Data Converter Basics

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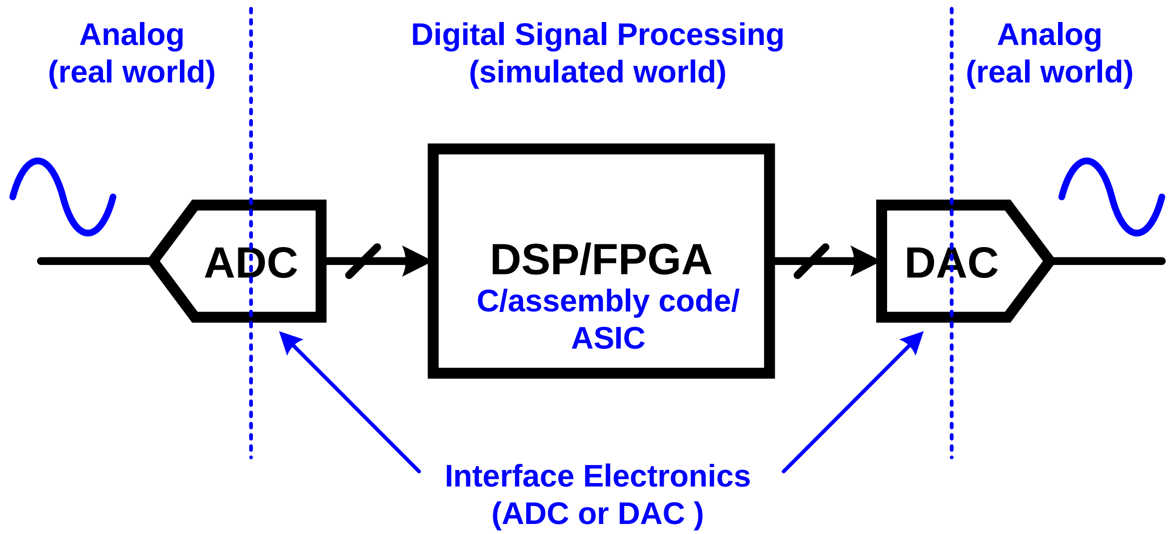
Data Converters

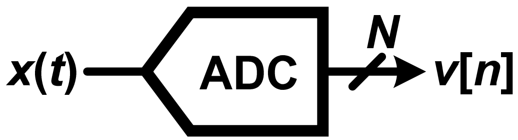


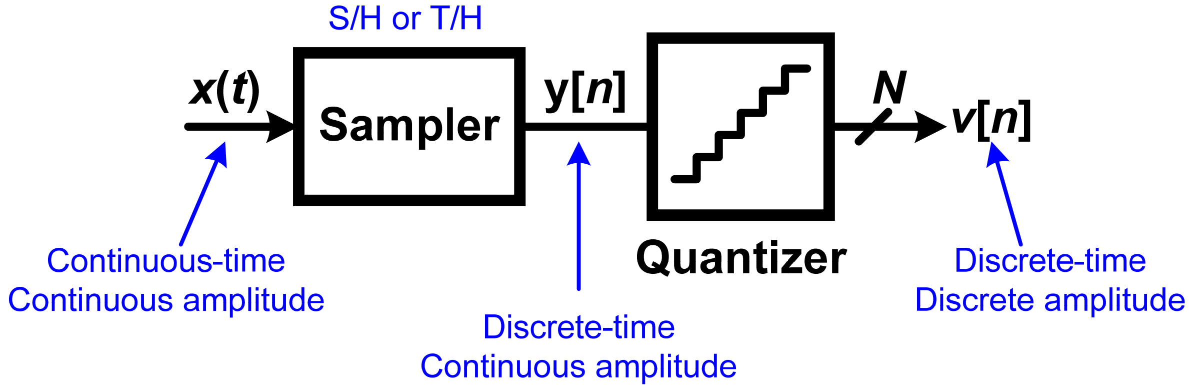
- Real world: Continuous-time, continuous-amplitude signals
- Digital world: Discrete-time, discrete-amplitude signal representation
- Interface circuits: ADC and DACs

Data Conversion Scenarios

- Any application using a sensor and/or an actuator
  - Wireless: RF Rx and Tx chain
  - Twisted pair: ADSL modem
  - Coaxial: Cable modem
  - Serial/Optical links: 10G+ ADC for modulation and equalization
  - Audio Recording: 24-bit stereo ADCs
  - Audio players: stored data to speaker (audio DAC)
  - HDD read channel: Magnetic disk to microprocessor
  - Biomedical applications (e.g. sensing blood glucose level and actuating the insulin pump),.........
- Speed and resolution requirements vary with the application.

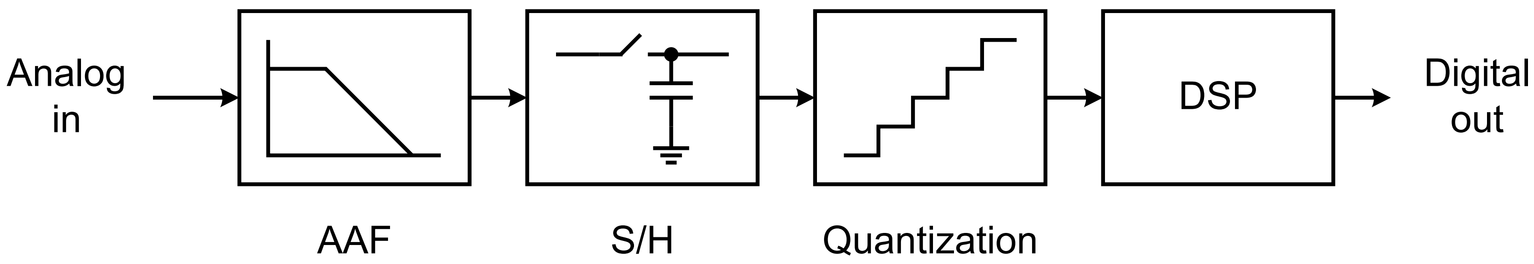
Analog-to-Digital Converter (ADC)



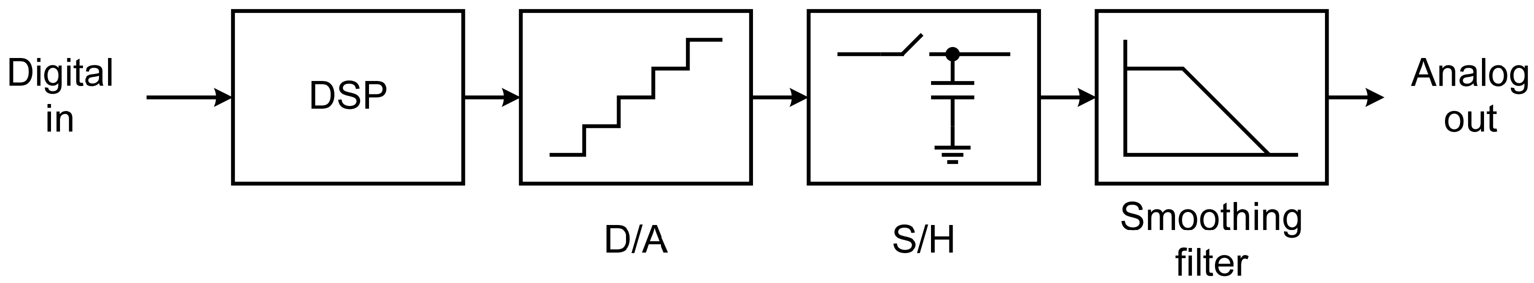


A/D and D/A Conversion

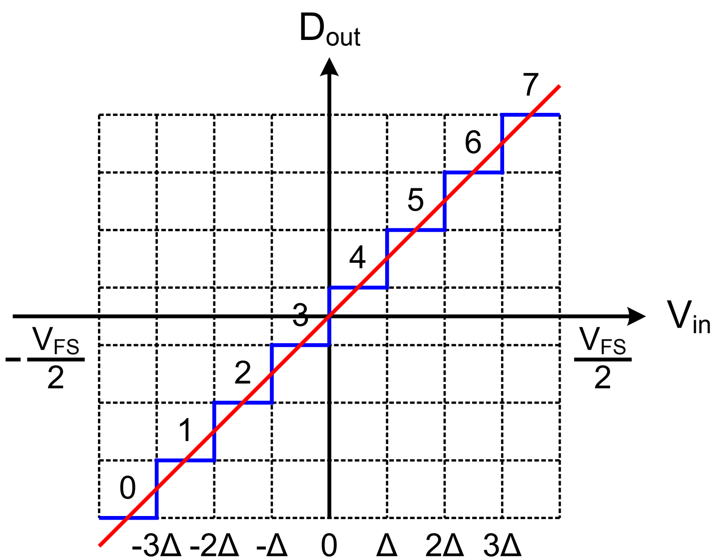
A/D Conversion



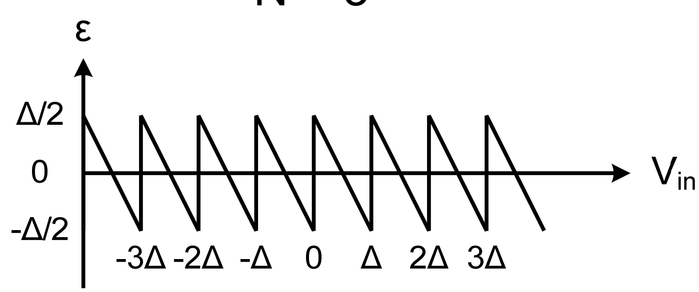
D/A Conversion



Quantization Error



N = 3



$$\Delta = \frac{V_{FS}}{2^N} = LSB$$

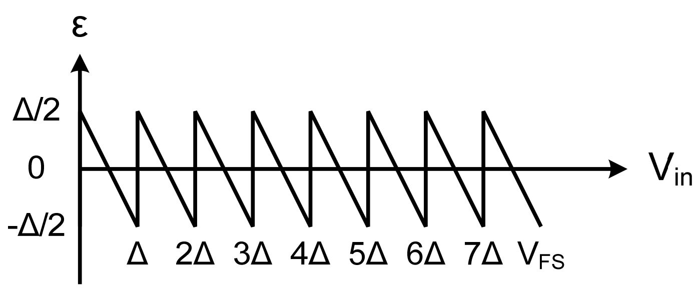
$$V_{in} \in [0, V_{FS}]$$

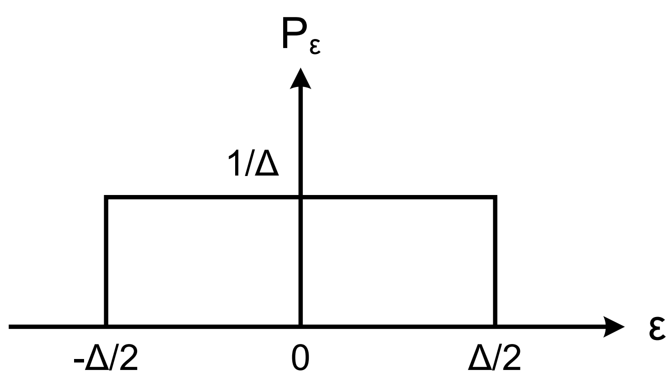
$$\varepsilon = D_{out}\Delta - V_{in} = D_{out}\left(\frac{V_{FS}}{2^N}\right) - V_{in}$$

$$-\frac{\Delta}{2} \leq \varepsilon \leq \frac{\Delta}{2}$$

"Random" quantization error is usually regarded as noise

Quantization Noise





Assumptions:

- N is large
- $0 \le V_{in} \le V_{FS}$ and $V_{in} >> \Delta$
- V_{in} is active
- ε is Uniformly distributed
- Spectrum of ε is white

$$\sigma_\varepsilon^2 = \int_{-\Delta/2}^{\Delta/2} \varepsilon^2 \cdot \frac{1}{\Delta} \cdot d\varepsilon = \frac{\Delta^2}{12}$$

Signal-to-Quantization Noise Ratio (SQNR)

Assume V_{in} is sinusoidal with $V_{p\text{-}p} = V_{FS}$,

$$SQNR = \frac{V_{FS}^2/8}{\sigma_\varepsilon^2} = \frac{\left(2^N\Delta\right)^2/8}{\frac{\Delta^2}{12}} = 1.5\times 2^{2N},$$

$$SQNR = 6.02\times N + 1.76\ \text{dB}$$

N (bits)	SQNR (dB)
8	49.9
10	62.0
12	74.0
14	86.0

- SQNR depicts the theoretical performance of an ideal ADC
- In reality, ADC performance is limited by many other factors:
  - Electronic noise (thermal, 1/f, coupling/substrate, etc.)
  - Distortion (measured by THD, SFDR, IM3, etc.)

