Delta-Sigma Analog-to-Digital Converters

Brief Overview

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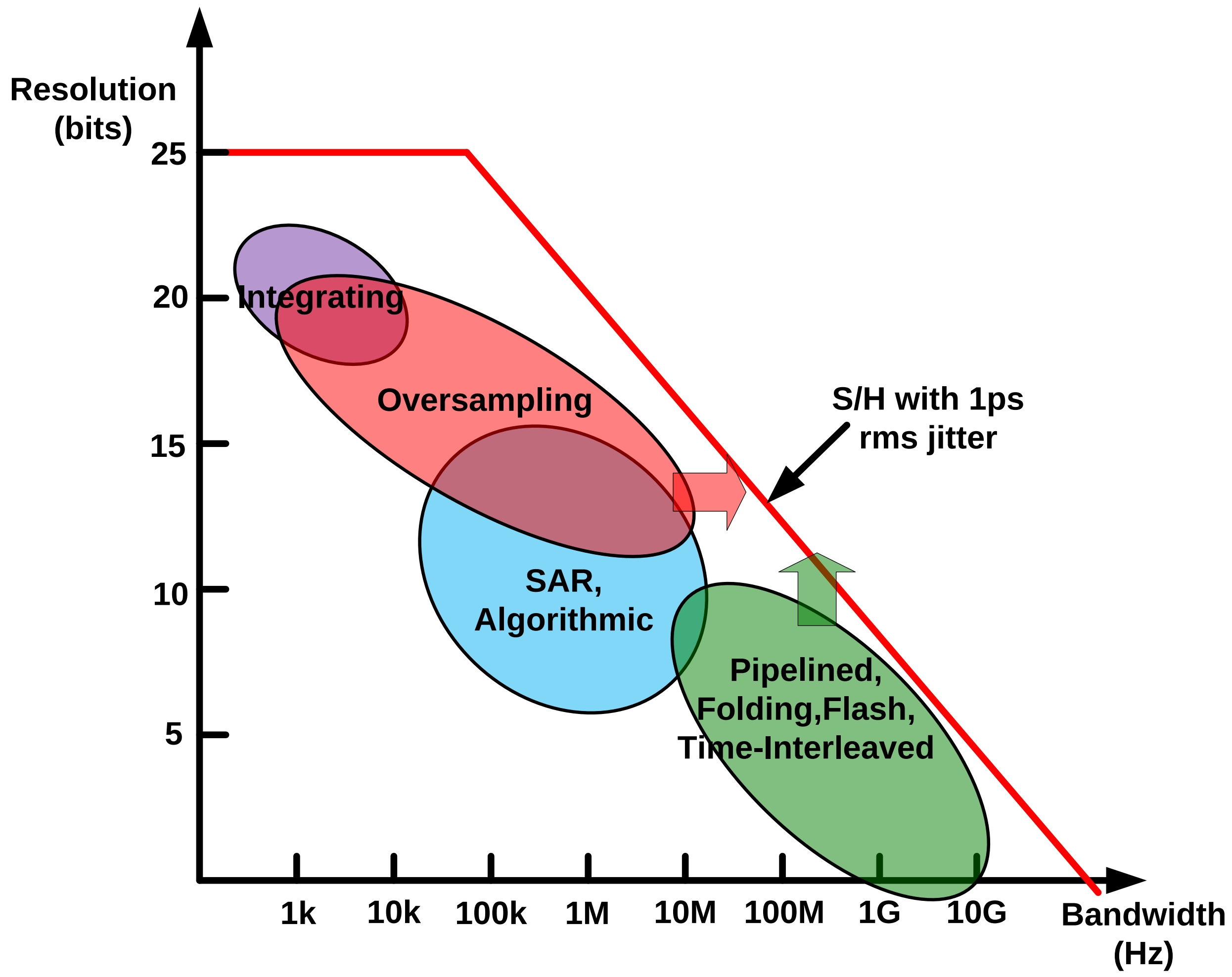
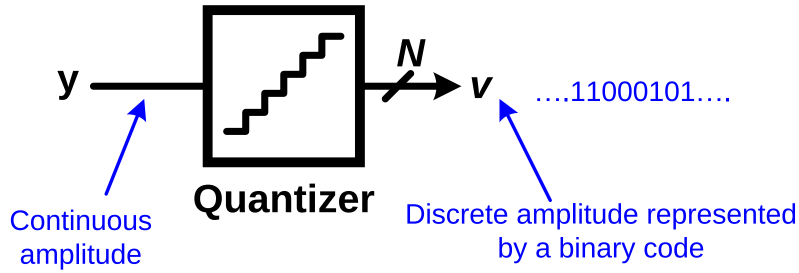
Analog to Digital Converter Architectures



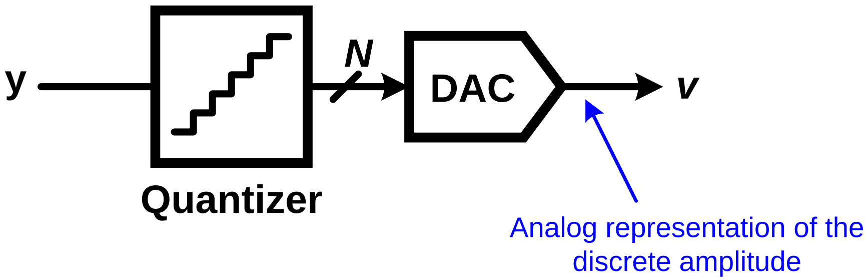
Why larger overlap on SAR and OS?

- Sometimes oversampling modulators are preferred over SAR. Why?
- Nyquist rate SAR converters can’s support complete bandwidth due to stringent requirements on Anti-aliasing Filters.
- Consider a data converter with 250 ksps output rate.
- For oversampled modulator OSR=64, input is sampled at 16 MHz.
- Nyquist rate SAR samples the input at 500 kHz.
- Requirements on anti-aliasing filter in the second case is very hard compared to oversampling case.

Quantization Basics

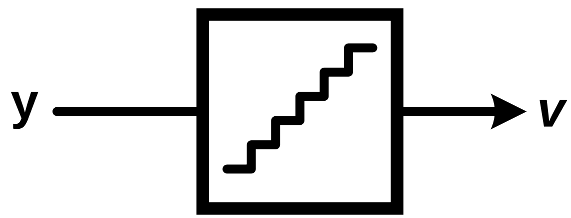


Modeling



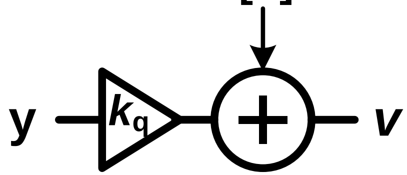
- Quantization error/noise: e[n] = v[n]–y[n]
- Least significant bit (LSB): Δ
- Quantizer has non-linear input-output characteristics
- Exact analytical modeling is difficult
  - Use a simplified model

Quantizer Modeling



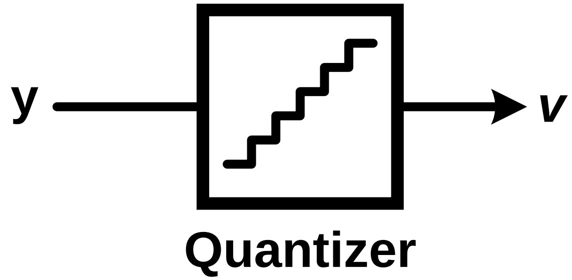
Quantizer

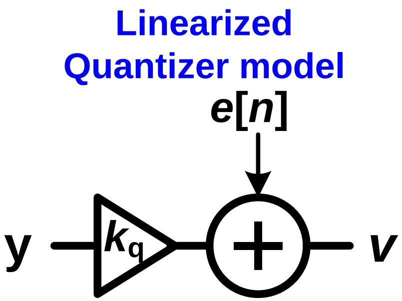
Linearized Quantizer model



- Quantization noise modeling can be simplified with the assumptions
  - Input (y) stays within the no-overload input range
  - e[n] is uncorrelated with the input y
  - Spectrum of e[n] is white
  - Quantization noise is uniformly distributed
- Linearized quantizer model
  - AWUN noise
- Quantization noise power: $$\sigma_e^2 = \frac{\Delta^2}{12}$$
- $SQNR = 6.02 \cdot N + 1.76$

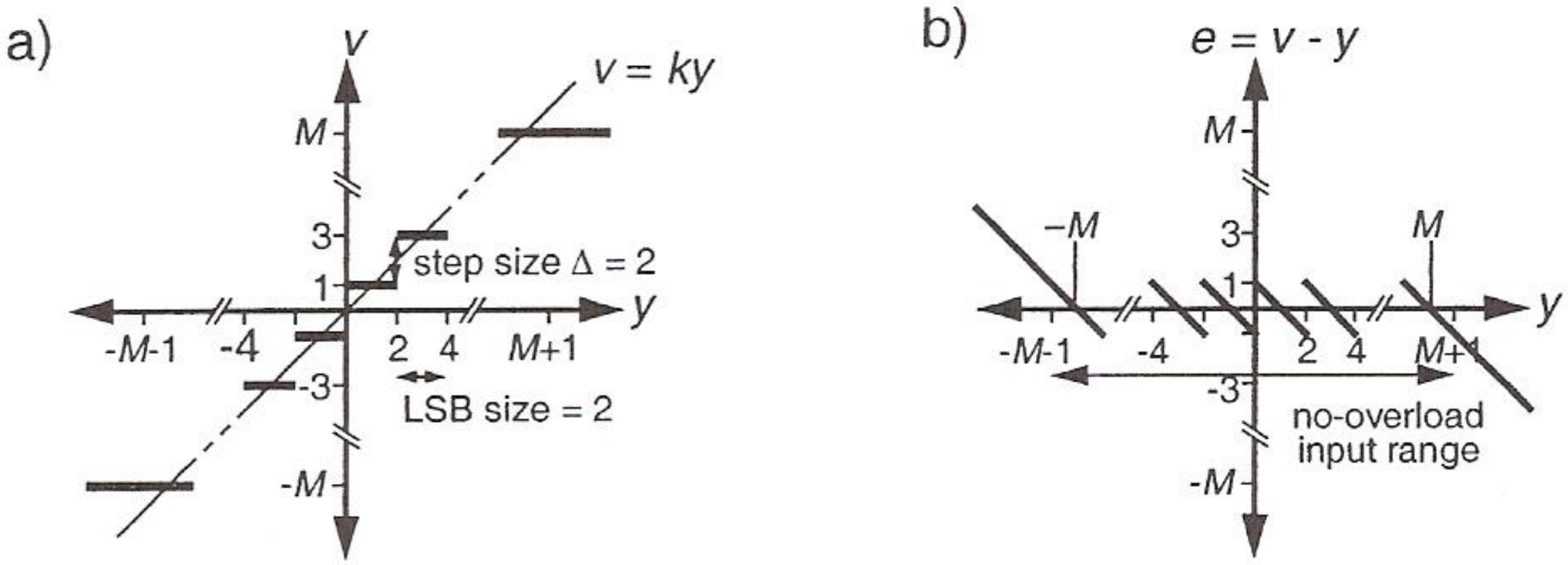
Quantizer Modeling





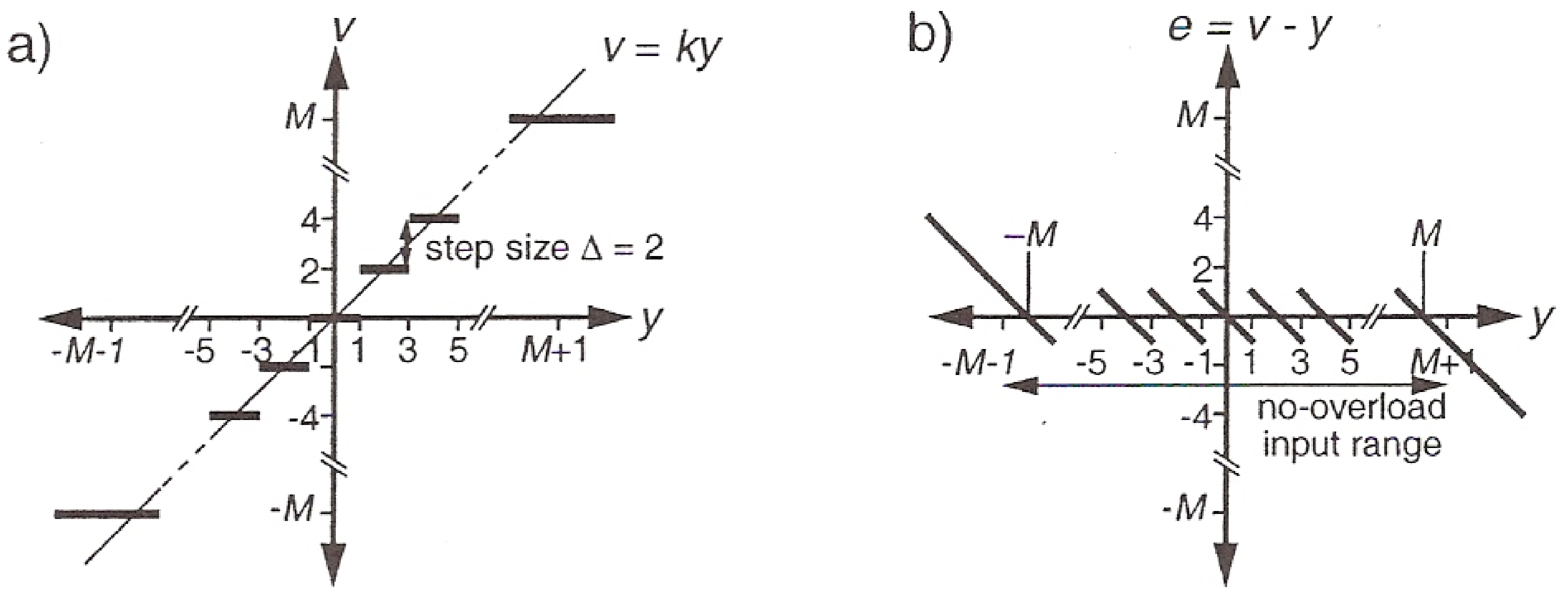
- Linear quantizer model holds true for large number of quantization levels and when the input is varying fast
- Linear quantizer model breaks down when
  - Input (y) is not varying fast
  - Input (y) is periodic with a frequency harmonically related to fs
  - Quantizer overload
- With quantizer overload, the effective gain of the quantizer drops as the input amplitude inscreases

Mid-Rise Quantizer (even number of levels)



