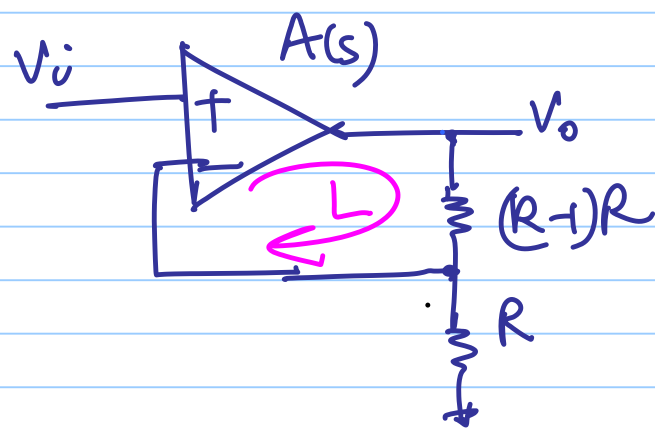
ECE 511 - Lecture 26

Note Title 4/28/2015

① gain



$$\frac{V_o}{V_i} = \frac{k\,L(s)}{1+L(s)}$$

closed-loop gain

loop gain

$$A(s) = \frac{A_0}{1+\frac{s}{\omega_0}}$$

at DC,

$$\frac{V_o}{V_i} = \frac{A_0}{1+\frac{A_0}{k}}, \quad L = \frac{A_0}{k}$$

$$= k \cdot \frac{1}{1 + \frac{k}{A_0}}$$

$$\frac{1}{1+x} \simeq 1 - x$$

$$|x| \ll 1$$

$$= k \cdot \left(1 - k \cdot \frac{1}{A_0}\right) = k(1 - \epsilon)$$

$$(1+x)^{-1}$$

relative gain error ϵ

$$\epsilon = k \cdot \frac{1}{A} \simeq \frac{1}{\text{loop-gain}}$$

for $k = 10$

$$\epsilon = \frac{10}{A}$$

for error $< 1\%$ $\Rightarrow$ $A > 1000$ $\Rightarrow$ $60\,dB$

② Small-signal BW

$$L(s) = \frac{A(s)}{k} \nearrow \frac{A_0}{1+s/\omega_0}$$

$$\frac{V_o}{V_i}(s) = \frac{kL(s)}{1+L(s)}$$

$$= \frac{\dfrac{A_0}{1+s/\omega_0}}{1+\dfrac{A_0/k}{1+s/\omega_0}} = \frac{A_0}{1+\dfrac{s}{\omega_0}+\dfrac{A_0}{k}} = \frac{A_0}{\left(1+\dfrac{A_0}{k}\right)+\dfrac{s}{\omega_0}}$$

$$= \boxed{\frac{A_0}{\left(1+\dfrac{A_0}{k}\right)}} \cdot \frac{1}{1+\dfrac{s}{\underbrace{\omega_0\left(1+\dfrac{A_0}{k}\right)}}}$$

DC gain

$$= A_{CL} \cdot \frac{1}{1 + \frac{s}{\omega_{u,loop}}}$$

$$\omega_{u,loop} = \omega_o \left(1 + \frac{A_o}{k}\right)$$

$$\simeq \omega_o \cdot \frac{A_o}{k} \quad \text{for} \quad \frac{A_o}{k} \gg 1$$

$$= \frac{(\omega_o \cdot A_o)}{k}$$

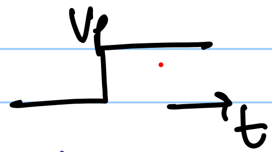
$f_{un} = f_{3dB} \times k = \text{const.}$

