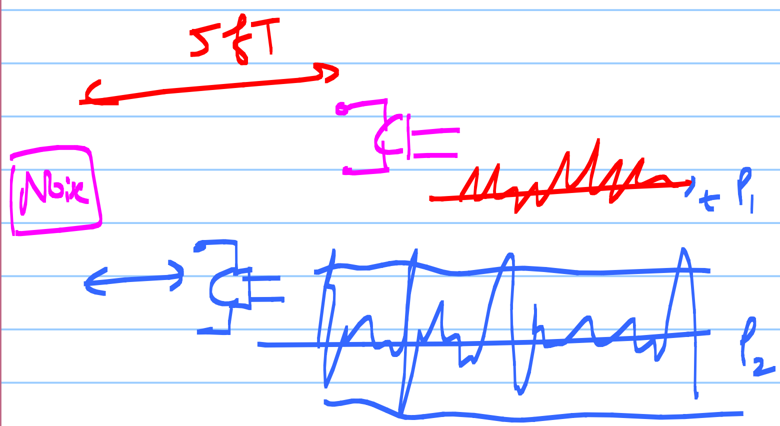
ECE 614 – Lecture 9

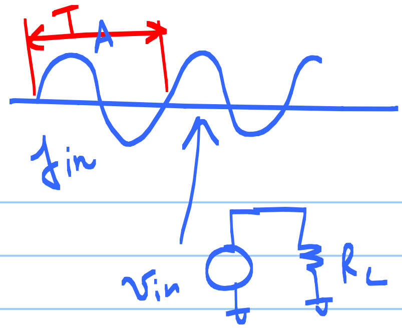
Note Title 9/25/2012



$$SNR = 10 \log_{10}\left(\frac{P_{sig}}{P_{noise}}\right)$$

$$= 10 \log_{10}\left(\frac{V_{s,rms}^2}{V_{n,rms}^2}\right)$$

$$= 20 \log_{10}\left(\frac{V_{s,rms}}{V_{n,rms}}\right)$$

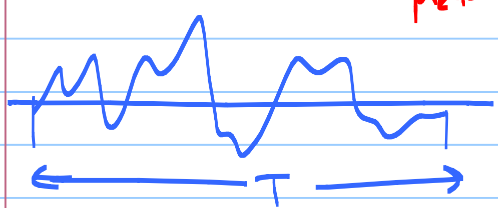


Periodic
Signal

$$P_{av} = \frac{1}{T}\int_{-T/2}^{T/2} \frac{v^2(t)}{R_L}\,dt = \frac{\mathcal{E}_T}{T}$$

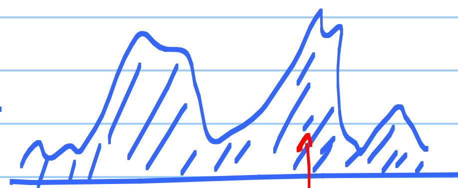
energy

$$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{x^2(t)}{R_L}\cdot dt \leftarrow W$$



Power meter

$(\ )^2 \Rightarrow$

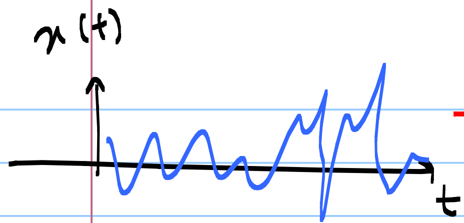


$P_{av} = \dfrac{\text{Area}}{T}$

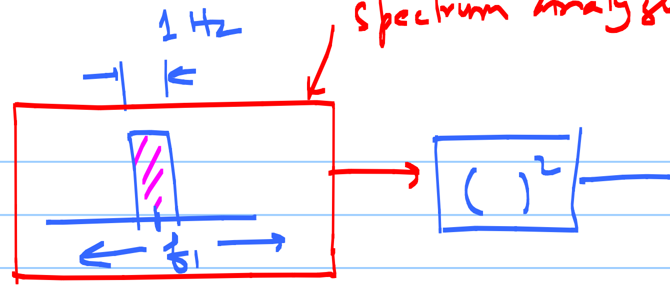
"drop R_L"