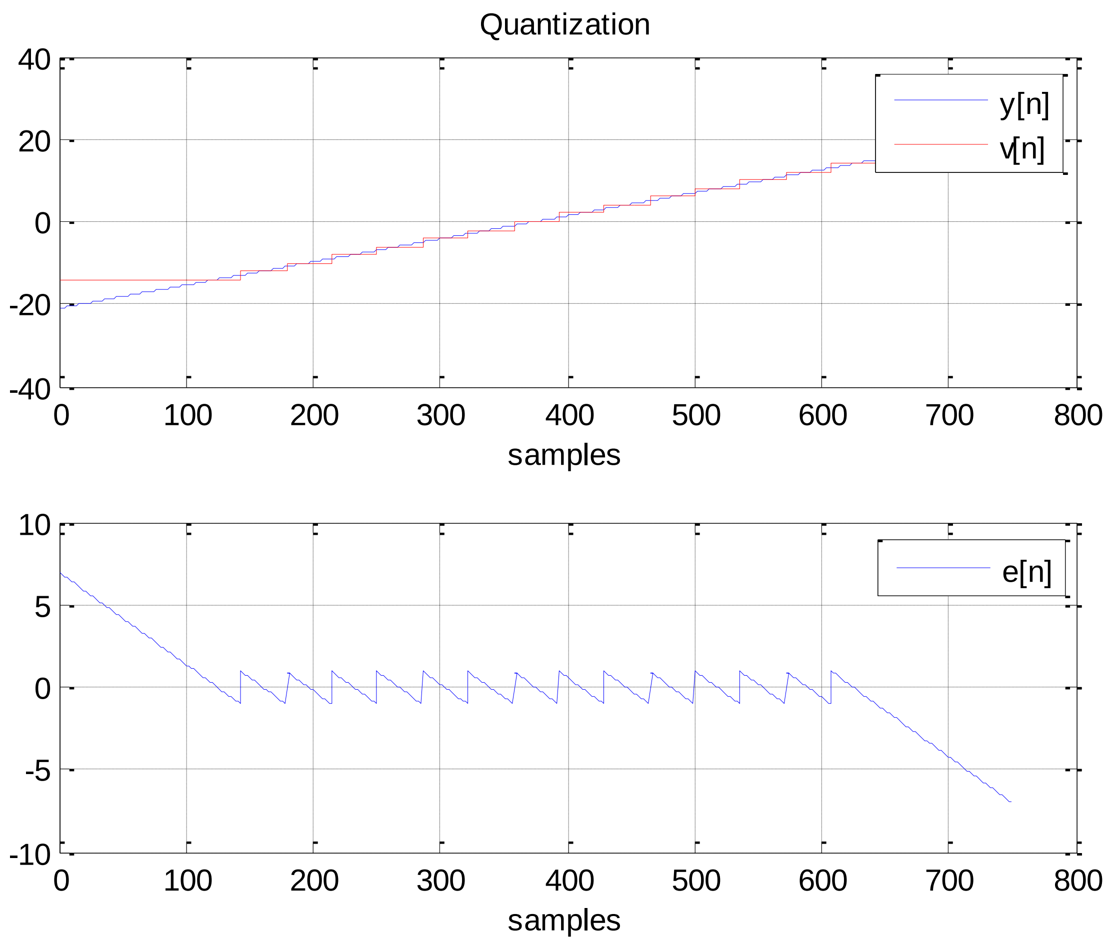
- Step rising at y=0 (mid-rise).
- In this figure (DSM toolbox model), LSB= Δ= 2
- M = Number of steps, (M is odd here)
  - Number of levels (nLev) = M+1, (even)
- Input thresholds: 0, ±2, ……, ±(M–1).
- Output levels: ±1, ±3, ……, ±M.

Mid-Tread Quantizer (odd number of levels)

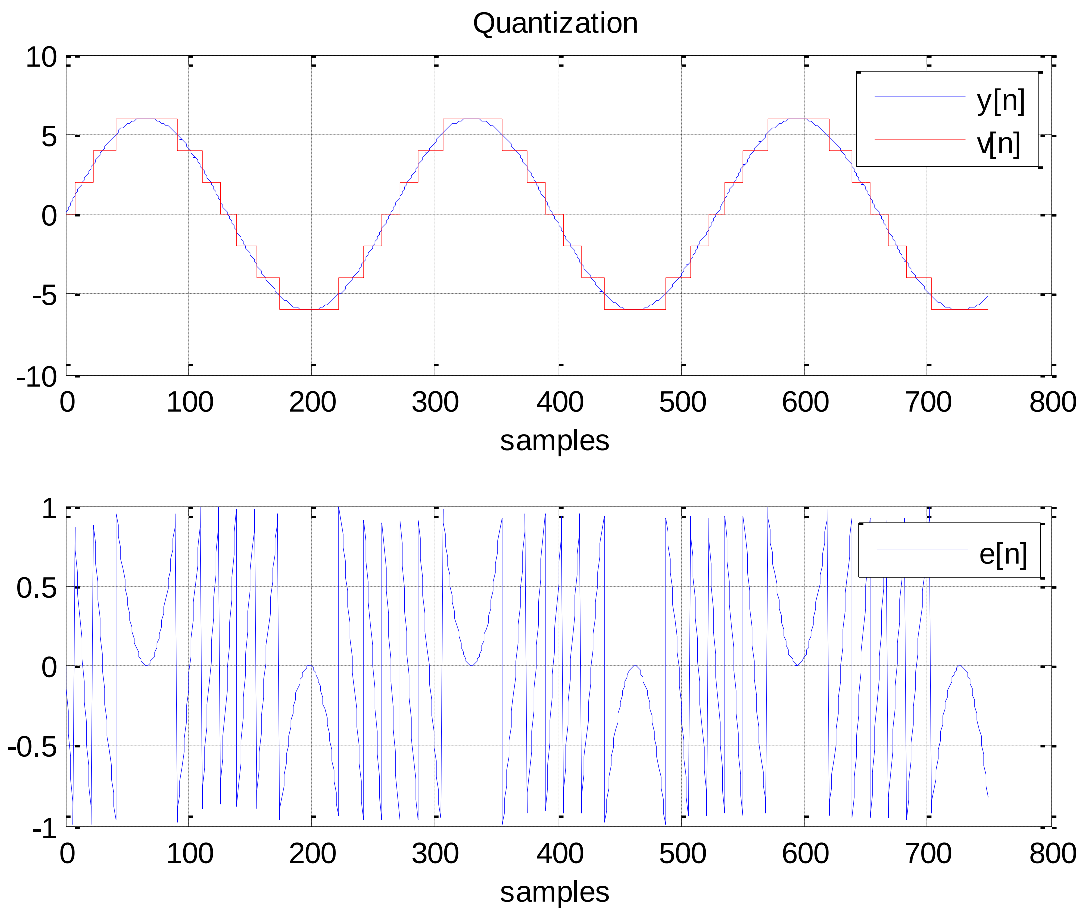


- Flat part of the step at y=0 (mid-tread).
- Here, LSB= Δ= 2
- M = Number of steps, (M is even here)
  - Number of levels (nLev) = M+1, (odd)
- Input thresholds: 0, ±2, ……, ±(M–1).
- Output levels: 0, ±2, ±4, ……, ±M.

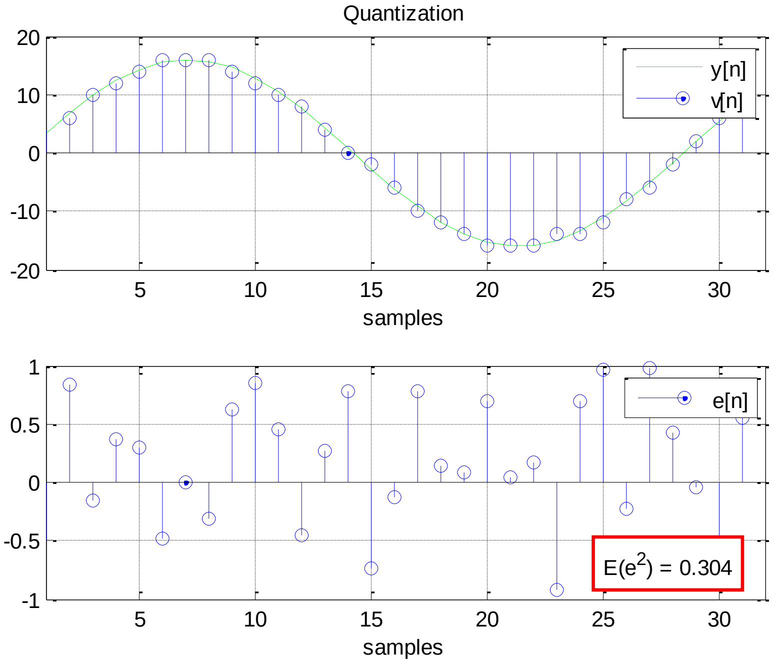
Quantizer Characteristics : Slow ramp input

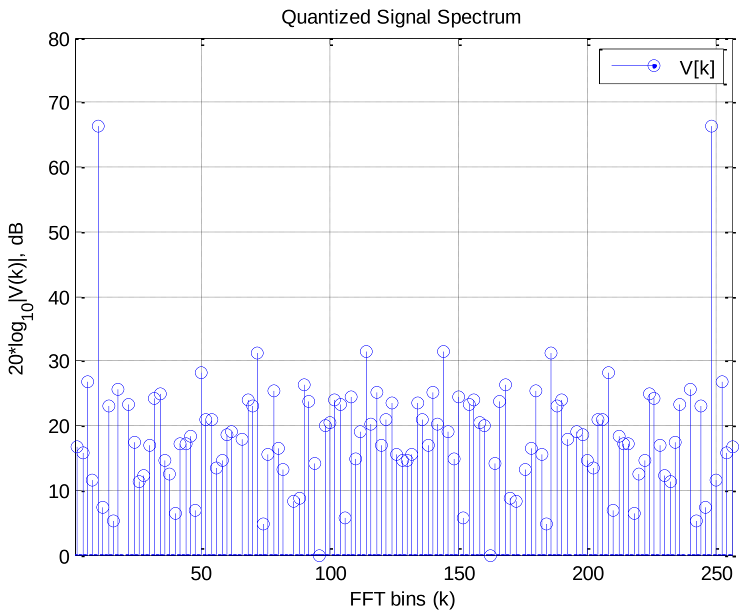


Quantizer Characteristics : Sine input



Quantization Noise : Example 1

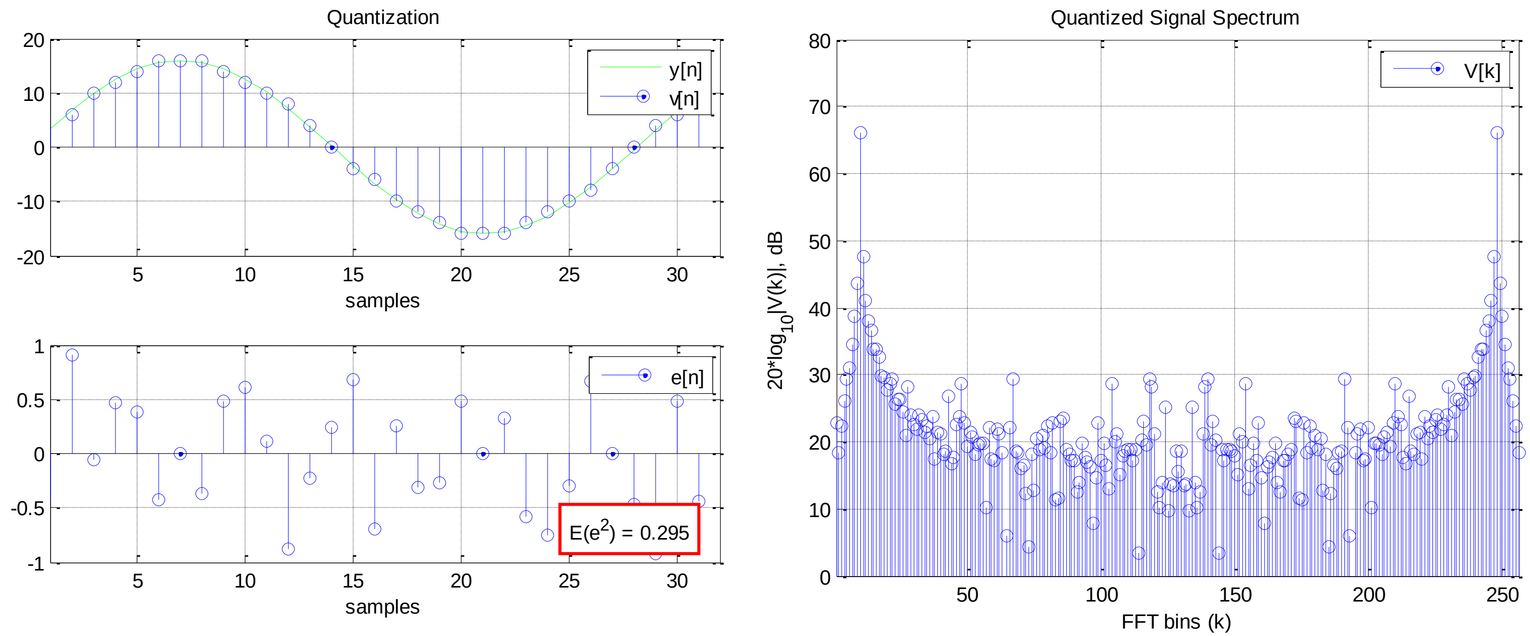




nLev=17, Δ=2, f_{in}/f_s = 9/256 :

•$E(e^2)$ = 0.304 ≈ $\Delta^2/12$

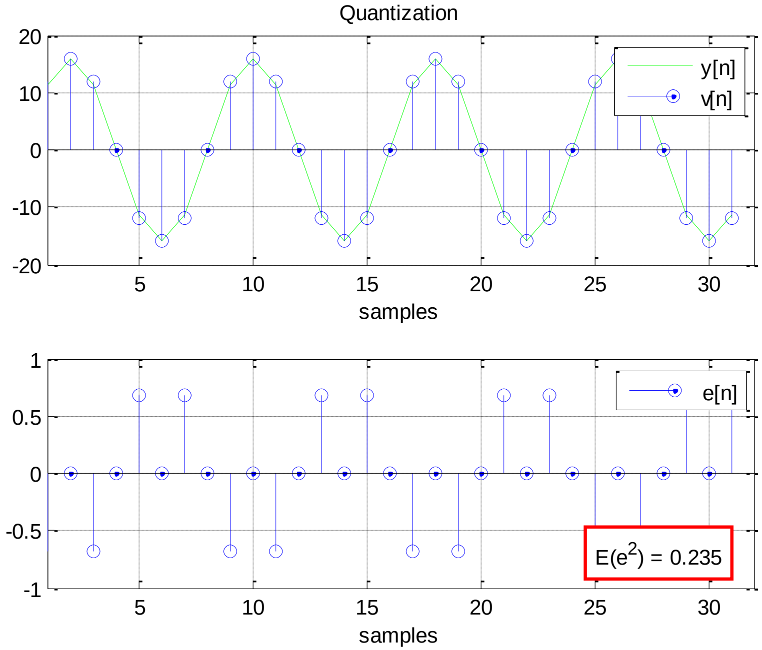
Quantization Noise : Example 1 contd.