GBW tradeoff

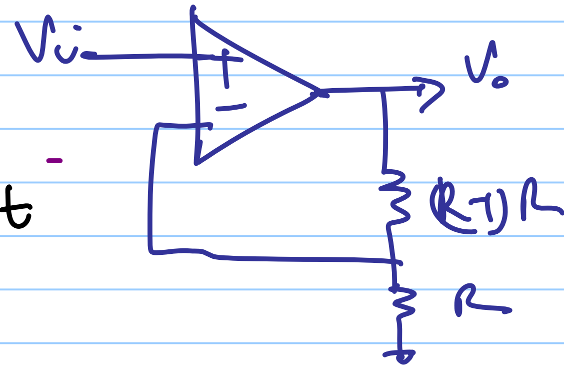
$$= \frac{\omega_{un}}{k}$$

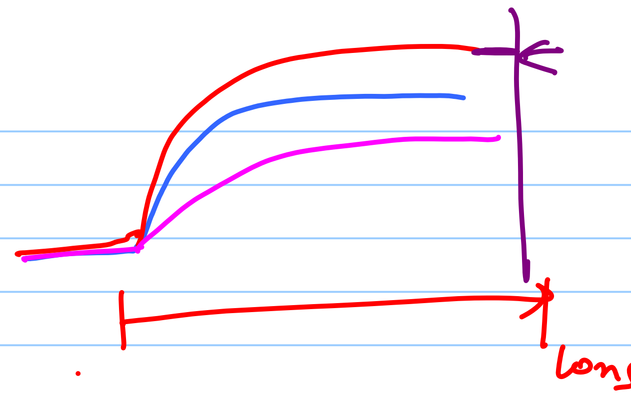
3-dB Bandwidth

③ Settling!



$V_p u(t)$





Assuming linear settling (small-signal)

$$V_{out}(s) = \frac{V_p}{s} \times \frac{k L(s)}{1 + L(s)}$$

linear settling

$$v_{out}(t) = \underbrace{k V_p}_{\text{final value}} \cdot \underbrace{\left(1 - e^{-\omega_{u,loop} t}\right)}_{\text{settling}} \cdot u(t)$$

for 99% accuracy in settling in a time T_s.

$$1 - e^{-\omega_{u,loop} \cdot T_s} = 0.99$$

$$\tau = \frac{1}{\omega_{u,loop}}$$

$$T_{99\%} = \tau \cdot \ln(100)$$

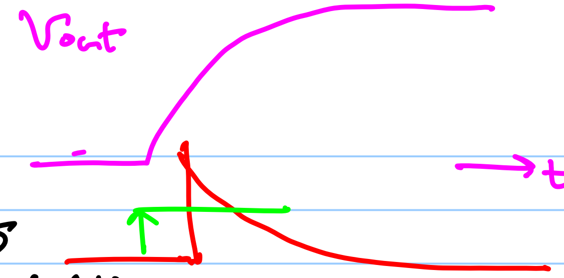
$$= 4.7\tau = \frac{4.7}{\omega_{u,loop}}$$

Settling accuracy $\rightarrow \omega_{u,loop} \rightarrow \omega_{un}$

$k \rightarrow \omega_{un}$

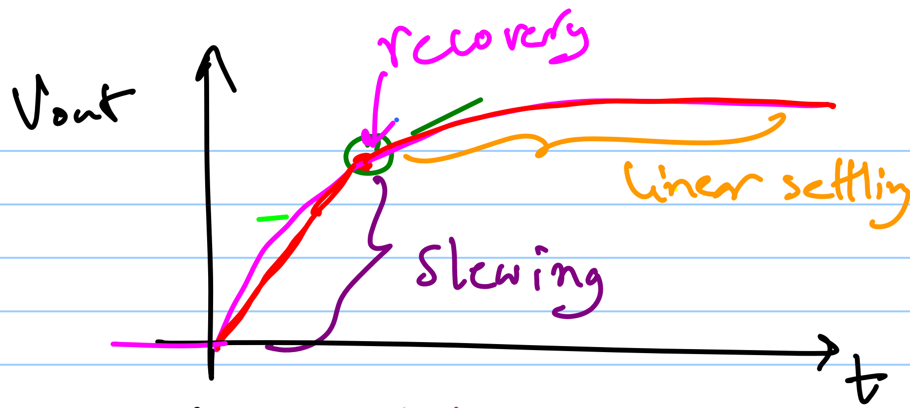
$\omega_{un} \rightarrow \frac{g_{m1}}{C_c}$

