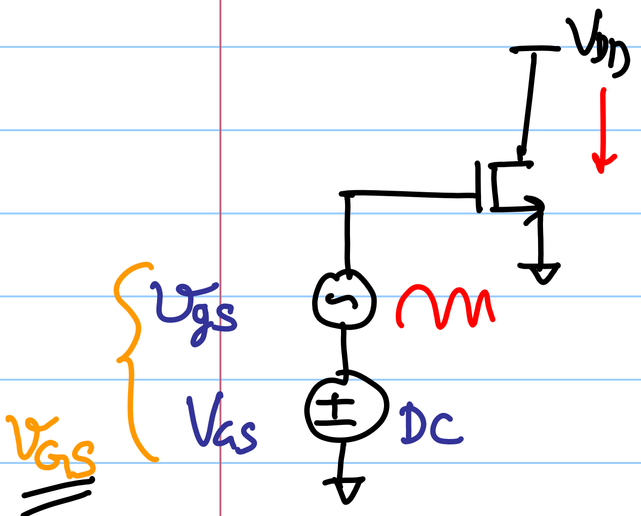
ECE 511 – Lecture 2

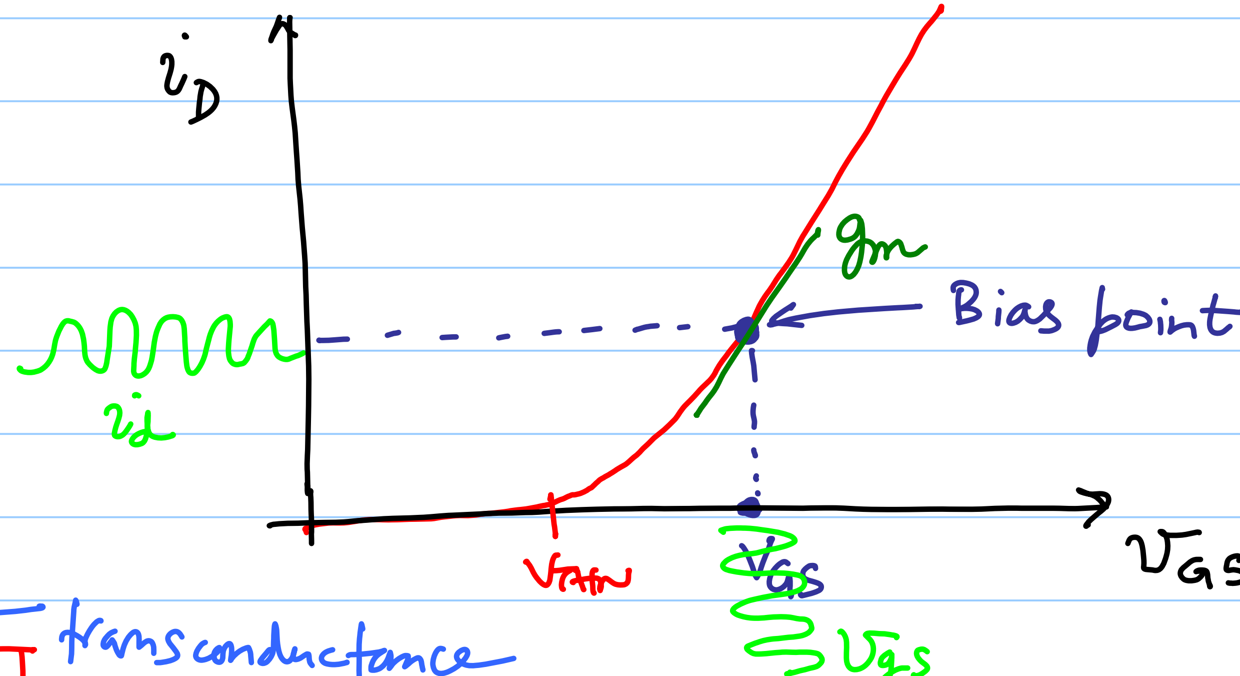
Note Title 1/15/2015

NMOS in Saturation

Bias point → (V_{GS}, I_D)