Spectral Estimation Techniques

Spectral Estimation

- Spectral estimation of signals is performed using DFT/FFT

$$V[k]=\sum_{k=0}^{N-1} v[n]W_N^{nk}, W_N = e^{-\frac{j2\pi}{N}}$$

- For a periodic signal $v_{in}(t)=A\sin(2\pi f_{in}t)=\mathrm{Im}\left\{e^{j2\pi f_{in}t}\right\}$, sampled with frequency f_s
  - sampled signal $v_{out}[n]$ is periodic only when all the harmonics satisfy

$$e^{\frac{j2\pi k f_{in}(n+N)}{f_s}} = e^{\frac{j2\pi k f_{in} n}{f_s}}$$

  - or

$$2\pi \frac{f_{in}}{f_s} N = 2m\pi$$

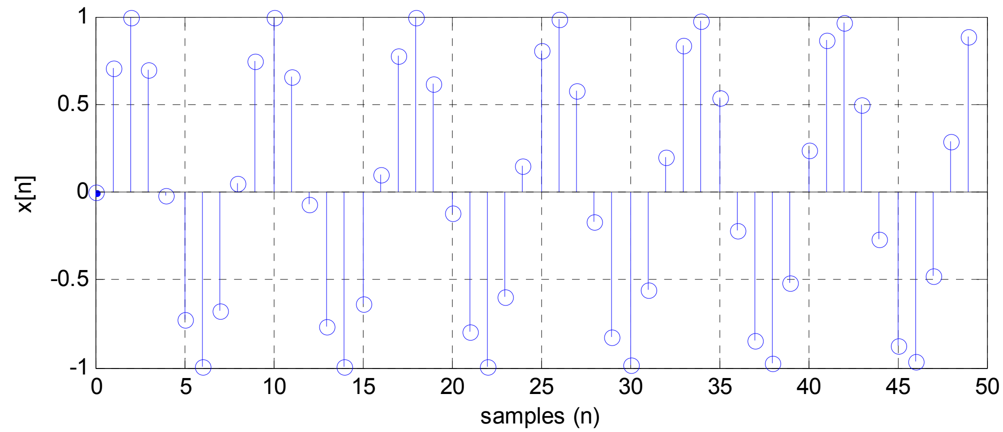
  - which implies

$$\boxed{\frac{f_{in}}{f_s} = \frac{m}{N}}$$

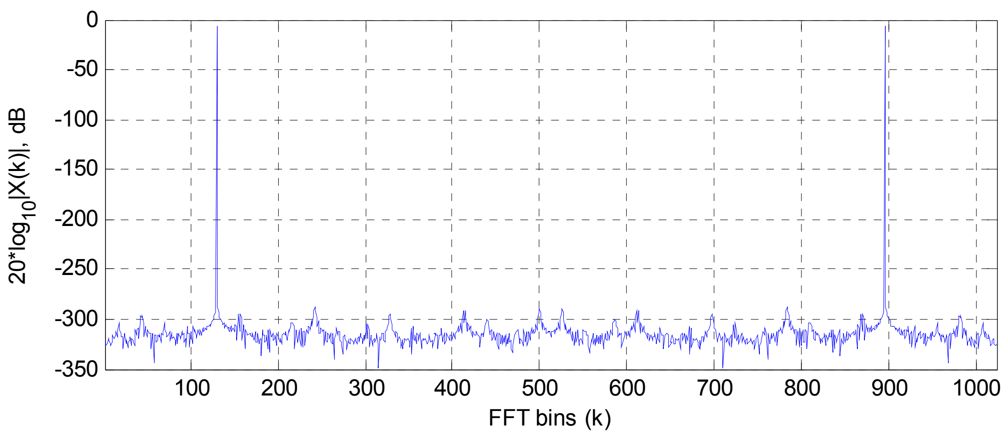
  - *m* integral cycles of signal (f_{in}) should fit in *N* cycles of f_s
    - Coherent sampling
    - *N*- FFT bins and f_s/N is the FFT resolution (bin size)

Coherent Sampling

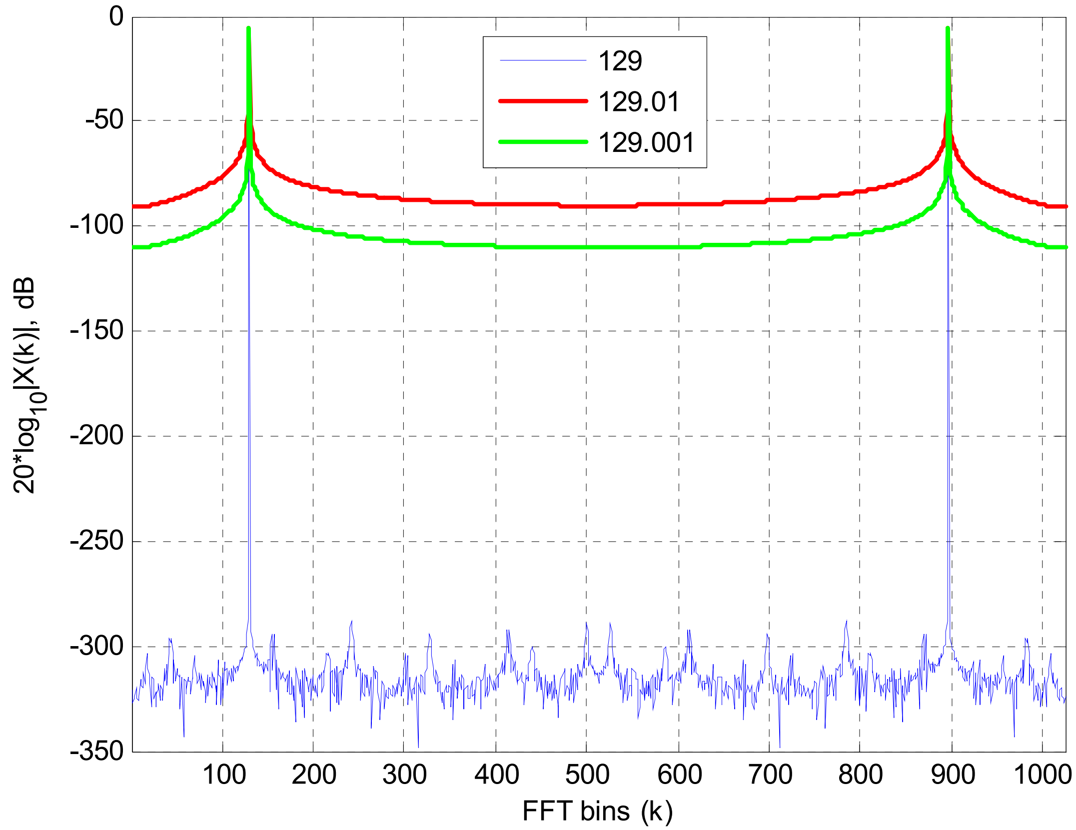
$$x[n] = sin(2\pi \frac{f_{in}}{f_s} n)$$



$$\frac{f_{in}}{f_s} = \frac{129}{1024}$$



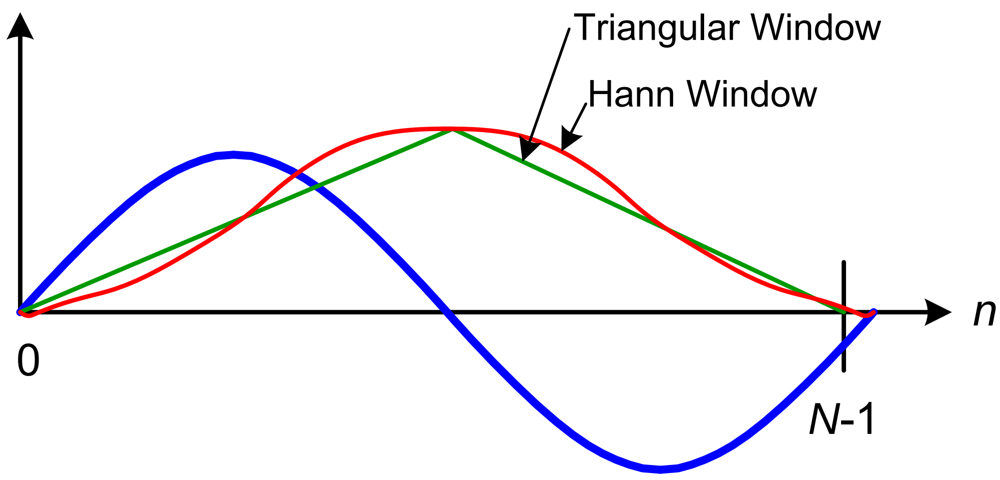
Non-Coherent Sampling : FFT leakage



$$\frac{f_{in}}{f_s} = \frac{129}{1024}, \frac{129.01}{1024}, \frac{129.001}{1024}$$

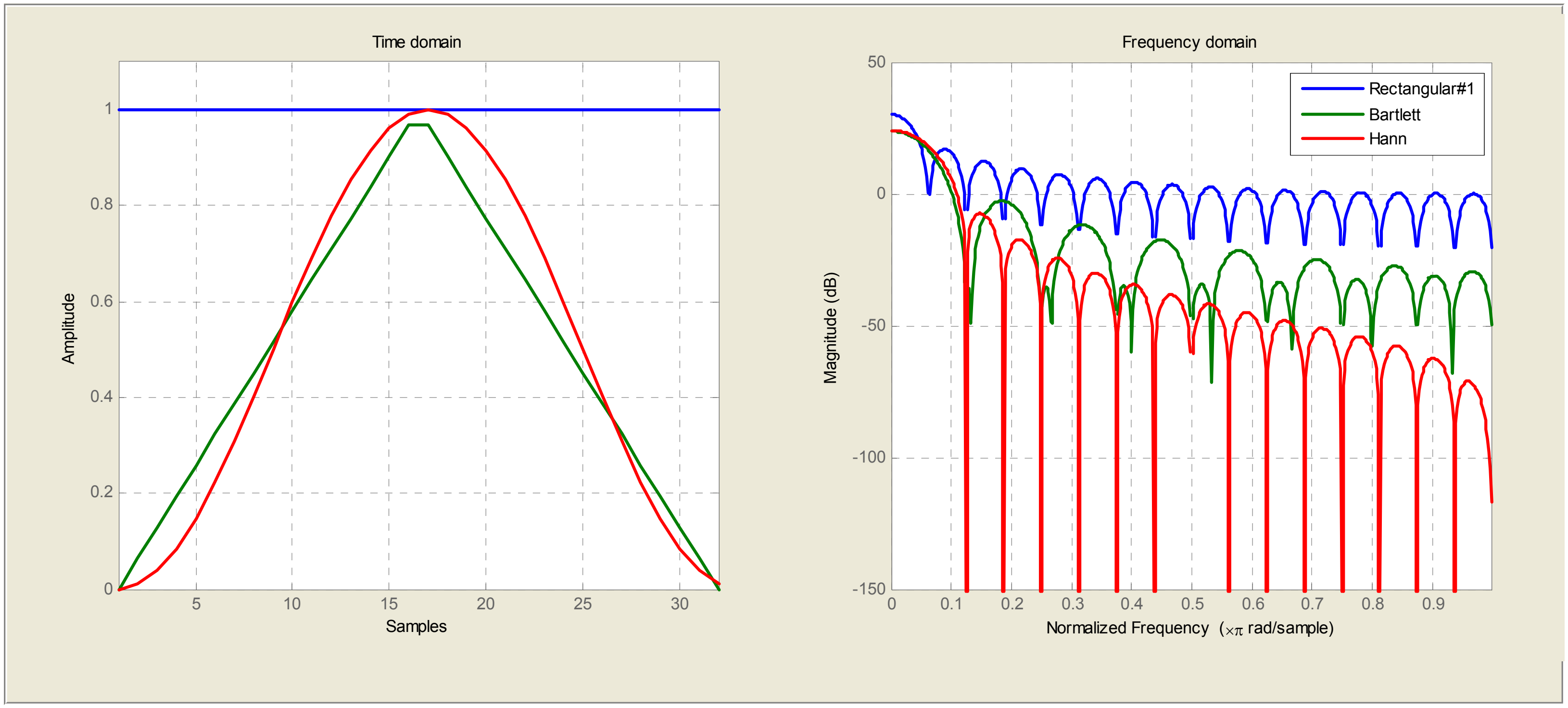
Spectral Windows

- Spectral windows for reducing FFT leakage due to temporal discontinuity
  - Lesser emphasis on the end of FFT frame and more importance to the signal in the middle
- Several FFT windows available in MATLAB
  - Triangular (Bartlett), Hann, Hanning, Blackmann-Harris, etc.



