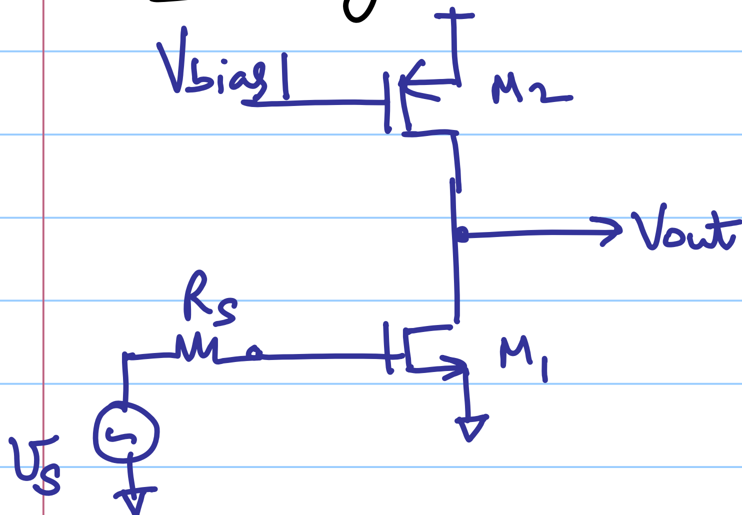
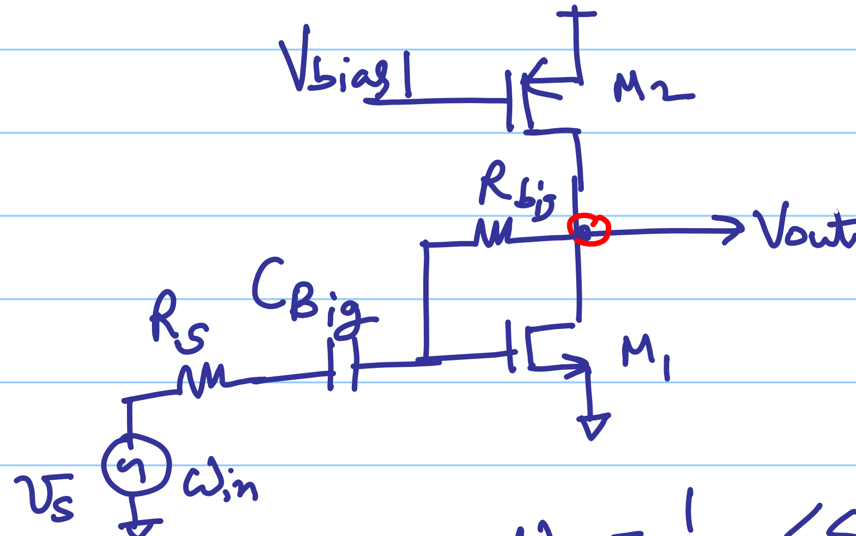
ECE 511 - Lecture 16

Note Title 3/12/2015

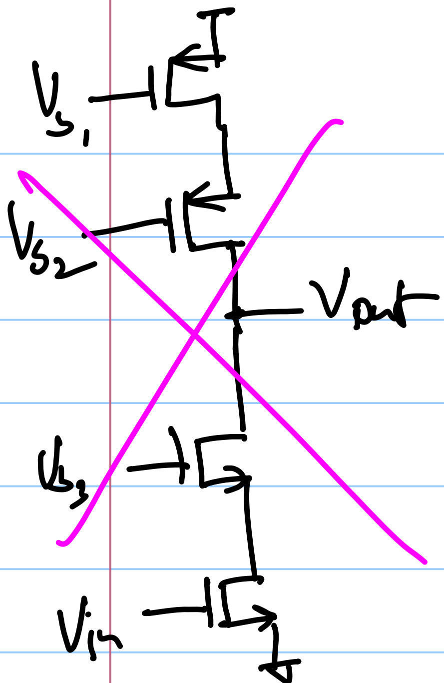
CS Stage



Exact Biasing (Simulation)