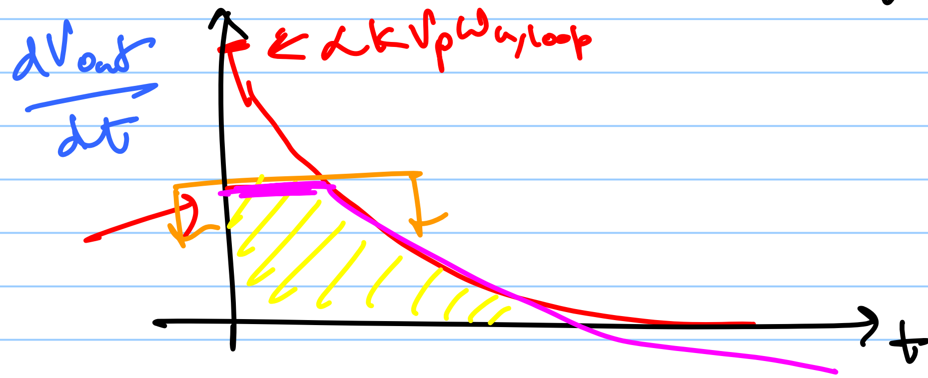
$$V_{out}(t) = kV_p \left(1 - e^{-\omega_{u,loop} t}\right) u(t)$$

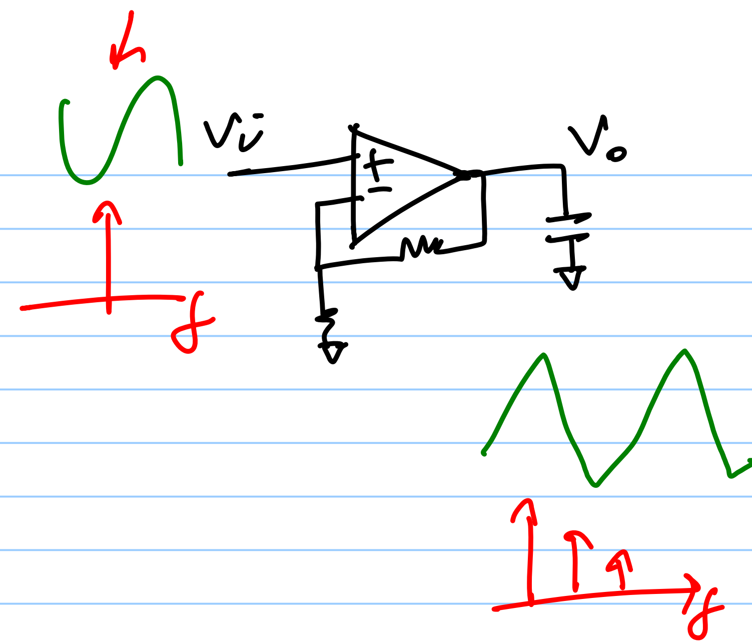


$$\frac{dV_{out}}{dt}(t) = \left(+\, kV_p \cdot \omega_{u,loop} \cdot e^{-\omega_{u,loop} t}\right) u(t)$$