Spectral Windows – Bartlett and Hann

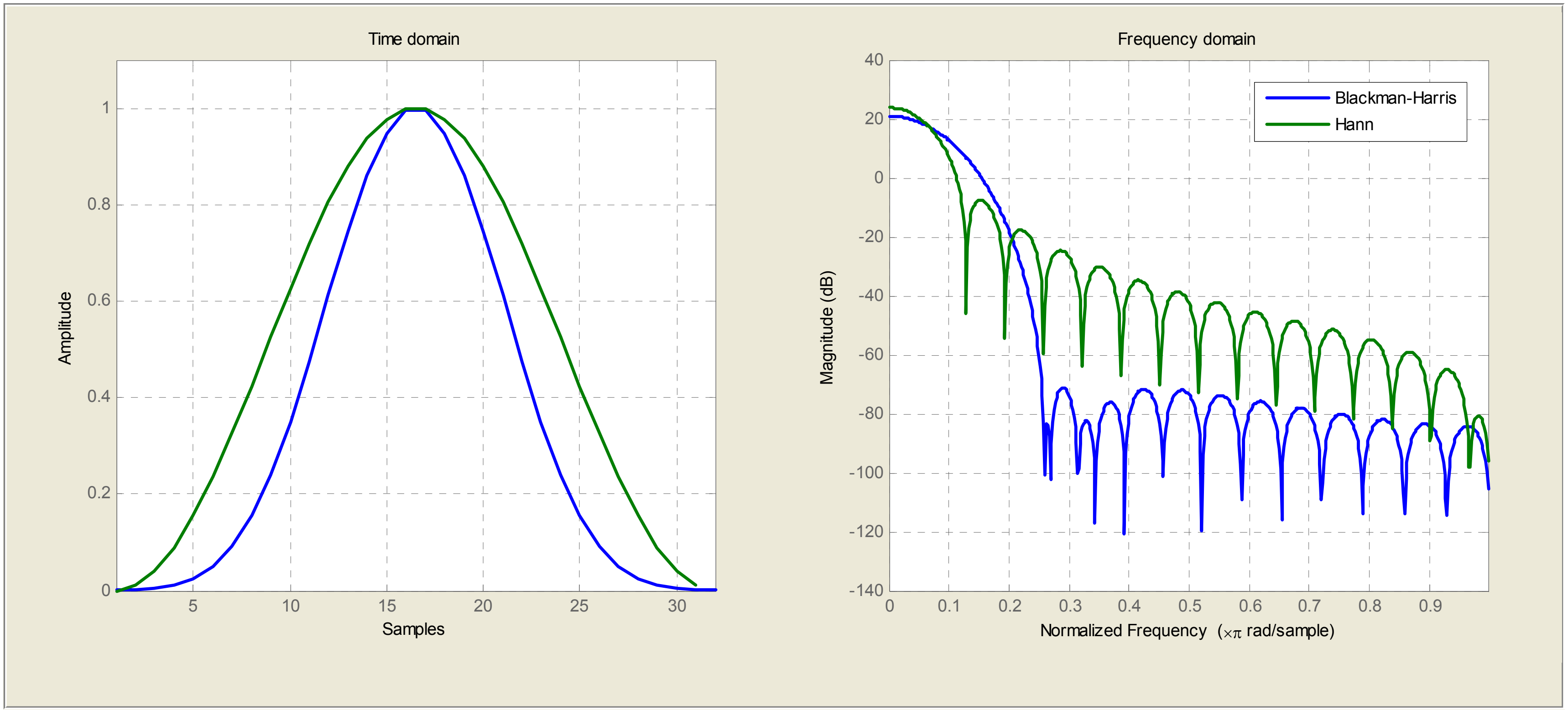
```
% Compare Rect, Bartlett and Hann windows
L = 32;
wvtool(rectwin(L), bartlett(L), ds_hann(L));
```



- Hann window provides larger side-lobe suppression (30 dB)
  - Signal tone occupies 3 FFT bins

Spectral Windows contd.

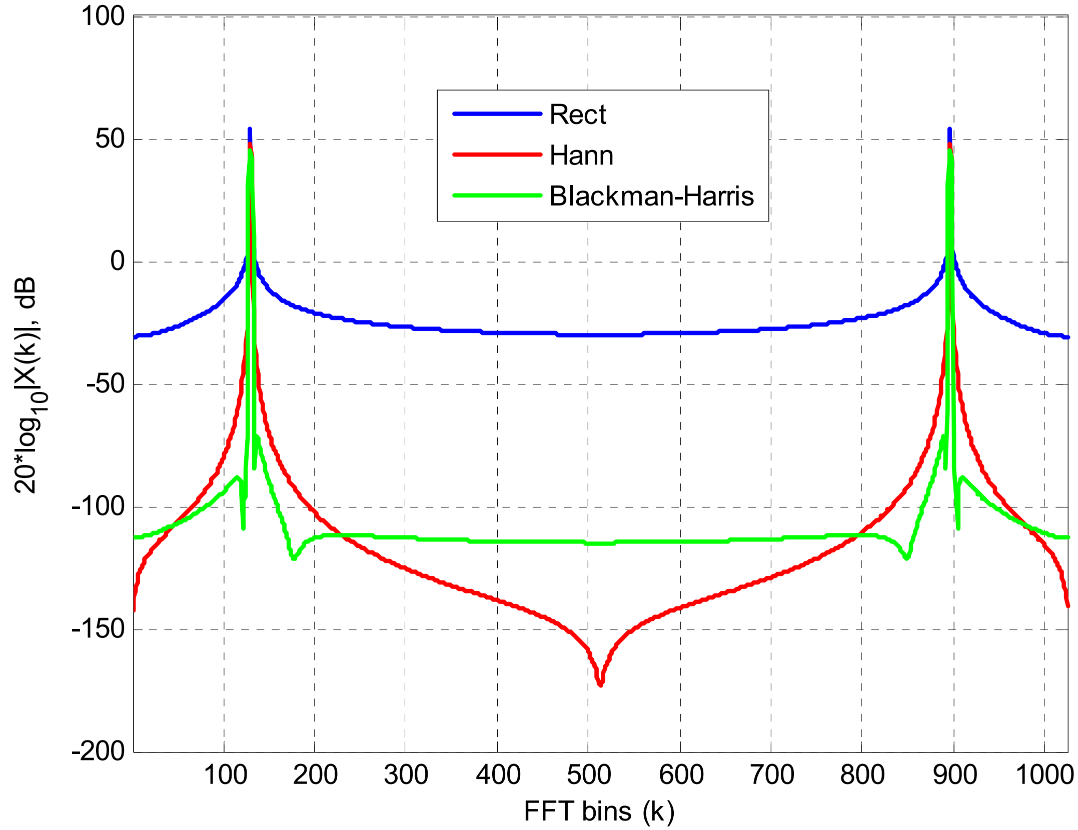
```
% Compare Blackman-Harris and Hann windows
L = 32;
wvtool(blackmanharris(L), ds_hann(L));
```



- Blackmann-Harris provides largest sidelobe suppression (50 dB)
  - Signal tone occupies 5 FFT bins

FFT with Windowing

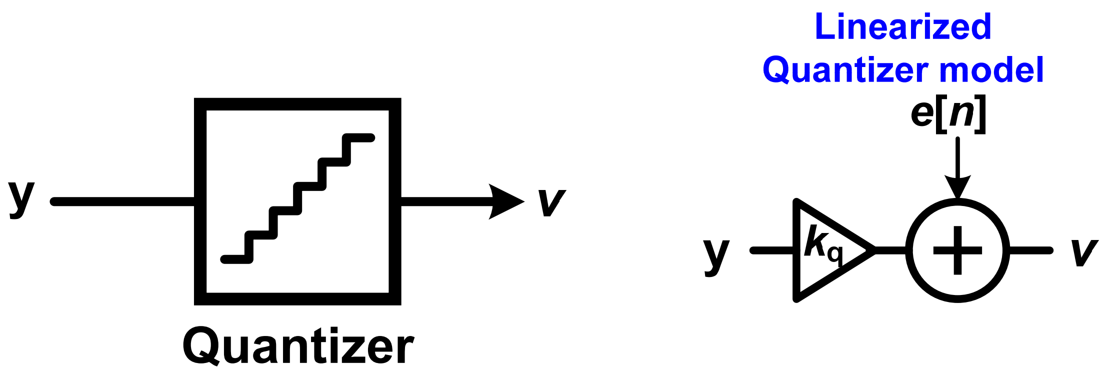
$$\frac{f_{in}}{f_s} = \frac{129.01}{1024}$$



- Hann is suitable for simulated data with exact or near coherent sampling
- Blackmann-Harris is best for experimental data where coherent sampling can't be enforced

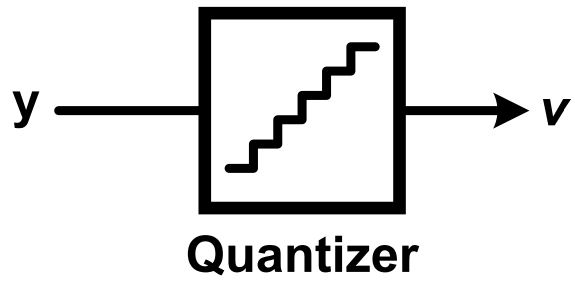
Quantizer Spectrum and Noise-Shaping

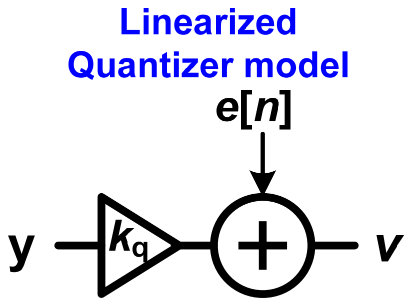
Quantizer Modeling



- Quantization noise modeling can be simplified with the assumptions
  - Input (y) stays within the no-overload input range
  - e[n] is uncorrelated with the input y
  - Spectrum of e[n] is white
  - Quantization noise is uniformly distributed
- Linearized quantizer model
  - AWUN noise
- Quantization noise power: $$\sigma_e^2 = \frac{\Delta^2}{12}$$
- $SQNR = 6.02 \cdot N + 1.76$

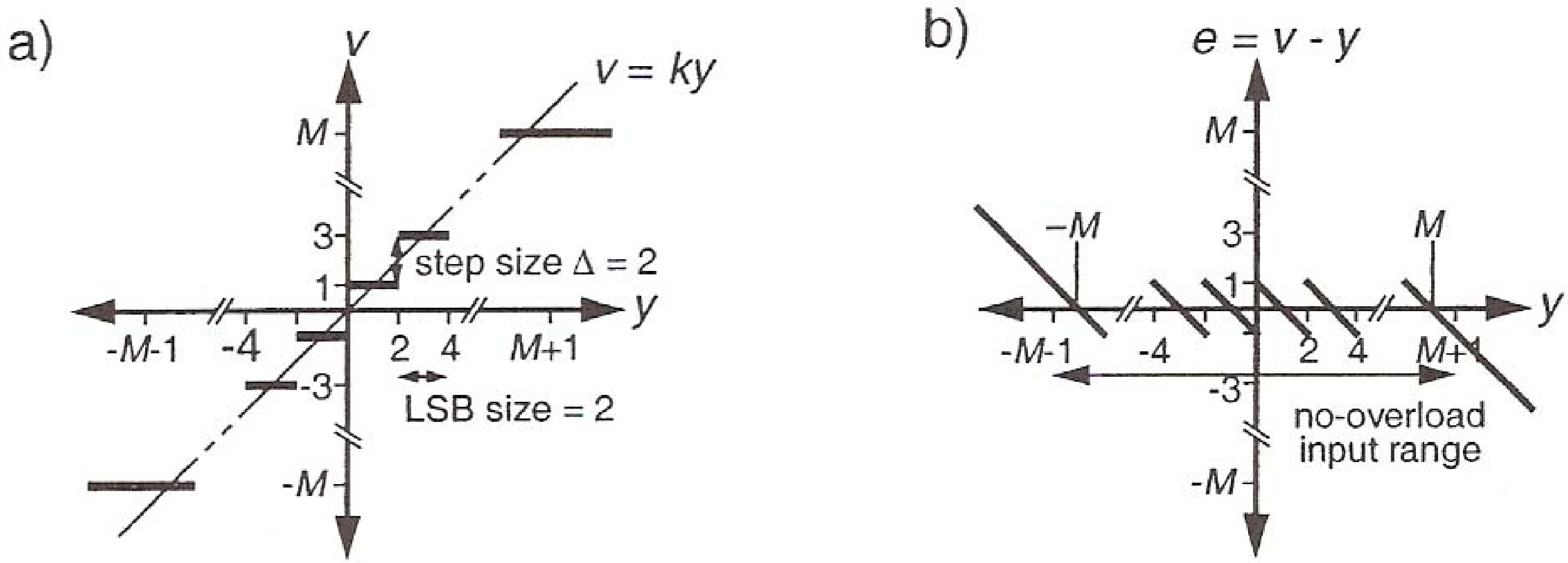
Quantizer Modeling Contd.





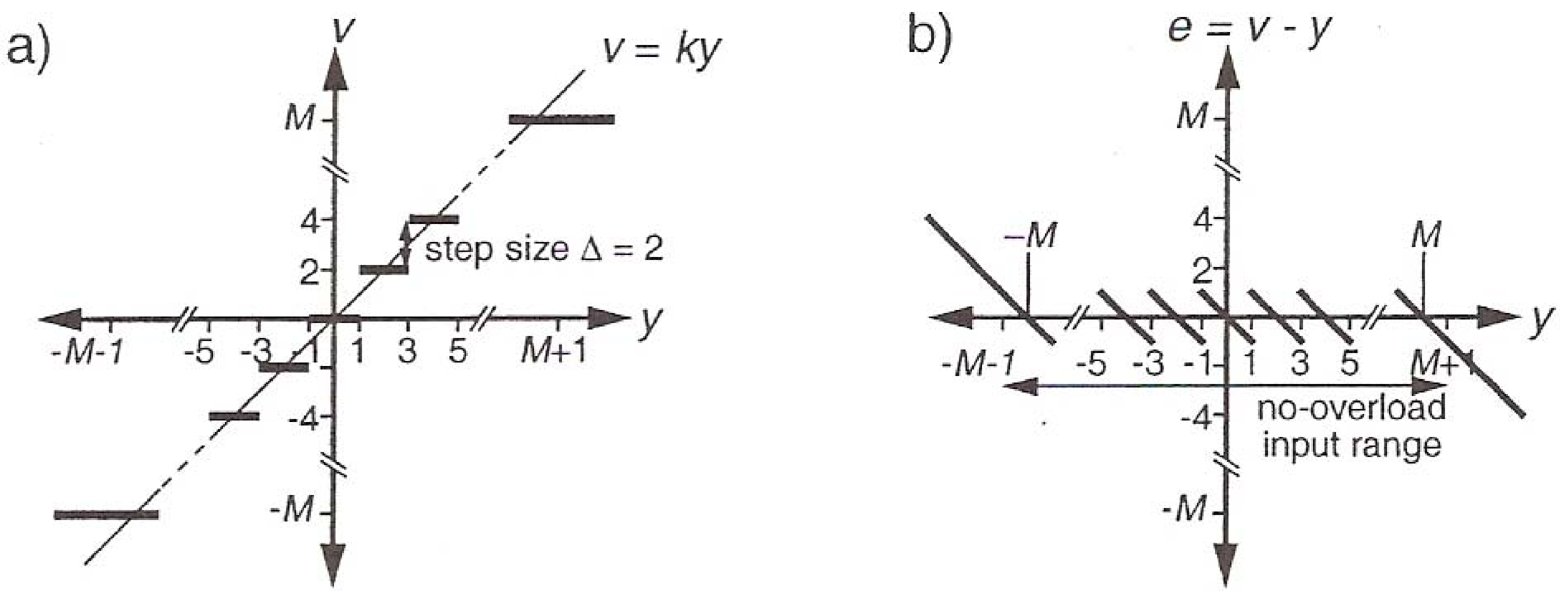
- Linear quantizer model holds true for large number of quantization levels and when the input is varying fast
- Linear quantizer model breaks down when
  - Input (y) is not varying fast
  - Input (y) is periodic with a frequency harmonically related to fs
  - Quantizer overload
- With quantizer overload, the effective gain of the quantizer drops as the input amplitude increases

