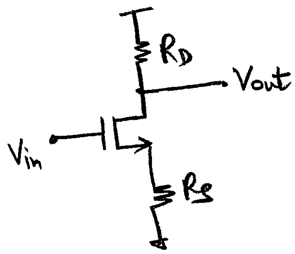
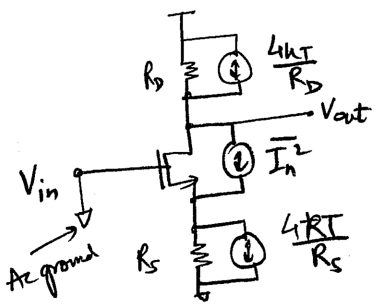
HW3 Solutions

Problem 3 :

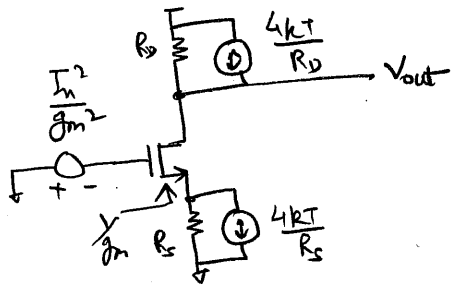
(a)



$$A_v = -\frac{g_m R_D}{1+g_m R_S}$$



$\Rightarrow$



$$\overline{I_n^2} = \frac{8}{3} kT g_m$$

for the I_{n,R_S} noise current it is divided between $\frac{1}{g_m}$ and R_S

$\Rightarrow$ current flowing into transistor and buffered to the output $\Rightarrow$ $\left(\frac{R_S}{R_S+1/g_m}\right) \cdot I_{n,R_S}$

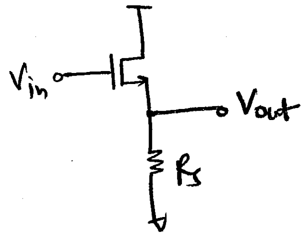
$$\Rightarrow \overline{V_{n,out}^2} = \frac{4kT}{R_D} \times R_D^2 + \frac{\overline{I_n^2}}{g_m^2} \times \left(\frac{g_m R_D}{1+g_m R_S}\right)^2 + \left(\frac{R_S}{R_S+1/g_m}\right)^2 \times \frac{4kT}{R_S} \times R_D^2$$

$$= 4kTR_D + \frac{8}{3} kT \cdot \frac{1}{g_m} \cdot \left(\frac{g_m R_D}{1+g_m R_S}\right)^2 + 4kTR_S \cdot \left(\frac{g_m R_D}{1+g_m R_S}\right)^2$$

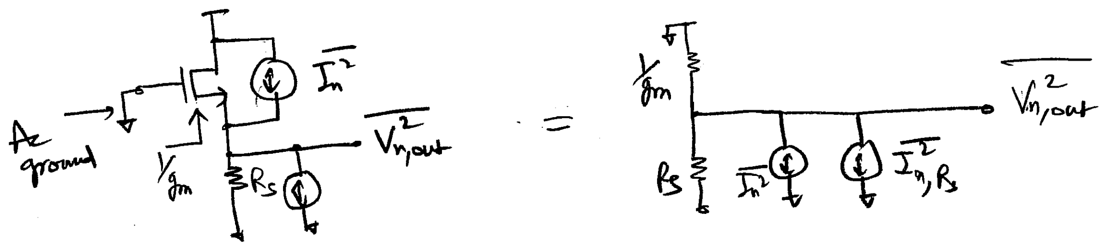
$$\Rightarrow \boxed{\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{A_v^2} = \frac{8}{3} kT \cdot \frac{1}{g_m} + 4kTR_S + 4kT R_D \left(\frac{1+g_m R_S}{g_m R_D}\right)^2}$$

$\Rightarrow$ to minimize $\overline{V_{n,in}^2}$, $g_m \uparrow$, $R_S \downarrow$, $R_D \uparrow$

(b)



$$A_v = \frac{R_S}{\frac{1}{g_m} + R_S} = g_m \cdot \left(\tfrac{1}{g_m} \| R_S\right)$$