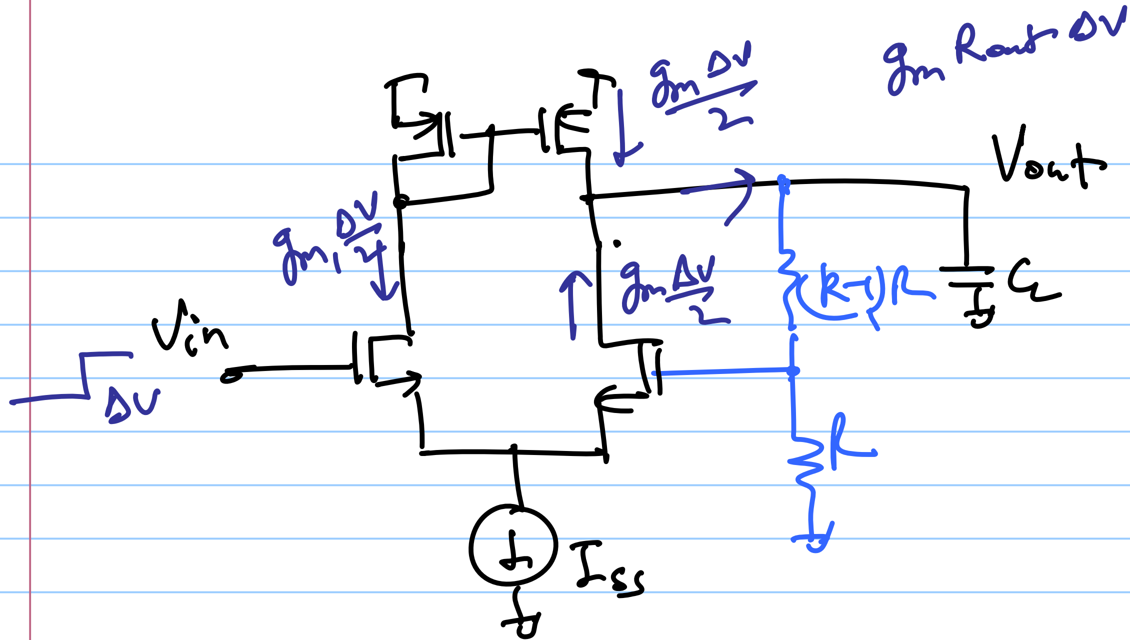
$$I = C_L \cdot \frac{dV_{out}}{dt}$$







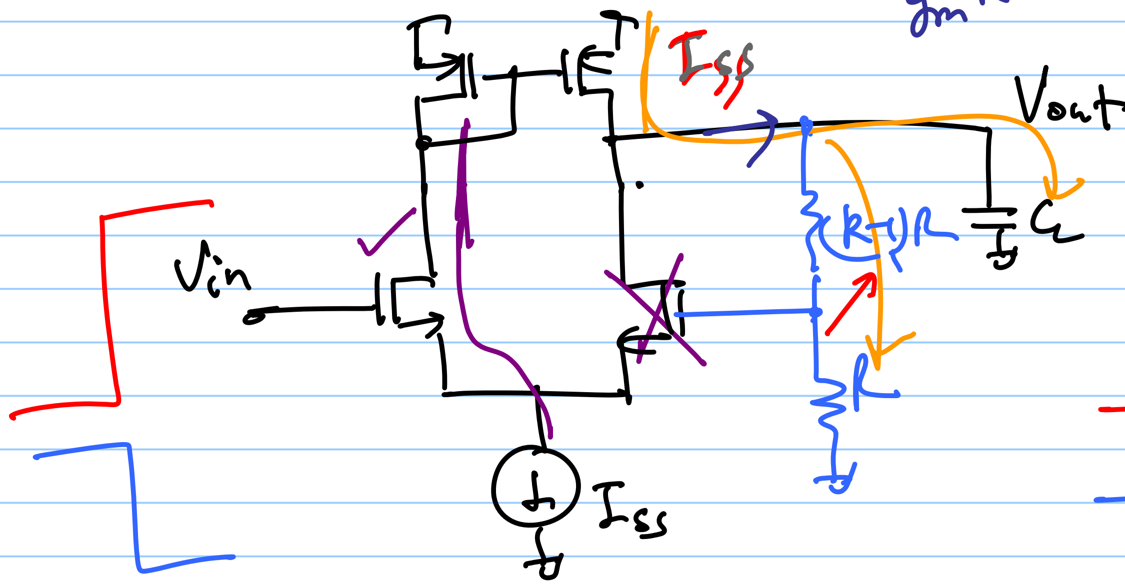
$$SR = \max\left(\frac{dV_{out}}{dt}\right)$$

