Assignment 5

ECE 615 − Mixed Signal IC Design

Due on Thursday, October 24, 2013.

Problem 1 Chebyshev NTF Design: This problem explores the use of inverse-Chebyshev high-pass response for NTF design.

1. Explore the MATLAB function for generating an inverse-chebyshev high-pass response.

```
[b,a] = cheby2(order,R,Wst, 'high')
```

`The inverse-chebyshev transfer function uses three parameters, viz, filter order, stop-band attenuation` R `and the stop-band edge-frequency` ω_{st}.

(a) Using an inverse-Chebyshev highpass filter response, determine the NTF of a fifth-order $\Delta\Sigma$ modulator with OSR=16, and OBG=3. You can achieve this by using any of the following design approaches:

i. Use a fixed $\omega_{st} = \frac{\pi}{OSR}$, and then iterate upon R to get the desired OBG.

ii. Use a fixed R (say equal to 60 dB), and then iterate upon $\omega_{st}$to achieve the desired OBG.

(b) Plot the impulse response, $h(n)$ of the NTFs and show that the designed NTF is realizable (i.e. there are no zero-delay loops). Also show the pole-zero plots for the NTF.

(c) Assuming a 4-bit (16-level) quantizer and an OSR=16, write a MATLAB code to simulate the $\Delta\Sigma$ modulator with the NTFs designed in part (a).

(d) Using your **estimateMSA** function created in problem 1, estimate the maximum stable amplitude (MSA) for the design in part (a) of this problem. Determine the peak in-band SQNR in dB. Show the relevant spectrum plots with an appropriately sized Hann window.

Problem 2 NTF Synthesis for low-OSR : This problem explores the limitations of the $\Delta\Sigma$ toolbox NTF synthesis algorithms.

1. Following MATLAB $\Delta\Sigma$ toolbox functions can be used for NTF synthesis (See toolbox demos: *dsdemo1* to *dsdemo4* and *dsexample1*) :

```
ntf = synthesizeNTF(order,OSR,opt,OBG,f0)
ntf = synthesizeChebyshevNTF(order,OSR,opt,OBG,f0)
```

(a) What are the possible values for the optimization parameter 'opt' and what do they stand for ?

(b) What happens if no value for the parameter OBG (or H_inf) is passed in the function call ?

(c) What does the function **rmsGain()** do ? Show that the in-band quantization noise can be expressed as

$$IBN = \frac{\sigma_H^2 \cdot \sigma_e^2}{OSR} \tag{1}$$

where σ_H is the rms noise gain in the signal-band, and $\sigma_e^2 = \frac{\Delta^2}{12}$ is the total quantization noise.

2. Design NTF of a fifth-order $\Delta\Sigma$ modulator with OSR=16, and OBG=3 using both the synthesis functions (**synthesizeNTF** and **synthesizeChebyshevNTF**) . Plot the magnitude responses and pole-zero plots for both the NTFs and compare them. Compute and compare the rms gains for both the NTFs. Which of the two synthesis functions performs better and why ?
3. Repeat part (2) for OSR=8 and OSR=4. What do you conclude from these experiments ?
4. Repeat (2) for a lower out of band gain value of OBG=1.2. What do you observe?

Problem 3 Bode's Sensitivity Theorem : We know that for a stable and minimum-phase NTF (all poles and zeros inside the unit circle), the Bode Sensitivity Theorem states that

$$\int_0^{\pi} log\left|NTF\left(e^{j\omega}\right)\right| \cdot d\omega = 0 \tag{2}$$

which implies that the total area bounded between the log-magnitude of the NTF and the 0-dB line is zero. This can also be interpreted as that the areas above the 0-dB line and below the 0-dB line are equal.

1. Demonstrate the validity of the Bode's Sensitivity Theorem for a fifth-order NTF with OSR=16, and OBG=3 synthesized using the toolbox. Show the two areas (above and below the 0-dB line) in the NTF spectrum using the **area()** command.
2. The amount of 'wiggling' in the time-domain output of a $\Delta\Sigma$ modulator, assuming a low-frequency input, can be defined as

$$\delta v[n] = v[n] - v[n-1] \tag{3}$$

which in z-domain is given by

$$\begin{aligned} \delta V(z) &= V(z) - V(z-1) = (1 - z^{-1})V(z) \\ &= (1 - z^{-1})STF(z)U(z) + (1 - z^{-1})NTF(z)E(z) \\ &\approx (1 - z^{-1})NTF(z)E(z) \end{aligned} \tag{4}$$

Simulate the NTF in part (1) using a 5-bit (32-level) quantizer and a sinusoidal input with MSA and OBG=2, 3 and 4. Plot the $\delta v[n]$ waveforms for the different OBG values. How does the amount of wiggling change as the OBG is increased ?

3. Using Bode's Sensitivity Theorem, show that

$$\int_{0}^{\pi} log\left|\left(1-e^{-j\omega}\right)\cdot NTF\left(e^{j\omega}\right)\right|\cdot d\omega = 0 \tag{5}$$

Plot the area above and below the 0-dB line for integral in Eqn. 4. The variance of the wiggling ($\sigma^2_{\delta v}$) is exponentially related to the area above the 0-dB line (C) as [3]

$$\sigma^2_{\delta v} \geq \frac{\triangle^2}{12\pi}\left(\pi-\omega_1\right) exp\left(\frac{2C}{\pi-\omega_1}\right) \tag{6}$$

where ω_1 is the 0-dB cross-over frequency as shown in Figure 1. What happens to the area C as the OBG is increased ? Can you interpret the trend in the variance of wiggling ($\sigma^2_{\delta v}$) as the OBG is increased ?

Note: As we will see later in the course, the variance of the time-domain wiggling ($\delta v[n]$), given by $\sigma^2_{\delta V}$, contributes to in-band jitter-noise in the continuous-time implementation of the NTF. Also for multi-bit quantizers, $\sigma^2_{\delta V}$ is a good estimator of the relative time-constants in the modulator loop-filter.

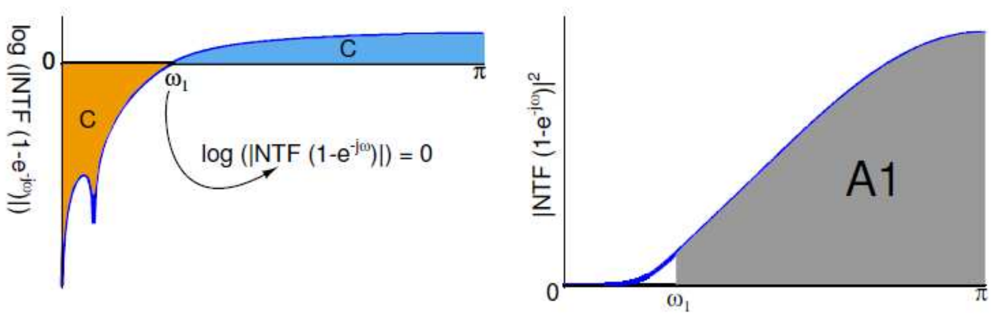

Figure 1: Log-magnitude and magnitude plots for $NTF\left(e^{j\omega}\right)\cdot\left(1-e^{-j\omega}\right)$. Here, ω_1 is the 0-dB cross-over frequency and the area A_1 corresponds to the wiggling variance $\sigma^2_{\delta V}$ [3].