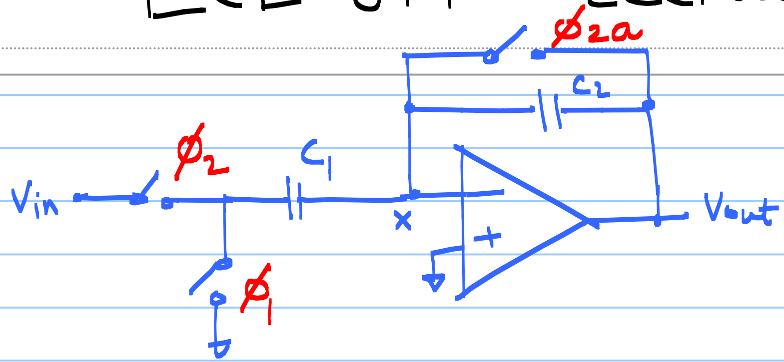
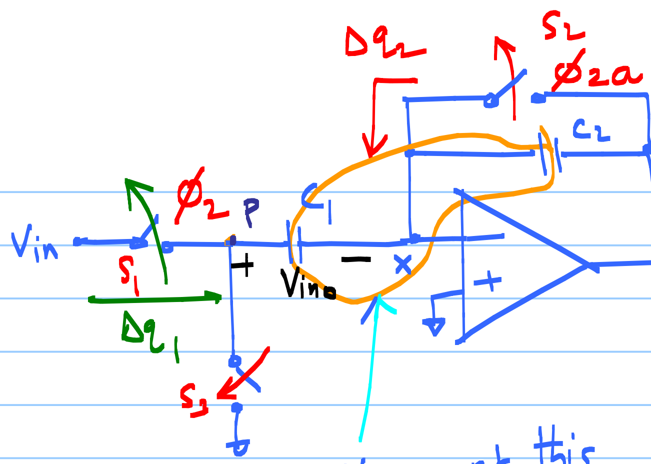
# ECE 614 – Lecture 17

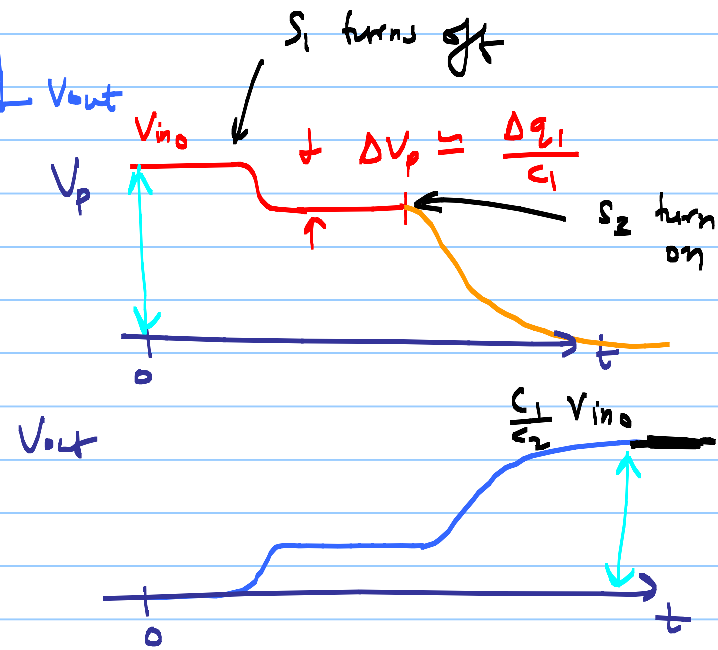
Note Title 10/30/2012




$$\boxed{V_{out} = +\frac{C_1}{C_2} V_{in}}$$




Charge at this surface is conserved




* intermediate perturbation due to charge injected by $S_1$, doesn't change the final $V_{out}$
  - ↳ charge at node-X is conserved

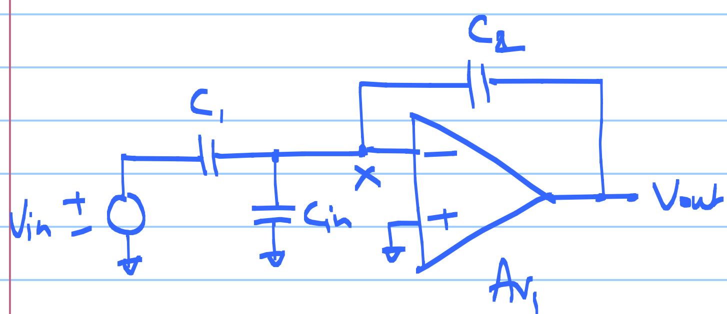
* used as a precise gain stage $2^M$, $M=1,2,\ldots$