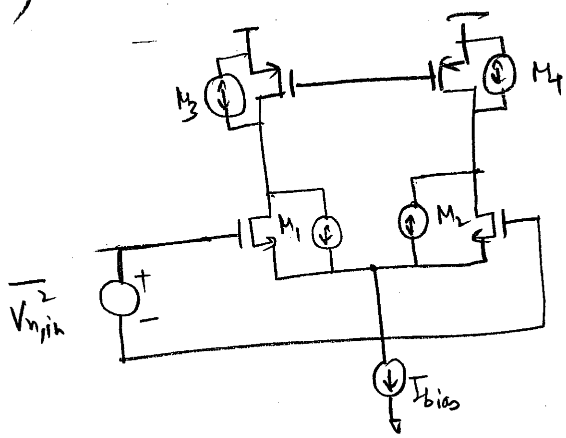
$$\overline{V_{n,out}^2} = \left(\overline{I_n^2} + \frac{4kT}{R_S}\right)\left(\tfrac{1}{g_m} \| R_S\right)^2$$

$$= \left(4kT \cdot \frac{2}{3} g_m + 4kT \cdot \frac{1}{R_S}\right)\left(\tfrac{1}{g_m} \| R_S\right)^2$$

$$\Rightarrow \boxed{\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{A_v^2} = 4kT \cdot \frac{2}{3} \frac{1}{g_m} + 4kT \cdot \frac{1}{g_m^2 R_S}}$$

$$\Rightarrow \overline{V_{n,in}^2} \downarrow \Rightarrow g_m \uparrow,\ R_S \uparrow$$

(c)

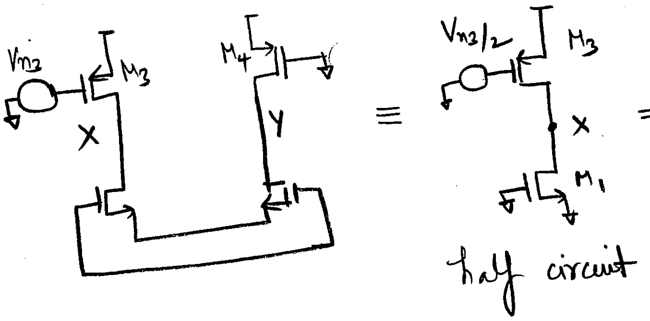


Noise current due to Ibias is CM at the output ⇒ no effect on $\overline{V_{n,in}^2}$

* short the input ports to find $\overline{V_{n,in}^2}$
* Reflect $\overline{I_{n1}^2}$, $\overline{I_{n2}^2}$, $\overline{I_{n3}^2}$ & $\overline{I_{n4}^2}$ to the input side to find $\overline{V_{n,in}^2}$

$$\overline{V_{n,in}^2}\Big|_{M_1,M_2} = 2\,\overline{V_{n1}^2} = 2\,\frac{\overline{I_{n1}^2}}{g_{m1}^2} = 2\times\frac{8}{3}kT\,\frac{g_{m1}}{g_{m1}^2} = 2\times\frac{8}{3}\frac{kT}{g_{m1}} \rightarrow (1)$$

for noise contribution from M3 & M4



$$\overline{V_{n,out}^2}\Big|_{M_3} = \overline{\left(g_{m3}(r_{o1}||r_{o3})\cdot 2\times\frac{V_{n3}}{2}\right)^2}$$
$$= g_{m3}^2(r_{o1}||r_{o3})^2\cdot\overline{V_{n3}^2}$$

$$\Rightarrow \overline{V_{n,out}^2}\Big|_{M_3,M_4} = g_{m3}^2(r_{o1}||r_{o3})^2\,\overline{V_{n3}^2} + g_{m4}^2(r_{o2}||r_{o4})\cdot\overline{V_{n4}^2}$$
$$= 2\times g_{m3}^2(r_{o1}||r_{o3})^2\,\overline{V_{n3}^2} \longrightarrow (2)$$