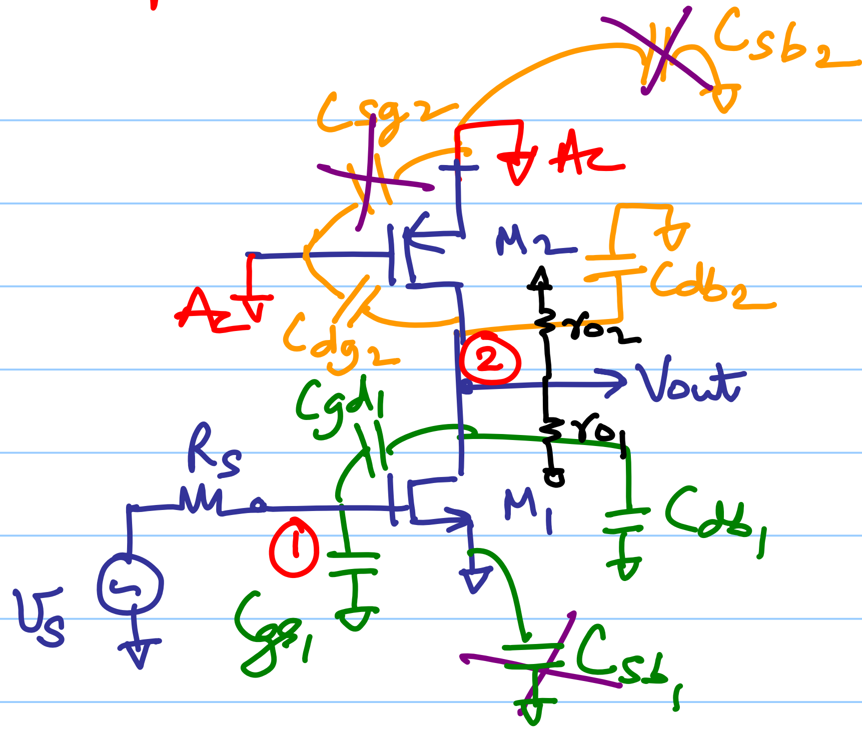


$$\omega_x = \frac{1}{RC} \ll \omega_{in}$$



V_{b_1}
V_{b_2}
V_{out}
V_{b_3}
V_{in}

AC picture with parasitic caps



low-frequency (DC) gain

$$A_v = -g_{m_1}(r_{o_1} \| r_{o_2})$$

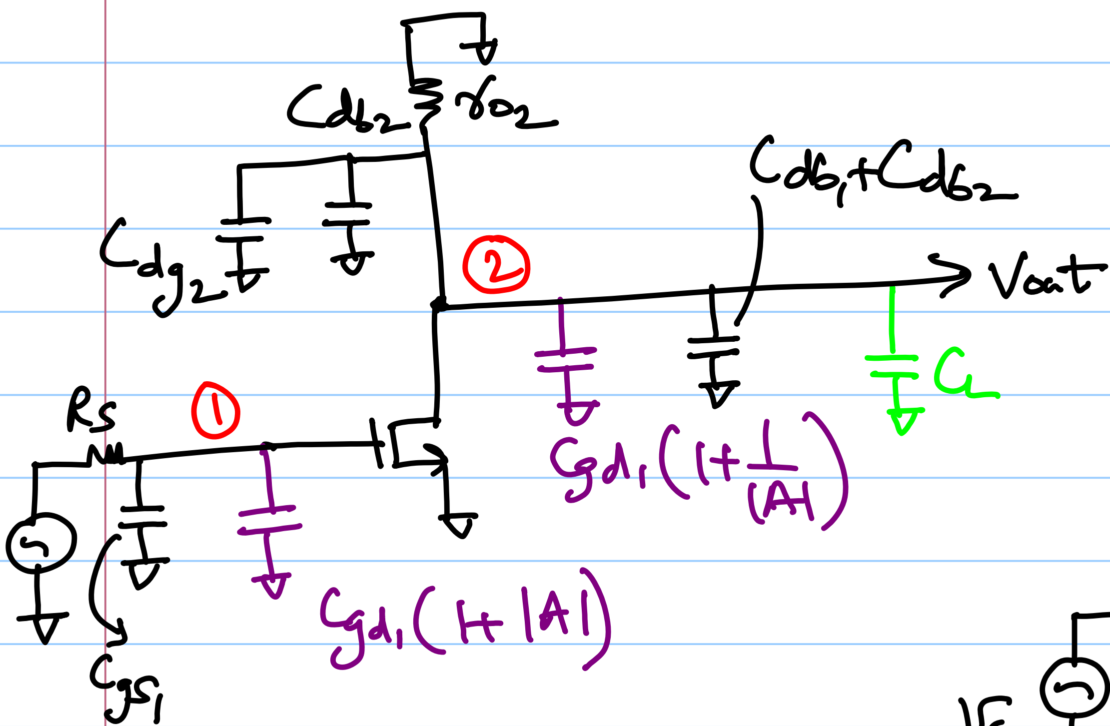
"grounded caps"