## precision Considerations

The nominal gain of this $+\frac{C_1}{C_2}$

0.1% matching

Layout technique




gain Error $\epsilon = \frac{1}{\beta A_v}$

$$(V_{out} - V_x) s C_2 = V_x s C_{in} + (V_x - V_{in}) s C_1 \longrightarrow ①$$

$$V_{out} = -A_v \cdot V_x \longrightarrow ②$$

Closed loop gain

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{C_1}{C_2 + \frac{C_2 + C_1 + C_{in}}{A_{v_1}}} \simeq \frac{C_1}{C_2}\left[1 - \frac{C_2 + C_1 + C_{in}}{C_2} \frac{1}{A_v}\right]$$

$$\beta = \frac{C_2}{C_2 + C_1 + C_{in}}$$

$$\text{gain error} = \frac{1}{\beta A_v}$$

* gain error increases with $\left(\frac{C_1}{C_2}\right)$

$\frac{C_1}{C_2} \uparrow, \beta \downarrow \Rightarrow \varepsilon \uparrow$

* $\beta = \frac{C_2}{C_2 + C_1 + C_{in}}$ ← minimize

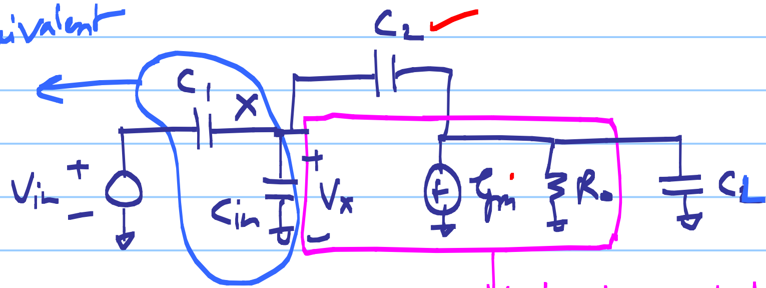
## Speed Considerations :

Time-constant during amplification phase

$$\tau_{amp} = \frac{C_L C_{eq} + C_L C_2 + C_{eq} C_2}{g_m C_2}$$

← similar to $\tau_{amp}$ of S/H when $C_{in} \leftarrow C_1 + C_{in}$

Thevenin equivalent $C_{eq}$




$\frac{V_{out}}{V_{in}}(s)$

$C_L \leftarrow$ load

$C_{eq} \leftarrow C_1 + C_{in}$

$$\beta = \frac{C_2}{C_1 + C_2 + C_{in}}$$ is smaller than $\beta$ of S/H

$f_{3dB} = \beta f_{un}$ is also smaller

$\Rightarrow$ slower than a S/H S/C circuit

* for larger nominal gain $\frac{C_1}{C_2} \uparrow$

$\Rightarrow f_{3dB} \downarrow$

$\rightarrow$ need faster opamp ($f_{un}$) for settling in given time $T_s/2$

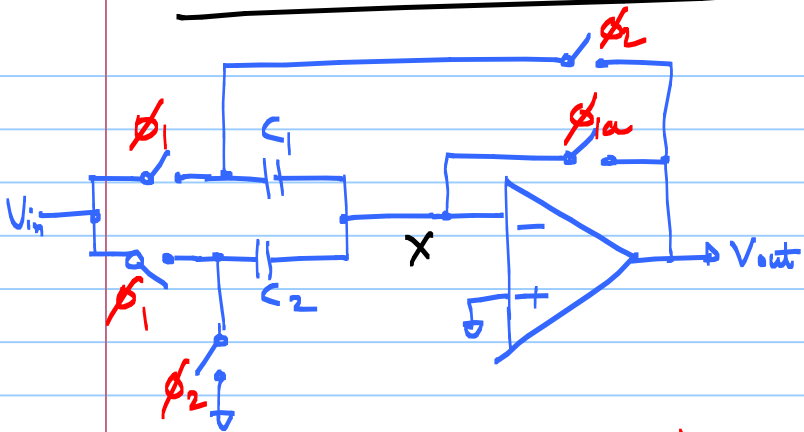
* lower $C_{in}$

* neglecting $C_{in}$

$$\frac{C_1}{C_2} = 2$$

* we have $\beta = \frac{C_2}{C_1 + C_2 + \cancel{C_{in}}} = \frac{C_2}{3C_2} = 1/3$