Mid-Rise Quantizer (even number of levels)



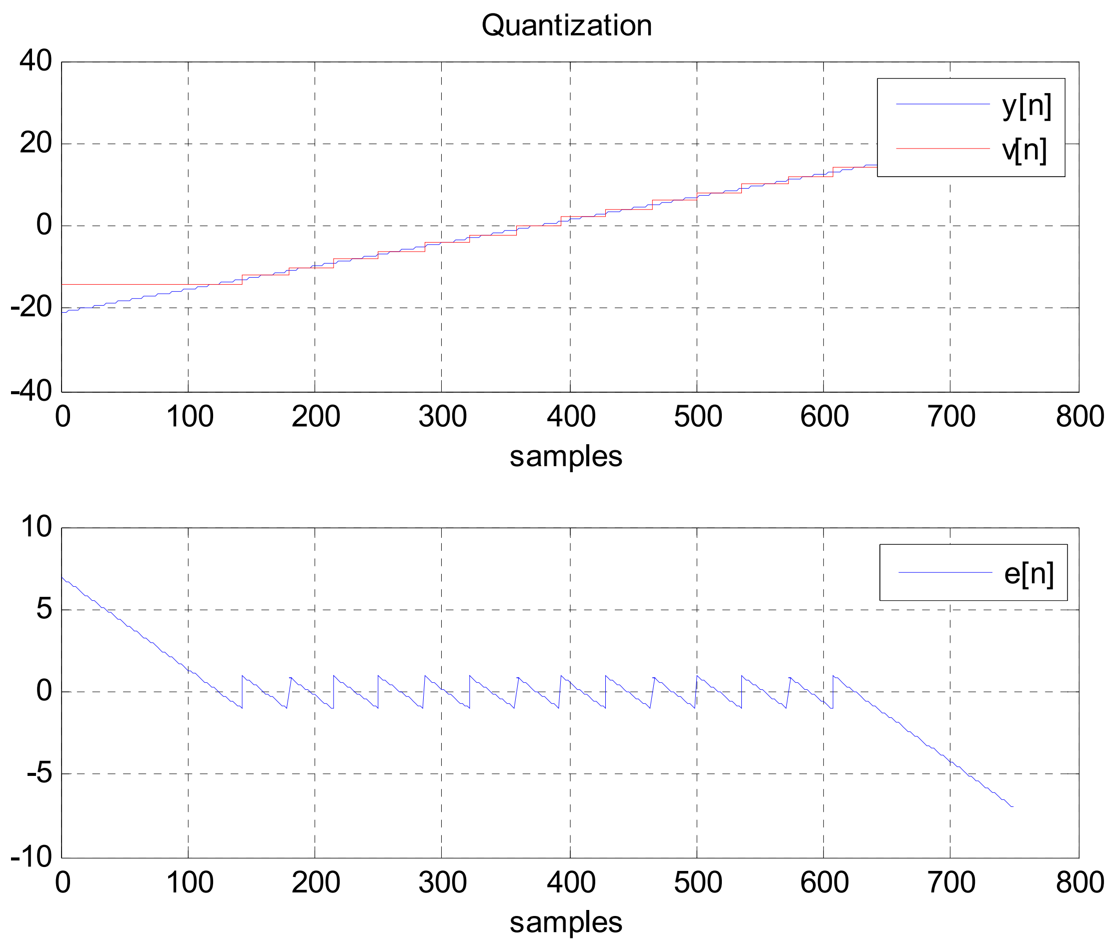
- Step rising at y=0 (mid-rise).
- In this figure, LSB= Δ= 2
- M = Number of steps, (M is odd here)
  - Number of levels (nLev) = M+1, (even)
- Input thresholds: 0, ±2, ……, ±(M–1).
- Output levels: ±1, ±3, ……, ±M.

Mid-Tread Quantizer (odd number of levels)

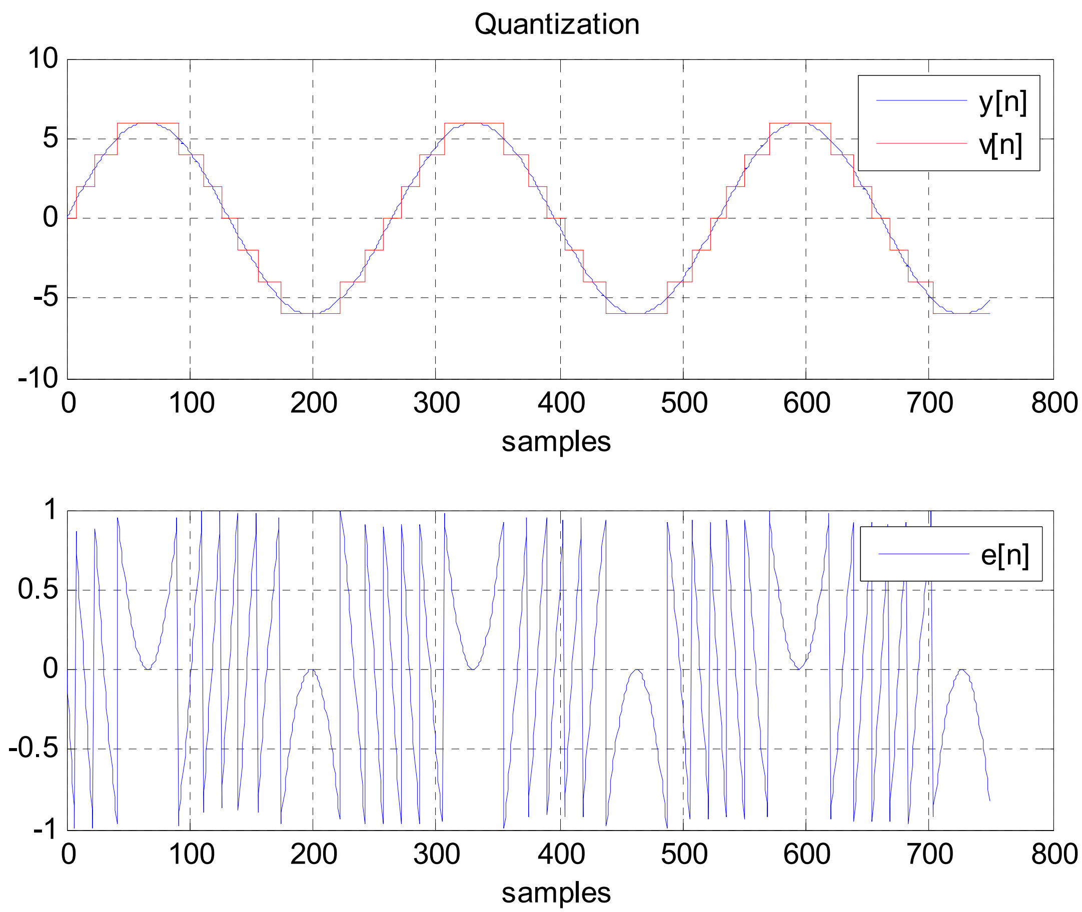


- Flat part of the step at y=0 (mid-tread).
- Here, LSB= Δ= 2
- M = Number of steps, (M is even here)
  - Number of levels (nLev) = M+1, (odd)
- Input thresholds: 0, ±2, ……, ±(M–1).
- Output levels: 0, ±2, ±4, ……, ±M.

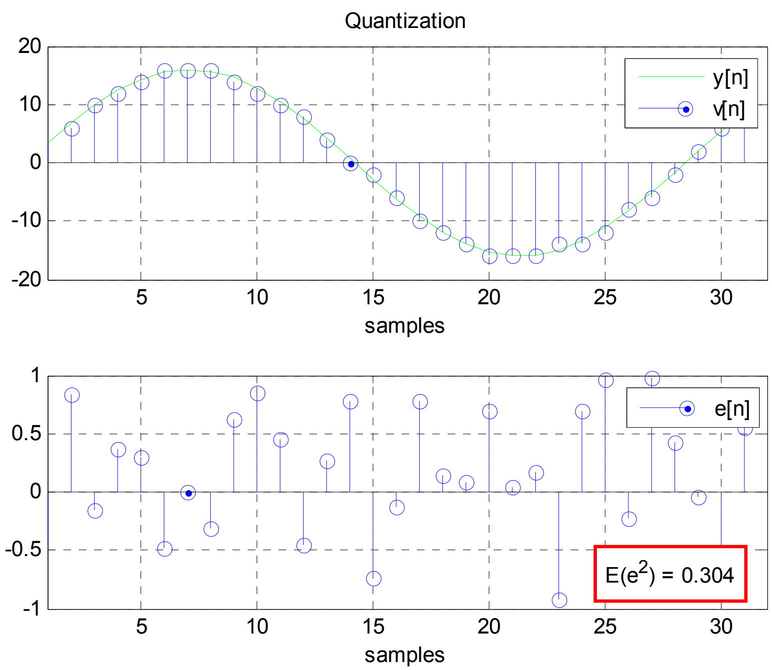
Quantizer Characteristics : Slow ramp input

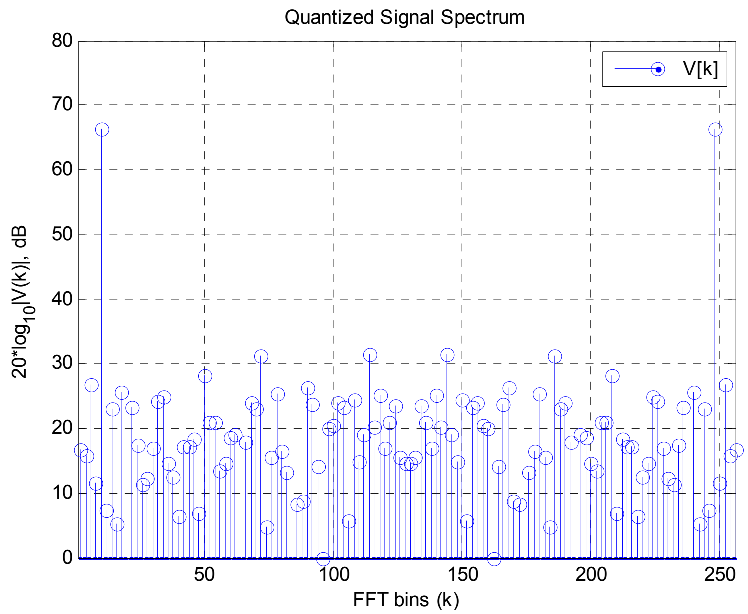


Quantizer Characteristics : Sine input



Quantization Noise : Example 1

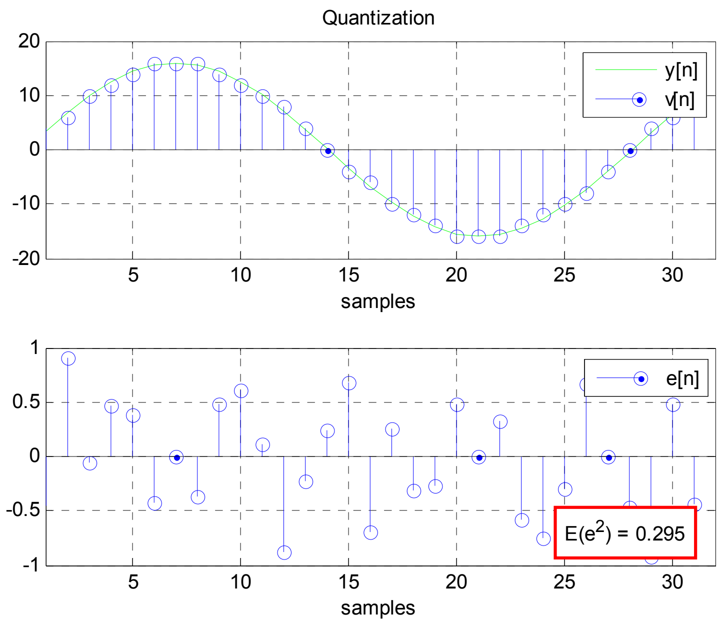


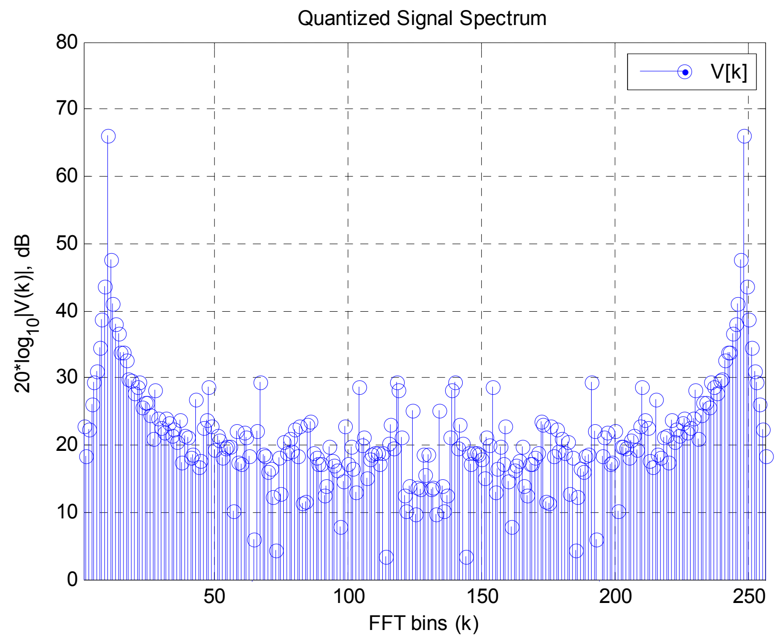


nLev=17, Δ=2, f_{in}/f_s = 9/256 :

•$E(e^2) = 0.304 \approx \Delta^2/12$

Quantization Noise : Example 1 contd.