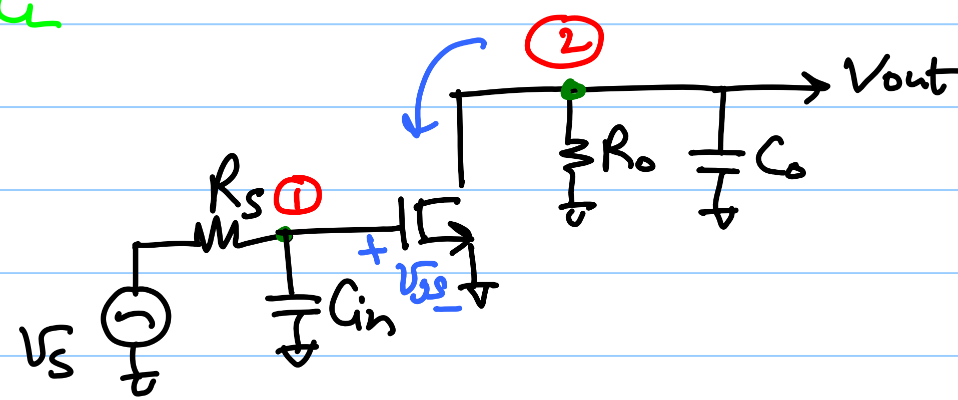
$C_{gs_1}, C_{db_1}, C_{dg_2}, C_{db_2}$

$C_{gd_1} \Rightarrow$ between input & output

(Miller Cap)

After Miller Simplification





$$R_o = r_{o_1} \| r_{o_2}$$

$$C_{in} = C_{gs_1} + C_{gd_1}(1+|A|)$$

$$C_o = C_{gd_1}\left(1+\frac{1}{|A|}\right) + \underbrace{C_{db_1}+C_{db_2}}_{C_{db}}$$

$$v_{gs} = \frac{v_s}{1+sR_sC_{in}}$$

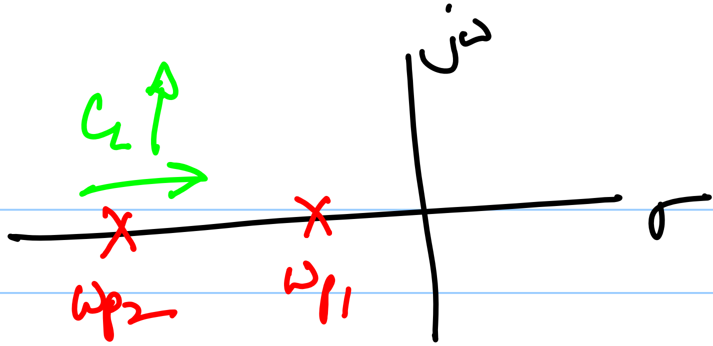
$$v_{out} = i_d\left(R_o \| \frac{1}{sC_o}\right) = -\frac{R_o}{(1+sR_oC_o)} \cdot g_{m1} \cdot \underbrace{\frac{v_s}{(1+sR_sC_{in})}}_{v_{gs}}$$

$$\Rightarrow \frac{v_{out}}{v_s}(s) = \frac{-g_{m1}(r_{o1} \| r_{o2})}{(1+sR_oC_o)(1+sR_sC_{in})}$$

$$\boxed{\frac{v_{out}}{v_s}(s) = \frac{A_v}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}}$$

$$\omega_{p1} = \frac{1}{R_sC_{in}} = \frac{1}{R_s[C_{gs1} + (1+|A|)C_{gd1}]}$$

$$\omega_{p2} = \frac{1}{R_oC_o} = \frac{1}{(r_{o1} \| r_{o2})(C_{gd1} + C_{db} + C_L)}$$