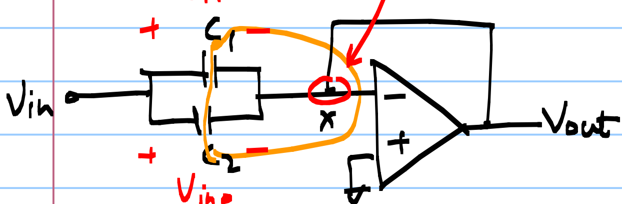
# Precision Multiply x2 circuit




$C_1 = C_2$

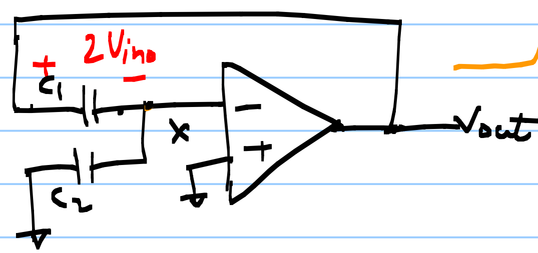
$\boxed{\phi_1}$

charge conservation at this node




$Q = 2C_1 V_{ino}$

$\boxed{\phi_2}$




Here,

$$\beta = \frac{C_1}{C_1 + \underset{\substack{\downarrow \\ C_1}}{C_2} + C_{in}} = \frac{C_1}{2C_1 + C_{in}}$$

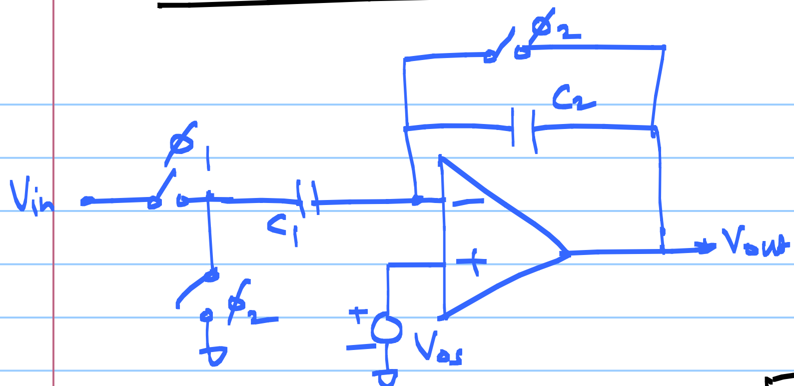
for $C_{in} \approx 0$ $\quad \beta = \frac{1}{2}$

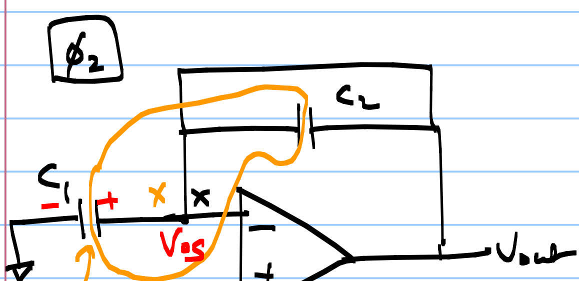
↳ relaxes precision and settling requirements on the opamp

↳ trade-off ⇒ 2x caps in the sampling phase

# Effect of Offset

$$V_{out} = -\frac{C_1}{C_2} V_{in}$$

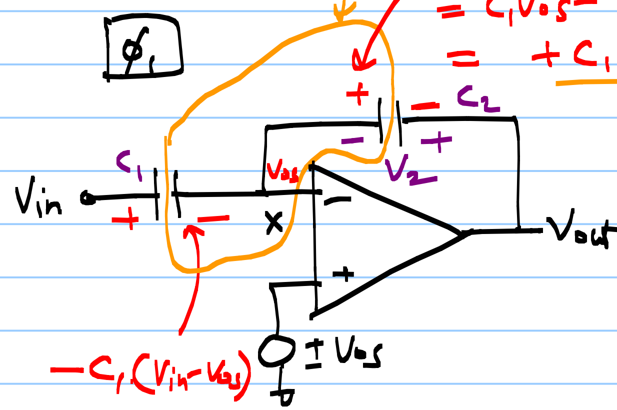



$Q = C_1 V_{os}$

total charge on this surface $= C_1 V_{os}$

charge on this plate
$= C_1 V_{os} - [-C_1(V_{in} - V_{os})]$
$= +C_1 V_{in}$




$-C_1(V_{in} - V_{os})$

$$+C_1 V_{in} = -C_2 (V_2)$$

$$\Rightarrow \quad V_2 = -\frac{C_1}{C_2} V_{in}$$

$\Rightarrow$ $\boxed{V_{out} = \underline{\underline{V_{os}}} - \frac{C_1}{C_2} V_{in}}$

