Assignment 1

ECE 615 − Mixed Signal IC Design (Fall 2013)

Due on Thursday, September 26, 2013.

Problem 1: Anti-alias filter design with oversampling : Assume the AAF is an N^{th} order Butterworth filter with a transfer function given by

$$H(s) = \frac{1}{1 + \left(\frac{s}{\omega_{3dB}}\right)^N} \tag{1}$$

and thus its magnitude response is

$$|H(f)| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3dB}}\right)^{2N}}} \tag{2}$$

where $f_{3dB}(\omega_{3dB})$ is the 3-dB bandwidth (angular frequency). The input signal bandwidth is $f_B = 1$ MHz. The specification for the AAF states that the attenuation in the first "alias-band" should be at least 60 dB. Also, the acceptable attenuation in the signal band is less than 0.5 dB. In class, we discussed that it is practically impossible to design a brick-wall AAF so that the sampling frequency (f_s) could be equal to the Nyquist rate (i.e. $f_s = 2f_B$= 2 MHz).

1. If the sampling rate (f_s) is 4 MHz, what is the oversampling ratio (OSR) in this case? What is the minimum order (N_{min}) required of the AAF? What is the bandwidth (f_{3dB}) of the AAF ? For the minimum order, N_{min}, how much the filter bandwidth can vary while still meeting the attenuation specification?
2. The sampling rate is now increased to 64 MHz. What is the OSR ? What is the minimum order (N_{min}) required of the AAF? What is the bandwidth (f_{3dB}) of the AAF ? For the minimum order, N_{min}, how much the filter bandwidth can vary while still meeting the attenuation specification?

Hint: For analog filter design, explore the following commands in MATLAB :

```
[n,Wn] = buttord(Wp,Ws,Rp,Rs, 's');
[z,p,k] = butter(n, Wn,'s');
[b,a] = zp2tf(z,p,k);
```

Problem 2 DTFT and DFT:

1. Consider the sequence

$$r[n] = \begin{cases} 1, & 0 \leq n \leq N \\ 0, & otherwise \end{cases} \tag{3}$$

   (a) Sketch $r[n]$, determine the DTFT $R\left(e^{j\omega}\right)$ and sketch its magnitude and phase.
   (b) Find DFT $R[k]$ of the sequence $r[n]$ and sketch its magnitude and phase response. Show that $R[k]$ can be obtained by sampling $R\left(e^{j\omega}\right)$ in frequency domain as $\omega = \frac{2\pi}{N}$.
   (c) Confirm your DTFT and DFT plots with MATLAB for $N = 8$.
   (d) Notice that $r[n]$ is a ractangular window. What is the mainlobe width and the first sidelobe depth ?

Hint:

```
r = [1 1 1 1 1 1 1 1];  % 8-point rect signal
fvtool(r);              % plot DTFT
R=fft(r)                % find DFT
```

Problem 3 Spectral Windows: Some commonly used FFT spectral windows available in MATLAB are Hann (or Hanning), Bartlett, Hamming, Blackman and Blackman-Harris.

1. Look up the MATLAB documentation on spectral windows (*command: doc window*). List the equations of the discrete sequences $w[n]$ for each of these windows.
2. Using the command *wvtool(hann(N), blackman(N),...., rectwin(N))* overlay the time-domain and frequecy domain plots for all of the windows, along with the rectangular window (rectwin). Here, the variable $N = 64$ is the window length.
3. Compare the first side-lobe depth of these windows and rank them in the increase order of side-lobe suppression.
4. Find the number of non-zero FFT bins for these windows, including the rectangular window.
5. Which of these windows will you prefer for plotting FFT of the results obtained from a simulation (where the ratio of the input and sampling frequency can be precisely controlled)? Which window is the best for processing experimental results where the relation between input and sampling frequency can not be precisely controlled? Explain.
6. In the text book, there is another window called $Hann^2$ window which is given by

$$w[n] = \begin{cases} \left(\frac{1-cos\left(\frac{2\pi n}{N}\right)}{2}\right)^2, & 0 \leq n \leq N \\ 0, & otherwise \end{cases} \tag{4}$$

   Using MATLAB, compare the $Hann^2$ window with the Hann window. How much is the first side-lobe suppression? What are the number of non-zero FFT bins? Is this window better than Blackman-Harris window for side-lobe suppression?