$\underbrace{I_D}_{DC} + \underbrace{i_d}_{AC} = i_D$





transconductance

$$\boxed{i_d = g_m v_{gs}}$$

$$i_D = \frac{k'_n}{2}\frac{W}{L}\left(v_{GS} - V_{THN}\right)^2$$

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{v_{GS} = V_{GS}} \leftarrow$$

$$= \underbrace{k'_n \cdot \frac{W}{L}}_{\beta_n}\left(V_{GS} - V_{THN}\right)$$

$$g_m = \beta_n\left(V_{GS} - V_{THN}\right) = \beta V_{OV} \longrightarrow (1)$$

$$I_D = \frac{\beta_n}{2}\left(V_{GS} - V_{THN}\right)^2 \longrightarrow \beta_n = \frac{2I_D}{\left(V_{GS} - V_{THN}\right)^2}$$

$$\Rightarrow \quad V_{GS} - V_{THN} = \sqrt{\frac{2I_D}{\beta_n}}$$

$$\Rightarrow g_m = \beta_n \cdot \sqrt{\frac{2I_D}{\beta_n}} = \sqrt{2\beta_n I_D} \longrightarrow (2)$$

$$g_m = \beta_n \cdot (V_{GS} - V_{THN})$$

$$= \frac{2I_D}{(V_{GS}-V_{THN})^2} \times (V_{GS}-V_{THN}) = \frac{2I_D}{V_{GS}-V_{THN}} \longrightarrow ③$$

$$g_m \begin{cases} = \beta V_{ov} \\ = \sqrt{2\beta I_D} \\ = \dfrac{2I_D}{V_{ov}} \end{cases}$$