$$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\, dt \qquad V^2$$

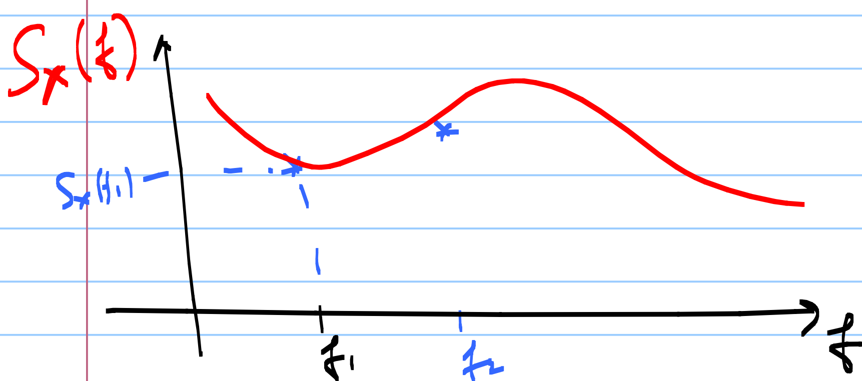
rms voltage for noise = $\boxed{\sqrt{P_{av}} = V_{rms}} \leftarrow V$



Spectrum Analyzer



$|x_{f_1}(t)|^2$



$$S_x(f_1) = \frac{|x_{f_1}(t)|^2}{T}$$

$\frac{V^2}{Hz}$

Power Spectral Density $\frac{V^2}{Hz}$

$x(t) \longrightarrow S_x(f) \Rightarrow \frac{V^2}{Hz}$

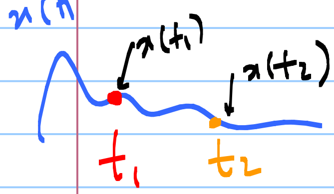
Voltage Spectral Density $VSD = \sqrt{S_x(f)} \Leftarrow \frac{V}{\sqrt{Hz}}$

Eg. amp @ 100 MHz $V_{SD} = \frac{3nV}{\sqrt{Hz}}$

$\downarrow$

1 Hz BW $\Rightarrow P_{av} = (3n)^2$

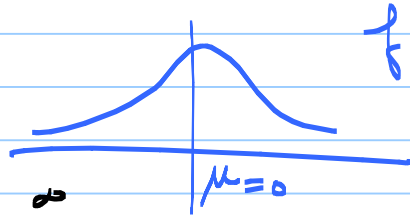
$x(t)$ ← random process



$x(t_1)$, $x(t_2)$ ← random variables

$$f_x(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma}}$$

gaussian

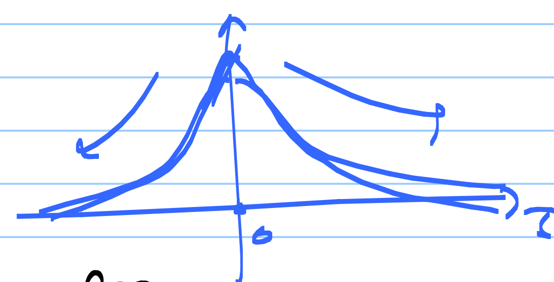


$$E\big(x(t_1)\cdot x(t_2)\big) = \int_{-\infty}^{\infty} x_1(t)\cdot x_2^*(t)\,dt = E\big(x(t)\,x(t-\tau)\big)$$