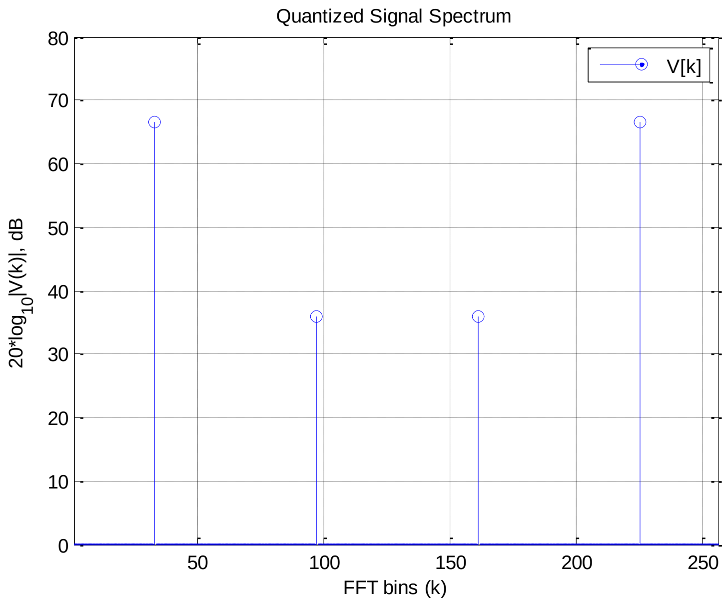


nLev=17, Δ=2, f_{in}/f_s = 9.1/256 :

- $E(e^2) = 0.295 \approx \Delta^2/12$
- Notice the FFT leakage.

Quantization Noise : Example 1 contd.



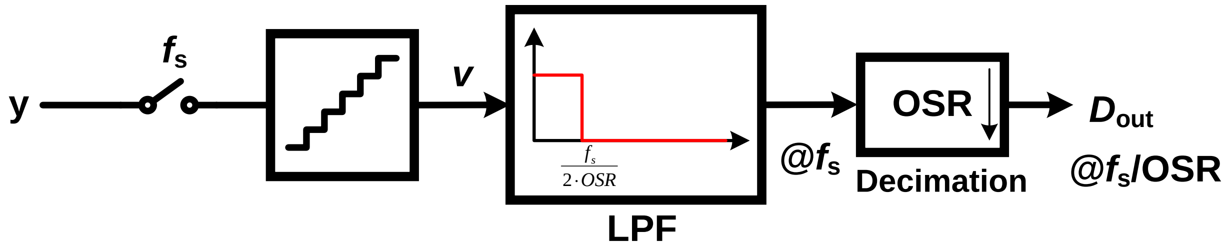


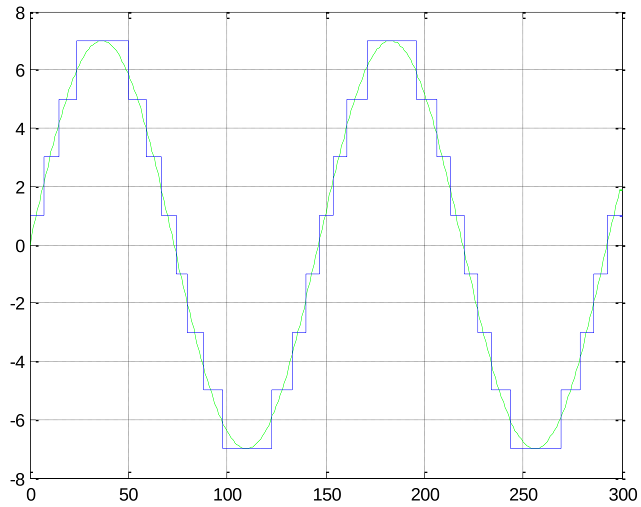
nLev=17, Δ=2, $f_{in}/f_s = 32/256 = 1/8$:

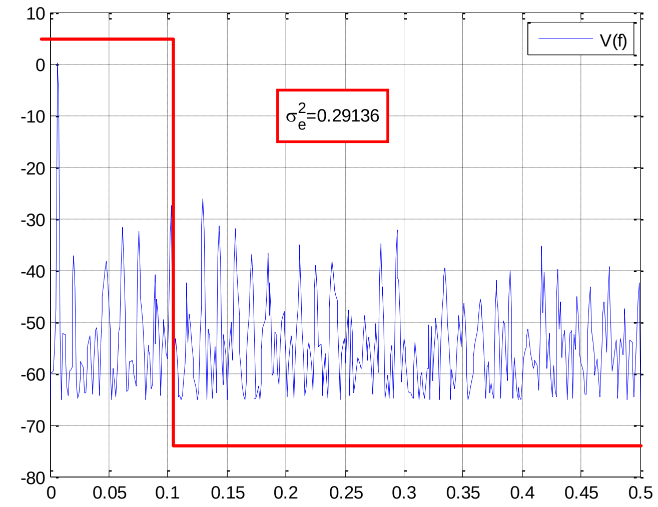
- $E(e^2) = 0.235 < \Delta^2/12$
    - Quantization *noise* approximation not valid

Oversampling and Noise-Shaping

Oversampling

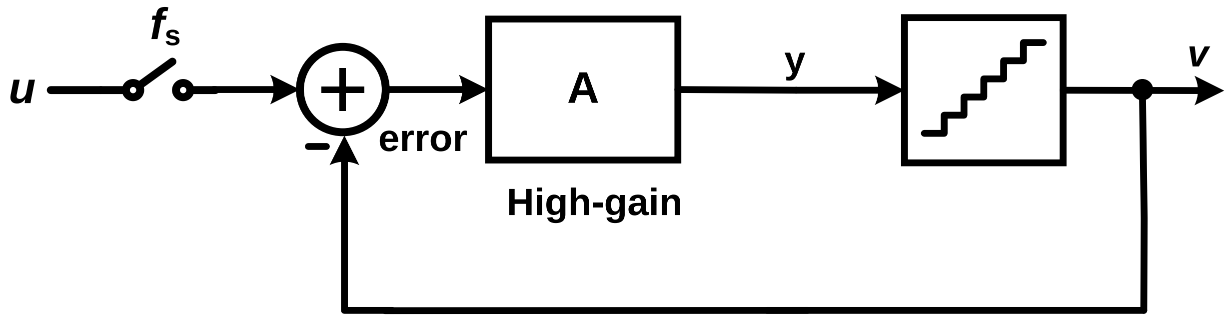






- Oversampling ratio (*OSR*)
- Conversion bandwidth: $f_B = f_s/2 \cdot OSR$
- $SQNR = 6.02 \cdot N + 1.76 + 10 \cdot \log_{10}(OSR)$
- 0.5 bits increase in resolution per doubling in OSR

Oversampling with Feedback

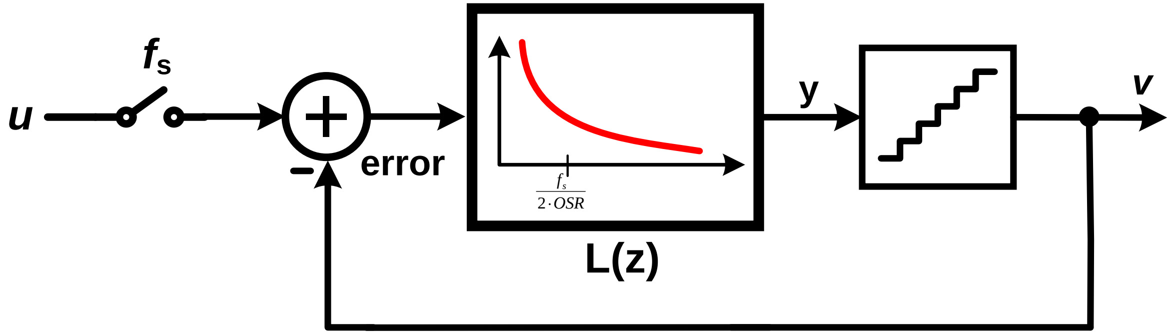


- Can we use feedback with high loop-gain ($A \cdot k_q$) to reduce the error e=|u-v| ?
- Since quantizer output can not be equal to the input

$$A \cdot k_q \rightarrow \infty \Rightarrow e = |u - v| \rightarrow \infty$$

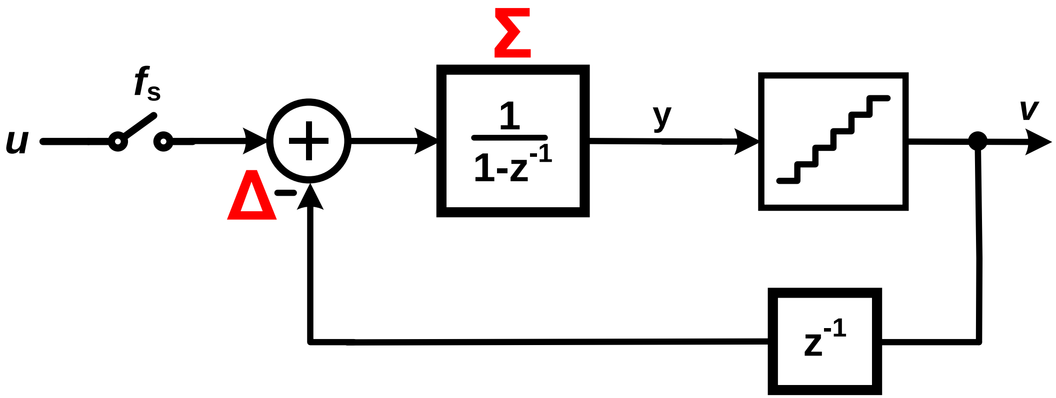
- The loop will be unstable as the error gets unbounded

Oversampling with Feedback



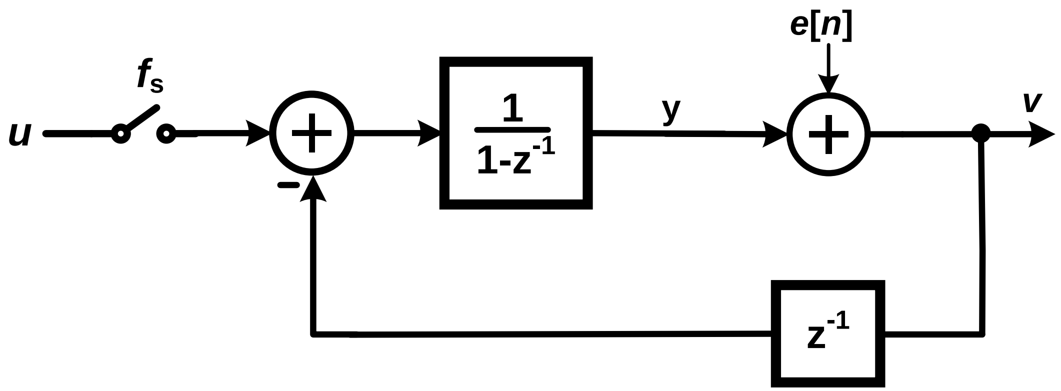
- Use large loop-gain in the signal band and small loop-gain at higher frequencies
- At low frequencies $e = |u - v| \to 0$
- At high frequencies, low loop-gain stabilizes the loop
- *L*(*z*) is the loop-filter
- This feedback arrangement is called a (noise) **modulator**

First-order Modulator



- Differencing (Δ) followed by an accumulator (Σ)
  - ΔΣ modulator
- At low frequencies $e = |u - v| \rightarrow 0$

First-order Noise Shaping

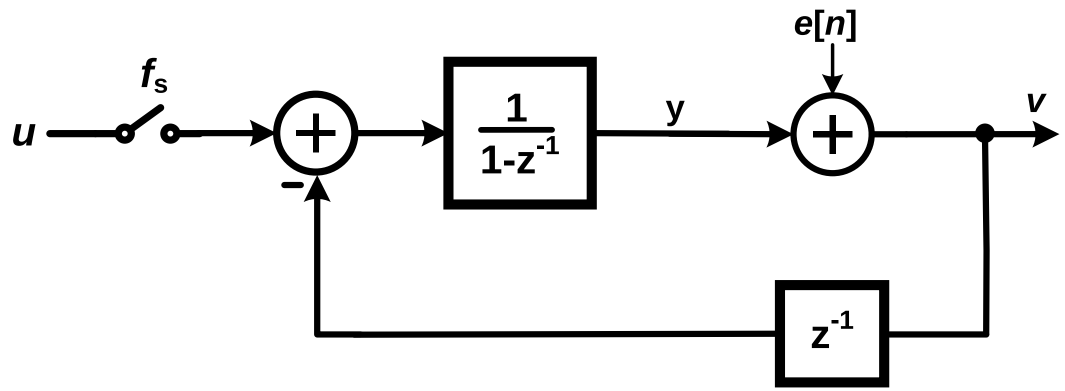


- Linearized model for the modulator

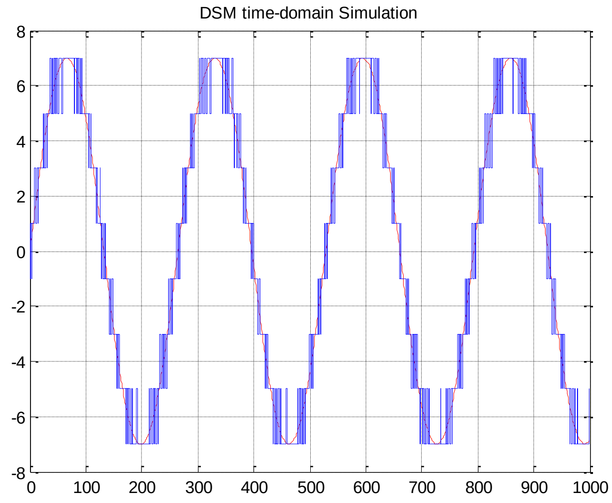
$$V(z) = \underbrace{z^{-1}}_{\text{STF}} U(z) + \underbrace{\left(1 - z^{-1}\right)}_{\text{NTF}} E(z)$$

- Noise transfer function (NTF)
  - $(1\text{-}z^{-1})$: first-order differentiator
  - High-pass shaping of quantization noise
- Signal transfer function (STF)
  - Unit delay

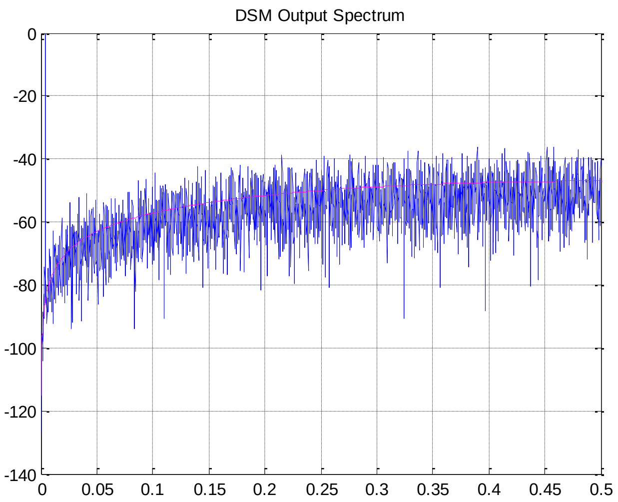
First-order ΔΣ Modulator



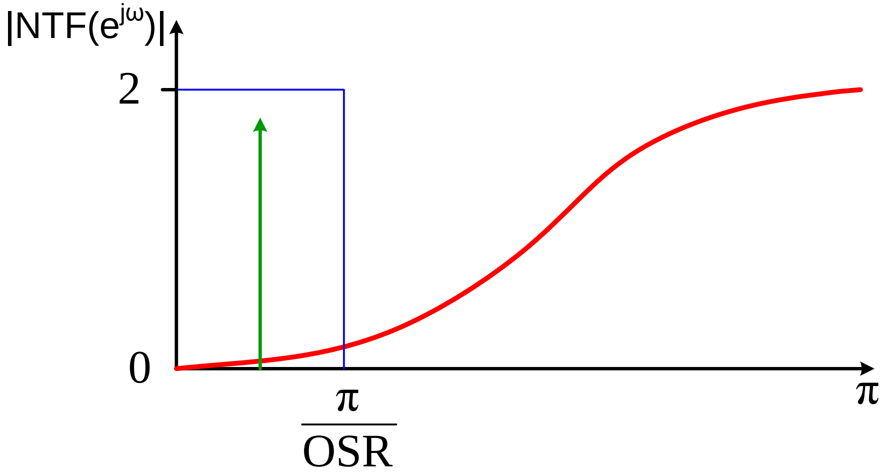
v[n]



