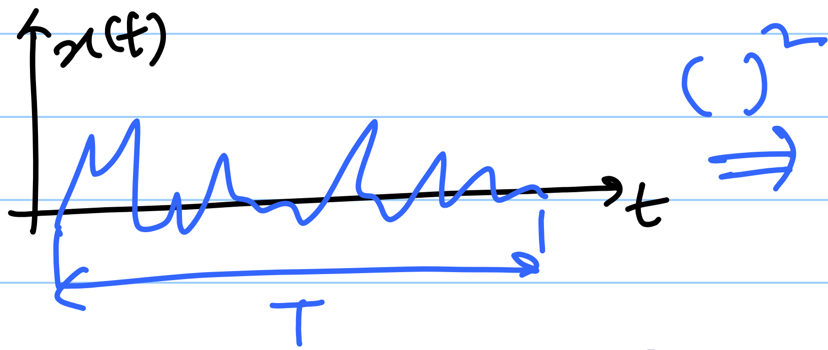
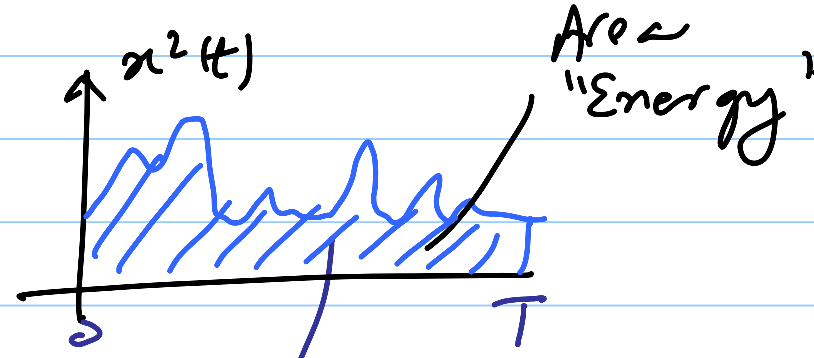
ECE 614 - Lecture 9.

Note Title 9/23/2014

Noise:





$$P_{av} = \frac{\text{Area}}{T}$$

$$P_{av} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

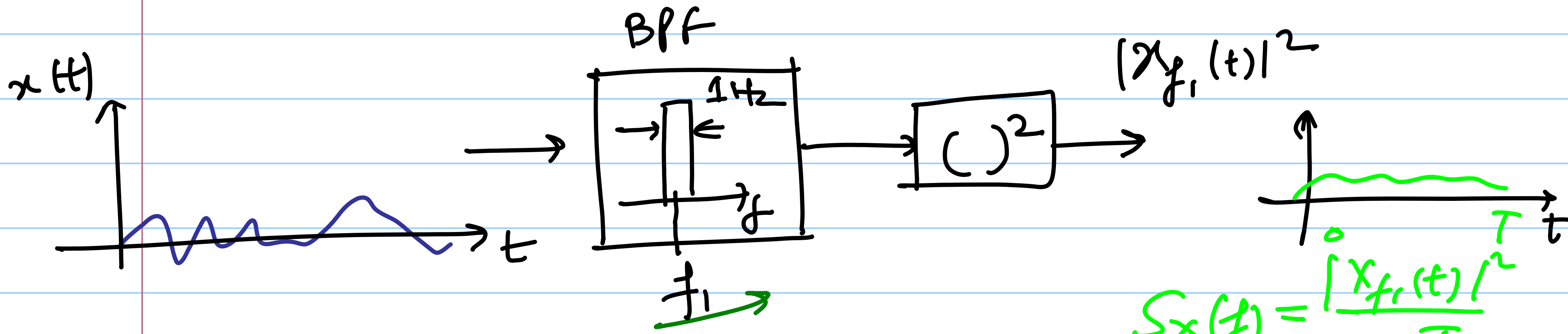
$R_L = 1\Omega$

P_{av} is expressed in V^2, instead of W

* rms voltage of noise $\Rightarrow$ $V_{rms} = \sqrt{P_{av}}$

Noise spectrum:

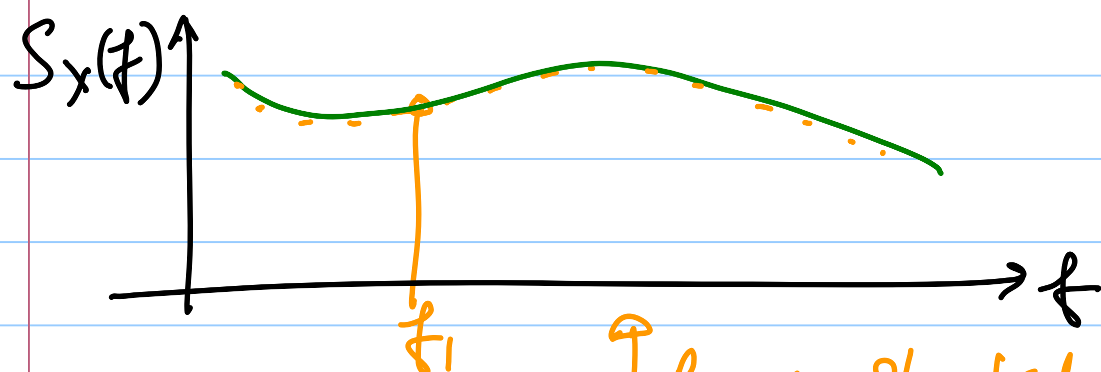
Noise → Random process



$$S_x(f) = \frac{|x_{f_1}(t)|^2}{T}$$

average power in 1Hz bandwidth at frequency f_1

Sweep f_1 from 0 to ∞

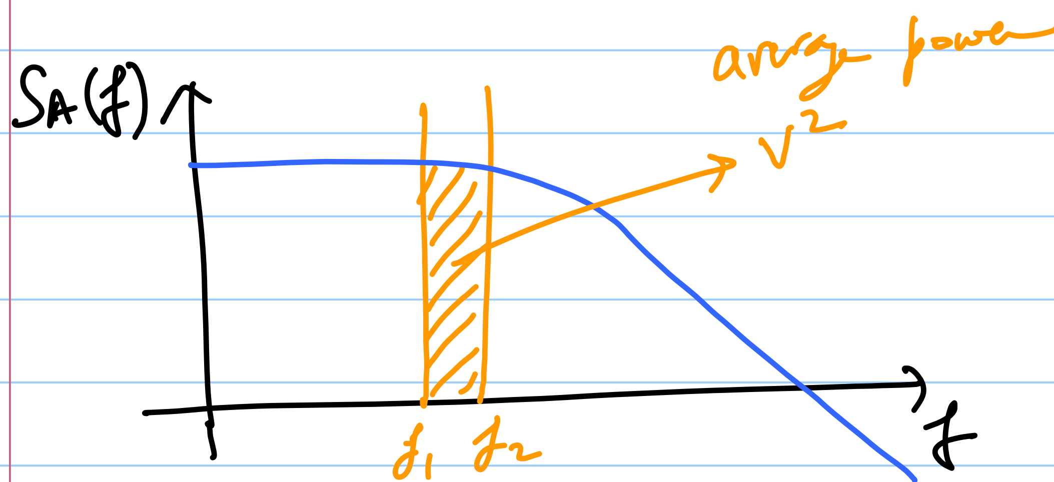


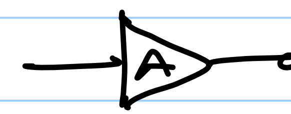
Power Spectral Density (PSD)

* most of the noise sources have a predictable spectrum.

PSD: $S_x(f) \Rightarrow \dfrac{V^2}{Hz}$

$VSD \triangleq \sqrt{S_x(f)} \Rightarrow \dfrac{V}{\sqrt{Hz}}$



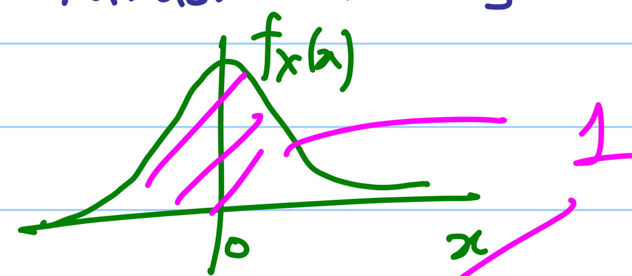


Mathematically:

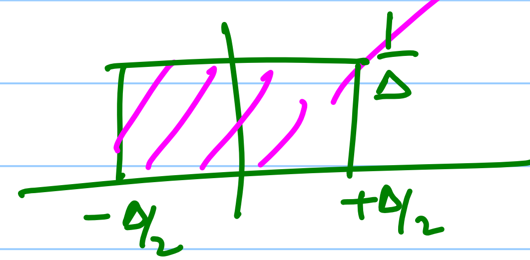
$x(t) \leftarrow$ random process, noise

$x(t_1), x(t_2) \ldots, x(t_n) \longrightarrow$ random variables

Gaussian



Uniform



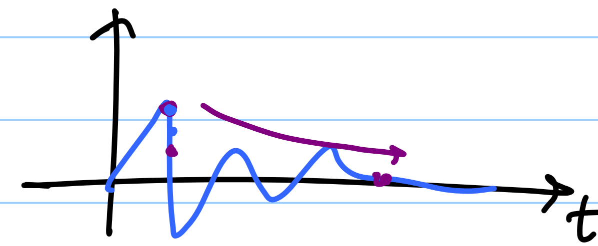
Wide Sense "Stationary" Random Process

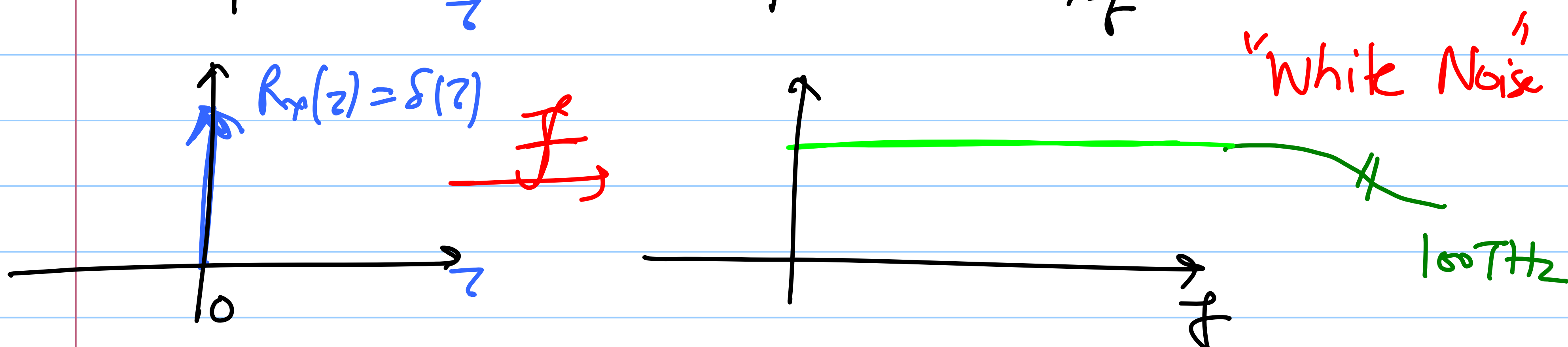
$$E\left(x(t_1) \cdot x(t_2)\right) = E\left[x(t_1) \cdot x(t_1 - \tau)\right]$$

Autocorrelation of a random process

$$R_x(\tau) = E\left[x(t) \cdot x(t-\tau)\right]$$

$$\mathcal{F}\left(R_x(\tau)\right) = S_x(f)$$



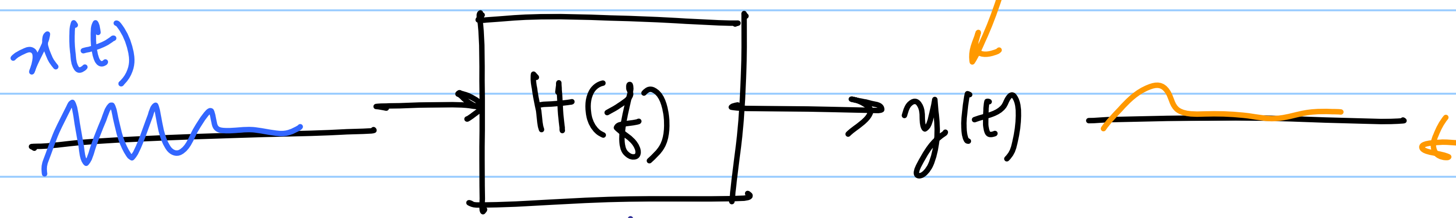


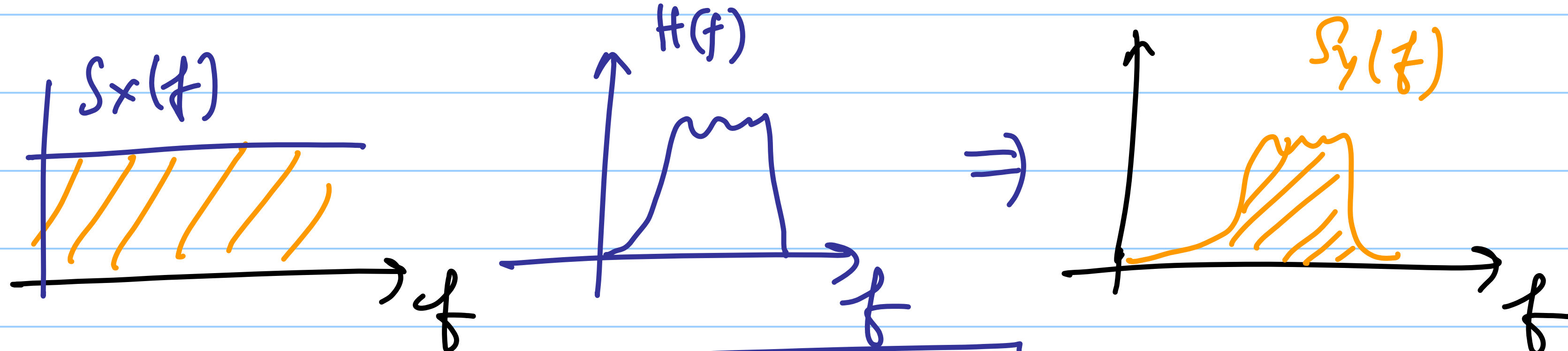
Therom:

L.T.I.