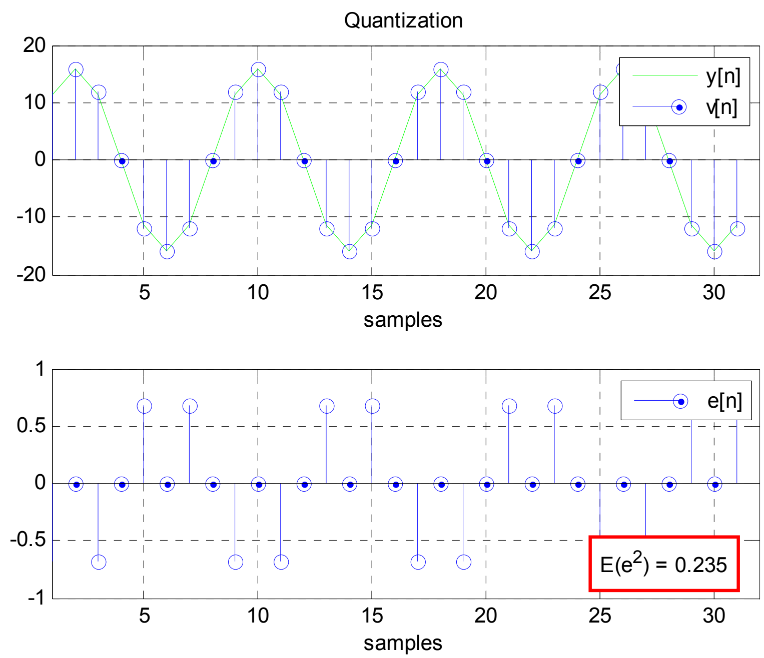


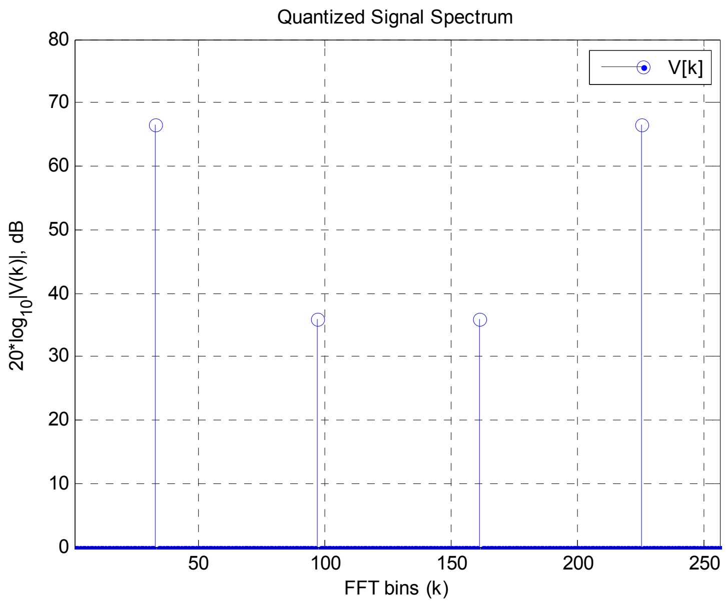
nLev=17, Δ=2, f_{in}/f_s = 9.1/256 :

•$E(e^2)$ = 0.295 ≈ $\Delta^2/12$

•Notice the FFT leakage.

Quantization Noise : Example 1 contd.

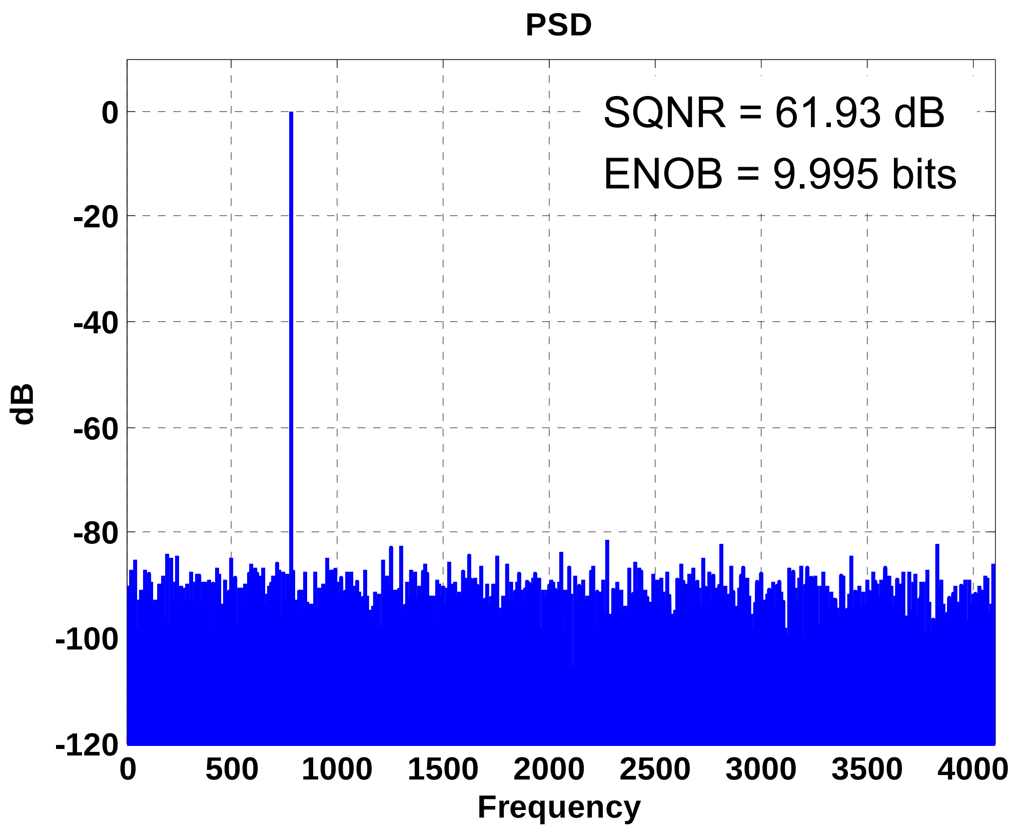




nLev=17, Δ=2, f_{in}/f_s = 32/256 = 1/8 :

- $E(e^2) = 0.235 < \Delta^2/12$
  - Quantization *noise* approximation not valid

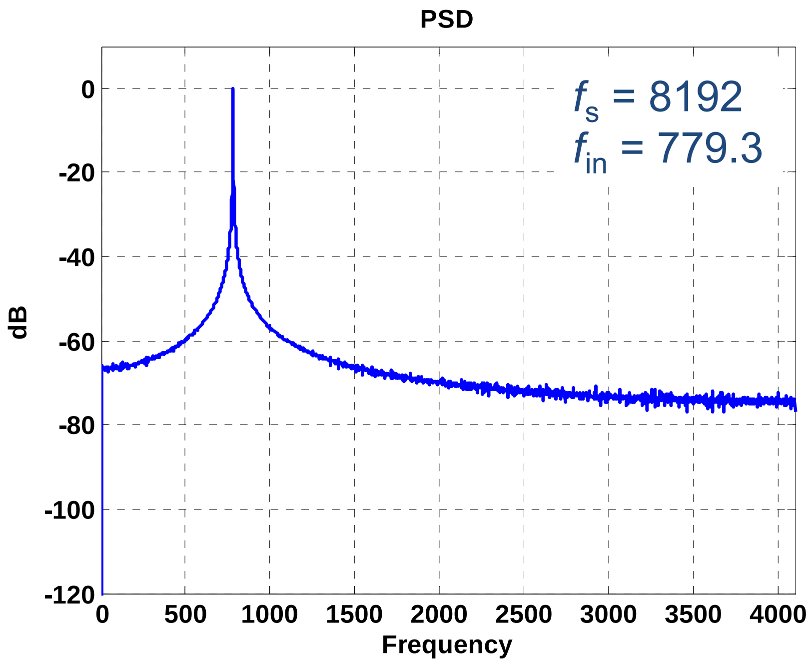
FFT Spectrum of Quantized Signal

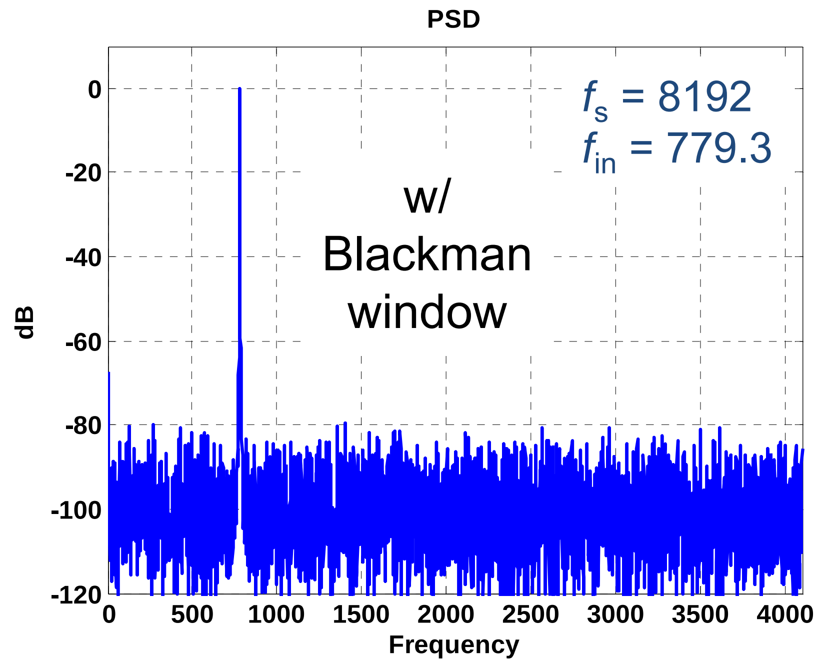


- N = 10 bits
- 8192 samples, only $f = [0, f_s/2]$ shown
- Normalized to V_{in}
- $f_s = 8192$, $f_{in} = 779$
- f_{in} and f_s must be incommensurate