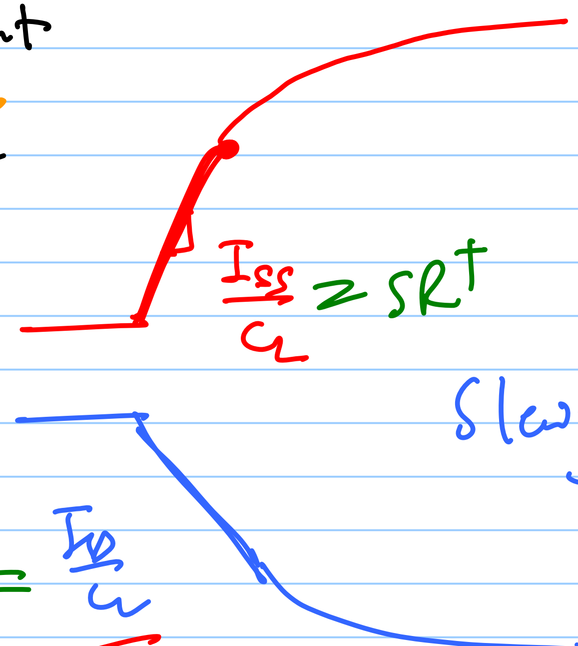


$g_m R_{out} \Delta V$
V_{out}
$\frac{g_m \Delta V}{2}$
$g_{m1} \frac{\Delta V}{2}$
$\frac{g_m \Delta V}{2}$
$(k-1)R$
C_L
V_{in}
ΔV
R
I_{SS}

$R \to \infty$

$g_m R_{out} \Delta V$





$\frac{I_{SS}}{C_L} = SR^{+}$

Slewing

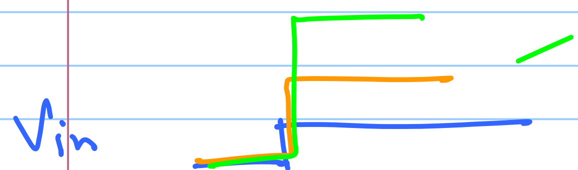
$SR^{-} = \frac{I_{SS}}{C_L}$

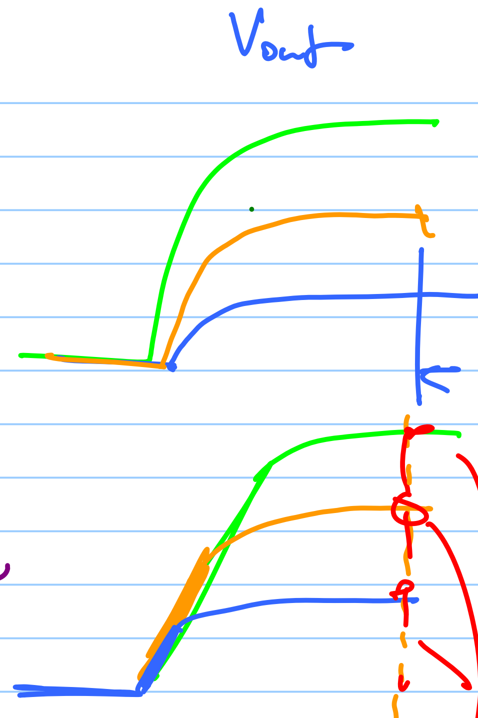
In two-stge
with class-AB
output

$$SR \simeq \frac{I_{SS}}{C_c}$$

finite settling time

$k = 1$





$\left(1-e^{-\omega_{u,loop}\beta T_s}\right)$ ← same settling error

linear settling

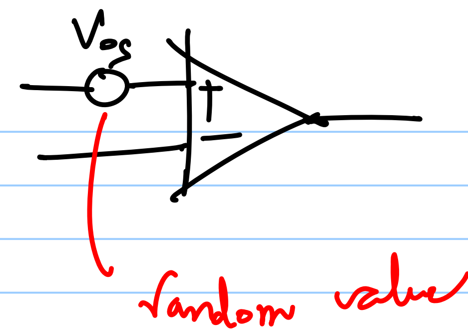
slew-limited settling

final values have input-dependent scaling factor

* Typically for mixed-signal circuits slewing should occur < 25% of settling period