$$\beta = KP_n \cdot \frac{W}{L}$$

$\beta_n V_{ov}$

$\frac{W}{L}$ constant

$\sqrt{2\beta_n I_D}$

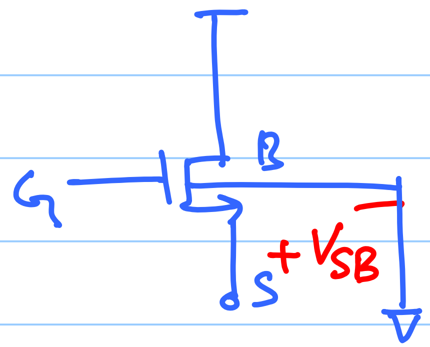
$\frac{W}{L}$ constant

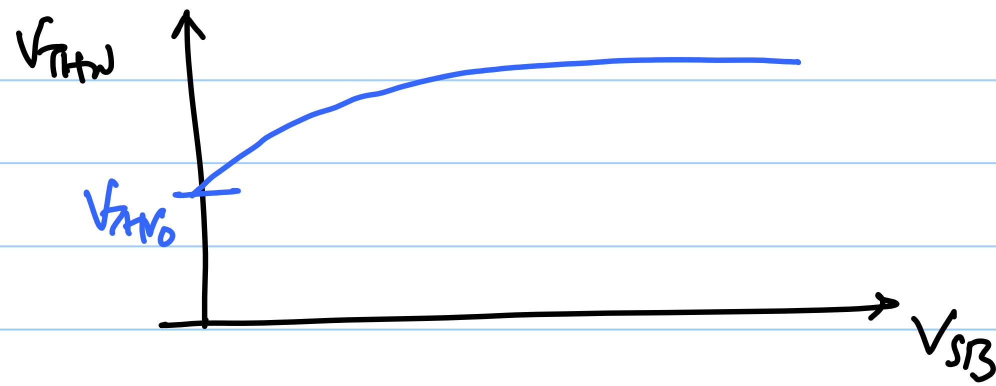
$\frac{2I_D}{V_{ov}}$

I_D constant

Body-effect

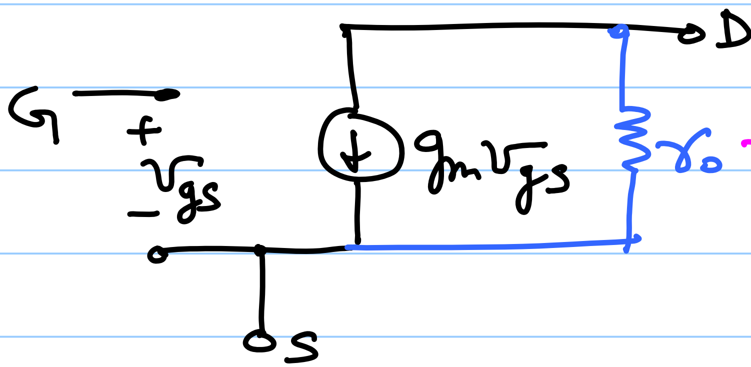
$$V_{THN}(V_{SB}) = V_{THN_0} + \gamma_n \left(\sqrt{2|V_{fp}| + V_{SB}} - \sqrt{2|V_{fp}|} \right)$$





Small-signal model of a NMOS

$$i_d = g_m v_{gs}$$

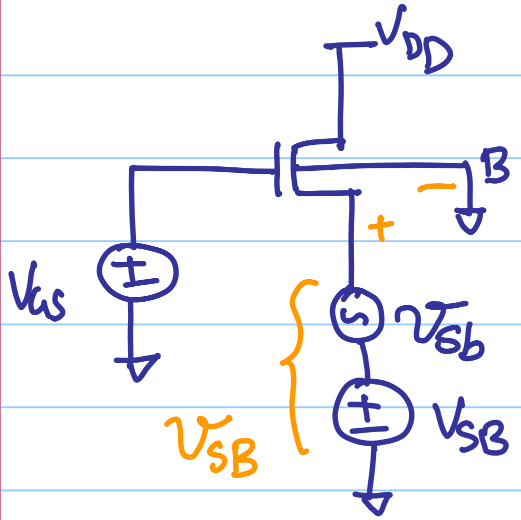


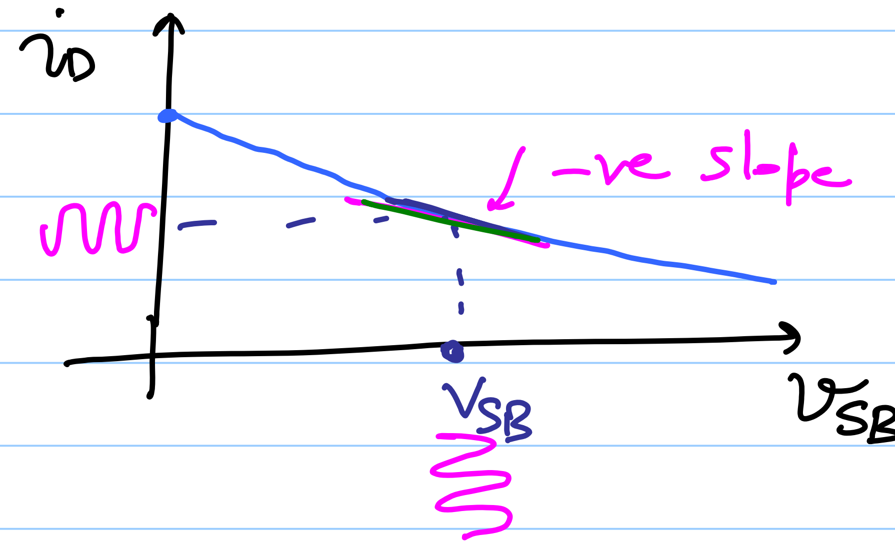
$$r_o = \left(\left. \frac{\partial i_D}{\partial v_{DS}} \right|_{v_{DS} = V_{DS}} \right)^{-1}$$

