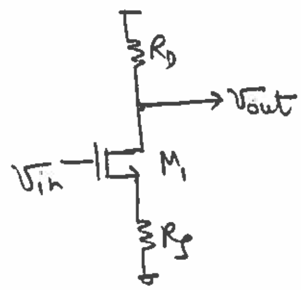
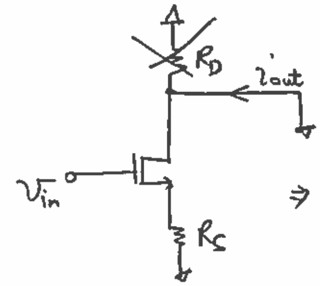
HW # 6

Problem 1:

(a)

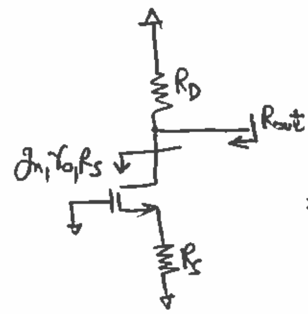


Find G_m



$$\Rightarrow G_m = \frac{g_{m1}}{1+g_{m1}R_S}$$

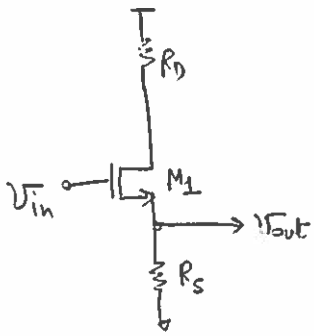
R_{out}



$$\Rightarrow R_{out} = g_{m1}r_{o1}R_S \parallel R_D$$
$$\simeq R_D$$

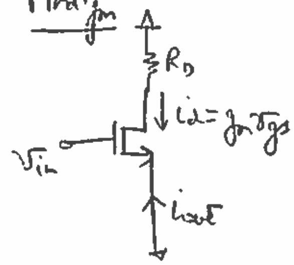
$$\Rightarrow \quad A_v = -G_m R_{out} = \frac{-g_{m1}R_D}{1+g_{m1}R_S}$$

(b)



(i) Using $A_v = G_m R_{out}$ Method:

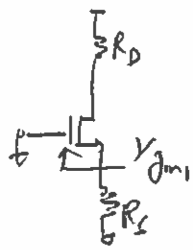
Find G_m:



$\Rightarrow i_{out} = -g_{m1} v_{gs} = -g_{m1} v_{in}$

$\Rightarrow \boxed{G_m = -g_{m1}}$

Find R_{out}:



$\gamma = 0$

Assuming $R_D \ll r_o$, impedance looking up the source $= \frac{1}{g_{m1}}$

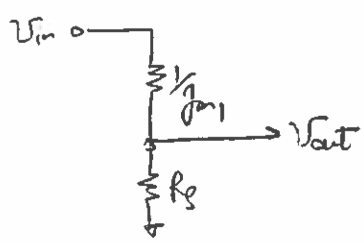
$\Rightarrow R_{out} = \frac{1}{g_{m1}} \| R_s$

$$\Rightarrow A_v = -G_m R_{out} = +g_{m1} \times \frac{1}{g_{m1}} \| R_s$$

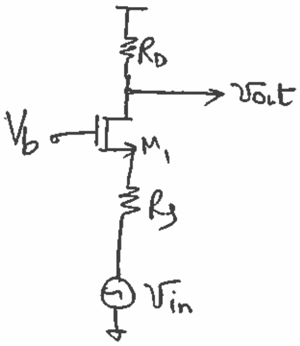
$$= \frac{\cancel{g_{m1}} \cdot \frac{1}{\cancel{g_{m1}}} \cdot R_s}{\frac{1}{g_{m1}} + R_s} = +\frac{R_s}{\frac{1}{g_{m1}} + R_s}$$

* Also, it can be observed that V_{out} is resulting due to a voltage divider formed using $1/g_{m1}$ & R_s

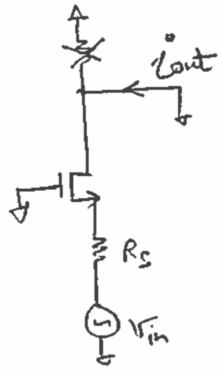
$\triangleq$

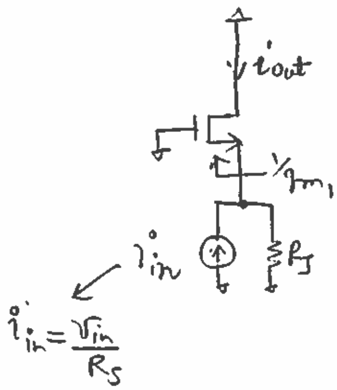


(c)