$\tau = t_1 - t_2$

$$R_x(\tau) = E\big(x(t)\,x(t-\tau)\big)$$

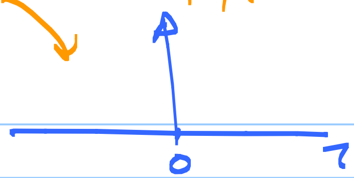
auto correlation



$$\mathcal{F}\big(R_x(\tau)\big) = S_x(f)$$ ← PSD

Real

$R_x(\tau)$



$\delta(t)$

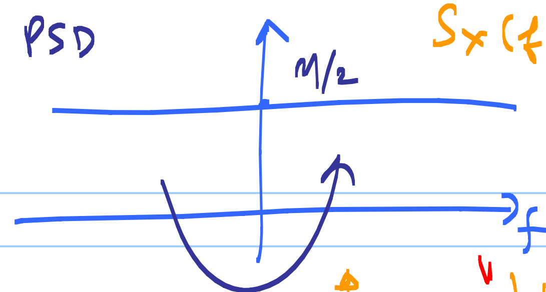
$\mathcal{F}$

PSD

$S_x(f)$

$\frac{V^2}{Hz}$

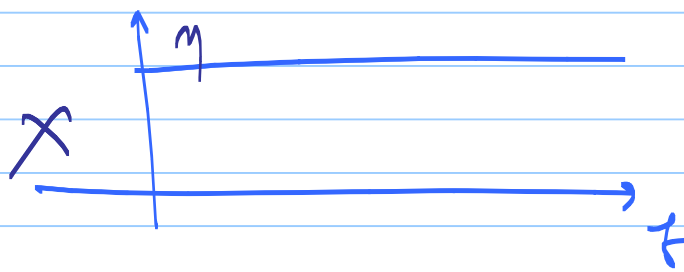
Flat



"White"

$\frac{1}{2}$ Fold

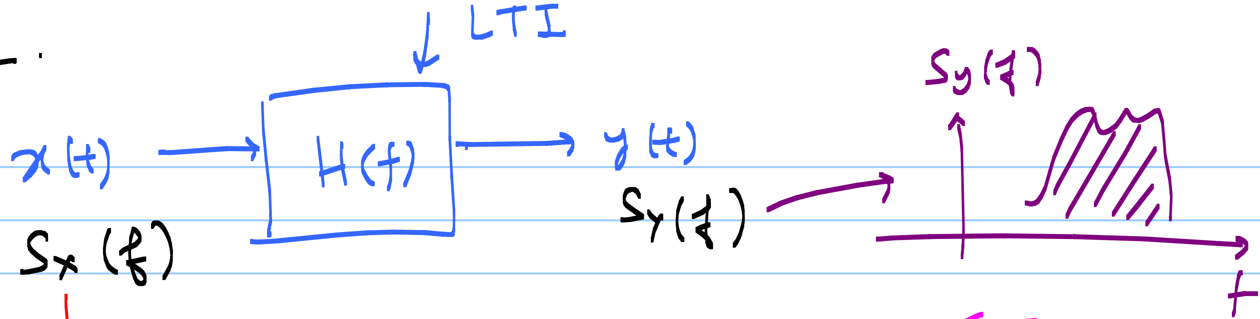
Even



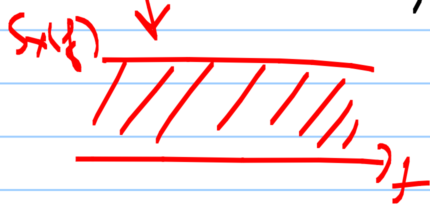
One-sided Spectrum

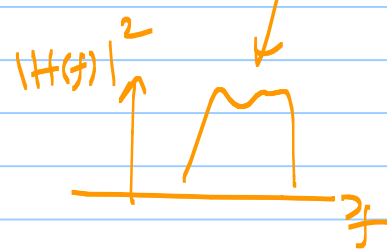
$(f > 0)$

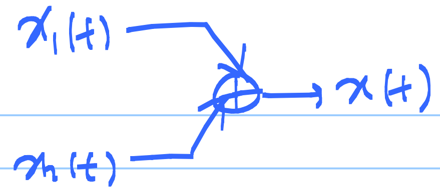
Theorem.



$$S_y(f) = S_x(f) \cdot |H(f)|^2$$ Important







Can we add their avg power by superposition?

$$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left(x_1(t) + x_2(t)\right)^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x_1^2(t)\,dt + (\,.\,)\int_{-T/2}^{T/2} x_2^2(t)\,dt + (\,.\,)\int_{-T/2}^{T/2} 2x_1(t)x_2(t)\,dt$$

$$P_{av} = P_{av_1} + P_{av_2} + \underbrace{2\times \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x_1(t)\cdot x_2(t)\,dt}_{\text{Correlation term}}$$

$E(x_1 \cdot x_2)$

How similar $x_1(t)$ & $x_2(t)$ are?