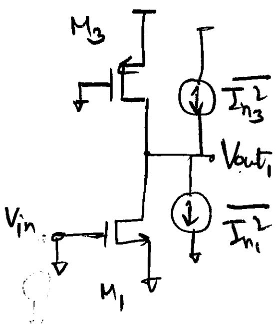
$\because A_v = g_{m1}\cdot(r_{o1}||r_{o3})^2$

$$\rightarrow \overline{V_{n,in}^2} = 2\,\overline{V_{n1}^2} + 2\cdot\frac{g_{m3}^2}{g_{m1}^2}\cdot V_{n3}^2$$

$$\boxed{V_{n,in}^2 = 8kT\left(\frac{2}{3g_{m1}} + \frac{2g_{m3}}{3g_{m1}^2}\right)}$$

Short-cut for differential circuits

* find input referred noise of the half circuit and multiply by 2

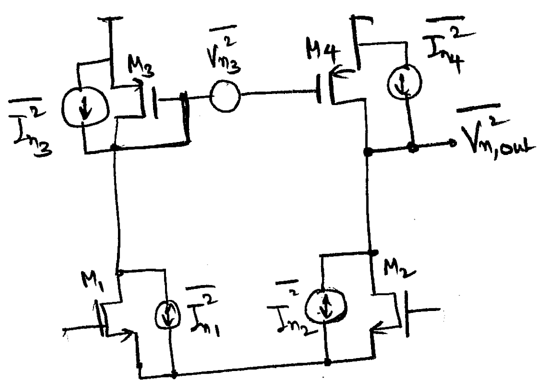


$$\Rightarrow \overline{V_{n,in}^2} = 2 \times \left(\frac{\overline{I_{n_1}^2}}{g_{m_1}^2} + \frac{\overline{I_{n_3}^2} \cdot (r_{o_1} \| r_{o_3})^2}{g_{m_1}^2 (r_{o_1} \| r_{o_3})^2} \right)$$

$$= 2 \times \left(\frac{8}{3} kT \frac{1}{g_{m_1}} + \frac{8}{3} kT \frac{g_{m_3}}{g_{m_1}^2} \right)$$

$$\boxed{\overline{V_{n,in}^2} = 8kT \left(\frac{2}{3 g_{m_1}} + \frac{2 g_{m_3}}{3 g_{m_1}^2} \right)}$$

(d)



Most of the noise current $\overline{I_{n_1}^2}$ and $\overline{I_{n_3}^2}$ flows into M_3 due to $\frac{1}{g_{m3}}$ impedance

⇒ This can be referred to the gate of M_3 as

$$\overline{V_{n_3}^2} = \left(\frac{\overline{I_{n_3}^2} + \overline{I_{n_1}^2}}{g_{m_3}^2}\right)$$

$g_{m_1} = g_{m2}$ & $g_{m3} = g_{m4}$
$r_{o_1} = r_{o_2}$ & $r_{o_3} = r_{o4}$

This noise voltage is then reflected into the output $\overline{V_{n,out}^2}$.

⇒

$$\overline{V_{n,out}^2} = \left[I_{n_2}^2 + I_{n4}^2 + \cancel{g_{m4}^2}\left(\frac{\overline{I_{n_3}^2} + \overline{I_{n_1}^2}}{\cancel{g_{m_3}^2}}\right)\right](r_{o3}\|r_{o4})^2$$

$$= \frac{8}{3}kT\left[g_{m2} + g_{m4} + g_{m3} + g_{m1} \right](r_{o3}\|r_{o4})^2$$

$$\Rightarrow \overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{g_{m_1}^2 (r_{o3}\|r_{o4})^2} = \boxed{\frac{8}{3}kT\left[\frac{2}{g_{m1}} + \frac{2g_{m3}}{g_{m1}^2}\right]}$$

⇒ Same input referred noise as part (c)

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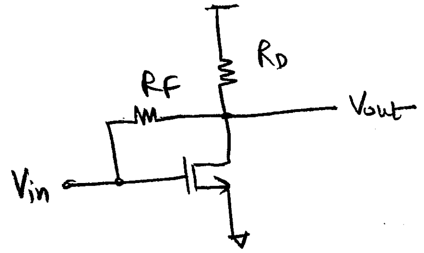
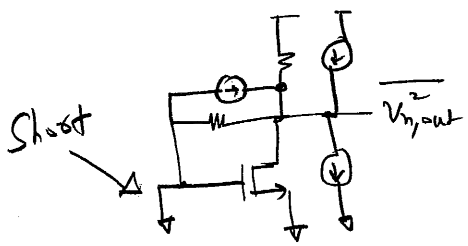