Linear Time Invariant

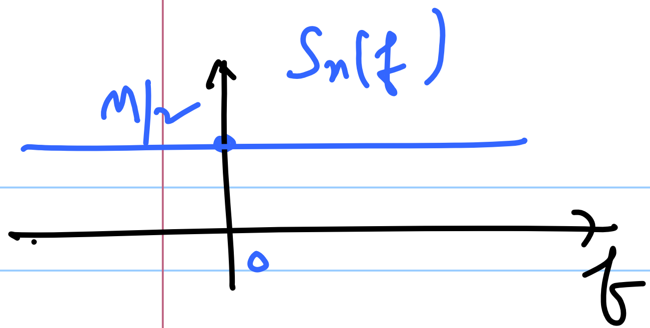
filtered noise



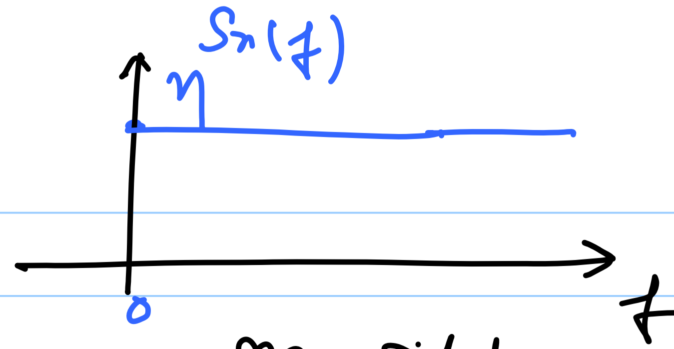


$$\boxed{S_y(f) = S_x(f) \cdot |H(f)|^2}$$

$\rightarrow H(f) \cdot H^*(f)$



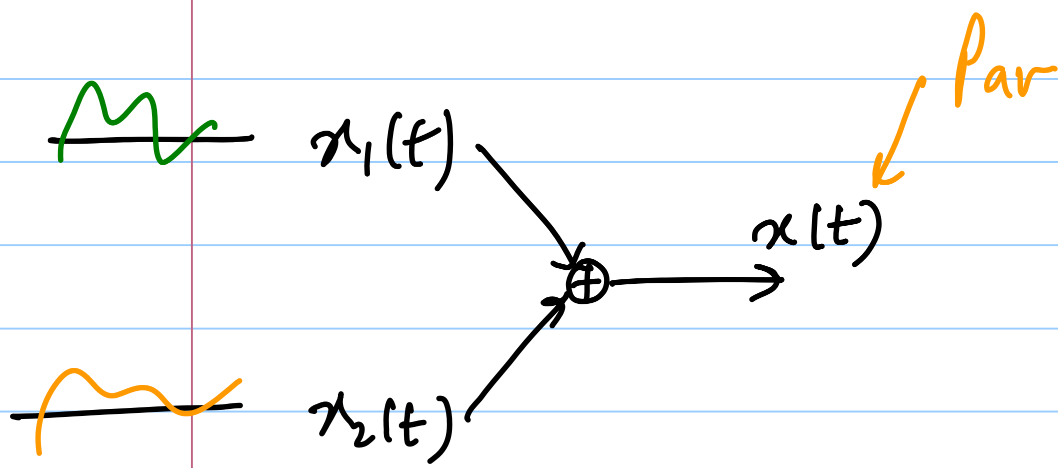
X



one-sided spectrum

Since $S_x(f)$ is even function,
$x(t)$ is real
we use only one-sided spectrum

Correlated and Uncorrelated Sources:



$$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left(x_1(t) + x_2(t)\right)^2 dt$$

$$= \frac{1}{T}\int x_1^2(t)\,dt + \frac{1}{T}\int x_2^2(t)\,dt + \underbrace{\frac{1}{T}\int_{-T/2}^{T/2} 2\,x_1(t)\cdot x_2(t)\cdot dt}_{\text{correlation between } x_1(t) \text{ and } x_2(t)}$$

$$= P_{av_1} + P_{av_2}$$

* If x_1 & x_2 are uncorrelated

$$\int_{-T_{0/2}}^{T_{0/2}} x_1(t)\, x_2(t)\, dt = 0$$

$$P_{av} = P_{av_1} + P_{av_2}$$

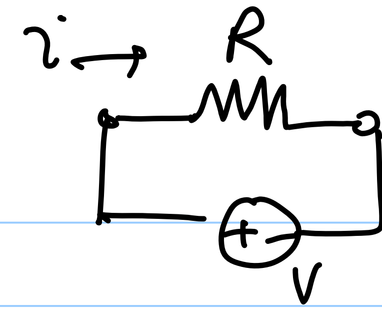
* Superposition of power holds for uncorrelated signals