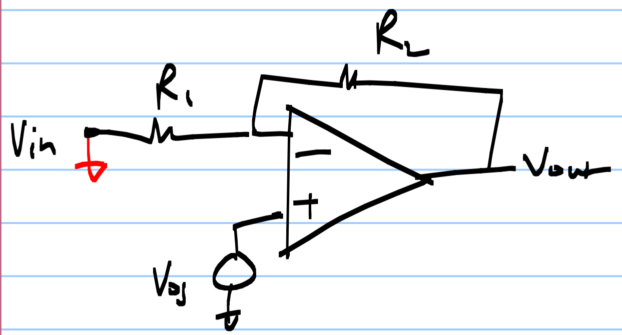
$V_{os} - \frac{C_1}{C_2} (\cancel{V_{in} + V_{os}})$

$\Rightarrow$ inherent input offset cancellation

↳ no offset in the voltage $V_2$

$\Rightarrow$ $V_{out} = -\frac{C_1}{C_2} V_{in} + \underline{\underline{V_{os}}}$ ← DC shift in the output

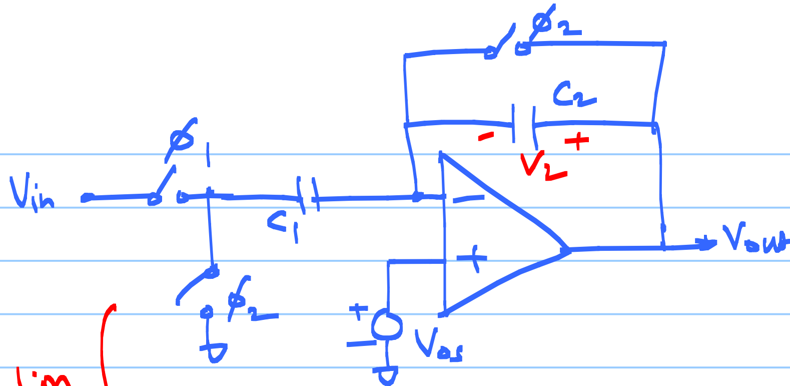
## 2nd CT amplifier case




$$V_{out} = -\frac{R_2}{R_1}V_{in} + V_{os}\left(1+\frac{R_2}{R_1}\right)$$

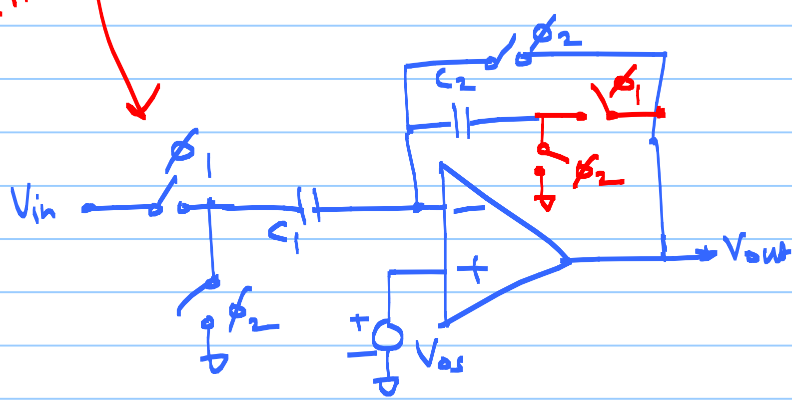
$$= -\frac{R_2}{R_1}\underbrace{(V_{in}-V_{os})} + \underbrace{V_{os}}$$

this offset effect is absent in the SC amplifier case




$$V_{out} = V_{os} + \underbrace{V_2}_{-\frac{C_1}{C_2} V_{in}}$$

modification




$$\begin{array}{r} -\cancel{V_{os}} \\ +\cancel{V_{os}} + V_{sig} \\ \hline V_{sig} \end{array}$$

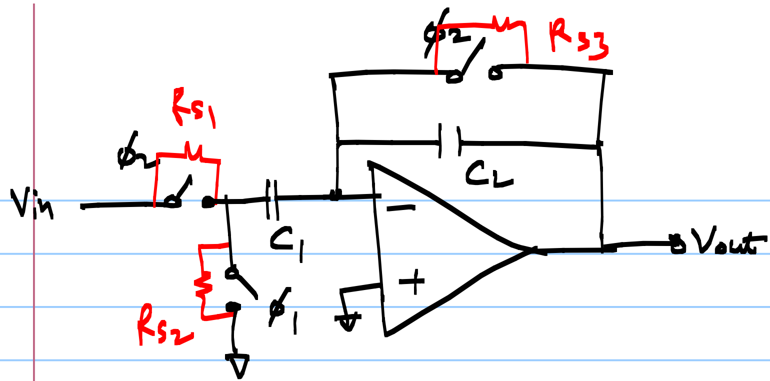
$\underline{\phi_2}$: charging $C_2$ to $V_{os}$ instead of 0