Step 1: find $\overline{V_{n,in}^2}$

* short the input to ground and equate the output noise of the circuits below



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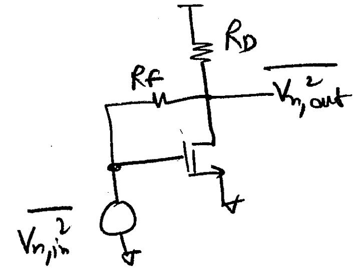


$$\overline{V_{n,out}^2} = \overline{V_{n,in}^2}\cdot A_v^2 \longrightarrow ②$$

Here $A_v = \left(-g_{m1} + \frac{1}{R_F}\right)\cdot(R_F \| R_D)$

$$\Rightarrow \overline{V_{n,out}^2} = \left(\overline{I_n^2} + \overline{I_{n,R_F}^2} + \overline{I_{n,R_D}^2}\right)(R_D\|R_F)^2$$

$$= 4kT\left(\frac{2}{3}g_m + \frac{1}{R_F} + \frac{1}{R_D}\right)(R_D\|R_F)^2 \longrightarrow ①$$

Equating ① + ②, we get

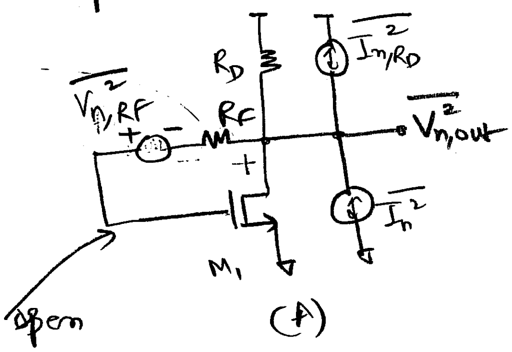
$$\overline{V_{n,in}^2} = \frac{4kT\left(\frac{2}{3}g_m + \frac{1}{R_F} + \frac{1}{R_D}\right)\cancel{(R_D\|R_F)^2}}{\left(-g_{m1} + 1/R_F\right)^2\cancel{(R_D\|R_F)^2}}$$

$$\boxed{\overline{V_{n,in}^2} = \frac{4kT\left(\frac{2}{3}g_m + 1/R_F + 1/R_D\right)}{\left(-g_{m1} + 1/R_F\right)^2}}$$

Ans.

step 2: find $\overline{I_{n,in}^2}$

* open the input and equate the output noise of the circuits below



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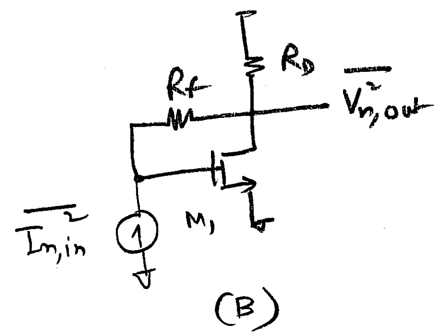


Circuit (A):

Sum the currents at the output node:

$$\frac{V_{n,out}}{R_D} + I_{n,M_1} - I_{n,R_D} + g_m\left(V_{n,out} - V_{n,RF}\right) = 0$$

$$\Rightarrow \quad V_{n,out} = \frac{-I_{n,M_1} + I_{n,R_D} + g_m V_{n,RF}}{(g_m R_D + 1)} \cdot R_D \longrightarrow ①$$

Circuit (B):

the transimpedance gain can be found as

$$R_T = \frac{V_{out}}{I_{in}} = -\frac{(g_m R_F - 1)}{(1 + g_m R_D)} \cdot R_D$$

$$\Rightarrow \quad V_{n,out} = I_{n,in} \times \frac{-(g_m R_F - 1)}{(1 + g_m R_D)} \cdot R_D \longrightarrow ②$$

from ① & ②

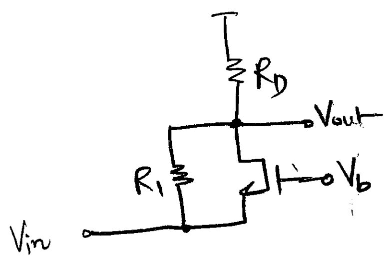
$$I_{n,in} = \frac{-I_{n,M_1} + I_{n,R_D} + g_m V_{n,RF}}{(-g_m R_F + 1)}$$

