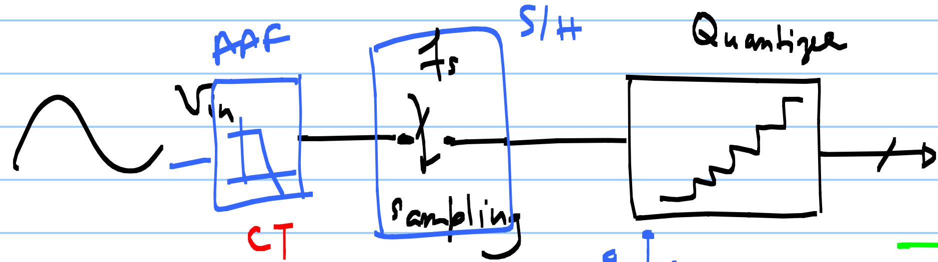
Note Title 11/29/2012

ECE 614 – Lecture 23

Nyquist Sampling Theorem

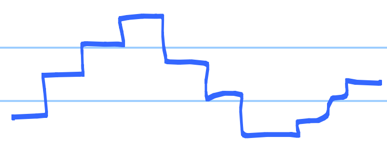
$BW \le \frac{f_s}{2}$

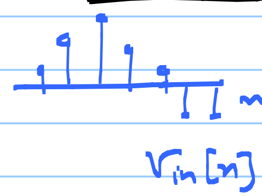
Analog to Digital Converters (ADC)

