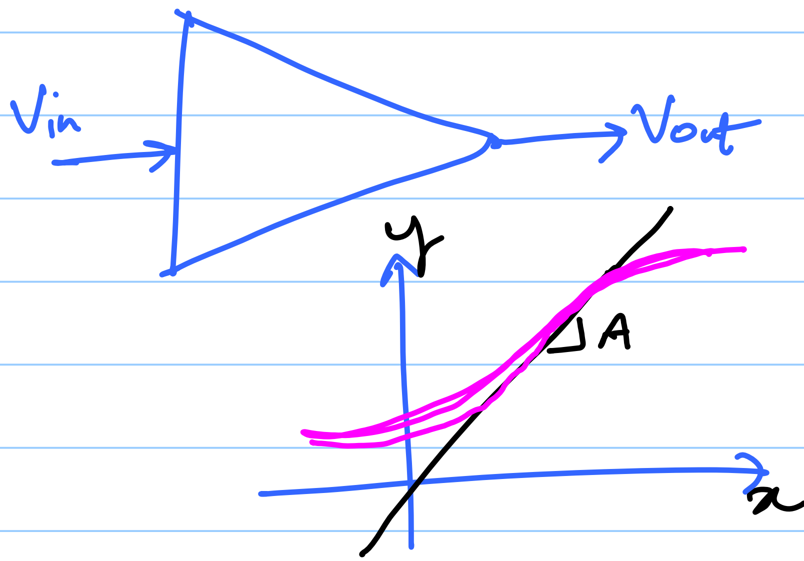
ECE 614 – Lecture 24

Note Title 11/20/2014

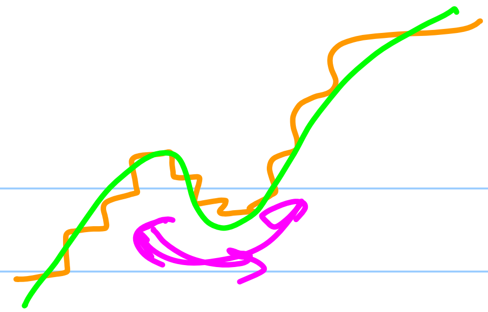


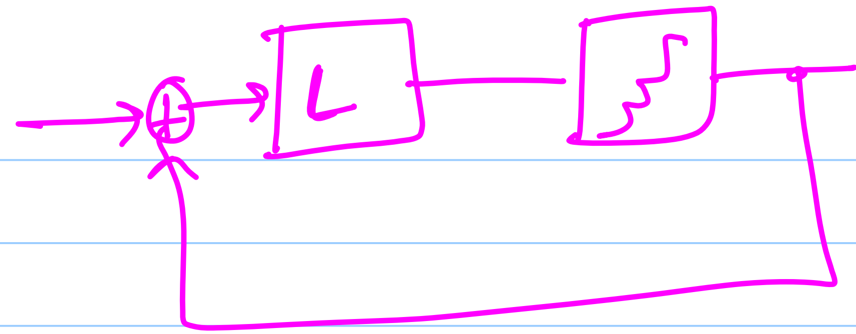
$y = Ax$: linear gain

$$y = \underbrace{\alpha_0 x - \alpha_3 x^3}_{\text{compressive NL}}$$

* Missing codes are not acceptable

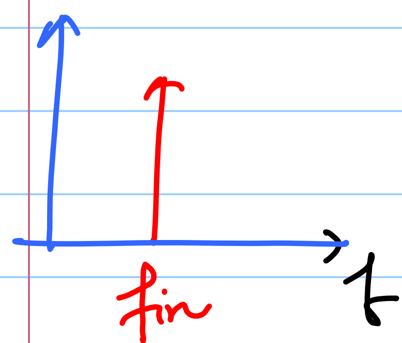
$$|DNL_i| < \tfrac{1}{2} LSB$$

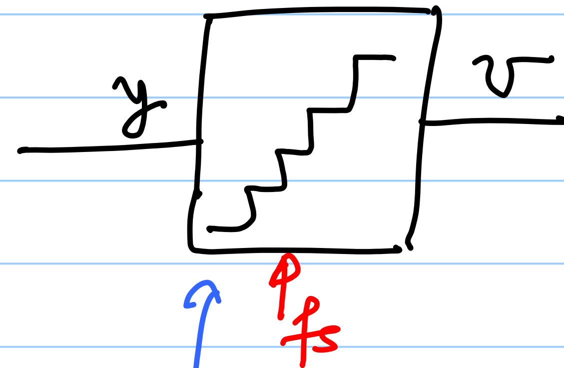



L

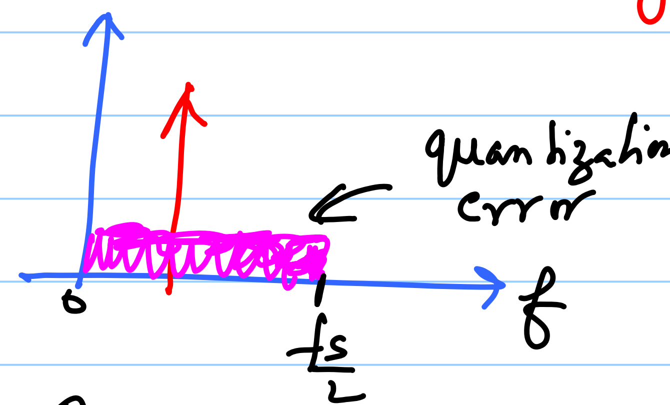
Quantization Error Model