First, G_m:





$\Rightarrow$ i_{in} divides between $\frac{1}{g_{m1}}$ & R_S

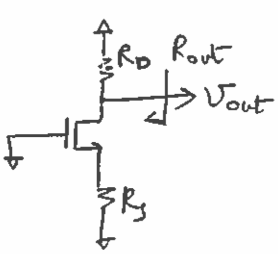
$$\Rightarrow i_{out} = -i_{in} \times \frac{R_S}{\frac{1}{g_{m1}} + R_S}$$

$$= -\frac{v_{in}}{\cancel{R_S}} \times \frac{\cancel{R_S}}{\frac{1}{g_{m1}} + R_S}$$

$$= -v_{in} \times \frac{1}{\frac{1}{g_{m1}} + R_S}$$

$$\Rightarrow \boxed{G_m = -\frac{1}{\frac{1}{g_{m1}} + R_S}}$$

Find R_{out}:

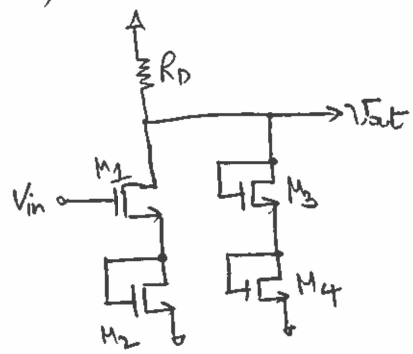


$$R_{out} = R_D \,||\, (g_{m1} r_{o1} R_S)$$
$$\simeq R_D$$

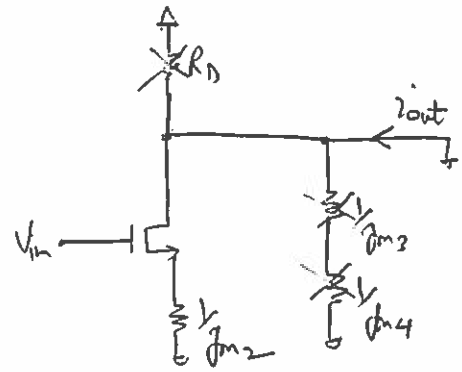
$$\Rightarrow A_v = -G_m R_{out} = +\frac{R_D}{\frac{1}{g_{m1}} + R_S} = +\frac{g_{m1} R_D}{1 + g_{m1} R_S}$$

cross check: for $R_S = 0$, $A_v = +g_{m1} R_D$ ✓

(d)

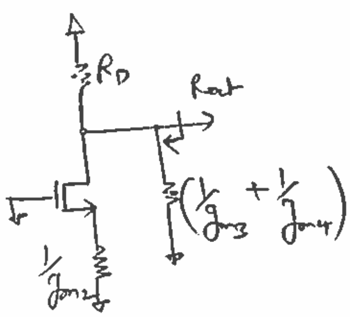


Find G_m:



$$\Rightarrow \quad G_m = \frac{g_{m1}}{1 + g_{m1} \times \frac{1}{g_{m2}}} = \boxed{\frac{g_{m1} g_{m2}}{g_{m1} + g_{m2}}}$$

Find R_{out}:

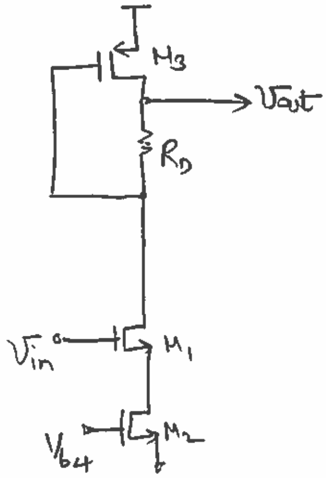


$$R_{out} = R_D \,\|\, \Big(\underbrace{g_{m1} r_{o1} \frac{1}{g_{m2}}}_{\text{large term}}\Big) \,\|\, \left(\frac{1}{g_{m3}} + \frac{1}{g_{m4}}\right)$$

$$\simeq R_D \,\|\, \left(\frac{1}{g_{m3}} + \frac{1}{g_{m4}}\right) \simeq \frac{1}{g_{m3}} + \frac{1}{g_{m4}}$$

$$\Rightarrow \quad A_1 = -G_m R_{out} \simeq -\frac{g_{m1} g_{m2}}{(g_{m1} + g_{m2})} \times \left(\frac{1}{g_{m3}} + \frac{1}{g_{m4}}\right)$$