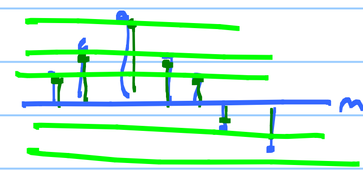


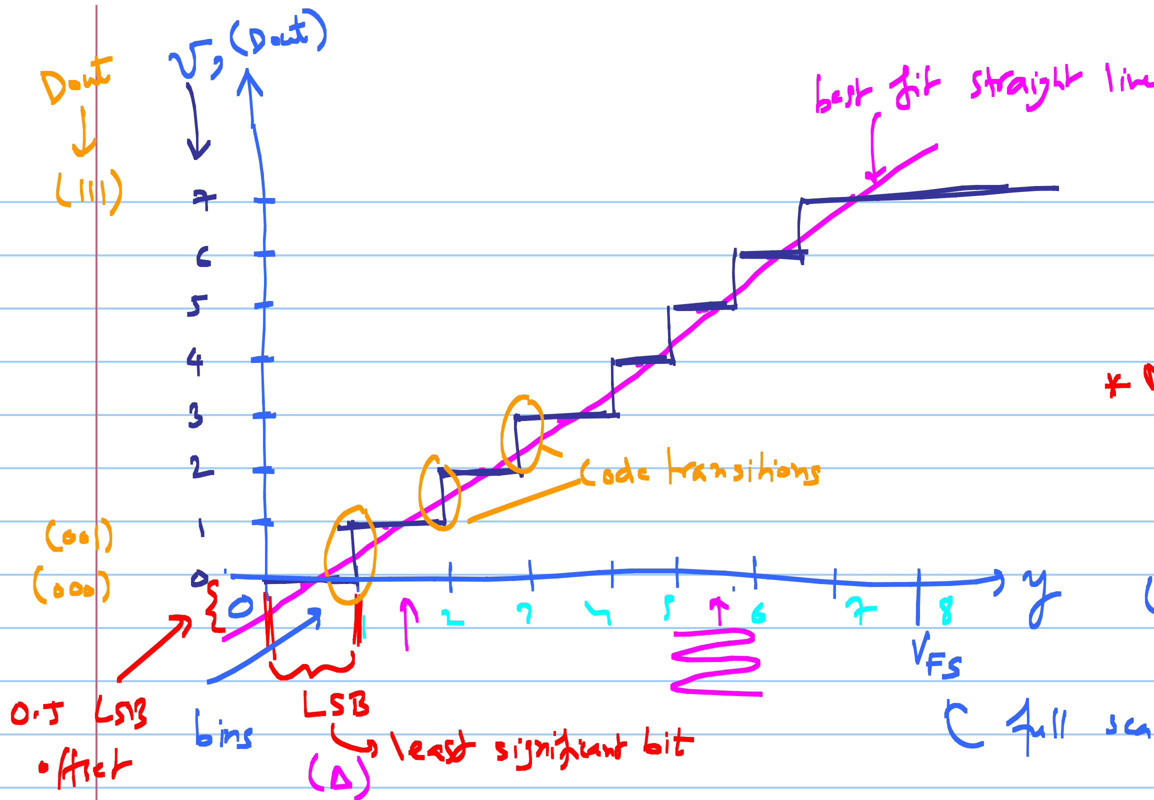
binary code

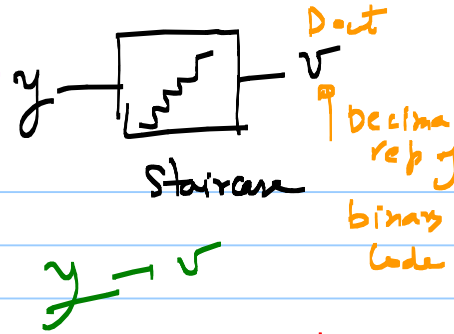
Amp is discrete







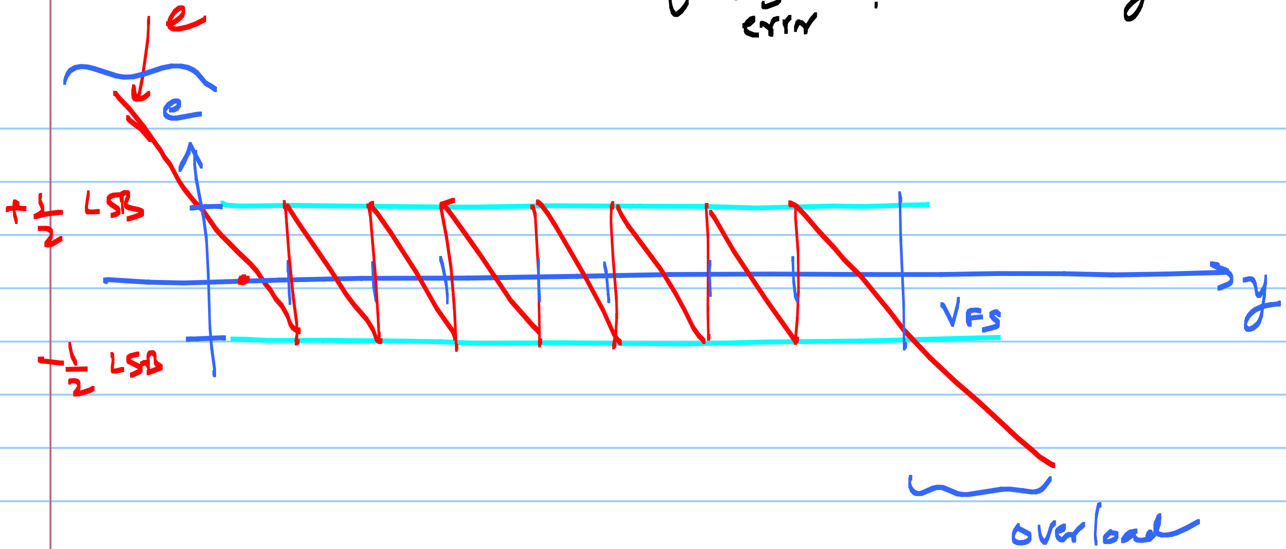


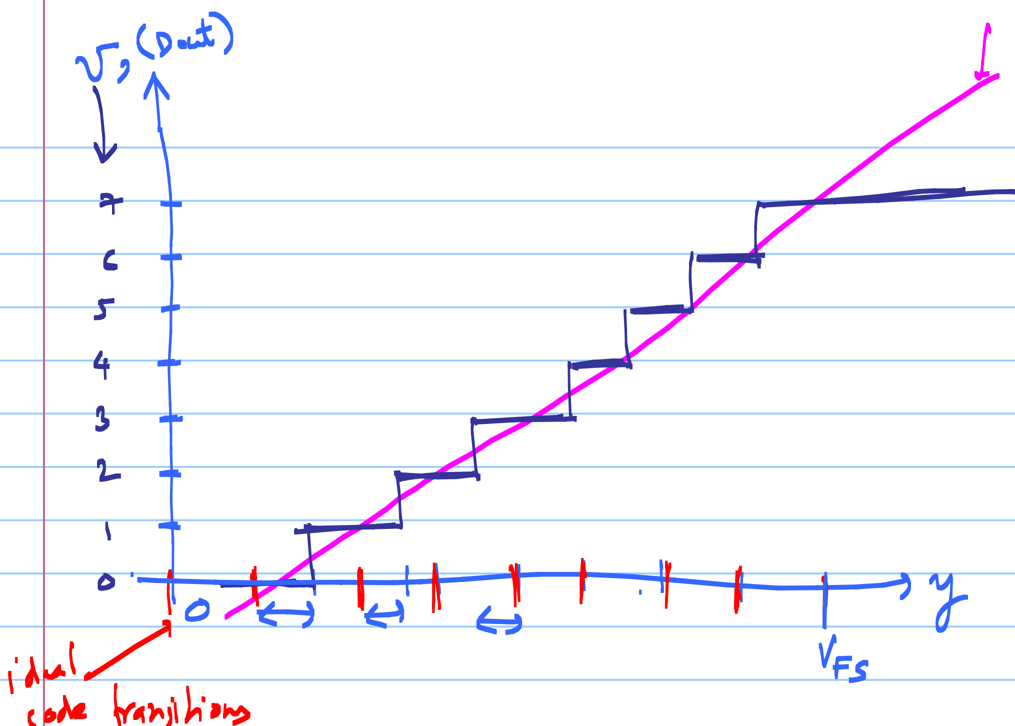


* Non-linear I/O characteristic