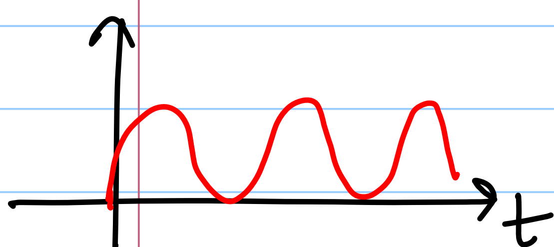
$f_{in} < \frac{f_s}{2}$ for no aliasing

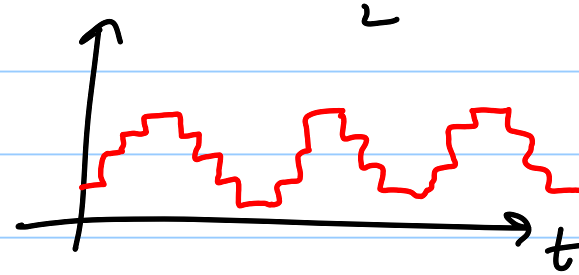


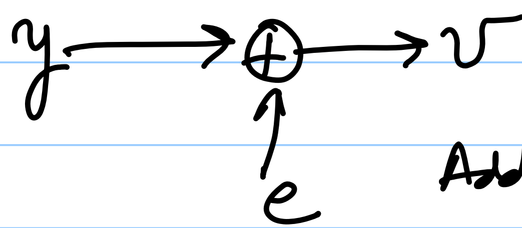


non-linear block







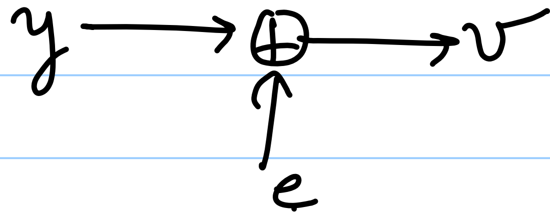


Additive noise model

Assumptions:

* Input is busy
* # of levels 2^N is very large

Assumes that $e[n]$ is uncorrelated with the input $y[n]$.



Quantization error :

$\Rightarrow$ noise

$\Rightarrow$ Uncorrelated with the input y

$\Rightarrow$ $e[n]$ is uniformly distributed

(I)