$S_v(f)$



First-order ΔΣ Modulator SQNR



- In-band quantization noise (IBN)

$$\frac{\Delta^2}{12}\cdot\frac{\pi^2}{3}\cdot OSR^{-3}$$

- *SQNR* = 6.02·*N* –3.4 + 30·log$_{10}$(OSR)
- 1.5 bits increase in resolution per doubling in OSR
- Out-of-band noise is filtered out using a digital filter

$$IBN = \int_0^{\frac{\pi}{OSR}} S_v(e^{j\omega})\cdot d\omega$$

$$= \frac{\Delta^2}{12}\int_0^{\frac{\pi}{OSR}} \left|NTF(e^{j\omega})\right|\cdot d\omega$$

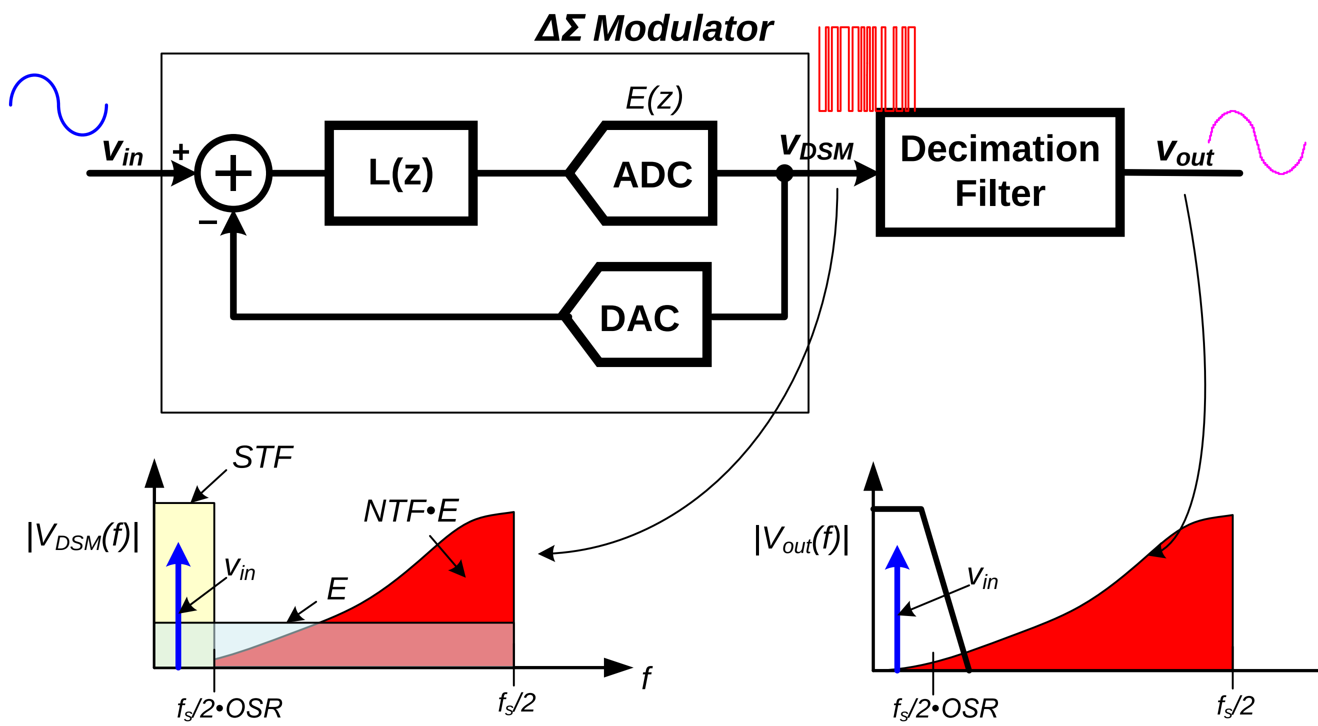
$$= \frac{\Delta^2}{12}\int_0^{\frac{\pi}{OSR}} 4\sin^2(\omega/2)\cdot d\omega$$

$$\approx \frac{\Delta^2}{12}\int_0^{\frac{\pi}{OSR}} \omega^2\cdot d\omega$$

$$= \frac{\Delta^2}{12}\cdot\frac{\omega^3}{3}\Bigg|_0^{\frac{\pi}{OSR}}$$

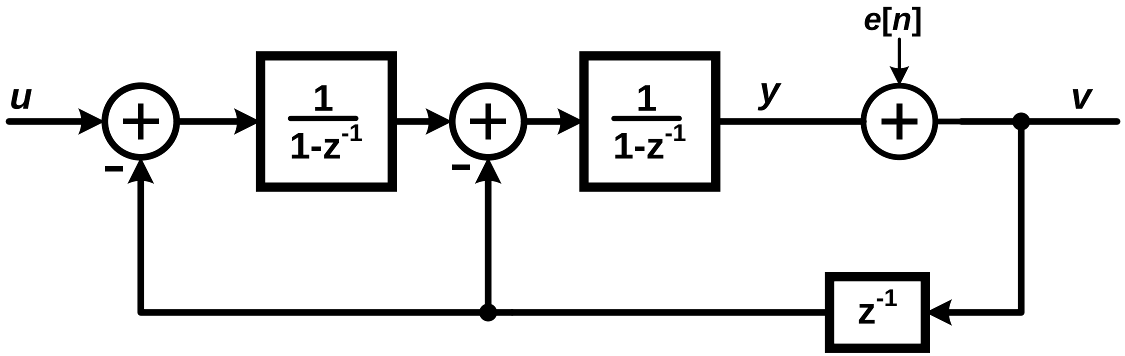
$$= \frac{\Delta^2\pi^2}{36\cdot OSR^3}$$

Delta-Sigma (ΔΣ) ADC



- Use oversampling ($f_s = 2 \cdot OSR \cdot BW$) to shape the quantization noise out of the signal band
- Use low-resolution ADC and DAC to higher much higher resolution
- Digitally filter out the out-of band shaped (modulated) noise
- Trades-off SQNR with oversampling ratio (OSR)

Second-Order Noise Shaping

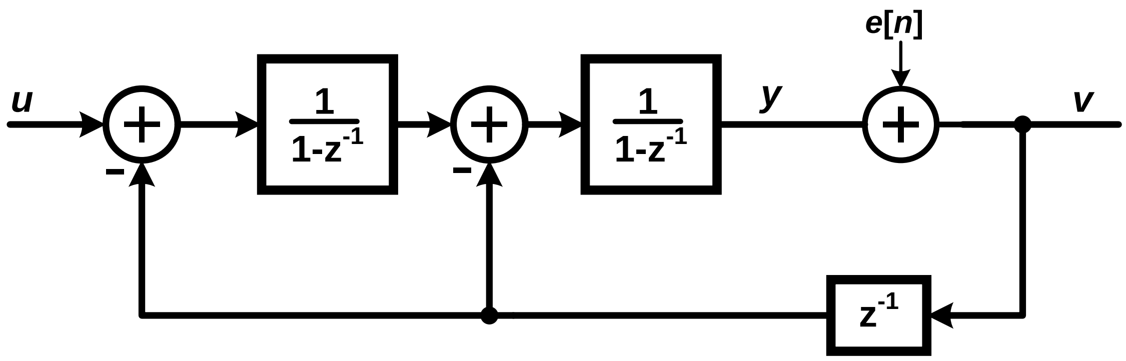


- Linearized model for the modulator

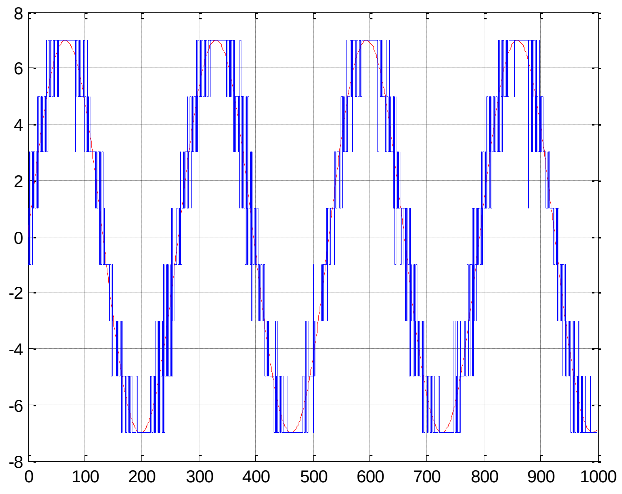
$$V(z) = \underbrace{z^{-1}}_{\text{STF}} U(z) + \underbrace{\left(1 - z^{-1}\right)^2}_{\text{NTF}} E(z)$$

- Second-order noise-shaping
- In-band quantization noise (IBN): $\frac{\Delta^2}{12} \cdot \frac{\pi^4}{5} \cdot OSR^{-5}$
  - 2.5 bits increase in resolution per doubling in OSR
- Can be extended to higher orders

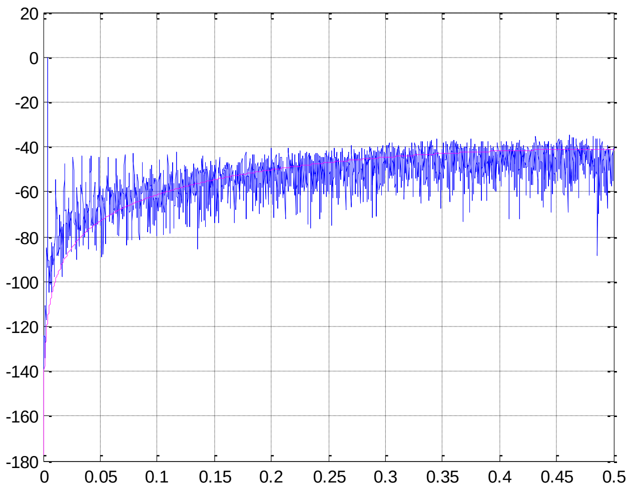
Second-order ΔΣ Modulator



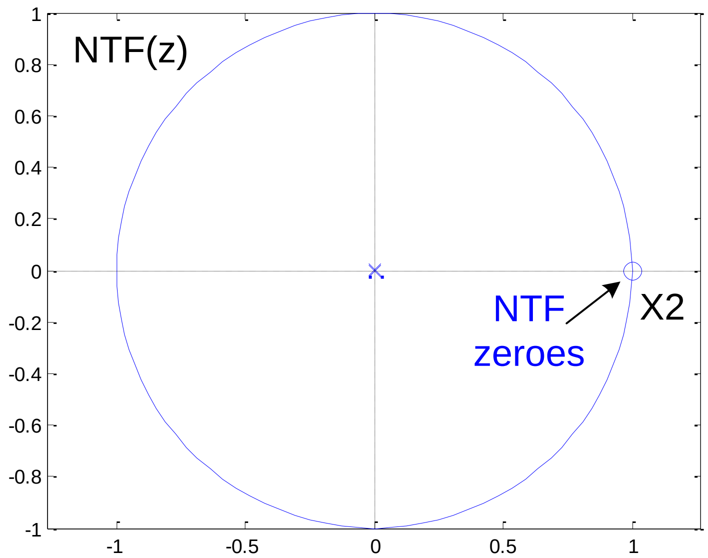
v[n]

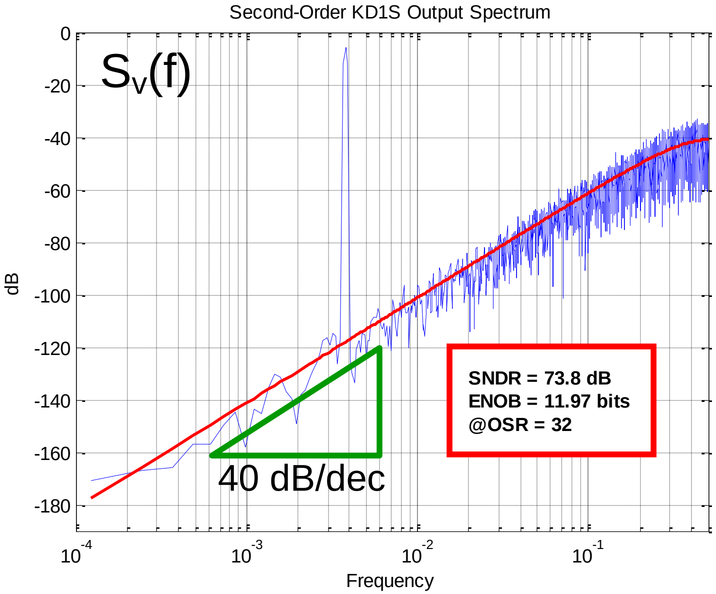


$S_V(f)$



Second-order ΔΣ Modulator

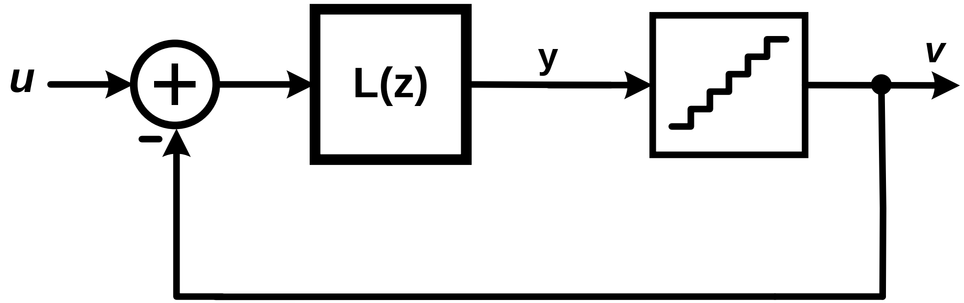




- $NTF(z) = (1-z^{-1})^2$
  - Two zeroes at DC
  - Out-of-Band Gain (OBG) (i.e gain at ω≈π) = 4