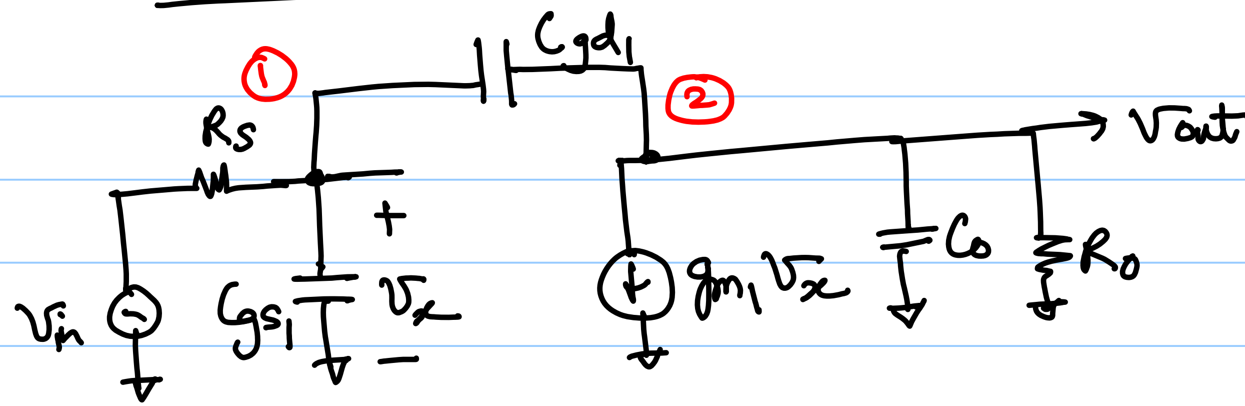


This analysis misses a zero (ω_z)

Equivalent Circuit

$R_o = r_{o1} \,||\, r_{o2}$

$C_o = C_{db} + C_{dg_2}$



@ 1 $\Rightarrow$

$$\frac{V_x - V_{in}}{R_S} + V_x C_{gs_1} s + (V_x - V_{out}) C_{gd_1} s = 0$$

@ 2 $\Rightarrow$

$$(V_{out} - V_x) C_{gd_1} \cdot s + g_{m_1} V_x + V_{out}\left(\frac{1}{R_o} + C_o \cdot s\right) = 0$$

Solve for V_x & substitute in 2

$$V_x = -\frac{V_{out}\left(C_{gd_1} s + \frac{1}{R_o} + sC_o\right)}{g_{m_1} - sC_{gd_1}}$$

$$\Rightarrow \frac{V_{out}}{V_{in}}(s) = \frac{(sC_{gd_1} - g_{m_1})R_o}{\boxed{R_S R_o \xi} s^2 + \boxed{[R_S(1+g_{m_1}R_o)C_{gd_1} + R_S C_{gs_1} + R_o(C_{gd_1}+C_o)]} s + \boxed{1}}$$

$$\xi = C_{gs_1} C_{gd_1} + C_{gs_1} C_o + C_{gd_1} C_o$$

1

2nd-order transfer function

$$N(s) = 0$$

$$\Rightarrow \quad sC_{gd_1} - g_{m_1} = 0$$

$$s = \frac{g_{m_1}}{C_{gd_1}}$$

R.H.P. zero

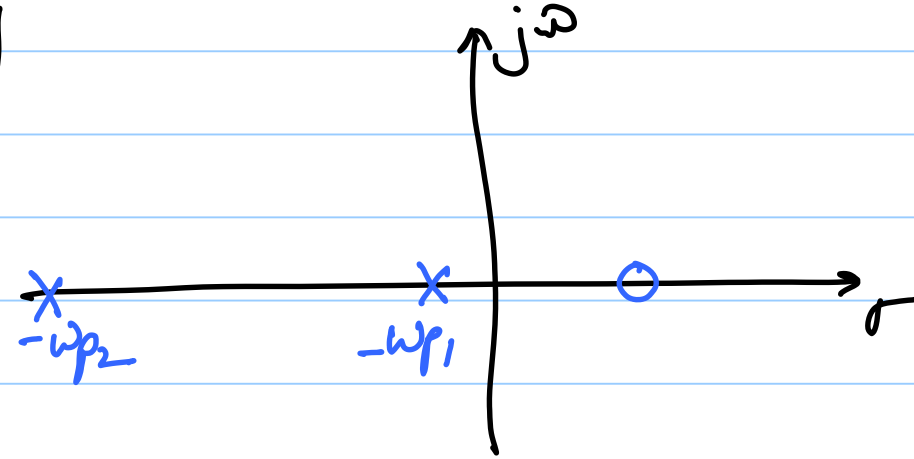
$$\Rightarrow \boxed{\omega_z = +\frac{g_{m_1}}{C_{gd_1}}}$$