Uncorrelated : $E(x_1 \cdot x_2) = 0$

$$P_{av} = P_{av_1} + P_{av_2}$$

if Correlated, $\Rightarrow$ account for the $\underbrace{E(x_1 \cdot x_2)}$

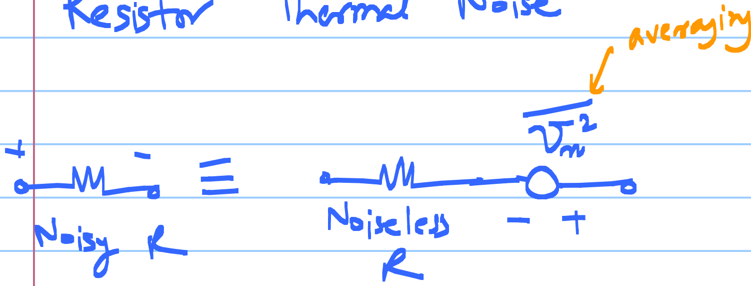
Types of Noise

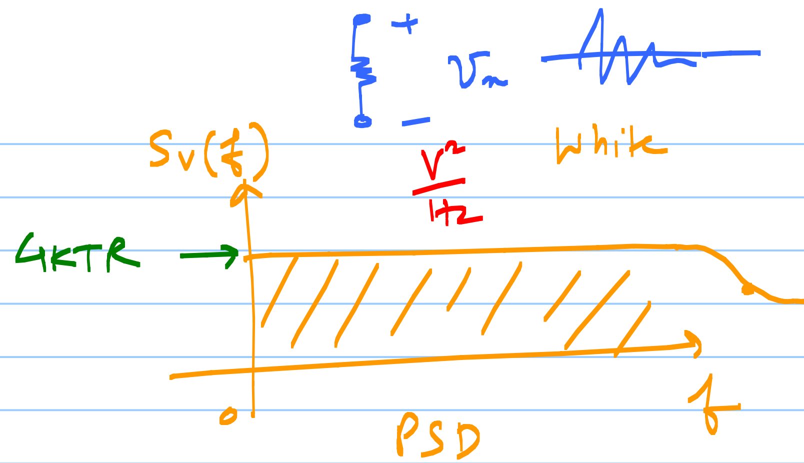
(1) device electronic noise

(2) environmental Noise ← Communication Designer

① Thermal Noise

Resistor Thermal Noise





$$S_v(f) = 4kTR, \quad f \geq 0$$

Boltzmann's $k = 1.38 \times 10^{-23}$ J/K

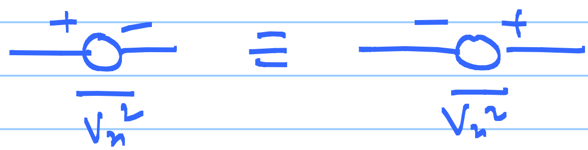
$\propto T$

$\propto R$

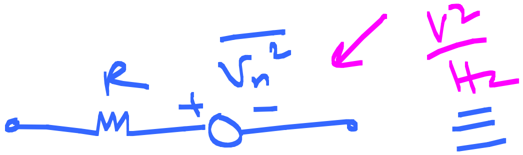
$$\overline{V_n^2} = 4kTR \cdot \Delta f$$