$$\text{ENOB} = \frac{\text{SQNR} - 1.76\ \text{dB}}{6.02\ \text{dB}}$$

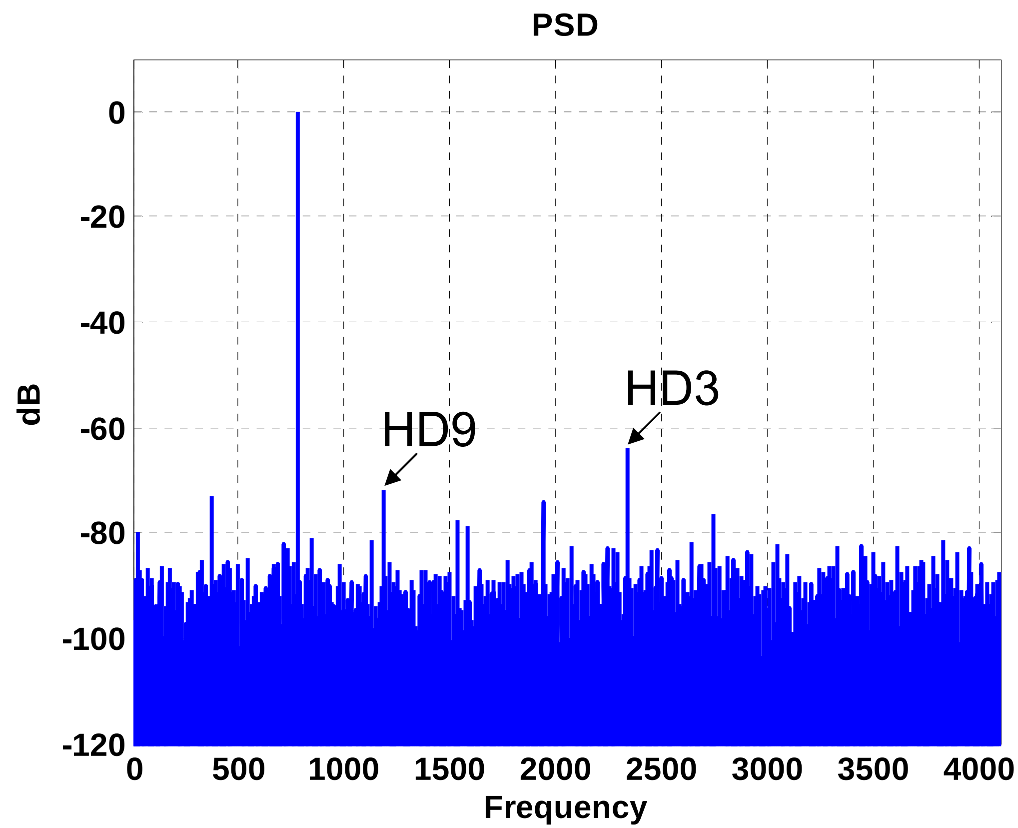
Spectrum Leakage





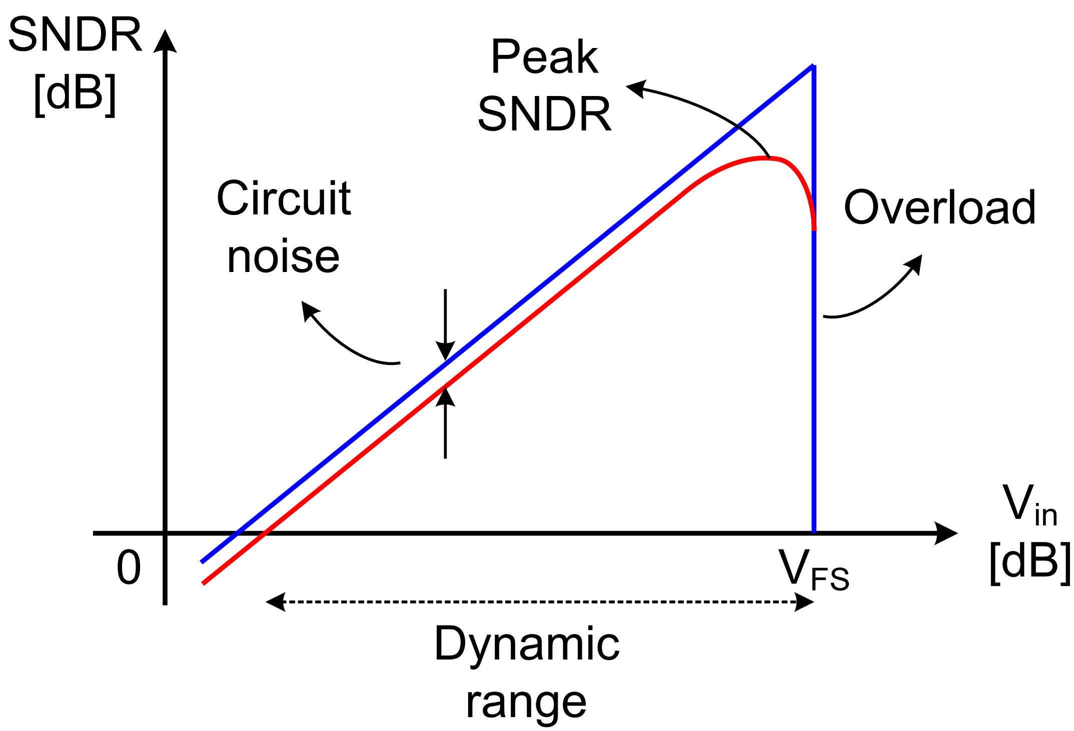
- TD samples must include integer number of cycles of input signal
- Windowing can be applied to eliminate spectrum leakage
- Trade-off between main-lobe width and sideband rejection for different windows

FFT Spectrum with Distortion



- High-order harmonics are aliased back, visible in [0, $f_s/2$] band
- E.g., HD3 @ 779x3+1=2338, HD9 @ 8192-9x779+1=1182

Dynamic Performance



SNR

$$= 10\mathrm{LOG}_{10}\left(\frac{V_{in}^{\,2}/2}{\Delta^2/12+\sigma_N^{\,2}}\right)$$

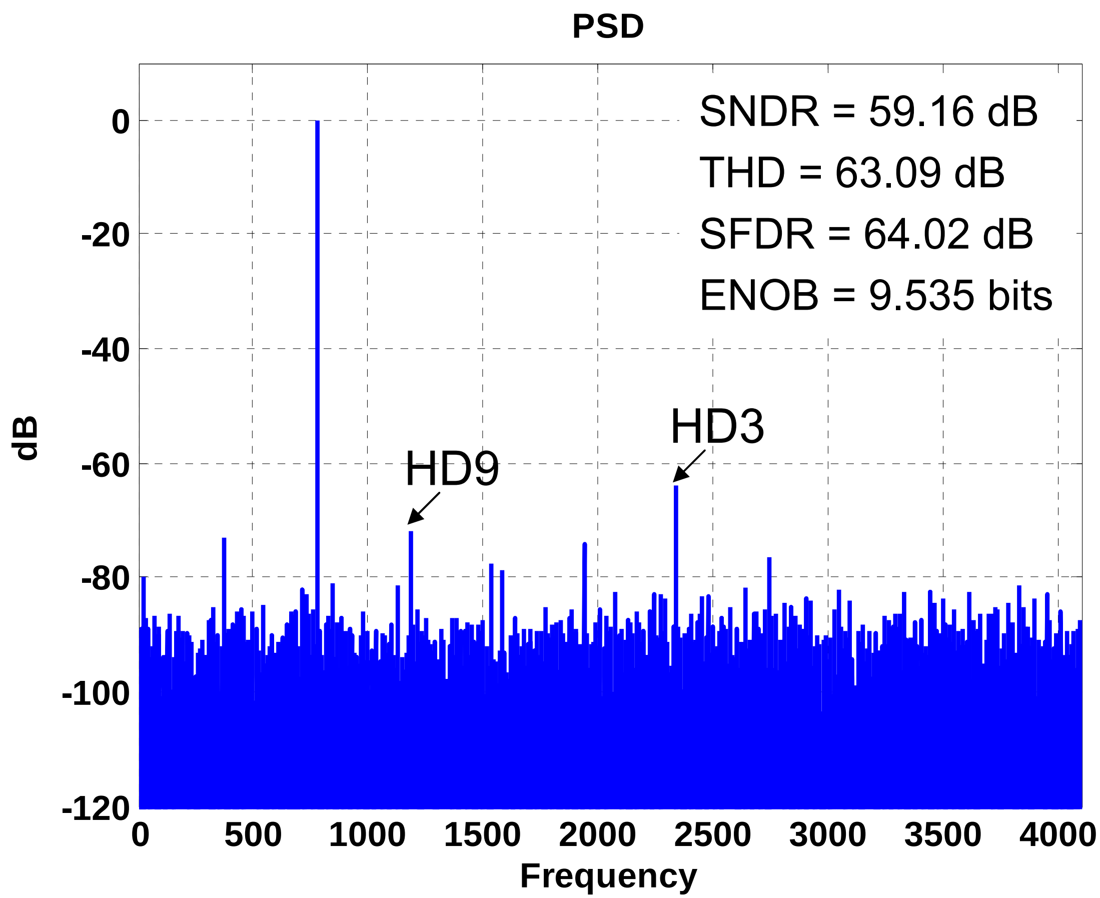
$$\propto V_{in}\,[\mathrm{dB}]$$

- Peak SNDR limited by large-signal distortion of the converter
- Dynamic range implies the “theoretical” SNR of the converter

Dynamic Performance Metrics

- Signal-to-quantization-noise ratio (SQNR)
- Total harmonic distortion (THD)
- Signal-to-noise and distortion ratio (SNDR or SINAD)
- Spurious-free dynamic range (SFDR)
- Two-tone intermodulation product (IM3)
- Aperture uncertainty (related to the frontend S/H and clock)
- Dynamic range (DR)

Evaluating Dynamic Performance



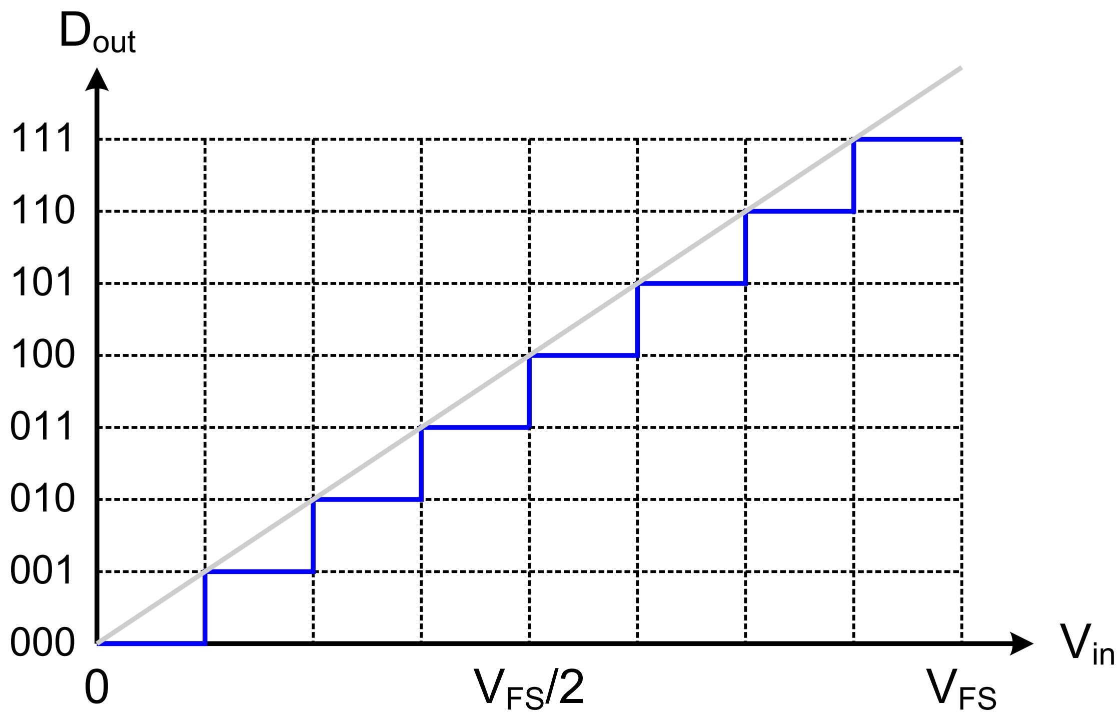
- Signal-to-noise plus distortion ratio (SNDR)
- Total harmonic distortion (THD)
- Spurious-free dynamic range (SFDR)

$$ENOB = \frac{SNDR - 1.76\,dB}{6.02\,dB}$$

Static Performance Metrics

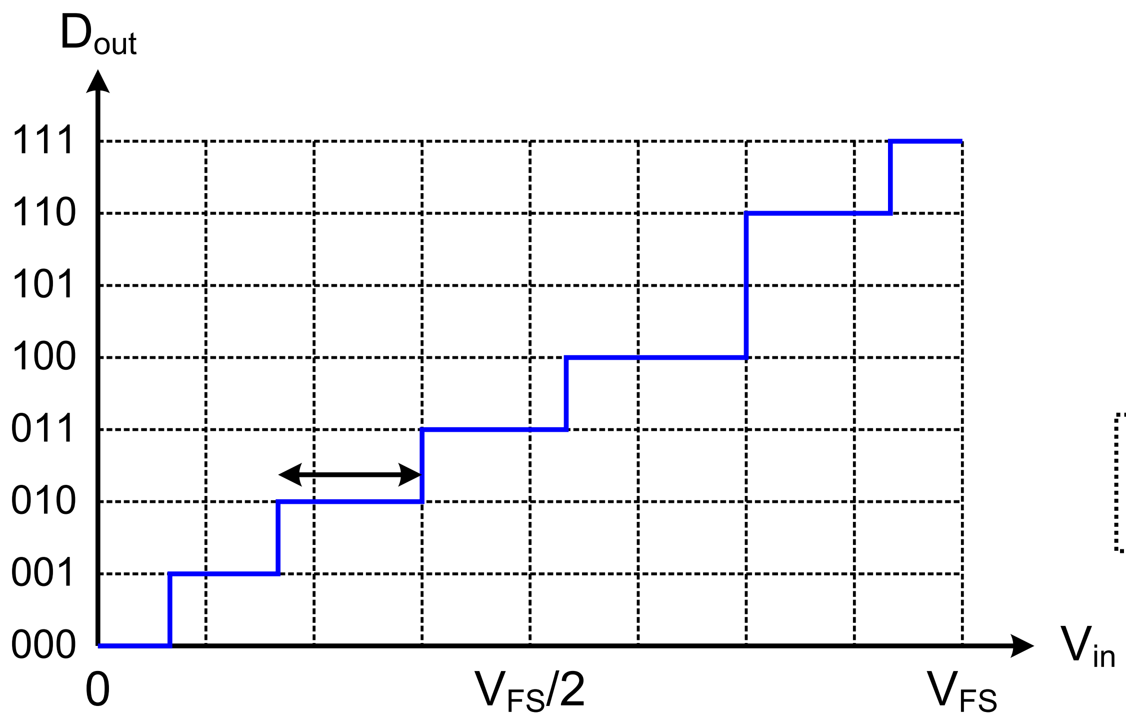
- Offset (OS)
- Gain error (GE)
- Monotonicity
- Linearity
  - Differential nonlinearity (DNL)
  - Integral nonlinearity (INL)

Ideal ADC Transfer Characteristic



Note the systematic offset! (floor, ceiling, and round)