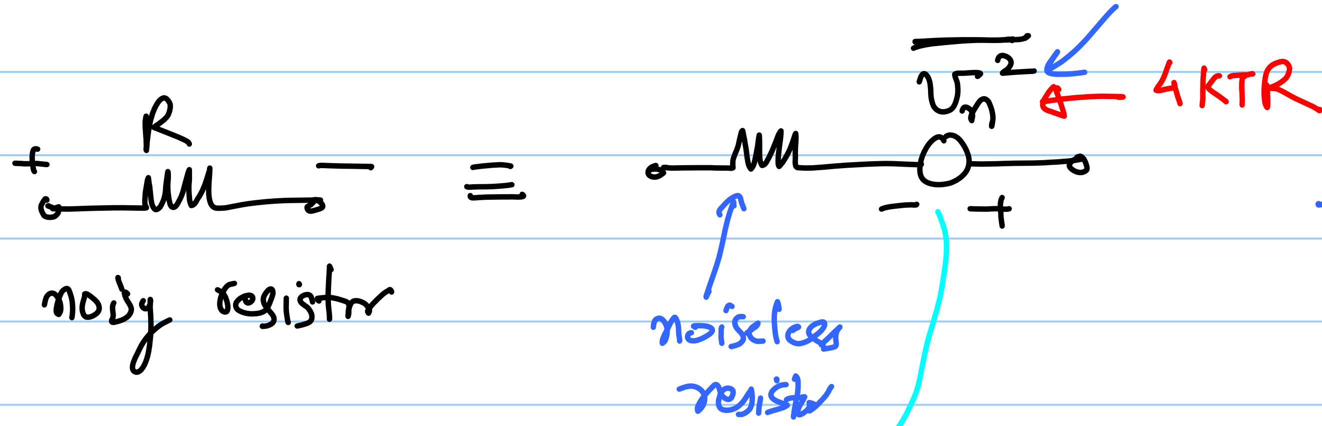
Types of Noise :

Analog signals are corrupted by two types of noise

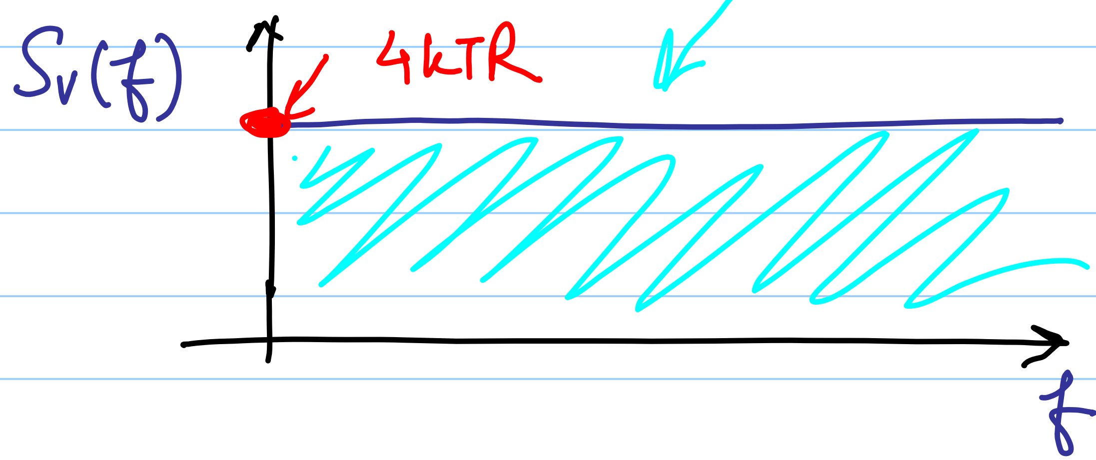
1. device/electronic noise
2. environmental noise

① Thermal Noise :





Since electron motion $\propto$ Temp



modeled as a series voltage source with one-sided PSD

$$S_v(f) = 4kTR \,, \quad f > 0$$

$\frac{V^2}{Hz}$

$\propto T$

$\propto R$

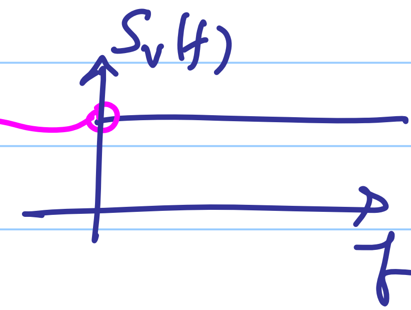
$k = 1.38 \times 10^{-23}\ J/K$

Some books use this notation

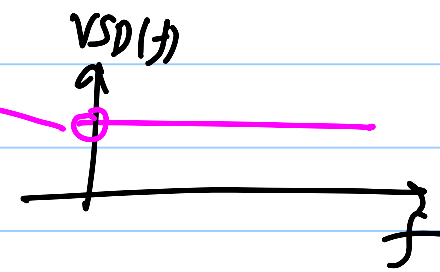
$$\overline{V_n^2} = 4kTR \cdot \underbrace{\Delta f}_{1\,\text{Hz}}$$