Higher-Order $\Delta\Sigma$ Modulators

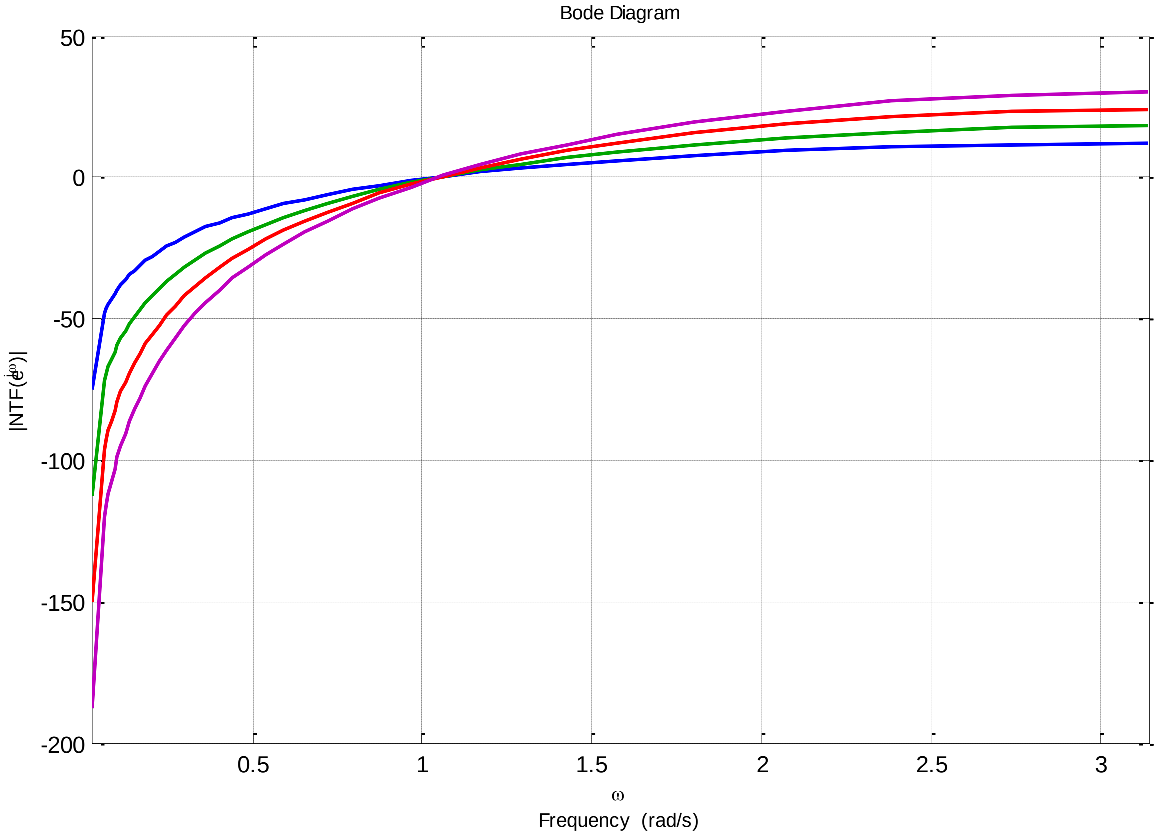
Higher-Order NTFs



$$V(z) = \underbrace{\frac{L(z)}{1+L(z)}}_{\text{STF}} U(z) + \underbrace{\frac{1}{1+L(z)}}_{\text{NTF}} E(z)$$

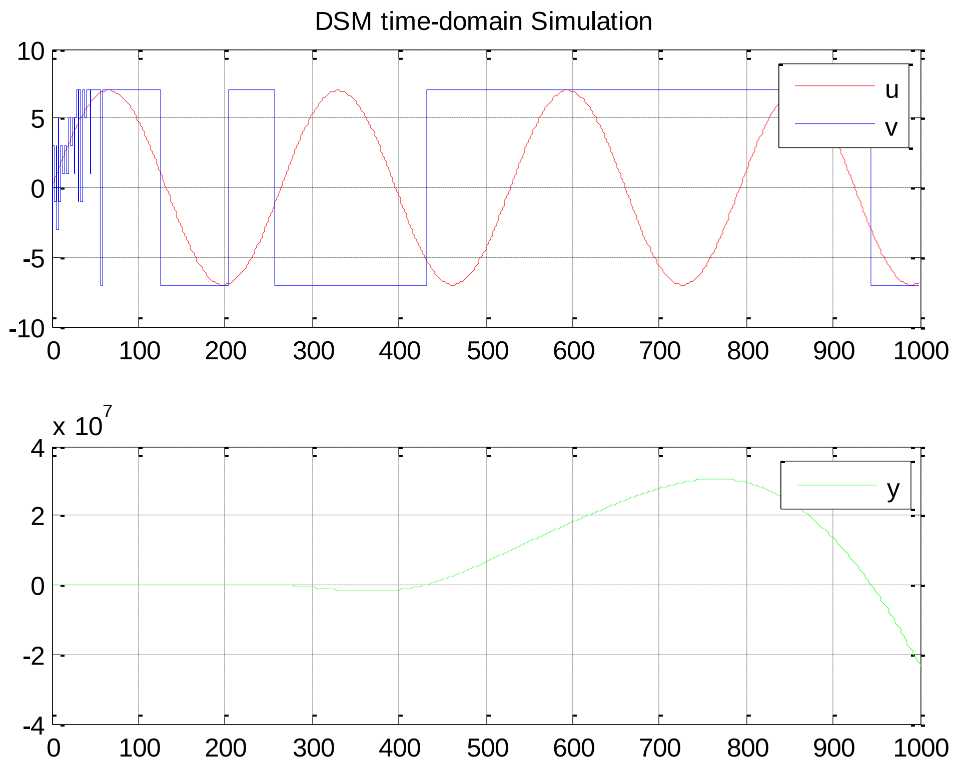
- Higher order noise shaping
  - Reduced in-band noise, higher SQNR
- For $NTF=(1-z^{-1})^{-N}$, in-band noise (IBN): $\frac{\Delta^2}{12} \cdot \frac{\pi^{2N}}{(2N+1)} \cdot OSR^{-(2N+1)}$
  - Ideally (N+1/2) bits increase in resolution per doubling in OSR

Higher-Order NTFs



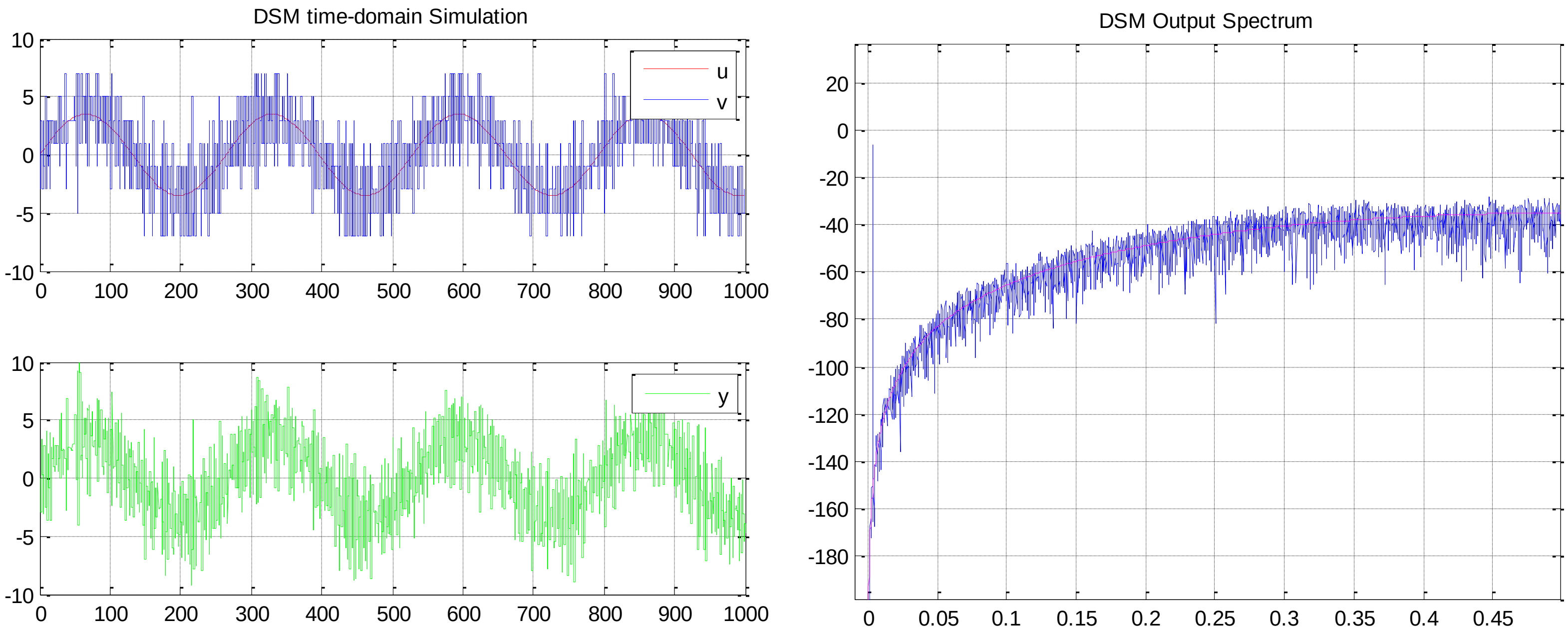
- NTF gain increases at high frequencies (around ω≈π)
- Can we go on increasing the order?

Third-order ΔΣ Modulator Example



- $NTF(z) = (1-z^{-1})^3$
- OBG = 8, Full-scale input.
- Unstable after few samples (look at quantize input (*y*) blowing up!).
  - Signature for ΔΣ instability
  - Worst case for a single-bit quantizer.

Third-order ΔΣ Modulator Example



- Stable for 50% of full-Scale amplitude
- Signal dependent stability
  - Need to develop intuition for modulator stability
  - Reference: Stability theory from the Yellow Bible of delta-sigma

Systematic NTF Design

- NTFs of the form $(1-z^{-1})^N$ have stability issues
  - The OBG (2^N) are too high
- A larger OBG causes more wiggling at the quantizer input
  - This saturates the quantizer for even smaller inputs
  - Irrecoverable quantizer saturation causes loop instability
- For high-OBG the maximum stable (input) amplitude (MSA) is small
- The stability is worse for low quantizer resolutions
- Thus we need to reduce OBG while maintaining high in-band noise shaping

Systematic NTF Design Procedure

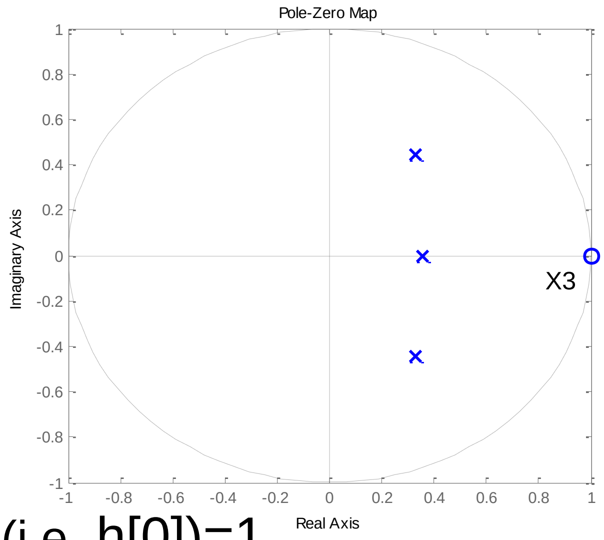
- Introduce poles into the NTF

- $$NTF(z) = \frac{(1-z^{-1})^N}{D(z)}$$

- NTF realizability criterion
  - No delay-free loops in the modulator
    - First sample of the NTF impulse response (i.e. h[0])=1

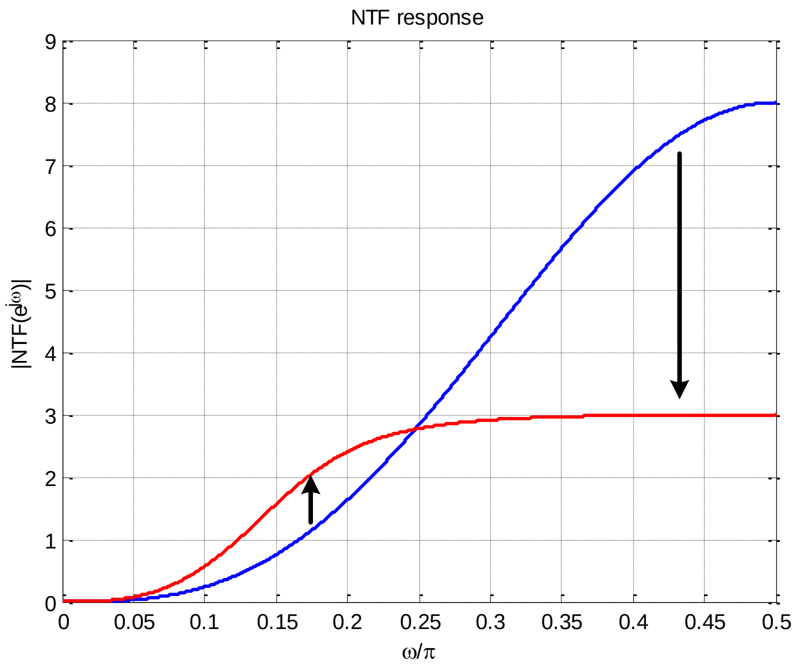
$\Rightarrow$ NTF(∞)=1

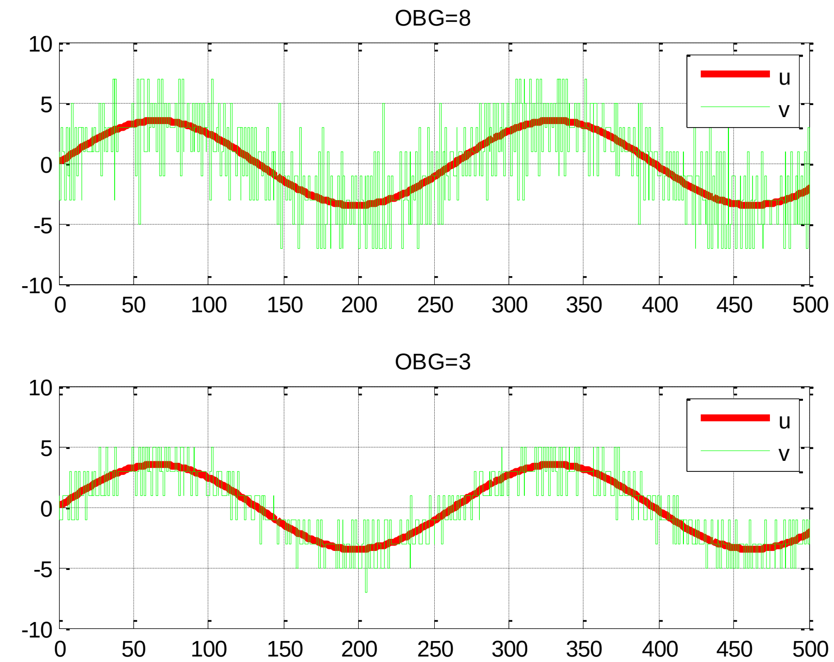
$\Rightarrow$ D(z=∞)=1



- Commonly used pole positions: Butterworth, Inverse Chebyshev and maximally flat poles (maxflat)