to emphasize that power per unit BW

$\Delta f = 1$ Hz

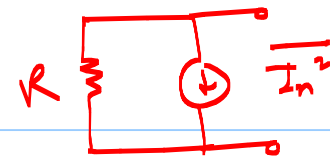


polarity doesn't matter



Thevenin

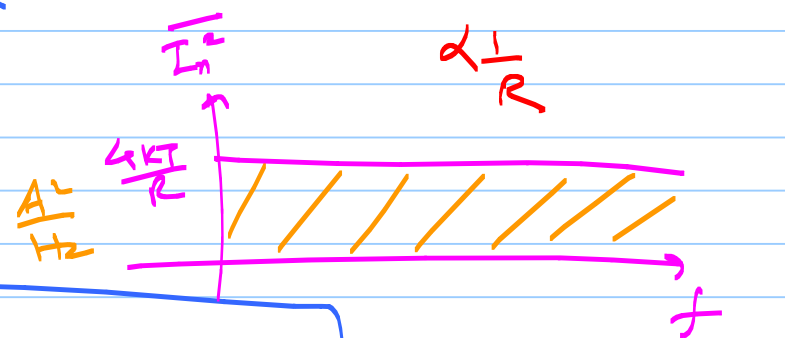
Norton

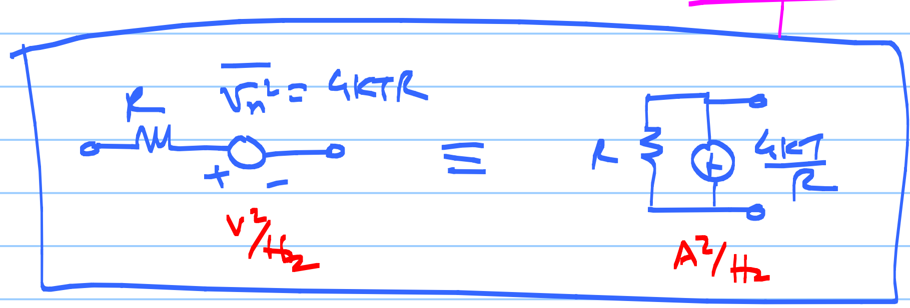


Noise Current PSD

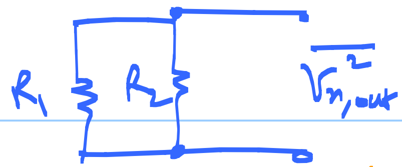
$$\overline{I_n^2} = \frac{\overline{V_n^2}}{R^2} = \frac{4kTR}{R^2} = \frac{4kT}{R} \qquad \frac{A^2}{Hz}$$

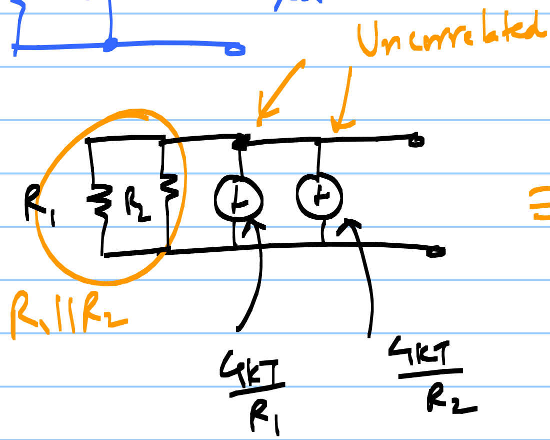
$$\propto \frac{1}{R}$$



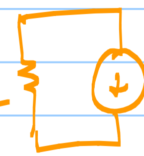


Ex.





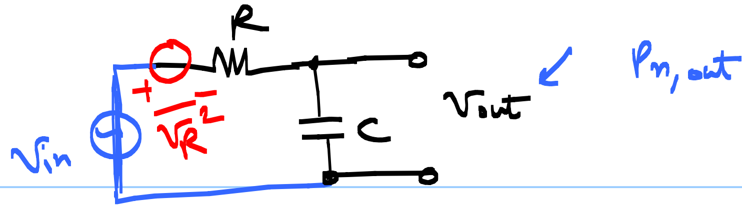
$\Rightarrow$ $R_1 \| R_2$



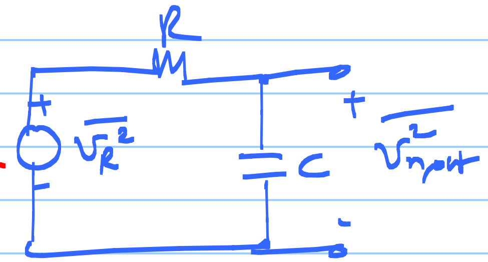
$$4kT\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