in $e \sim \left[-\frac{\Delta}{2}, +\frac{\Delta}{2}\right]$

$\Delta \Leftarrow$ LSB size or V_{LSB}

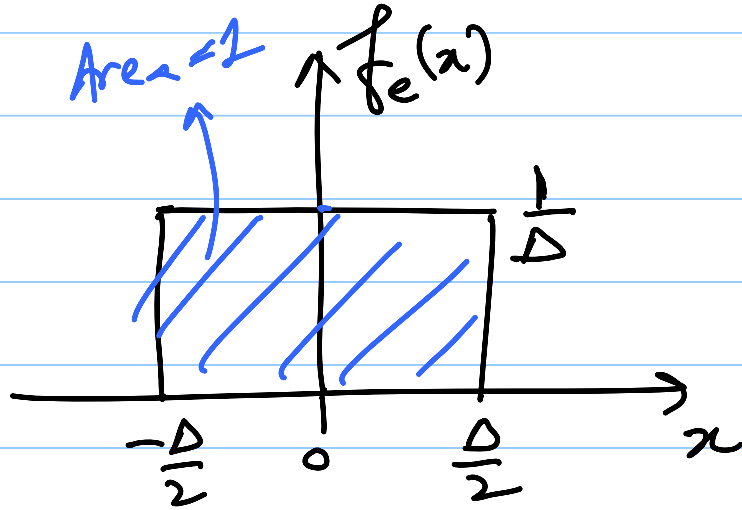
$\Rightarrow$ $e[n]$ is a white noise

(II)

white noise assumption

$$E\big[e[n]\,e[n-m]\big] = \sigma^2\delta[m]$$

AWGN in RF system



AWUN

$\hookrightarrow$ Spectrum is white

mean $\mu = 0$

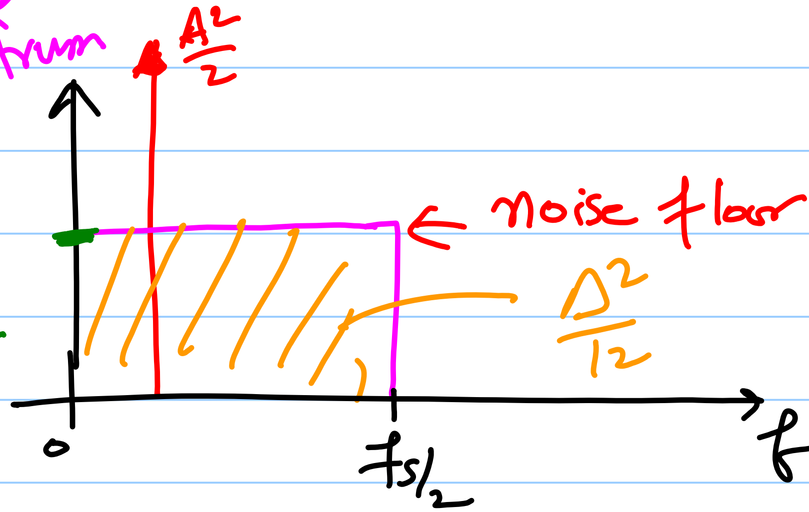
mean square value of e = $E[e^2] = \sigma^2$

$$\sigma^2 = \frac{1}{\Delta}\int_{-\Delta/2}^{+\Delta/2} x^2\,dx = \frac{\Delta^2}{12} \Leftarrow \text{noise power}$$

or Variance

or $(\text{rms})^2 \equiv \text{ms.}$

Quantization Noise Spectrum



$$\frac{\Delta^2}{6f_s} \Leftarrow \frac{\Delta^2}{12 \times \frac{f_s}{2}}$$

$N \uparrow$

$$\Delta = \frac{V_{FS}}{2^N} \downarrow$$

↳ noise floor $\downarrow$

SNR?

$y = A\sin(\omega t)$ is quantized with $LSB = \Delta$.

$$\text{signal power} = \frac{A^2}{2}$$

$$\text{noise power} = \frac{\Delta^2}{12}$$

for an N-bit ADC, full scale range $V_{FS} = 2^N \Delta$

max amplitude $\quad A_{max} = \frac{1}{2} V_{FS} = 2^{N-1}\Delta$

$$\frac{\text{maximum signal power}}{\text{noise power}} = \frac{\frac{A^2}{2}}{\frac{\Delta^2}{12}} = \frac{(2^{N-1}\Delta)^2}{2}$$

Signal to Quantization Noise Ratio

peak SQNR $= \dfrac{(2^{N-1}\Delta)^2}{2 \times \frac{\Delta^2}{12}} = 3 \times 2^{2N-1} \longrightarrow (1)$

SQNR in dB $= 10 \log_{10} \left(3 \cdot 2^{2N-1}\right)$ dB

$= (6.02N + 1.76)$ dB

$$\boxed{SQNR_{dB} = 6.02N + 1.76}$$

"full scale sine wave input"

for 1-bit increase in resolution

$\Rightarrow$ SQNR increases by 6 dB

SNDR → Signal to noise & distortion ratio

→ Quantization noise

→ Thermal noise

Effective number of bits

$$ENOB = N_{eff} = \frac{SNDR - 1.76}{6.02}$$