NTF Response with Poles





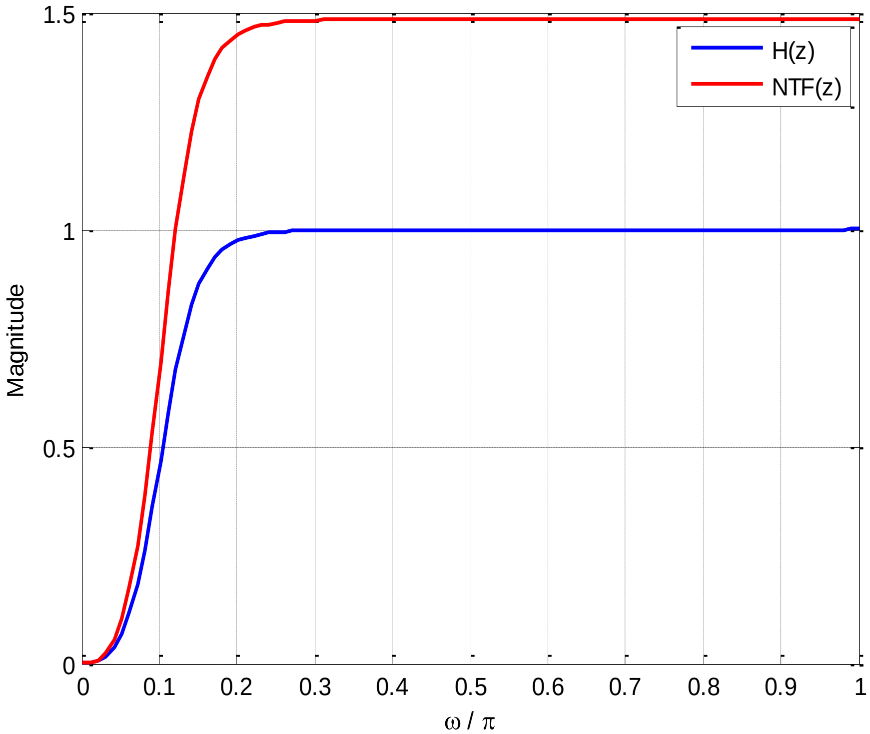
- ❑ Select appropriate OBG for the NTF to assure stability
- ❑ Trade-off between stability and increased in-band noise
  - MSA vs SQNR for a given order and quantizer resolution

Systematic NTF Design Example

- Specifications
  - SQNR > 120 dB
  - A signal bandwidth which results in an OSR = 64
    - Study optimal clock rate for the given process and quantizer design.
- Designer's Choice
  - Order = 3
  - Quantizer levels (nLev) = 16
  - Butterworth high-pass response for the NTF

- Use MATLAB for finding coefficients of the HPF response.
  - [b,a] = butter(order, ω_{3dB}, 'high')
  - The cutoff frequency ω_{3dB} specifies the transfer function.

Systematic NTF Design contd.

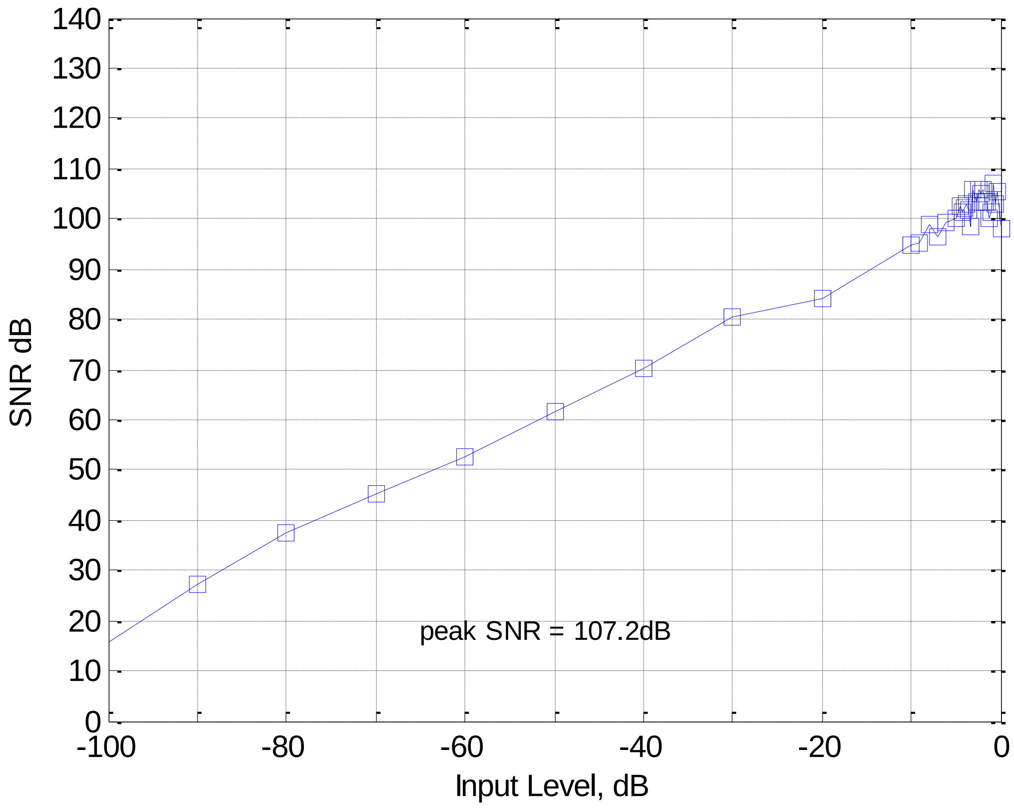


- Start with cutoff frequency $\omega_{3dB}=\pi/8$, for the butterworth HPF H(z).
- Derive a realizable NTF using $NTF(z)=H(z)/b_0$

Systematic NTF Design contd.

- Map the NTF response to a loop-filter architecture (details later)
- Simulate the modulator for all possible amplitudes and input tone frequencies.
- Compute the peak SQNR and MSA.
  - simulateDSM function in the toolbox.
  - Can use Risbo's method shown later

Systematic NTF Design contd.

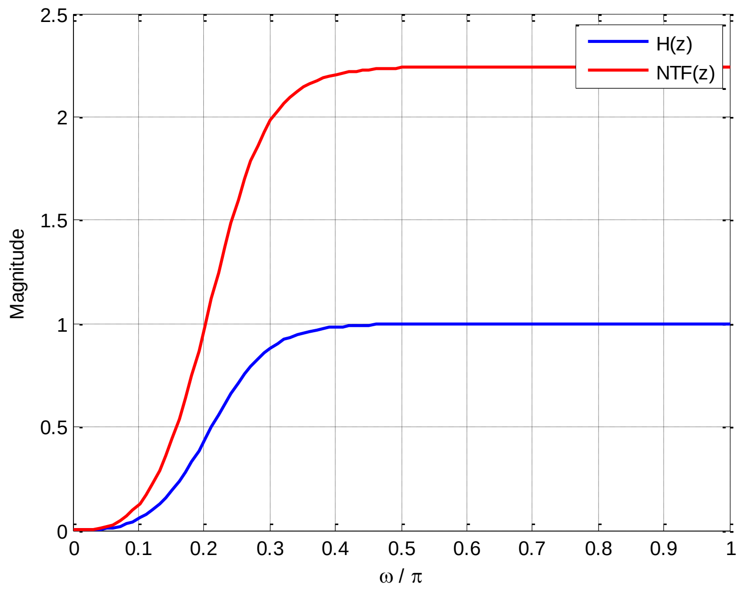


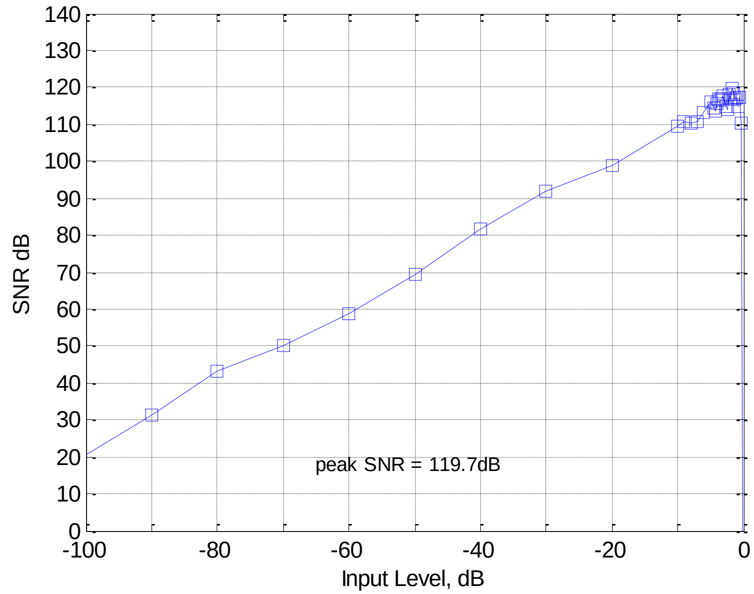
- Peak SNR = 107 dB
- MSA = 0.9

Systematic NTF Design contd.

- If SNR is not enough, repeat the entire procedure with a higher cutoff frequency for the Butterworth HPF
  - IBN ↓, SQNR ↑
  - OBG ↑ and MSA ↓
- If SNR is too high, repeat the entire procedure with a lower cutoff frequency for the Butterworth HPF
  - IBN ↑, SQNR ↓
  - OBG ↓ and MSA ↑

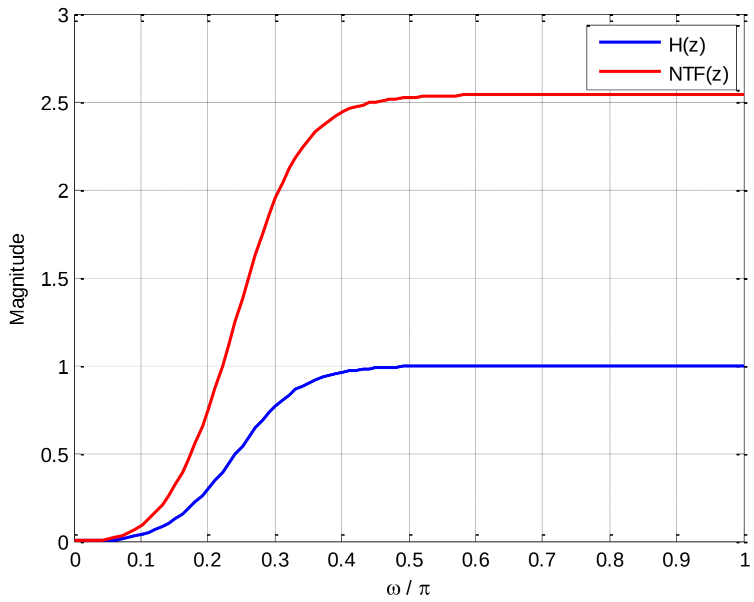
Systematic NTF Design contd.

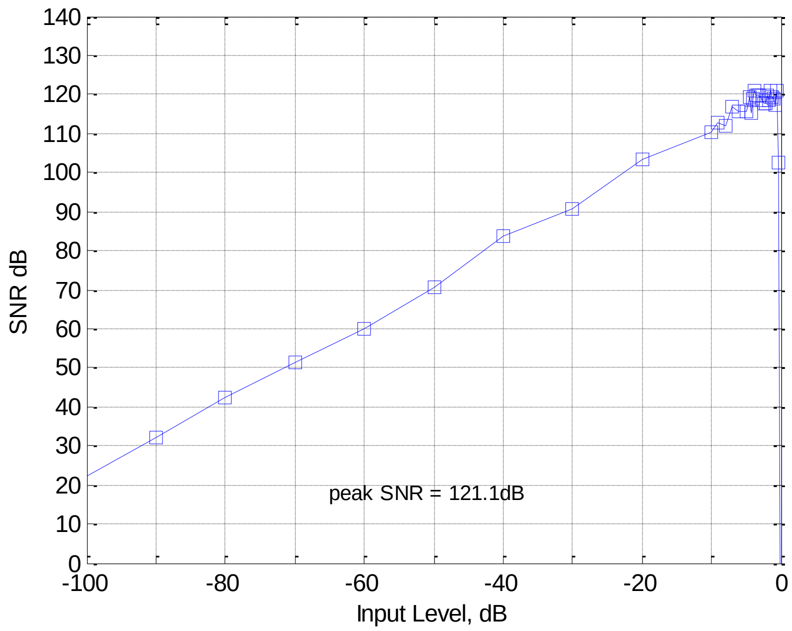




- $\omega_{3dB} = \pi/4$.
- Peak SNR = 119 dB, OBG = 2.25, MSA = 0.8

Systematic NTF Design contd.





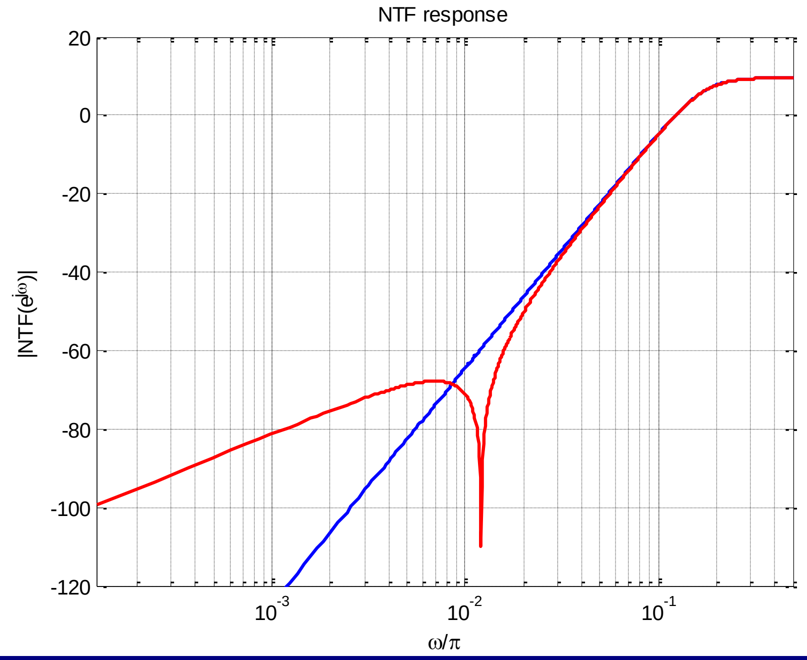
- $\omega_{3dB}=2\pi/7$.
- Peak SNR = 121 dB, OBG = 2.54, MSA = 0.8.
  - Design closed !