Computing the mean square values and assuming $g_m R_F >> 1$,

$$\overline{I_{n,in}^2} = \frac{\overline{I_{n,M_1}^2} + \overline{I_{n,R_D}^2} + g_m^2 \cdot \overline{V_{n,R_F}^2}}{g_m^2 R_F^2}$$

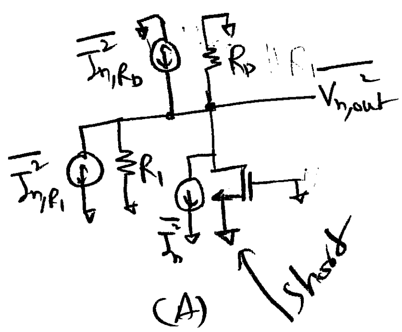
$$\boxed{\overline{I_{n,in}^2} = 4kT\left(\frac{2}{3} \cdot \frac{1}{g_m R_F^2} + \frac{1}{g_m^2 R_F^2 R_D} + \frac{1}{R_F}\right)} \text{ Ans.}$$

(b)



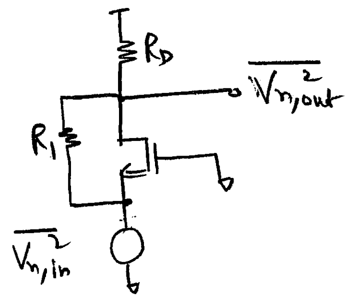
Step 1: find $\overline{V_{n,in}^2}$:

* short the input and equate $\overline{V_{n,out}^2}$



(A)

&



(B)

(A) $\rightarrow$ $\overline{V_{n,out}^2} = 4KT\left[\frac{2}{3}g_{m1} + \frac{1}{R_1} + \frac{1}{R_D}\right](R_1 \| R_D)^2 \longrightarrow ①$

(B) $\rightarrow$ Voltage gain $A_V = (g_{m1} + 1/R_1)(R_1 \| R_D)$

$\Rightarrow \quad \overline{V_{n,in}^2} = \dfrac{\overline{V_{n,out}^2}}{A_V^2} \longrightarrow ②$

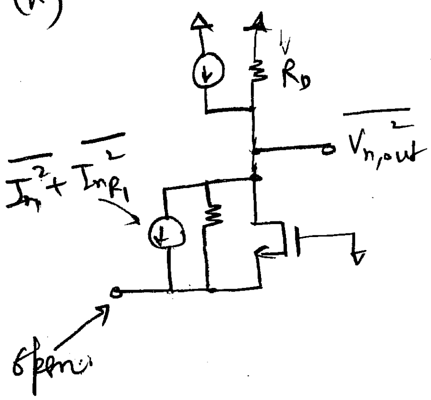
from ① & ②,

$$\boxed{\overline{V_{n,in}^2} = 4KT\left(\frac{1}{g_m + 1/R_1}\right)^2\left(\frac{2}{3}g_{m1} + 1/R_1 + 1/R_D\right)}$$ Ans

Step 2: Find $\overline{I_{n,in}^2}$

* Open the input and equate $\overline{V_{n,out}^2}$

(A)

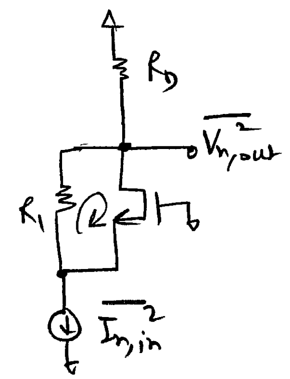


$\because$ current at the source of $M_1 \Rightarrow I_n + I_{nR_1} + I_D = 0$

The current $I_n + I_{n,R_1}$ doesn't flow into R_D

$$\Rightarrow \overline{V_{n,out}^2} = 4kT R_D \longrightarrow ①$$

(B)