DNL and Missing Code

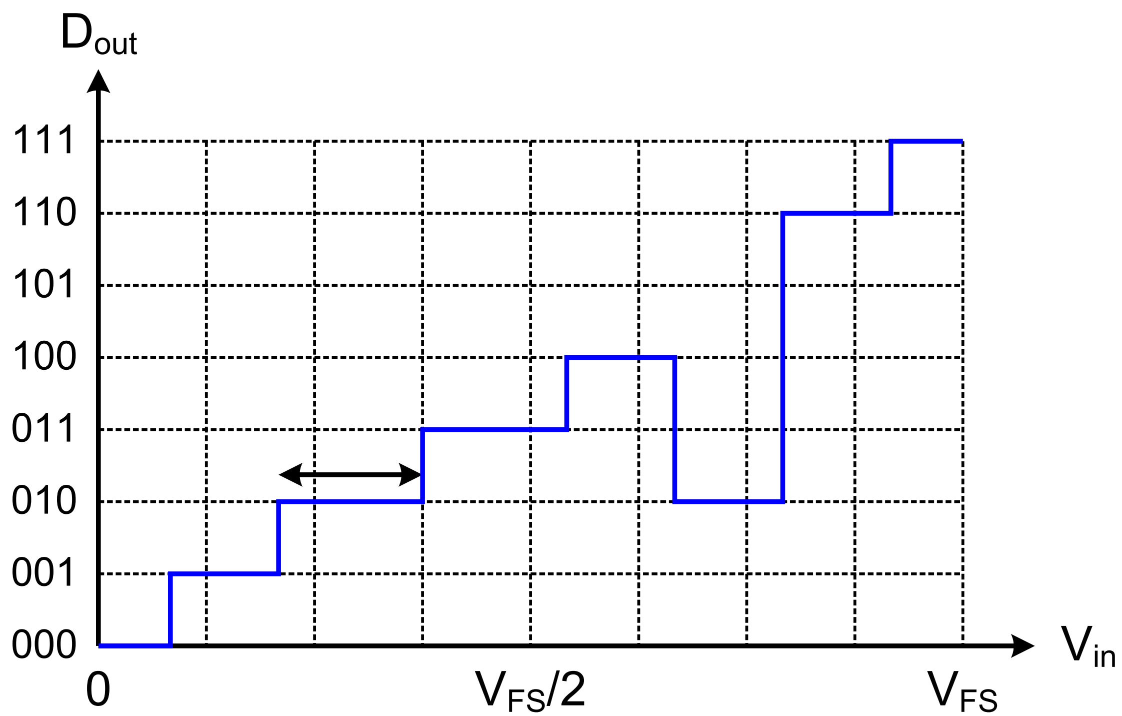


- DNL = ?
- Can DNL < -1?

$$DNL_i = \frac{i^{th}\ \text{Step Size} - \Delta}{\Delta}$$

DNL = deviation of an input step width from 1 LSB (= $V_{FS}/2^N = \Delta$)

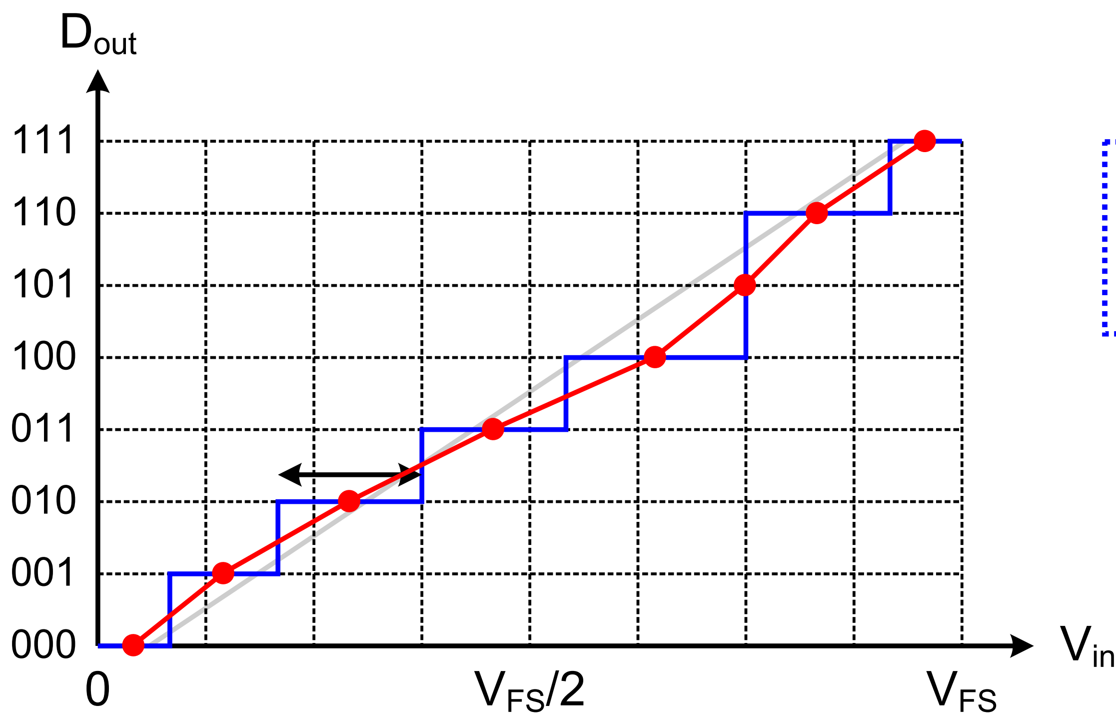
DNL and Nonmonotonicity



- DNL = ?
- How can we even measure this?

DNL = deviation of an input step width from 1 LSB ($= V_{FS}/2^N = \Delta$)

INL



$$INL_i = \sum_{j=0}^{i} DNL_j$$

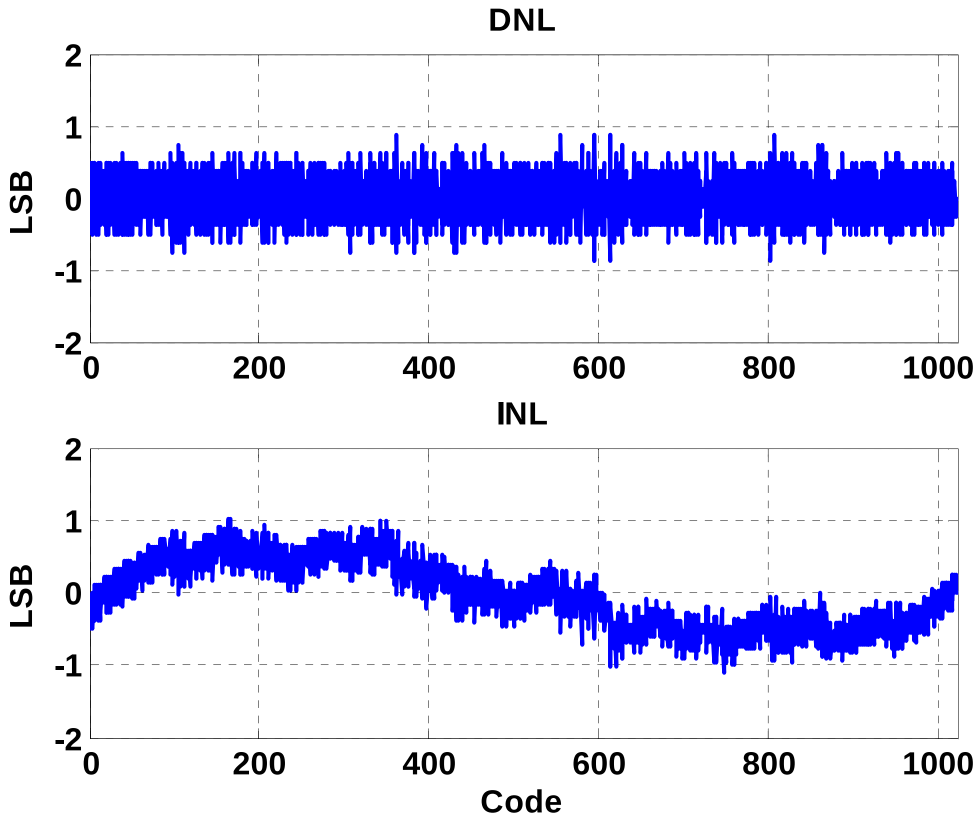
Any code

- Missing?
- Nonmonotonic?

INL = deviation of the step midpoint from the ideal step midpoint

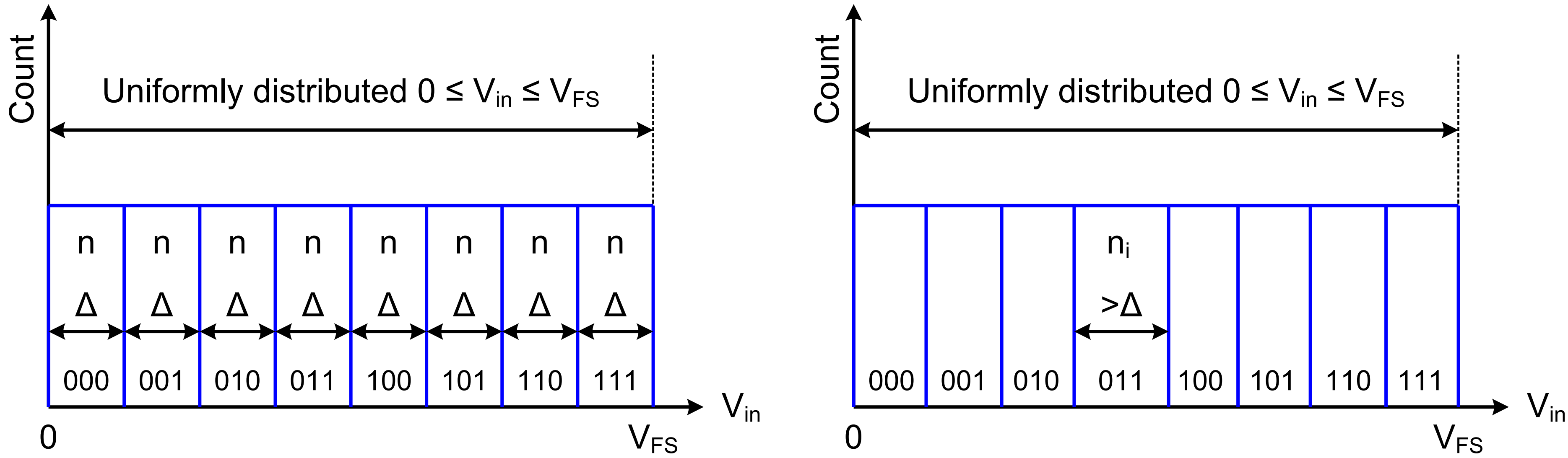
(method I and II …)

10-bit ADC Example



- 1024 codes
- No missing code!
- Plotted against the digital code, not V_{in}

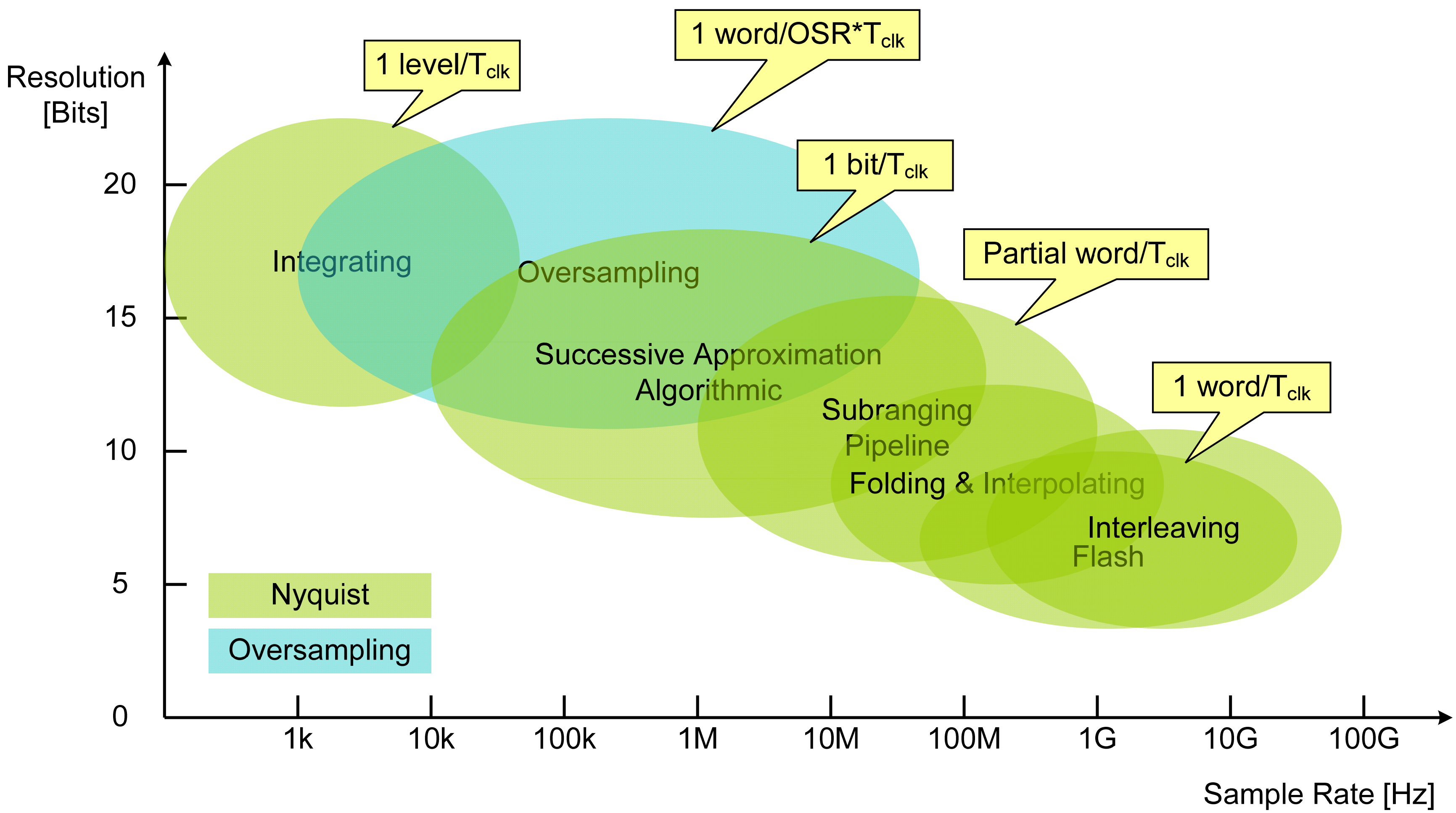
Code Density Test



$$DNL_i = \frac{i^{th}\ \text{Step Size} - \Delta}{\Delta} \approx \frac{n_i - \langle n_i \rangle}{\langle n_i \rangle}$$