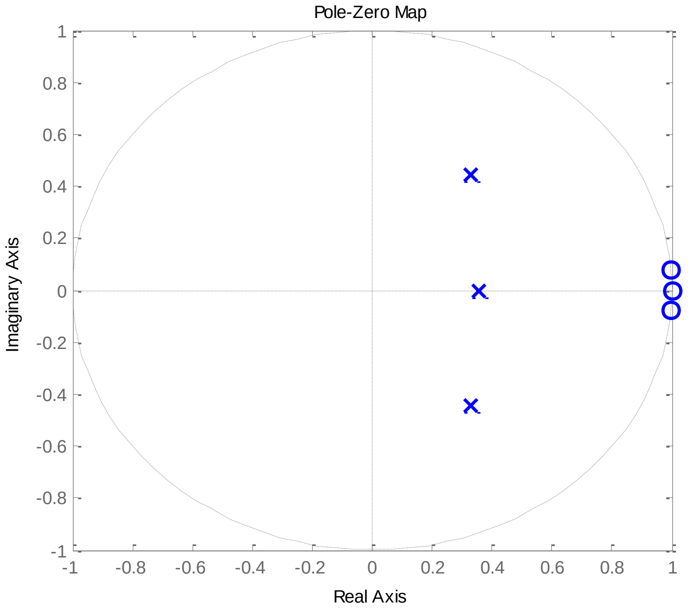
Systematic NTF Design contd.

- An advanced version of this iterative process is implemented as the function synthesizeNTF in the delta-sigma Toolbox.
  - Several ‘opt’ params for NTF zero (and pole) optimization
  - Use synthesizeChebyshevNTF for low OSR and low OBG designs.

NTF-Zero Optimization

- Spread zeros in the signal band to minimize in-band noise
  - Complex zeros on the unit circle
  - 8dB increase in SQNR for 3rd order modulator
- Bandwidth normalized NTF-zero locations obtained by toolbox function ds_optzeros(order, 1)
- Already implemented in synthesizeNTF function for opt=1

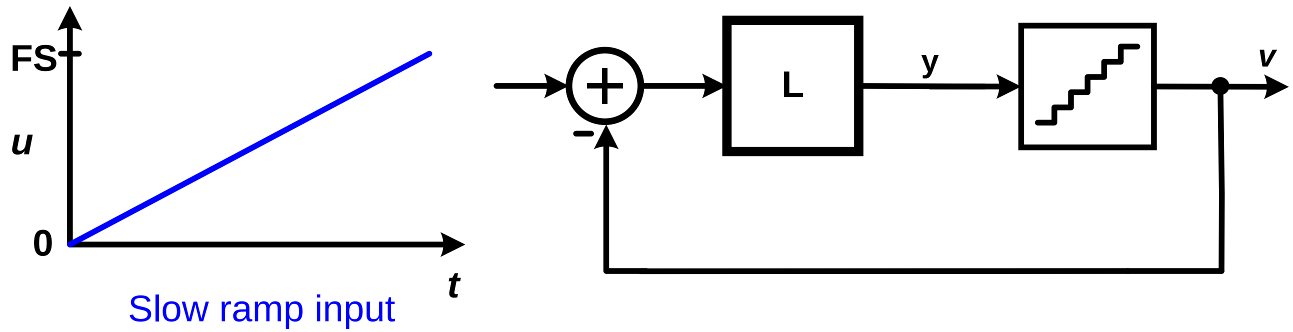




Estimating MSA (Maximum Stable Amplitude)

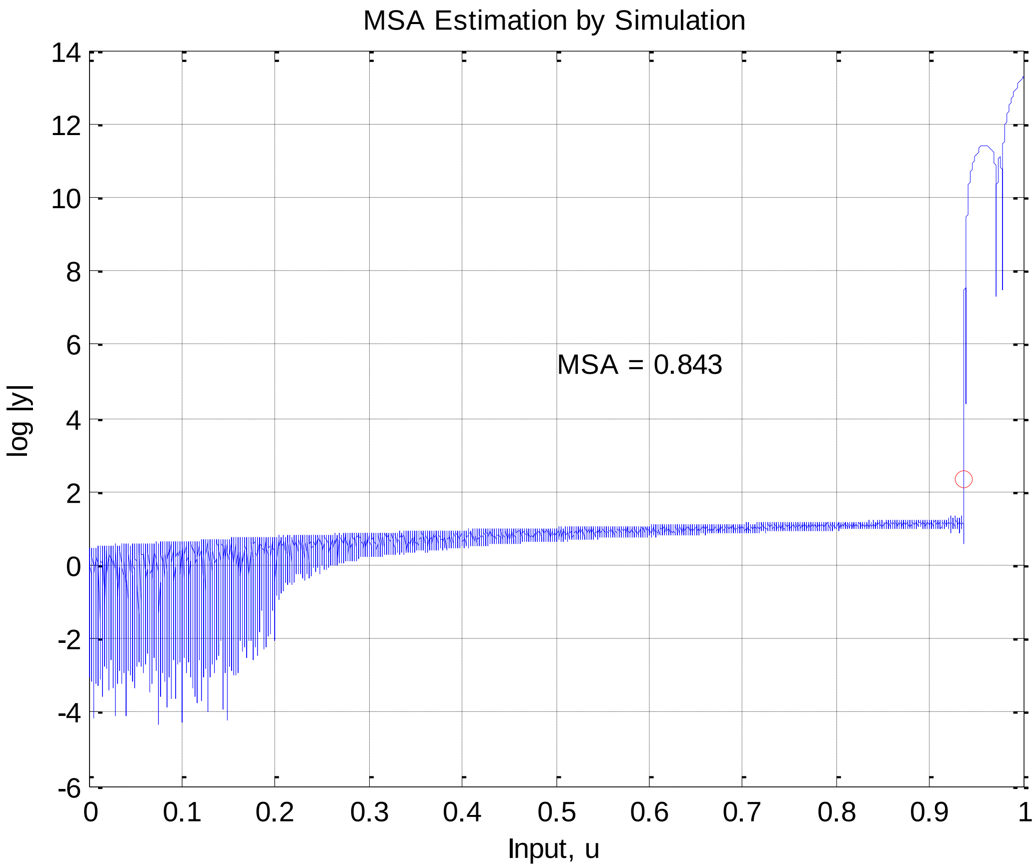
- MSA is found through extensive simulation
- Simulate for input sinusoids of varying amplitudes for all possible signal frequencies in the signal band.
  - For every input amplitude compute in-band SNR.
  - Beyond the MSA, the NTF poles move out of the unit circle.
  - Noise shaping is disrupted and the in-band SNR drops.
  - At this point the quantizer input (y[n]) blows up.
- simulateSNR function in the toolbox does exactly the same
- Time consuming and often impractical for iterative design

Estimating MSA using Risbo's Method



- Use a slow ramp input from 0 to FS value.
  - Plot $\log_{10}|y[n]|$. Observe where this plot blows up.
  - Take 90% of the input amplitude where $\log_{10}|y[n]|$ blows up as a conservative estimate for MSA.
  - Estimated MSA is close to that predicted by the sinewave input method.
- Much quicker than the sinewave technique (simulateSNR function)

Estimating MSA using Risbo's Method



Loop-Filter Architectures

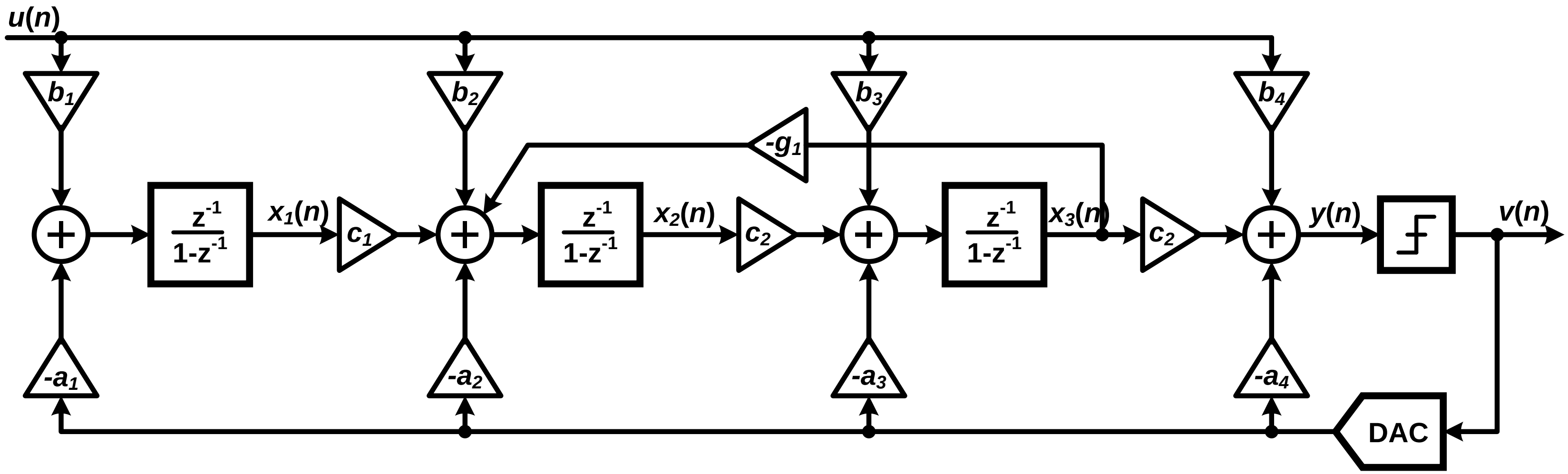
- Several loop-filter discrete-time architectures possible
- Toolbox function realizeNTF maps the synthesized NTF to loop-filter co-efficients

```
[a,g,b,c] = realizeNTF(H, form);
```

- Dynamic Range Scaling (DRS) performed to scale loop-filter states to a bounded value
  - Scaling performed using ABCD matrix representation of the loop-filter
  - See any introductory text on Linear Systems

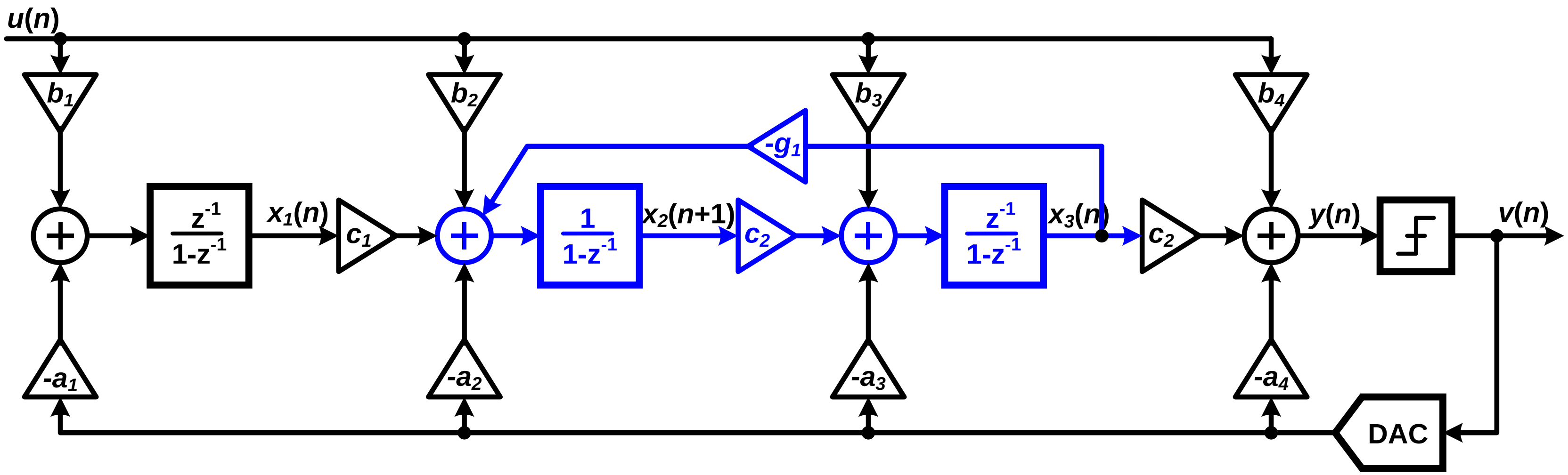
```
ABCD = stuffABCD(a,g,b,c,form);
[ABCDs umax] = scaleABCD(ABCD, nLev, f0, xLim);
[a,g,b,c] = mapABCD(ABCDs, form);
```

CIFB (Cascade of Integrators with Distributed Feedback)



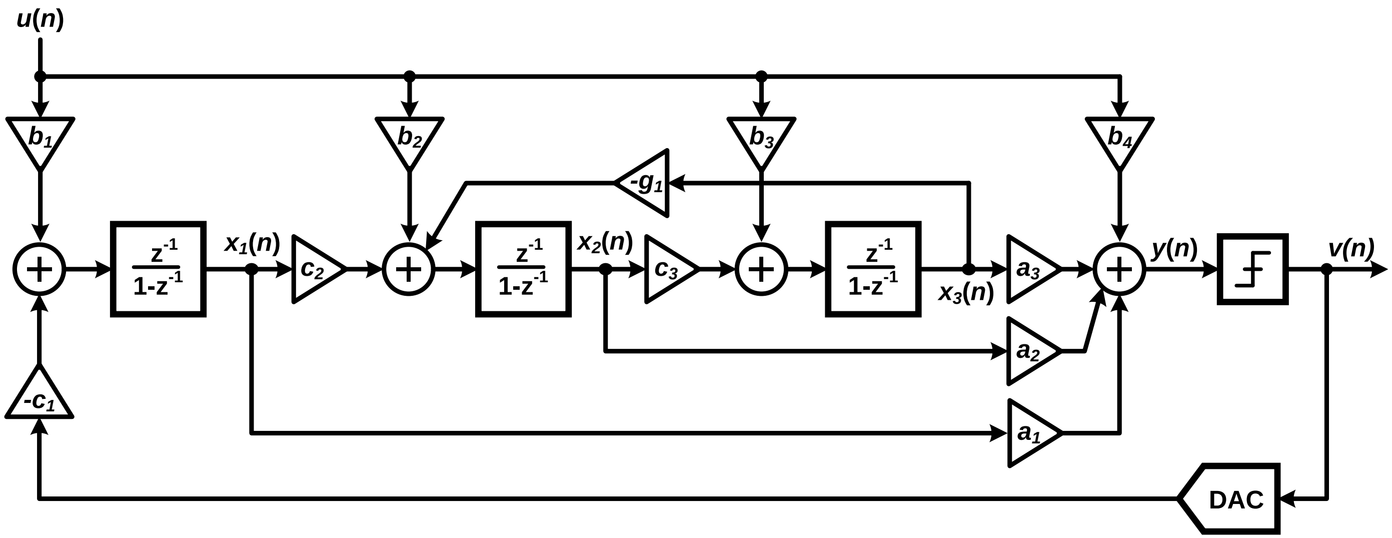
- Cascade of delaying integrators:
  - Feedback coefficients **a**'s realize the zeros of L_1 and thus the NTF and STF poles.
  - Feed-in coefficients **b**'s determine zeros of L_0 and thus the STF zeros.
  - State scaling coefficients **c**'s are used for dynamic range scaling.