* in an ideal quantizer, the code widths of all the bins = 1 LSB

quantization error : $e = v - y$





best fit straight line

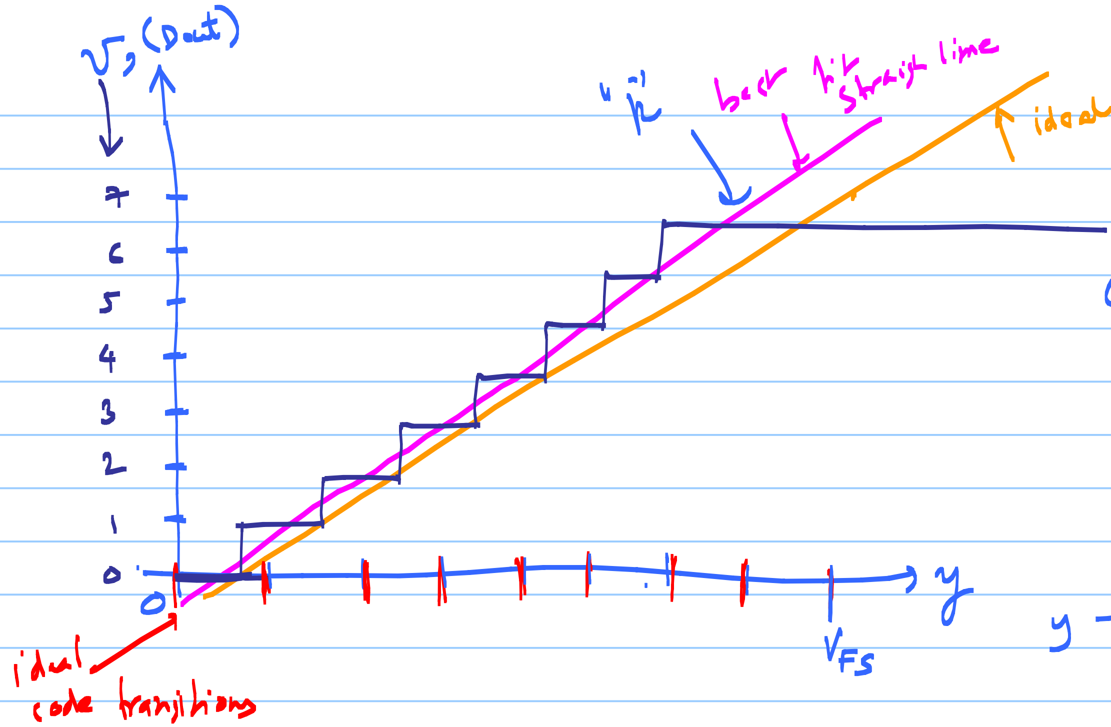
* LMS Algorithm curve fitting

Code widths are same = 1 LSB

$\Rightarrow$ Offset error

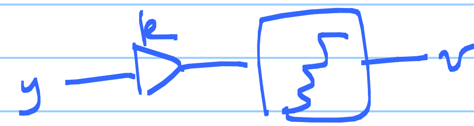
$\downarrow$