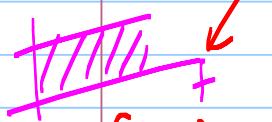
$$= \frac{4kT}{R_1 \| R_2}$$

$$\overline{v_{n,out}^2} = 4kT\,(R_1 \| R_2)$$

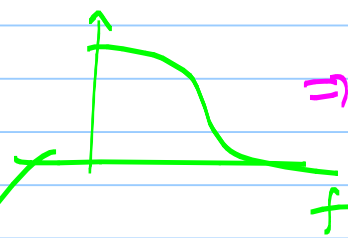


Noise circuit

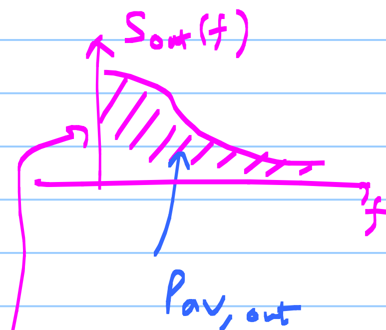




$S_R(f) = 4kTR$



$\Rightarrow$



$$S_{out}(f) = S_R(f) \cdot \left| \frac{V_{out}}{V_R}(f) \right|^2 = 4kTR \cdot \frac{1}{1 + (2\pi f RC)^2}$$

$$P_{n,out} = \int_0^\infty S_{out}(f)\,df = \int_0^\infty \frac{4kTR}{1+(\underbrace{2\pi f R c}_{x})^2}\, df$$

$x = 2\pi f R c$

$$= \frac{\overset{2}{\cancel{4}}\,kT\,\cancel{R}}{\cancel{2}\pi \cancel{R} c}\underbrace{\int_0^\infty \frac{1}{1+x^2}\,dx} \qquad = \frac{2kT}{\pi c} \cdot \underbrace{\tan^{-1}(x)\Big|_0^\infty}_{\pi/2}$$

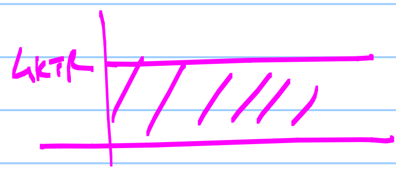
$$P_{n,out} = \frac{kT}{C} \qquad V^2$$

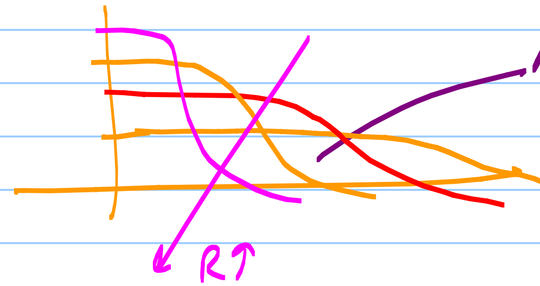
$$\overline{V}_{n,out,rms} = \sqrt{P_{n,out}} = \boxed{\sqrt{\frac{kT}{c}} \quad V}$$

$$4kT\cancel{R} \cdot \frac{1}{2\pi \cancel{R} C}$$

$R\uparrow \Rightarrow S_x \uparrow$

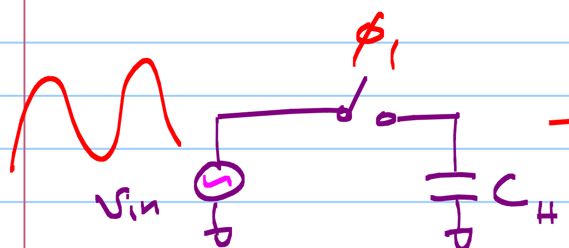
$f_{3d} \downarrow$

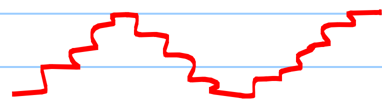




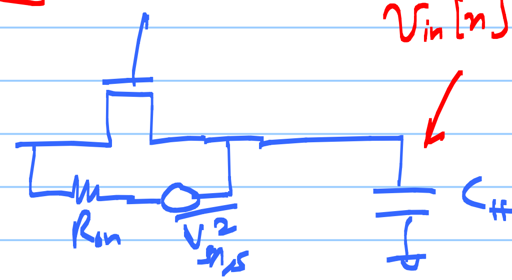
$$\text{Area} = \frac{kT}{C}$$

$$\sqrt{\frac{kT}{C}}$$





$v_{in}[n] + \overline{v_n}$



To reduce $\overline{v_n} \Rightarrow C_H \uparrow$