CRFB (Cascade of Resonators with Distributed Feedback)

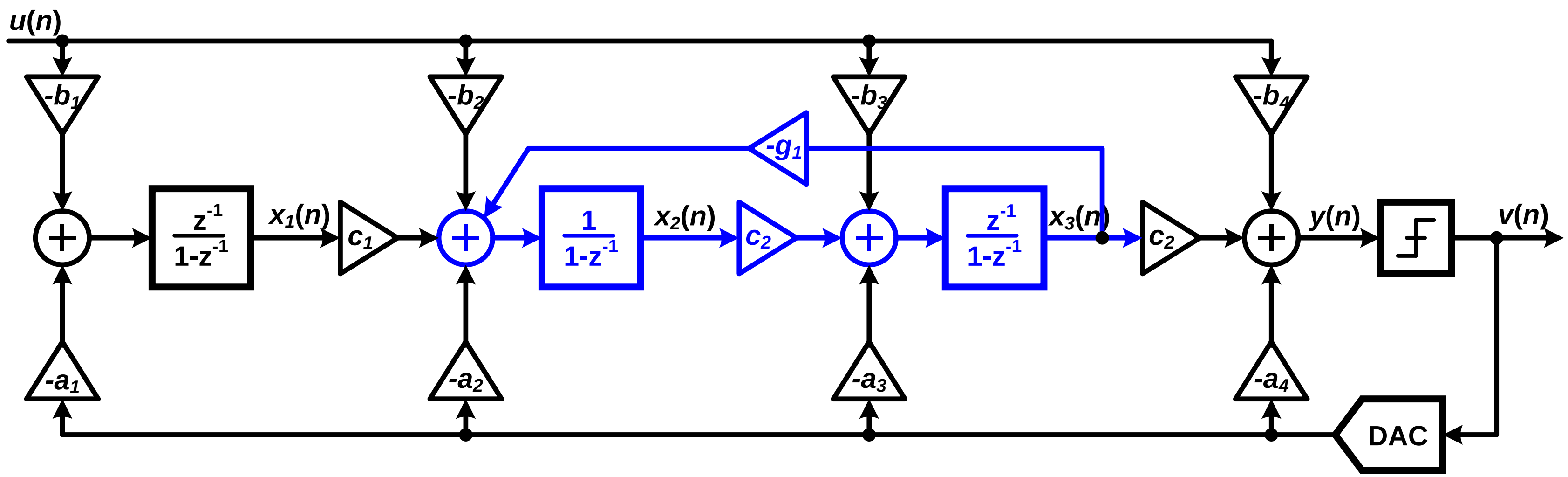


- ❑ Combine a non-delaying and a delaying integrator with local feedback around them, to form a stable resonator
  - ▪ Local feedback coefficients **g**'s realize the complex zeros in the NTF.
  - ▪ Implements NTF with complex zei $z_i = e^{\pm j\sqrt{g_1}}$
- ❑ For odd-order, use an integrator in the front to avoid noise coupling due to g

CIFF (Cascade of Integrators with Feed-Forward Summation)



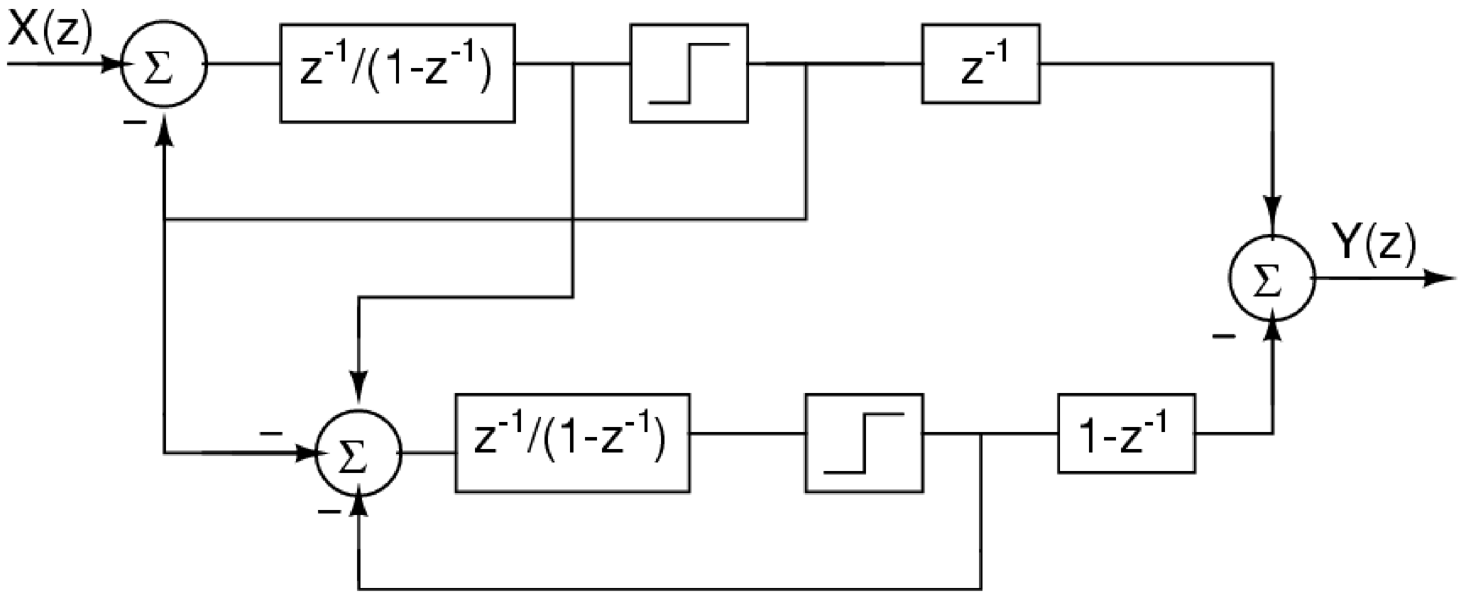
- Feedforward summation of states
- For $b_1 = b_{N+1} = 1$ and b_2 to $b_N = 0$, STF=1
  - Loop-filter only processes quantization noise, low power and distortion
- Feedforward loop-filters typically result in lower-power implementation



- Use resonators with feedforward summation
  - Implements NTF with complex zeros
- For odd-order, use an integrator in the front to avoid noise coupling due to g

ΔΣ Modulator Architectures

- Cascade/ MASH architecture:

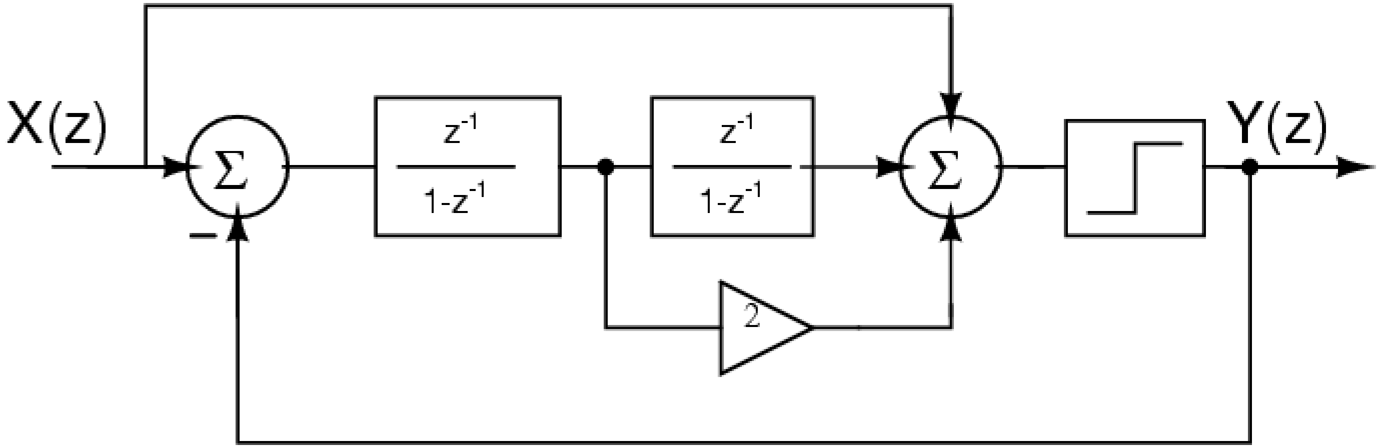


•Eg. Two first order modulators are used to implement second order modulator.

•Stability concerns are relaxed but mismatch in the two forward paths should be properly monitored.

ΔΣ Modulator Architectures

- Feedforward modulators

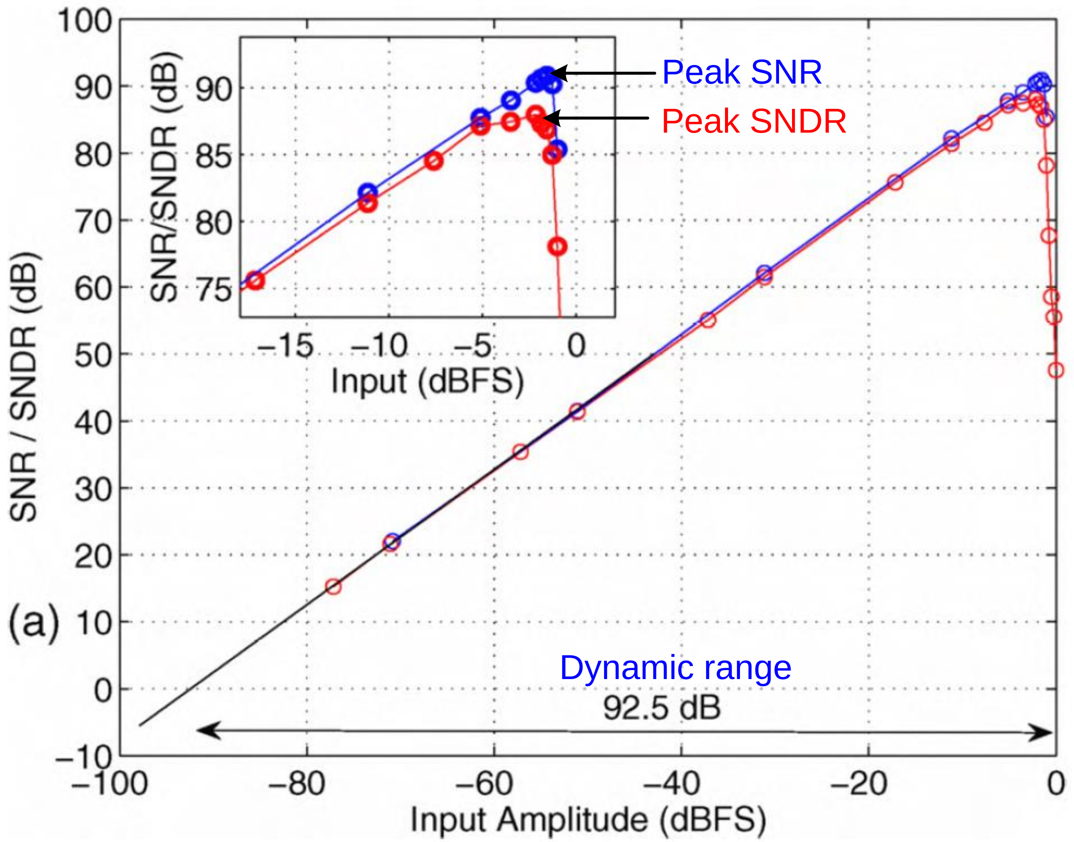


- Most popular architecture.
- Input signal is summed at Nth stage integrator output.
- Summation block may be required at higher order modulators.
- Multibit quantizer is necessary.

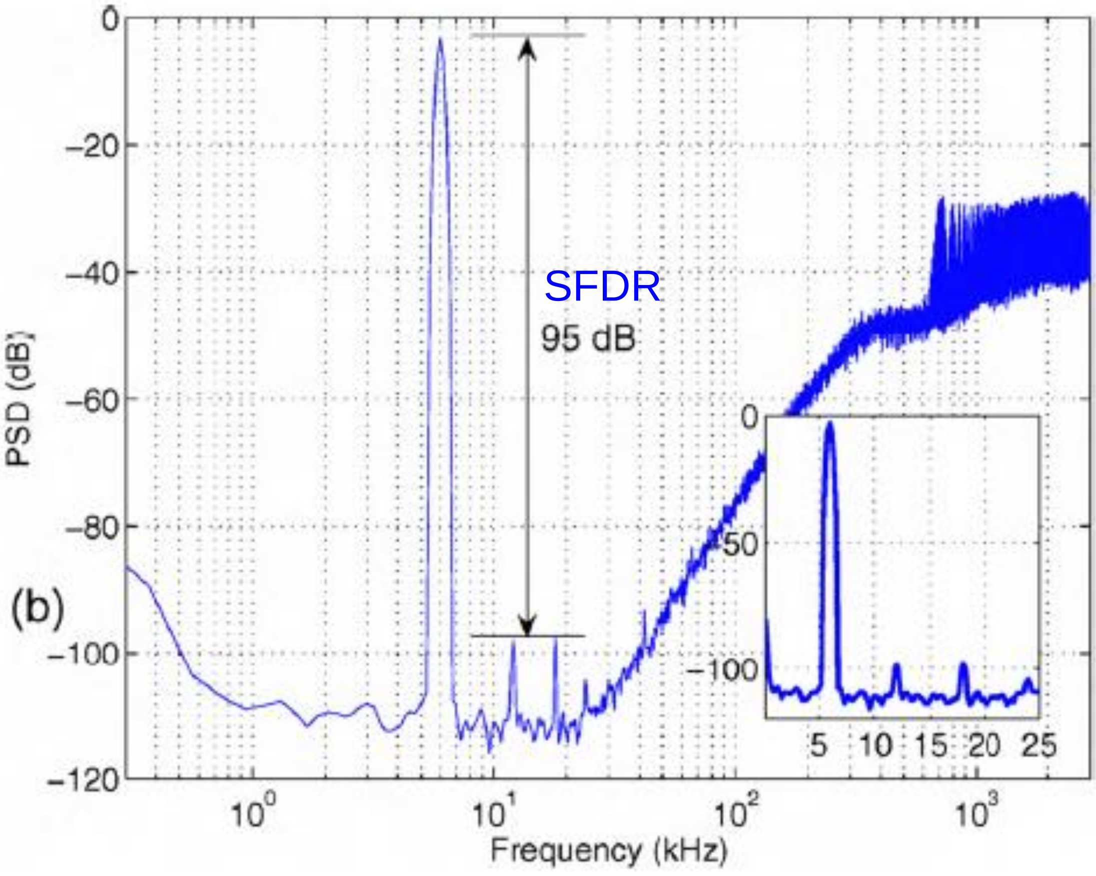
Key Terminologies :

- SQNR – Signal to quantization noise ratio
  - Thermal/electrical noise are not included.
- SNR - Signal to noise ratio
  - Distortion is not included.
- SDNR - Signal to noise and distortion ratio
  - All noise sources are included.
- ENOB – Effective number of bits (resolution)
  - This is very important than actual number of output bits
- Dynamic Range (DR)
  - Measured with input of the modulator shorted.
- Harmonic Distortion
  - THD is usually total harmonic distortion. Or Third??
- Spur Free Dynamic Range (SFDR)
  - Very key parameter in communication systems

Frequency Domain Measurements



Spurious (tone) Free Dynamic Range (SFDR)



References

Delta-Sigma Data Converters

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