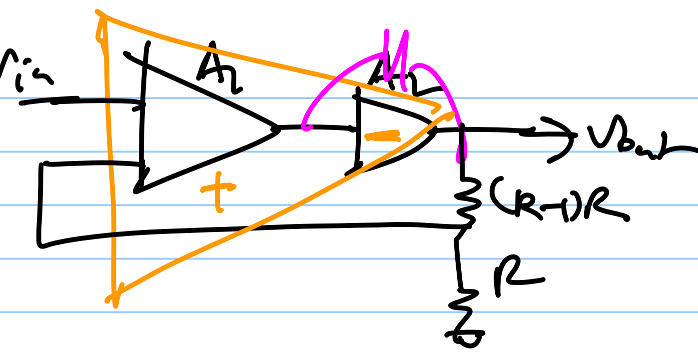
4 random offset ← mismatch

↳ estimate & cancel offset



5 Noise

Standard 2nd order response



with zero cancellation

Closed-loop response

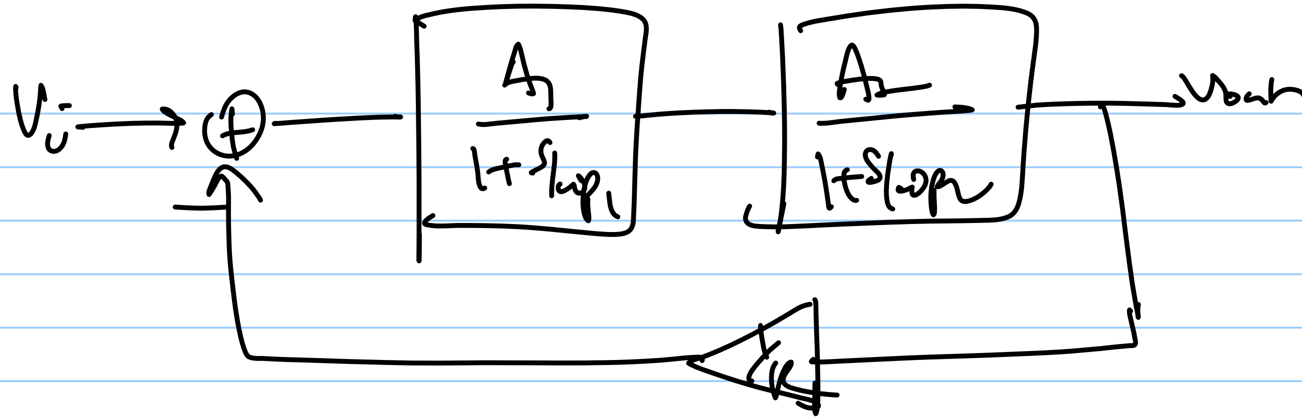
$$\frac{V_o}{V_i}(s) = \frac{k}{\left(\frac{s}{\omega_0}\right)^2 + \left(\frac{s}{\omega_0 Q}\right) + 1}$$

$$= \frac{k}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + 1}$$

damping factor

$$\zeta = \frac{1}{2Q}$$

Quality factor



$$\frac{v_o}{v_i} = \frac{K}{\left(1+\frac{K}{A_0}\right) + s\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)\frac{K}{A_0} + \frac{s^2}{\omega_{p1}\omega_{p2}}\cdot\frac{K}{A_0}}$$

$$\zeta = \frac{1}{2}\sqrt{\frac{K}{A_0}}\left(\sqrt{\frac{\omega_{p2}}{\omega_{p1}}} + \sqrt{\frac{\omega_{p1}}{\omega_{p2}}}\right)$$

for $\quad \frac{\omega_{p_2}}{\omega_{p_1}} = \frac{A_o}{k}$

$$\zeta = \frac{1}{2}\sqrt{\frac{k}{A_o}}\left(\sqrt{\frac{A_o}{k}} + \sqrt{\frac{k}{A_o}}\right)$$

$$= \frac{1}{2}x\left(x + \frac{1}{x}\right) \qquad x = \sqrt{\frac{A_o}{k}}$$

$$\leq 1 \qquad x + \frac{1}{x} \leq 2x$$

$\zeta > 1$ overdamped

$= 1$ critically damped

< 1 underdamped

faster settling is for $\zeta = \frac{1}{\sqrt{2}}$

$\Rightarrow \phi_m = 63.6^{\circ}$

for $\phi_m = 60^{\circ}$

$\hookrightarrow \zeta = 0.66$