$$= \frac{1}{\lambda I_{D,sat}}$$

$$L \downarrow \Rightarrow \lambda \uparrow \Rightarrow r_o \downarrow$$

Body-effect transconductance:





$$i_d = g_{mb} \cdot v_{sb}$$

$$g_{mb} = \left.\frac{\partial i_D}{\partial v_{SB}}\right|_{v_{SB}=V_{SB}}$$

$$= \left.\frac{\partial}{\partial v_{SB}}\left[\frac{K'_n}{2}\cdot\frac{W}{L}\left(V_{GS}-V_{THN}\right)^2\right]\right|_{v_{SB}=V_{SB}}$$

$$= \underbrace{K P_n \cdot \frac{W}{L}\left(V_{GS} - V_{THN}\right)}_{g_m} \cdot \left(-\frac{\partial V_{THN}}{\partial v_{SB}}\right)\Bigg|_{v_{SB}=V_{SB}}$$

$$= g_m \cdot \underbrace{\left(-\frac{\gamma}{2}\left(|2V_{fb}| + V_{SB}\right)^{-1/2}\right)}_{\eta}$$

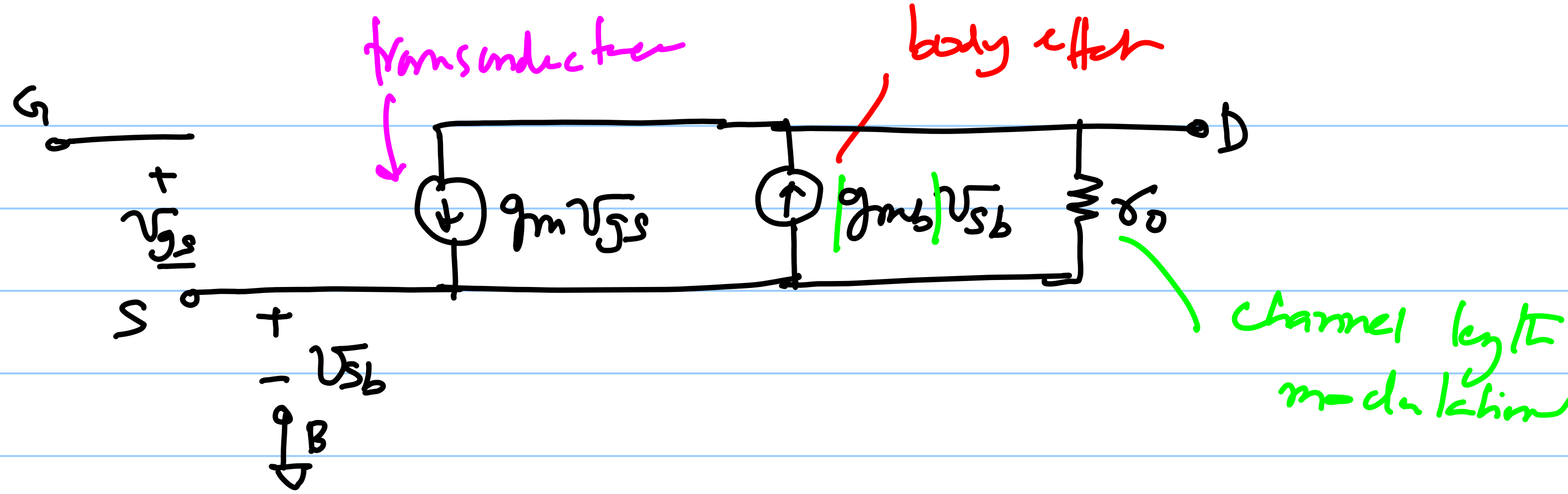
$$g_{mb} = \eta\, g_m \qquad\qquad \eta = \frac{-\gamma}{2\sqrt{2|V_{fb}| + V_{SB}}}$$

