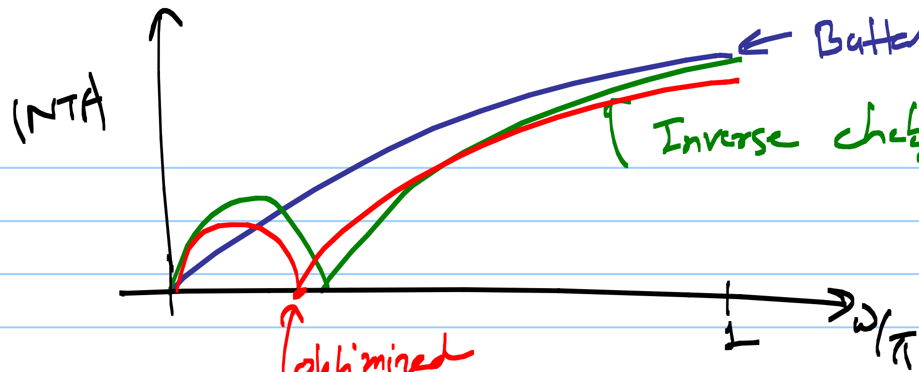
* properly choose pole to reduce OBG

$\Rightarrow$ MSA $\uparrow$ $\Rightarrow$ stability is improved

$\hookrightarrow$ higher IBN



$$\frac{(1-z^{-1})^3}{D(z)}$$



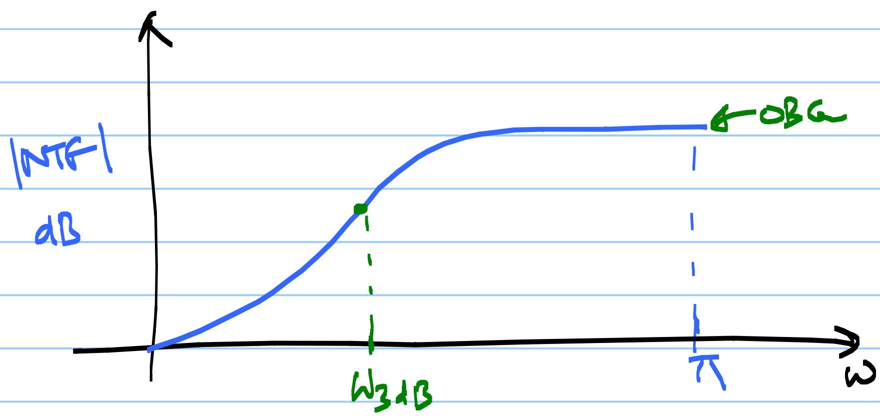
← Butterworth (all NTF zeros at DC)

Inverse chebyshev (complex NTF zeros)

optimized NTF zeros with a maximally flat all-pole $\frac{1}{D(z)}$ response

* MATLAB functions
  - butter
  - cheby2
  - maxflat

* Let's choose a Butterworth NTF response



$$\frac{(1-z^{-1})^3}{a_0 + a_1 z^{-1} + \cdots + a_3 z^{-3}}$$

3rd-order Butterworth response

We need $a_0 = 1$