benign

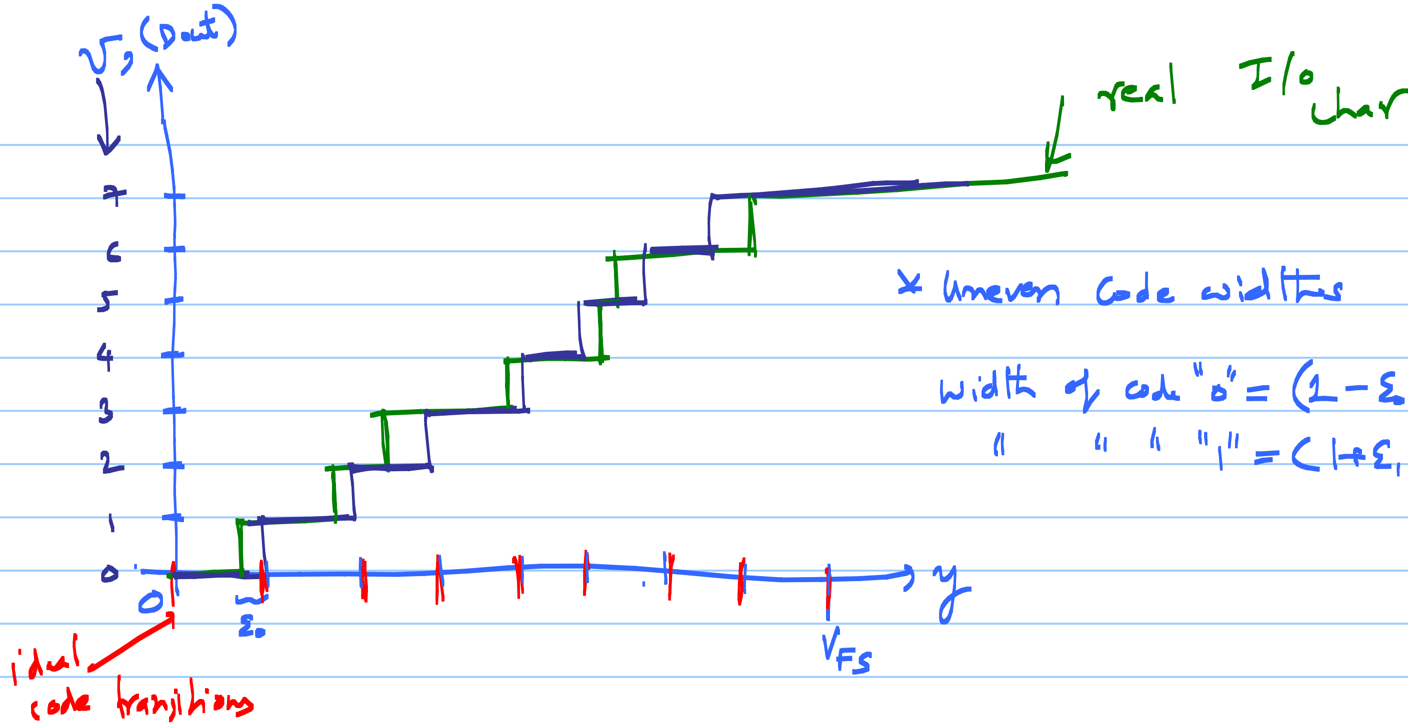


Code width are the same

< 1 LSB



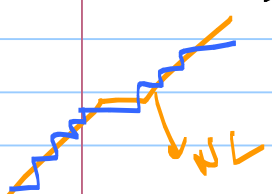
"gain error"
↳ benign



* Uneven code widths

width of code "0" = $(1-\varepsilon_0)$ LSB

" " " "1" = $(1+\varepsilon_1)$ LSB

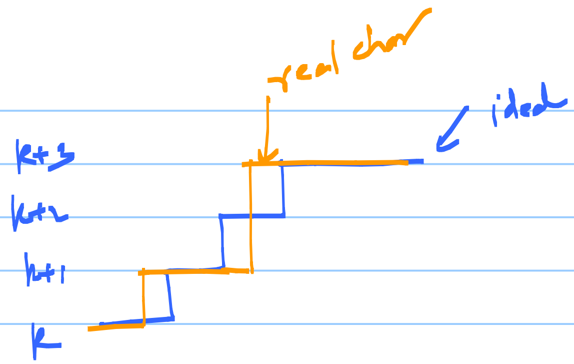


* Just specify the deviation from the 1 LSB

"Differential Nonlinearity" (DNL)

$$DNL(k) = \frac{\text{Width of Code "k"} - 1LSB}{1\,LSB}$$

* Specifying DNL defines the entire characteristic of the quantizer



DNL(k) = −0.3

DNL(k+1) = +0.7

DNL(k+2) = ? −1 missing code (code width = 0)