T.F. zero

$$f_z = \frac{g_{m_1}}{2\pi C_{gd_1}}$$

Assume that the poles are real and far apart

$$|\omega_{p_1}| \ll |\omega_{p_2}|$$



$$D(s) \triangleq \left(\frac{s}{\omega_{p_1}}+1\right)\left(\frac{s}{\omega_{p_2}}+1\right)$$

$$= \frac{s^2}{\omega_{p_1}\omega_{p_2}} + \left(\frac{1}{\omega_{p_1}}+\frac{1}{\omega_{p_2}}\right)s + 1$$

2

If $\omega_{p_1} \ll \omega_{p_2} \Rightarrow \frac{1}{\omega_{p_1}} \gg \frac{1}{\omega_{p_2}}$

Coeffⁿ of 's' term

$$\frac{1}{\omega_{p_1}} + \frac{1}{\omega_{p_2}} \simeq \frac{1}{\omega_{p_1}}$$

from ① & ②

$$\omega_{p_1} = \frac{1}{R_s \underbrace{(1+g_{m_1}R_o)\,C_{gd_1}}_{\text{Miller term } C_{in}} + R_s C_{gs_1} + \underbrace{R_o\left(C_{gd_1} + C_o\right)}_{\text{Extra term}}}$$

This is the contribution of node ② in ω_{p_1}

coeff of 's^2' $= \dfrac{1}{\omega_{p_1} \cdot \omega_{p_2}}$

$$\Rightarrow \quad \omega_{p_2} = \frac{1}{\omega_{p_1}} \cdot \frac{1}{R_S R_o \xi}$$

$$= \frac{1}{\omega_{p_1}} \cdot \frac{1}{R_S R_o \left[C_{gs_1} C_{gd_1} + C_{gs_1} C_o + C_{gd_1} C_o \right]}$$

$$= \frac{R_S(1 + g_m R_o) \cancel{C_{gd_1}} + R_S C_{gs_1} + R_o(\cancel{C_{gd_1}} + C_o)}{R_S R_o \left[C_{gs_1} C_{gd_1} + C_{gs_1} C_o + C_{gd_1} C_o \right]}$$

If $\quad C_{gs_1} \gg (1 + g_m R_o) C_{gd_1} + \dfrac{R_o}{R_S} \left(C_{gd_1} + \underline{\underline{C_o}} \right)$

(not always true)

$$\omega_{p2} = \frac{\cancel{R_s} \cancel{C_{gs1}}}{\cancel{R_s} R_o \left[\cancel{C_{gs1}} C_{gd1} + \cancel{C_{gs1}} C_o + \cancel{C_{gd1} C_o} \right]}$$