Ball casting problem: # of balls collected by each bin (n_i) is proportional to the bin size (converter step size)

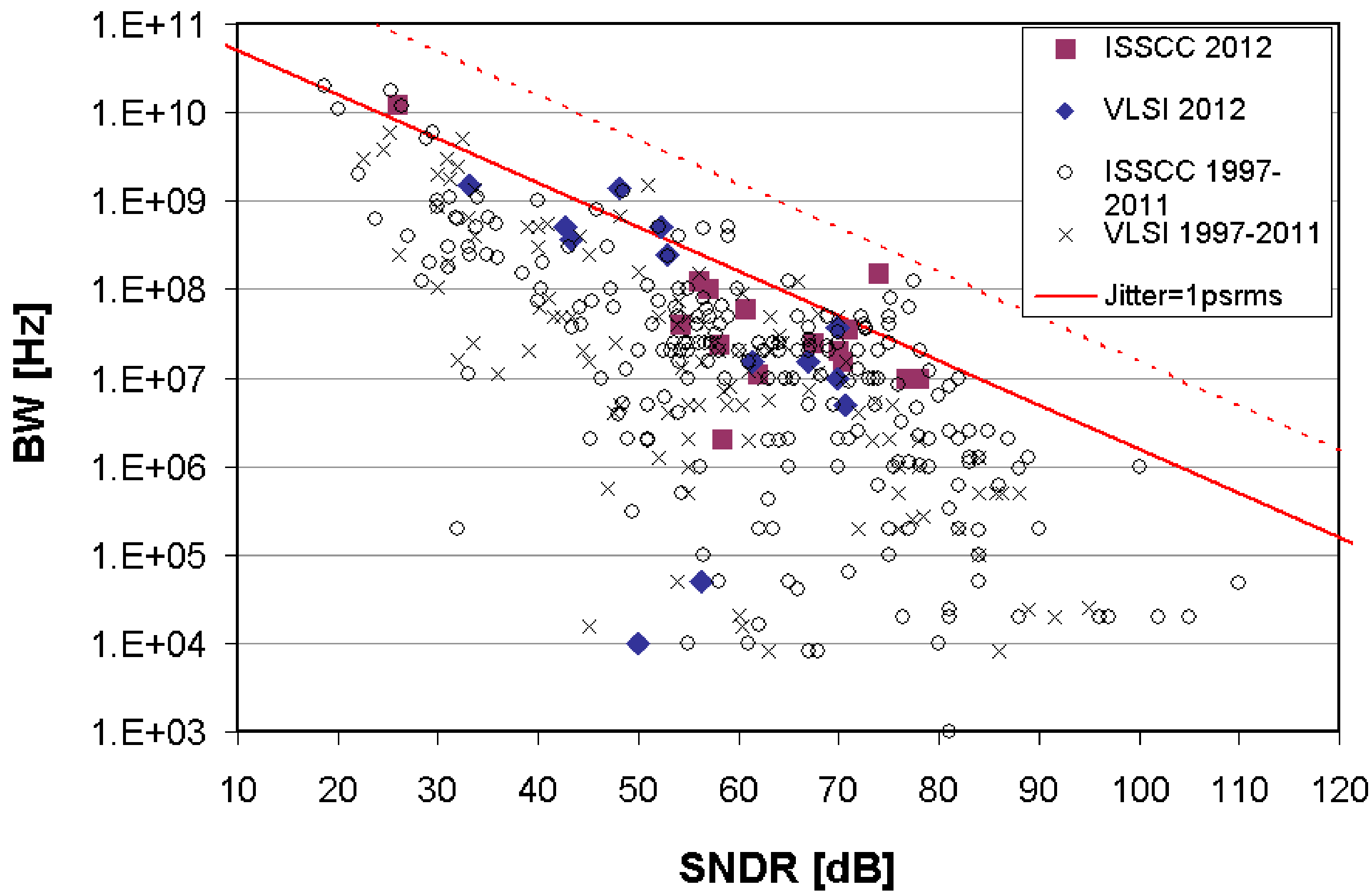
ADC Architectures



Nyquist-Rate ADC (N-Bit, Binary)

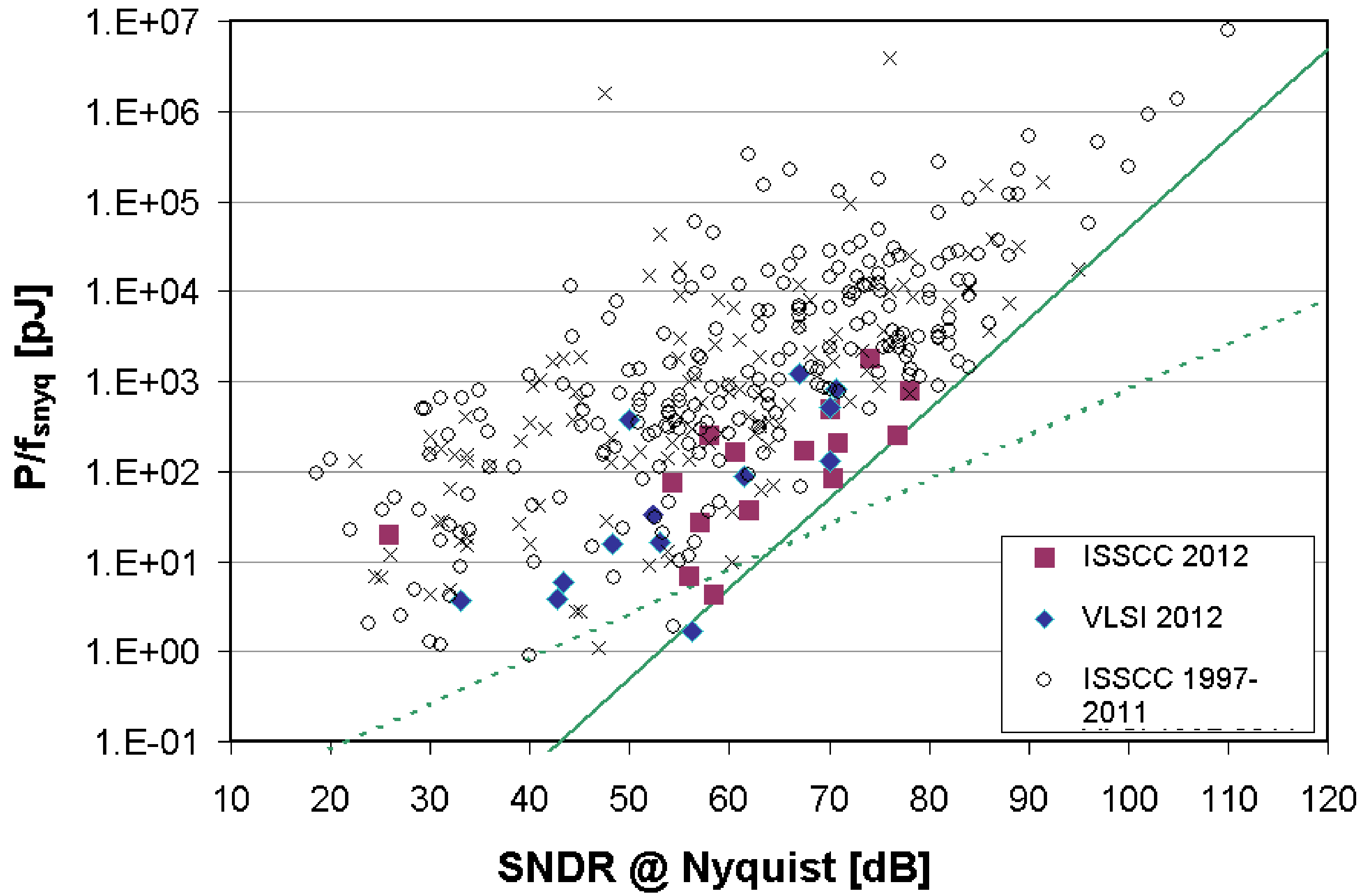
- Word-at-a-time (1 step)† ← fast
  - Flash
- Level-at-a-time (2^N steps) ← slowest
  - Integrating (Serial)
- Bit-at-a-time (N steps) ← slow
  - Successive approximation
  - Algorithmic (Cyclic)
- Partial word-at-a-time ($1 < M \leq N$ steps) ← medium
  - Subranging
  - Pipeline
- Others ($1 \leq M \leq N$ step)
  - Folding ← relatively fast
  - Interleaving (of flash, pipeline, or SA) ← fastest

ADC Survey: Aperture



http://www.stanford.edu/~murmann/adcsurvey.html

ADC Survey: Energy



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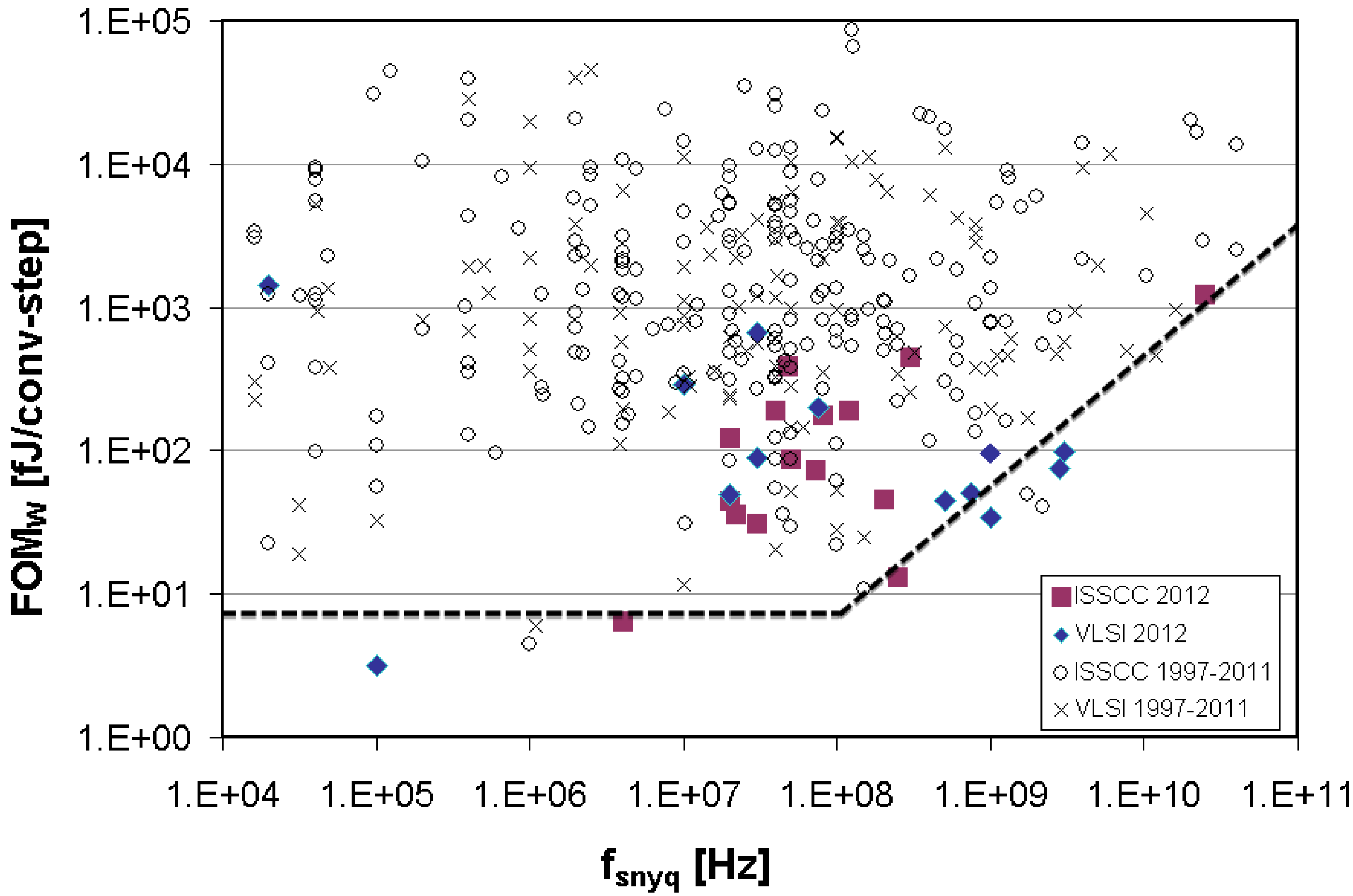
ADC Survey: Figure of Merit



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References

1. Rudy van de Plassche, “CMOS Integrated Analog-to-Digital and Digital-to-Analog Converters,” 2nd Ed., Springer, 2005.
2. M. Gustavsson, J. Wikner, N. Tan, *CMOS Data Converters for Communications*, Kluwer Academic Publishers, 2000.