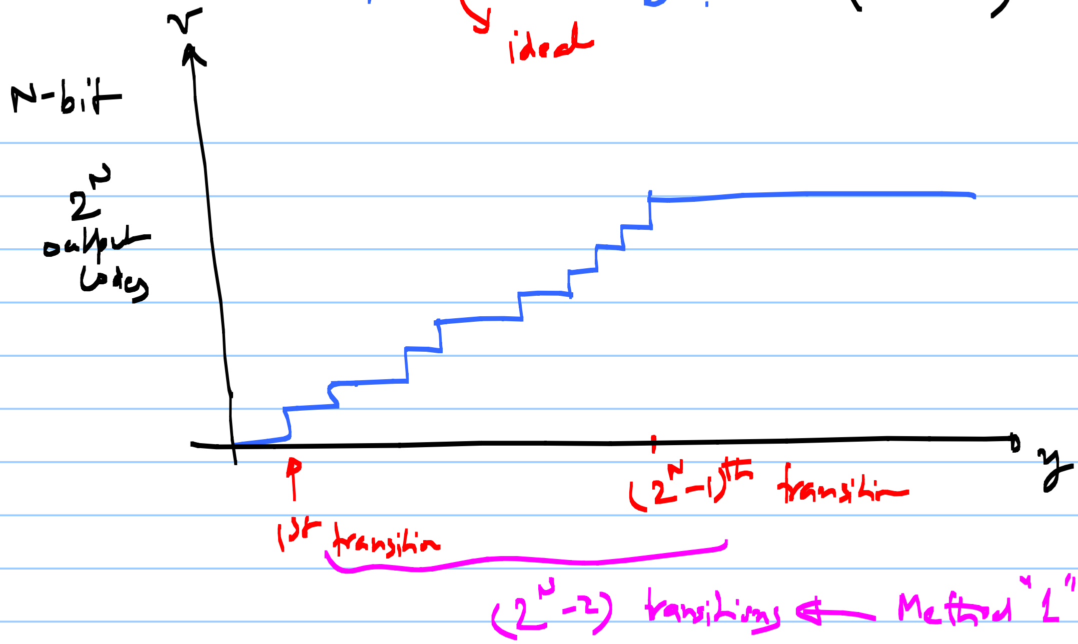
DNL(k+3) = +0.6

* Need to ensure that there are no missing codes

$|DNL| \leq 0.5$ LSB → sufficient to ensure no missing codes

$V_{LSB} = \frac{V_R}{2^N}$

* How do we define ideal LSB size? (N-bit)



Method II

① Best fit a straight line

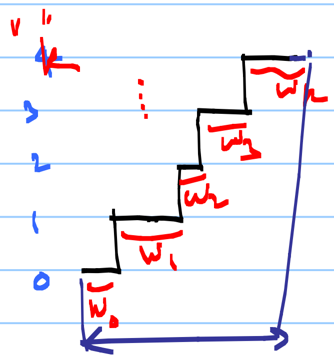
↳ remove the offset and gain errors

② from the slope → compute the ideal "LSB"

③ Compute DNL(k)

* INL (integral non-linearity)

INL(k) $\triangleq$ deviation in LSB of the the code transition wrt the ideal transition



* ideal transition for the k^{th} code $= k \cdot LSB$

* $$INL(k) = \frac{\sum_{i=0}^{k-1} W_i - k \cdot LSB}{1\ LSB}$$

* Rudy van de Plassche

* Difference in LSB b/w the measured transition at "k"th code w.r.t to the ideal transition of the k^{th} code (in terms of LSB)

$$= INL(k) = \sum_{i=0}^{k-1} W_i - k$$

&

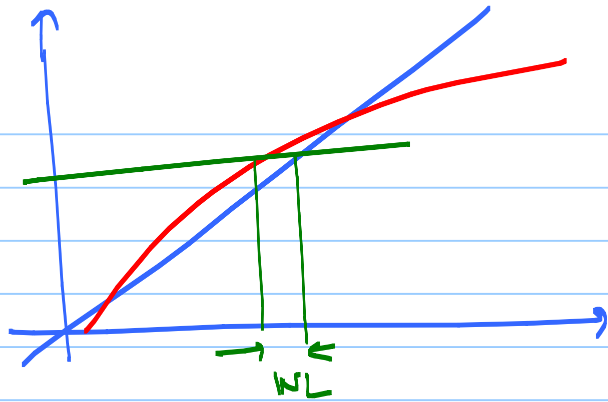
$$INL(k+1) = \sum_{i=0}^{k} W_i - (k+1)$$

$$\boxed{INL(k+1) - INL(k) = W_k - 1 = DNL(k)}$$

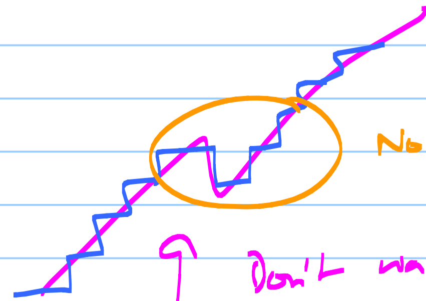
$$INL(k) = \sum_{i=0}^{k} DNL(k)$$

INL is the running sum of DNL

"Static characteristics"