$$\overline{V_{n,out}^2} = \overline{I_{n,in}^2}\, R_D^2 \longrightarrow ②$$

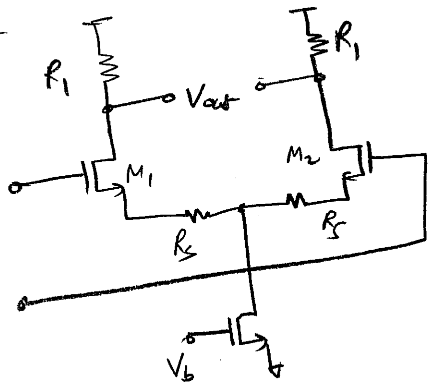
from ① & ②

$$\boxed{\overline{I_{n,in}^2} = \frac{4kT}{R_D}}$$ Ans.

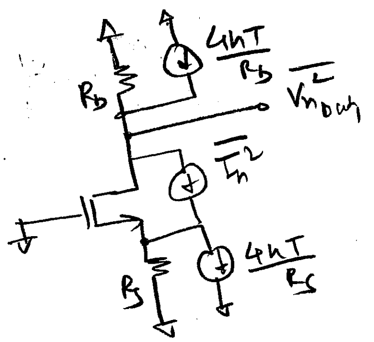
Extra Problem 1:

(a) M_3 doesn't contribute differential noise

$$|A_v| = \frac{g_{m1} R_1}{1 + g_{m1} R_s}$$



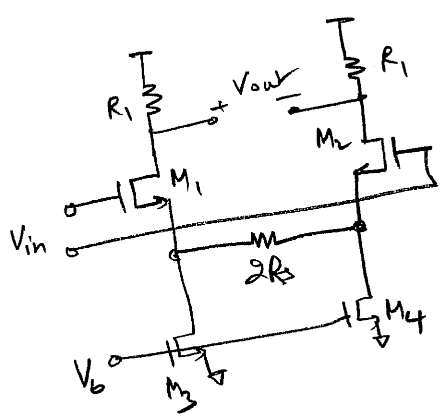
Half Circuit



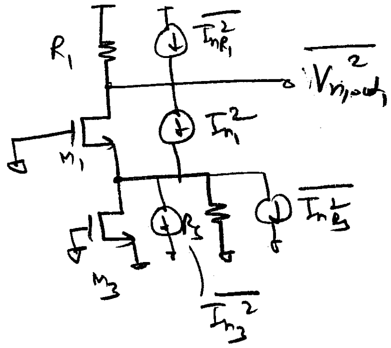
$$\overline{V_{n,out}^2} = 2 \times 4kT \times \left[\frac{1}{R_1} + \frac{2}{3} g_{m1} \left(\frac{g_m}{1 + g_m R_s} \right)^2 + \frac{1}{R_s} \left(\frac{R_s}{1/g_{m1} + R_s} \right)^2 \right] R_D^2$$

$$\Rightarrow \boxed{\overline{V_{n,in}^2} = 8kT \left[\frac{2}{3 g_{m1}} + R_s + \frac{1}{R_1} \left(\frac{1 + g_m R_s}{g_{m1}} \right)^2 \right]} \quad \text{Ans}$$

(b) Here M_3 & M_4 contribute differential noise



Half circuit



* I_{n3} is divided between R_s and $1/g_{m1}$

$$\overline{V^2_{n,out}(b)} = \overline{V^2_{n,out(a)}} + 2\times \frac{8}{3}kT g_{m3}\left(\frac{R_s}{R_s+1/g_{m1}}\right)^2 R_D^2$$

$\Rightarrow$

$$\overline{V^2_{n,in(b)}} = 8kT\left[\frac{2}{3g_{m1}} + R_s + \frac{1}{R_D}\left(\frac{1+g_{m1}R_s}{g_{m1}}\right)^2 + \frac{2}{3}g_{m3}R_s^2\right]$$

$$\Rightarrow \overline{V^2_{n,in(b)}} = \overline{V^2_{n,in,a}} + \underbrace{2(4kT)\left(\frac{2}{3}g_{m3}R_s^2\right)}_{\text{Extra noise contributed by } M_3 \,\&\, M_4}$$

* circuit (a) has large input CMR but suffers from higher input referred noise.
The gains of the circuits are equal.

Extra problem ②: See lecture notes.