$$\boxed{g_{mb} = -|\eta|\, g_m}$$

$$|\eta| < 1$$

the $-ve$ sign indicates the opposing nature of

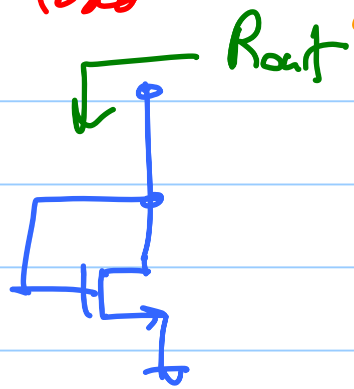
$V_{SB} \uparrow \Rightarrow i_D \downarrow$

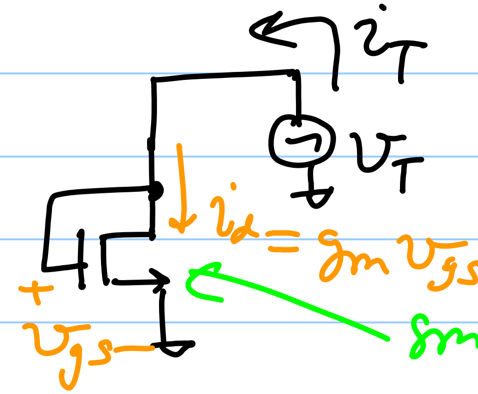


transconductance
body effect
G
D
S
B
+
v_{gs}
+
−
v_{sb}
$g_m v_{gs}$
$|g_{mb}| v_{sb}$
r_o
channel length modulation

diode load

small-signal impedance looking into the drain

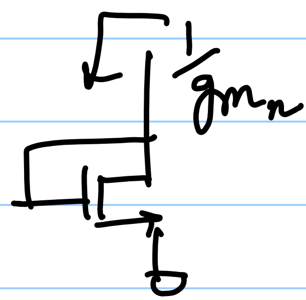


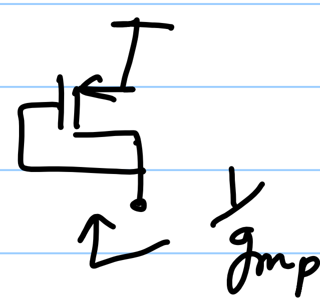


$$R_{out} = \frac{v_T}{i_T}$$

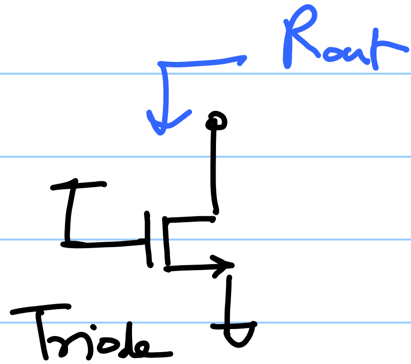
$$i_T = i_d = g_m v_{gs}$$

$$= \frac{\cancel{v_{gs}}}{g_m \cancel{v_{gs}}} = \frac{1}{g_m}$$





$\rightarrow \beta(V_{GS} - V_{thn})$



$$R_{out} = R_{ch} = \frac{1}{KP_n \frac{W}{L}(V_{GS} - V_{THN})}$$

much larger

$$\ll \frac{1}{g_m}$$

Example of small-signal analysis

① Given a circuit, draw the DC picture

↳ short ac voltage sources
open ac currents

Caps will open

Find all bias conditions for all devices

② Use DC values from ①, to calculate

small-signal parameters (g_m, r_o, g_{mb} ...)

↳ linearization about the bias point

③ Draw AC picture

↳ replace devices by their small-signal models

↳ short DC voltage sources
open DC current sources

Solve for the AC values in circuit

4. Superposition of DC & AC quantities