Ex. 50Ω Resistor at $T = 300°K = 27°C$

PSD: $$\overline{v_n^2} = 4kTR = 8.28 \times 10^{-19}\ \frac{V^2}{Hz}$$

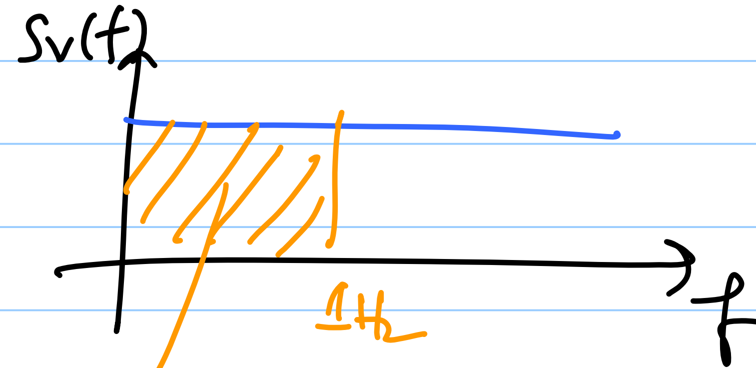


VSD: $$\overline{v_n} = \sqrt{\overline{V_n^2}} = 0.91\ \frac{nV}{\sqrt{Hz}}$$

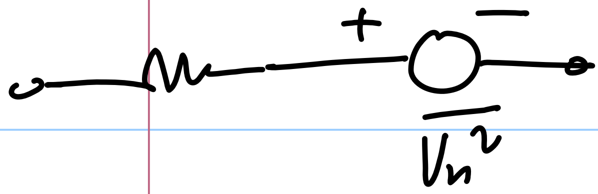


find rms noise in 1kHz BW

PSD



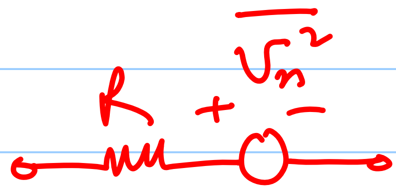
$$V_{rms} \leftarrow \sqrt{(\quad)}$$



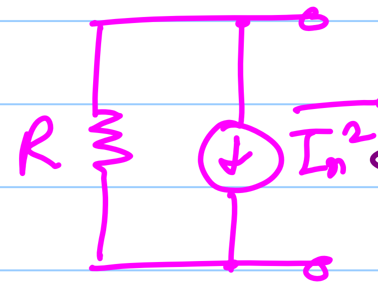
zero mean

Thevenin

Norton



$\equiv$



noise current

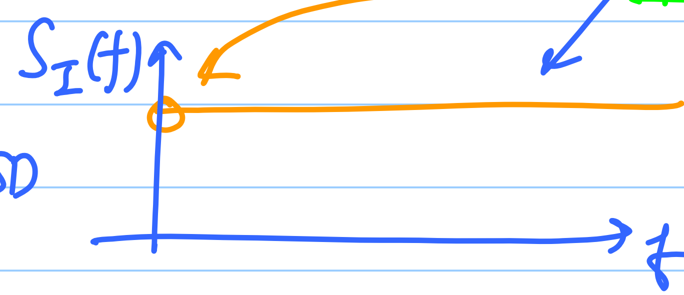
noise voltage

$$\boxed{\overline{V_n^2} = 4kTR}$$

$\frac{V^2}{Hz}$

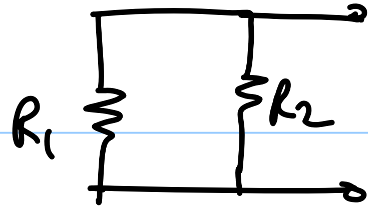
$\frac{A^2}{Hz}$

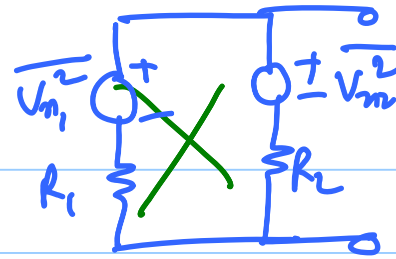
$$\overline{I_n^2} = \frac{\overline{V_n^2}}{R^2} = \frac{4kTR}{R^2} = \boxed{\frac{4kT}{R}}$$

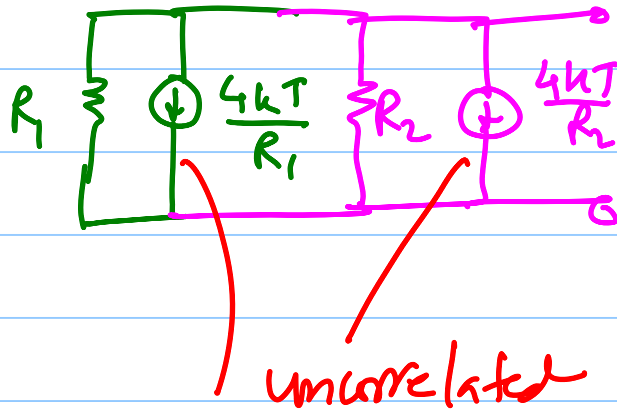


$\frac{A^2}{Hz}$

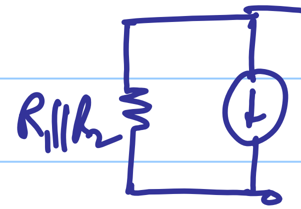
Ex:





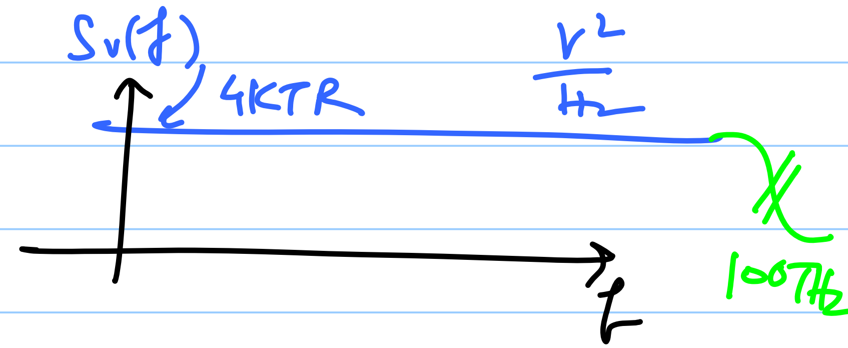


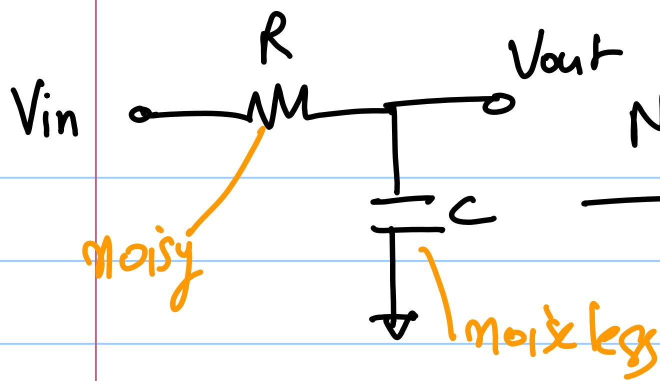
$\Rightarrow$



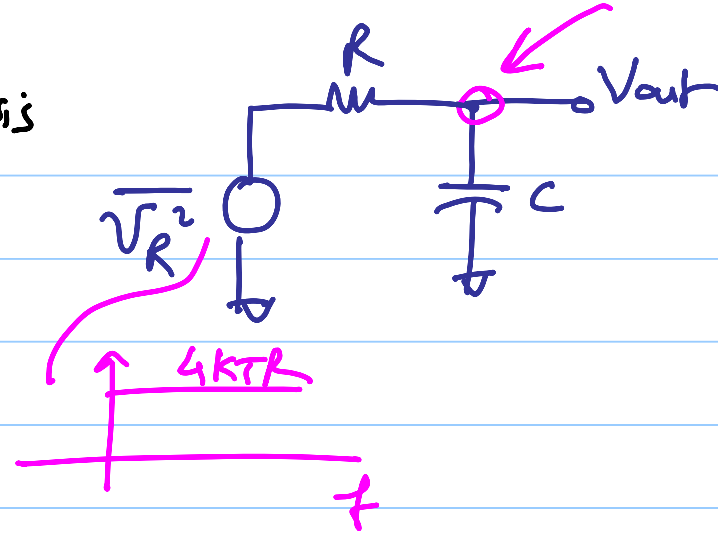
$$4kT\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$\frac{4kT}{R_1\|R_2}$$

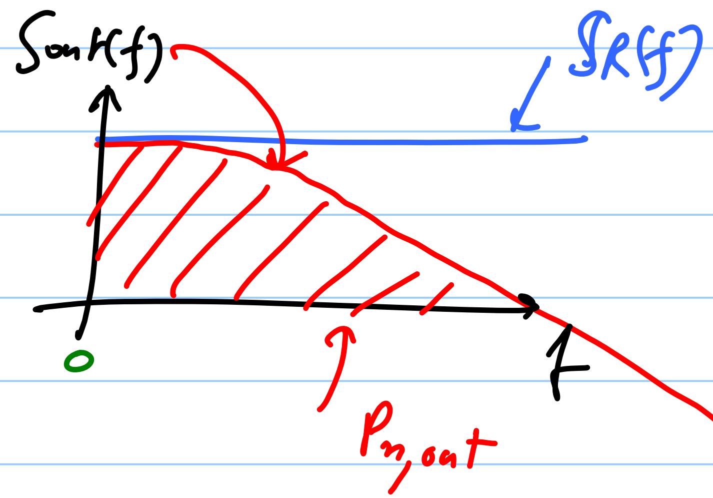




Noise analysis $\longrightarrow$



$$\frac{V_{out}}{V_R}(s) = \frac{1}{1+sRC}$$



PSD:

$$S_{out}(f) = S_R(f)\cdot\left|\frac{V_{out}}{V_R}(f)\right|^2$$

$$= 4KTR\cdot\frac{1}{1+(2\pi fRC)^2}$$

$$P_{n,out} = \int_0^\infty S_{out}(f)\,df = \int_0^\infty \frac{4KTR}{1+(2\pi fRC)^2}\,df$$

