ECE 614 - Lecture 24

\[ y = Ax : \text{linear gain} \]

\[ y = \alpha_0 x - \alpha_3 x^3 \]

* Missing codes are not acceptable

\[ |\text{DNL}_i| < \frac{1}{2} \text{ LSB} \]
Quantization Error Model

Assumptions:

- Input is busy
- # of levels $2^N$ is very large
Assume that $e[n]$ is uncorrelated with the input $y[n]$.

\[ y \xrightarrow{D} u \]

- **Quantization error:**
  - $\Rightarrow$ noise
  - $\Rightarrow$ uncorrelated with the input $y$
  - $\Rightarrow$ $e[n]$ is uniformly distributed

$e \in \left[ -\frac{\Delta}{2}, \frac{\Delta}{2} \right]$

$\Delta \leq$ LSB size $\Rightarrow V_{\text{LSB}}$
AWGN in RF system

\[ \mathbb{E}[\epsilon[n] \epsilon[n-m]] = \sigma^2 \delta[m] \]

\[ \text{AWGN spectrum is white} \]

\[ \text{mean } \mu = 0 \]

\[ \text{mean square value of } \epsilon = \mathbb{E}[\epsilon^2] = \sigma^2 \]

\[ \sigma^2 = \frac{1}{\Delta} \int_{-\Delta/2}^{+\Delta/2} x^2 \, dx = \frac{\Delta^2}{12} \]

\[ \text{noise power} \]

\[ \sigma^2 \text{ (rms)} \approx \text{m.s.} \]
\[ \frac{\Delta^2}{6\Delta} \leq \frac{\Delta^2}{12 \times \Delta_{\text{max}}} \]

Quantization Noise Spectrum

\[ \frac{A_l}{2} \]

\[ \frac{\Delta^2}{12} \] noise floor

\[ \Delta = \frac{V_{\text{FS}}}{2^n} \]

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noise floor

\[ \downarrow \]
SNR?

\[ y = A \sin(\omega t) \text{ is quantized with } \text{LSB} = \Delta. \]

\[ \text{Signal power} = \frac{A^2}{2} \]
\[ \text{Noise power} = \frac{\Delta^2}{12} \]

For an N-bit ADC, full scale range \( V_{fs} = 2^{N} \Delta \)

\[ A_{\text{max}} = \frac{1}{2} V_{fs} = 2^{N-1} \Delta \]

\[ \text{Maximum signal power} = \frac{A^2}{2} = \frac{(2^{N-1} \Delta)^2}{2} \]
\[ \text{Noise power} = \frac{\Delta^2}{12} \]
Signal to Quantization Noise Ratio

\[ \text{peak SQNR} = \frac{(2^{n-1} \Delta)^2}{2 \Delta^2} = 3 \times 2^{2n-1} \]

\[ \text{SQNR in dB} = 10 \log_{10} \left( 3 \times 2^{2n-1} \right) \text{ dB} \]

\[ = (6.02n + 1.76) \text{ dB} \]

\[ \text{SQNR}_{\text{dB}} = 6.02n + 1.76 \]

"full scale sine wave input"

for 1-bit increase in resolution

\[ \Rightarrow \text{SQNR increases by 6 dB} \]
SNDR → Signal to noise & distortion ratio

Effective number of bits

\[ \text{ENOB} = \text{Neff} = \frac{\text{SNDR} - 1.72}{6.02} \]