→ Large INL

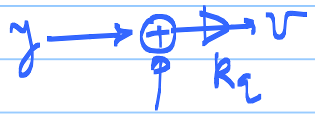


Not-monotonic

Don't want this in feedback control

Dynamic characteristics

linear model for the quantizer



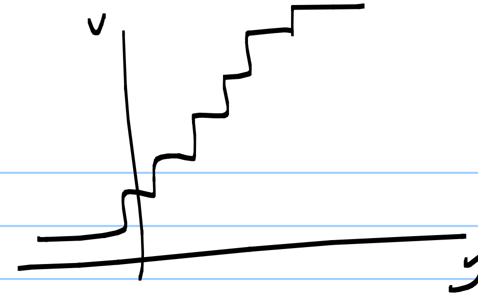
additive white noise

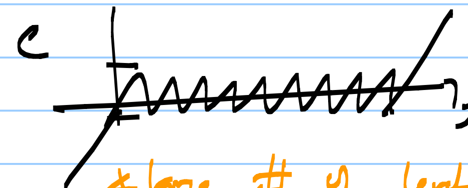
(Quantization Noise)

additive error

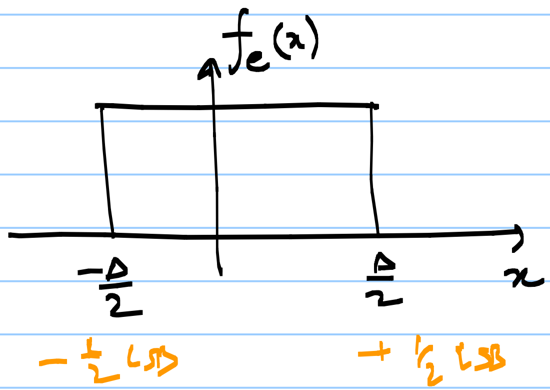
↳ error → white noise

↓ not correlated with signal



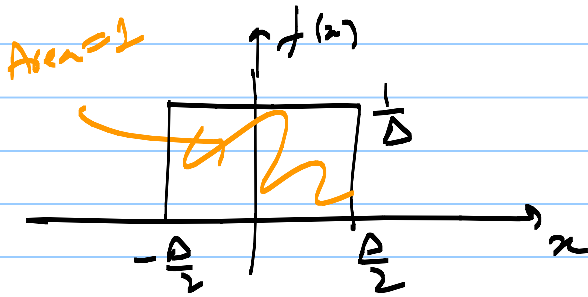


* large # of levels
* input is busy
* no overloading



* e(n) is uniformly distributed

mean square error $= \dfrac{\Delta^2}{12}$ ← LSB size



mean square error

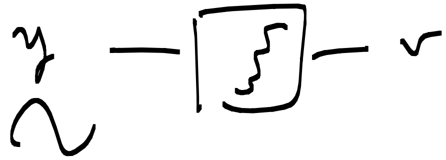
$$\sigma_e^2 = \int_{-\Delta/2}^{\Delta/2} x^2 f_e(x)\, dx$$

$$= \frac{\Delta^2}{12}$$

quantization noise power

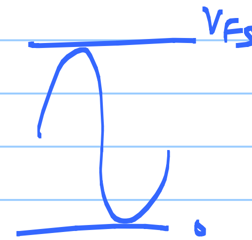
$LSB \rightarrow \Delta$

$y = A\sin(\omega_0 t)$



$\longrightarrow$ SQNR $\Rightarrow$ $\begin{cases} \text{Signal power} \Rightarrow \frac{A^2}{2} \\ \text{Noise power} \end{cases}$

signal to quantization noise ratio



full scale range $= 2^N \Delta$

$A_{max} = \frac{V_{FS}}{2} = 2^{N-1}\Delta$

* max signal power $= \frac{A_{max}^2}{2} = \frac{(2^{N-1}\Delta)^2}{2}$
* noise power $\Rightarrow \frac{\Delta^2}{12}$

$$\text{peak SQNR} = \frac{\frac{(2^{N-1}\Delta)^2}{2}}{\frac{\Delta^2}{12}} = 3\cdot 2^{2N-1}$$

$\frac{max}{peak}$ $SQNR_{dB} = 6.02\,N + \underbrace{1.76}_{}$ dB

resolution in # bits

rms $\frac{1}{\sqrt{2}}$

N ⇒ +1

SQNR ↑ ⇒ +6 dB