$x = 2\pi fRC$

$$\frac{dx}{2\pi RC} = df$$

$$= \frac{4kT\cancel{R}}{2\pi \cancel{R} C} \int_0^{\infty} \frac{dx}{1+x^2}$$

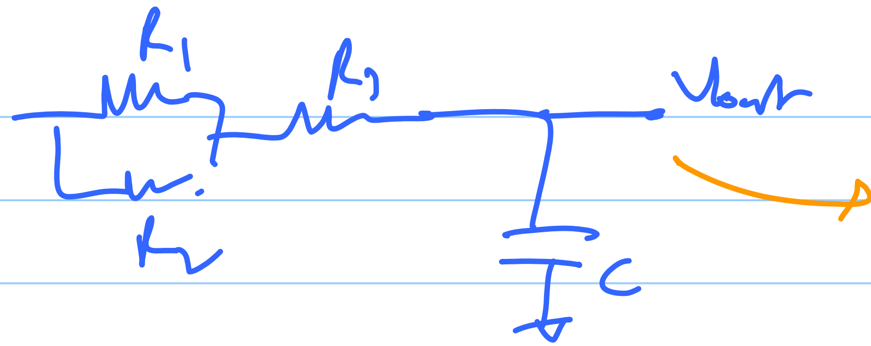
$$= \frac{2kT}{\pi C} \tan^{-1}(x) \Big|_0^{\infty} \longrightarrow \pi/2$$

$$= \frac{\cancel{2}kT}{\cancel{\pi} C} \times \cancel{\pi}/\cancel{2}$$

$$P_{n,out} = \frac{kT}{C} \quad V^2$$

$$\overline{V_{n,out,rms}} = \sqrt{P_{n,out}} = \sqrt{\frac{kT}{C}} \quad V$$

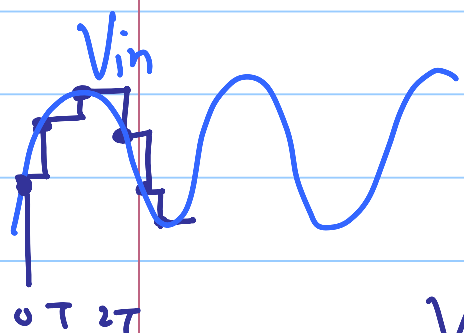
KT-over-C noise

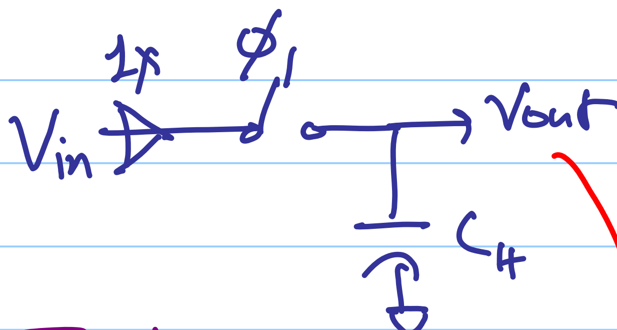


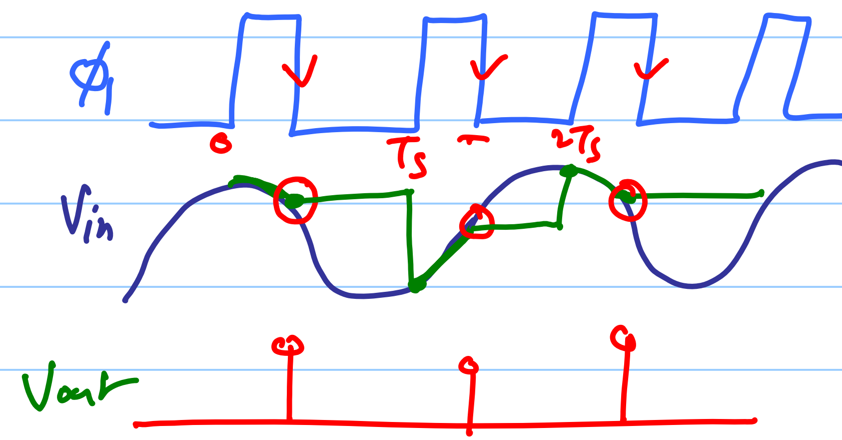
$\sqrt{\frac{kT}{C}}$

Ideal T/H

Track-and-hold

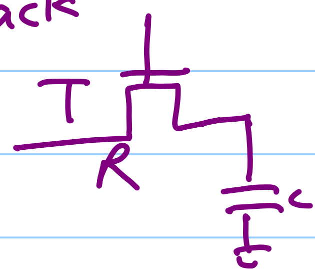






$V_{out}[nT_s]$

Track



rms noise in the output

$$\sqrt{\frac{kT}{C_H}}$$

C_H (pF)	$\overline{V_{n,out,rms}} = \sqrt{\frac{kT}{C}}$
0.1	200 µV
1	64 µV
10	20 µV
100	6 4 µV