$\underline{\phi_1}$: $V_{out} = -\frac{C_1}{C_2} V_{in} \longrightarrow$ offset is completely removed

# Thermal Noise

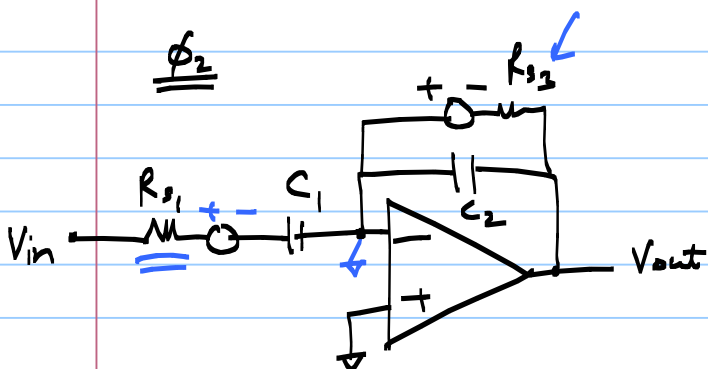
* Assume that the switches are noisy & the opamp $\overline{V_{n,in}^2}$ is negligible

* Noninverting amplifier




$\overline{V_{o,noise,rms}}$

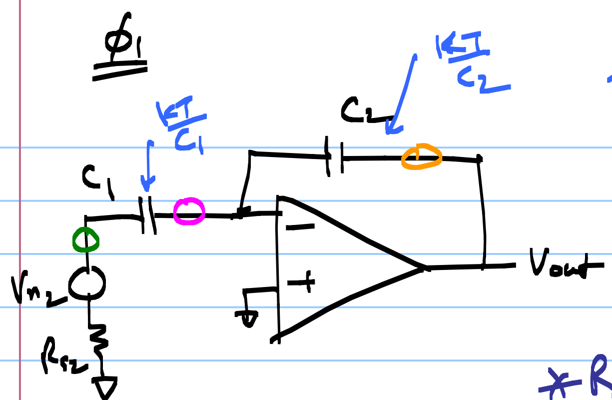
**$\phi_2$**




$R_{s_1} \Rightarrow C_1$ has a noise variance of $\frac{kT}{C_1}$

$R_{s_3} \Rightarrow C_2$ has noise variance $\frac{kT}{C_2}$

$\underline{\underline{\phi_1}}$




$* \underline{R_{s_1}} \Rightarrow$ In $\phi_2 \rightarrow \frac{kT}{C_1}$ is amplified by $\left(\frac{C_1}{C_2}\right)^2$

$\Rightarrow \frac{kT}{C_1}\left(\frac{C_1}{C_2}\right)^2$ ← ①

$* R_{s_3} \Rightarrow$ In $\phi_2 \rightarrow \frac{kT}{C_2}$ ← ② is held

$* R_{s_2} \Rightarrow$ Results in noise variance $\frac{kT}{C_1}$ on $C_1$ during $\phi_1$ phase

$\Downarrow$

$\frac{kT}{C_1}\left(\frac{C_1}{C_2}\right)^2$ at the output ← ③

Total output noise variance $= 2\frac{kT}{C_1}\left(\frac{C_1}{C_2}\right)^2 + \frac{kT}{C_2}$

$$= \frac{kT}{C_1}\left[2\left(\frac{C_1}{C_2}\right)^2 + \frac{C_1}{C_2}\right] \longrightarrow (1)$$

$\Rightarrow$ Input referred noise variance $= \dfrac{\frac{kT}{C_1}\left[2\left(\frac{C_1}{C_2}\right)^2 + \frac{C_1}{C_2}\right]}{\left(\frac{C_1}{C_2}\right)^2}$

$$= \frac{kT}{C_1}\left[2 + \frac{C_2}{C_1}\right]$$

for gain of 2 $\Rightarrow \frac{C_1}{C_2} = 2$

$\Rightarrow$ input-referred noise variance $= 2.5\,\frac{kT}{C_1}$

$$\overline{V_{in,noise,rms}} = \sqrt{2.5\,\frac{kT}{C_1}}$$