$$\omega_p \simeq \frac{1}{R_o (C_{gd1} + C_o)}$$

Valid only when C_{gs1} dominates the response

* not always true

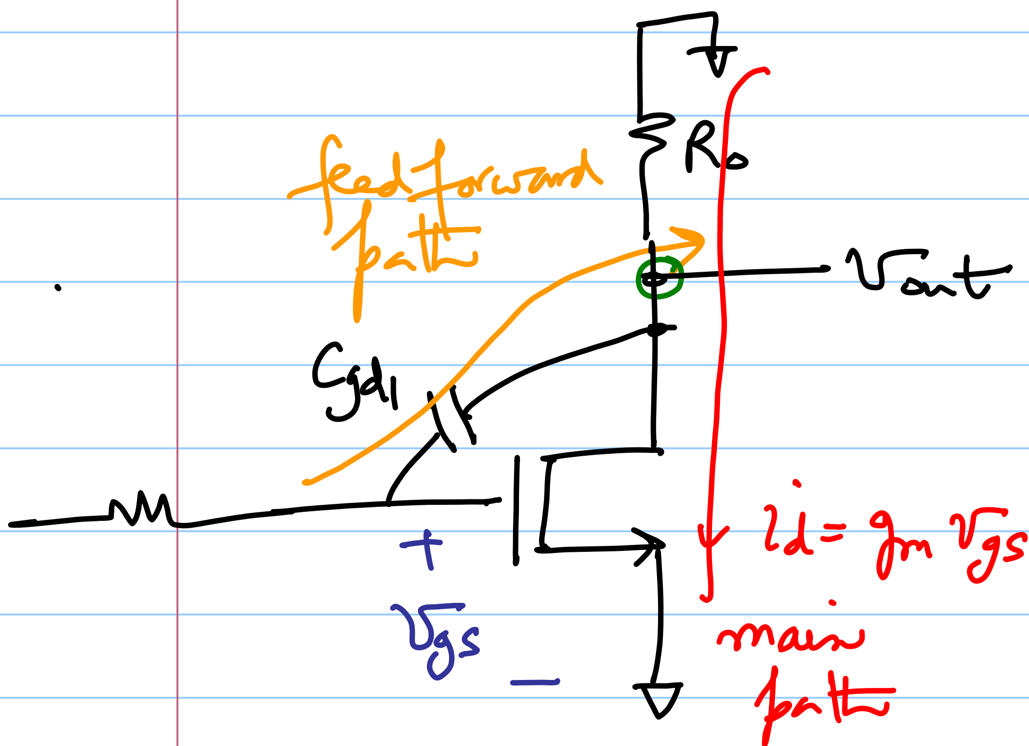
$$\frac{V_{out}}{V_{in}}(s) = \frac{A_v \left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$

$\boxed{\text{zero}}$

$$\omega_z = \frac{g_{m1}}{C_{gd1}}$$

"RHP"

$$s = -\frac{g_{m1}}{C_{gd1}}$$

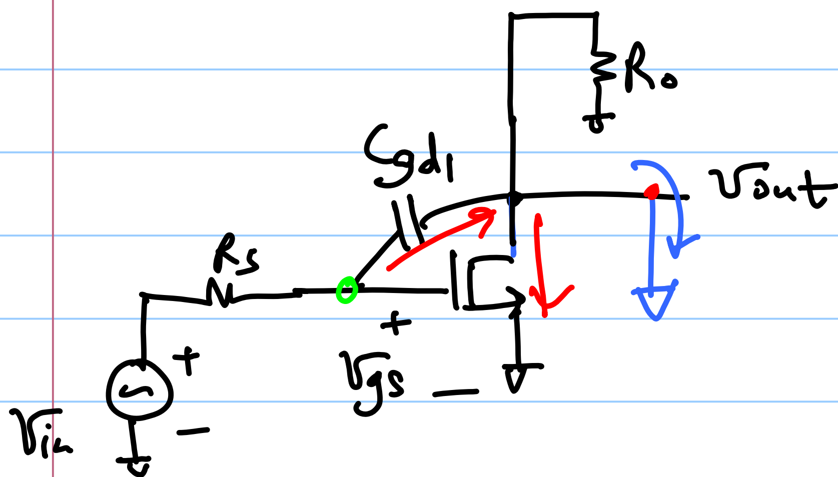


C_{gd1} provides a feedforward path that conducts the input signal to the output at high frequencies

↳ signal thru C_{gd1} adds to the main path in the opposite phase

↳ RHP zero

Quick Derivation of z_{∞}



@ $s = s_z = j\omega_z$

$$\frac{v_{out}}{v_{in}}(s) = 0$$

$\Rightarrow$ $v_{out}(s_z) = 0$

$\Rightarrow$ o/p can be shorted to ground

Currents through M_1 & C_{gd_1} are equal and opposite

$$\Rightarrow \quad v_{gs} C_{gd_1} \cdot s_z = g_m v_{gs}$$

$$s_z = \frac{g_{m_1}}{C_{gd_1}} \Rightarrow$$

zero is at $\frac{g_{m1}}{C_{gd1}}$

@ $s = j\omega_z$

$$\frac{V_{out}}{V_{in}}(s) = \frac{\left(1 - \frac{s}{\omega_z}\right)}{D(s)}$$

$V_{out}(j\omega) = 0$