(e)



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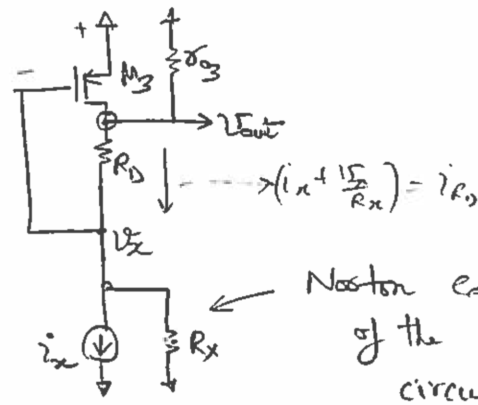


Norton equivalent of the input side of the circuit

$$i_x = \frac{g_{m1} v_{in}}{1 + g_{m1} r_{o2}}, \quad R_x = g_{m1} r_{o1} r_{o2}$$

<u>KCL @ v_x</u>

$$v_x + \left(i_x + \frac{v_x}{R_x}\right) R_D = v_{out} \longrightarrow (1)$$

<u>KCL @ v_{out}</u>

$$\underbrace{\left(g_{m3} v_x + \frac{v_{out}}{r_{o3}}\right)}_{\text{current due to } M_3} + \underbrace{\left(i_x + \frac{v_x}{R_x}\right)}_{i_{R_D}} = 0 \longrightarrow (2)$$

Solve (2) for v_x $\Rightarrow$ $v_x = -\dfrac{i_x + \frac{v_{out}}{r_{o3}}}{g_{m3} + 1/R_x}$

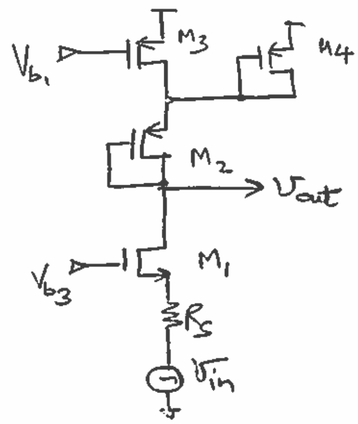
Substitute v_x in (1), and solve

$$\frac{v_{out}}{v_{in}} = \frac{\frac{g_{m1}}{1 + g_{m1} r_{o2}}\left(g_{m3} R_D - 1\right)}{\frac{1}{R_x} + g_{m3} + \frac{1}{r_{o3}}\left(1 + \frac{R_D}{R_x}\right)}$$

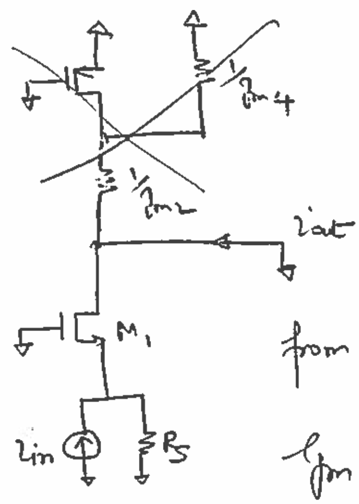
$$\simeq \frac{g_{m1}}{1 + g_{m1} r_{o2}} \times \frac{\left(g_{m3} R_D - 1\right)}{g_{m3} + 1/r_{o3}} \simeq \underbrace{\frac{g_{m1}}{1 + g_{m1} r_{o2}} \times \frac{\left(g_{m3} R_D - 1\right)}{g_{m3}}}$$

Ans.

(f)



Find G_m :



from part (c)

$$G_m = \frac{-1}{1/g_{m1} + R_S}$$

find R_{out} :

$$R_{down} = g_{m1} r_{o1} R_S$$

$$R_{up} = \left(r_{o3} \,\|\, \frac{1}{g_{m4}}\right) + \frac{1}{g_{m2}}$$

$$\simeq \frac{1}{g_{m2}} + \frac{1}{g_{m4}}$$

$$\Rightarrow R_{out} = (g_{m1} r_{o1}) R_S \,\|\, \left(\frac{1}{g_{m2}} + \frac{1}{g_{m4}}\right)$$

$$= \frac{1}{g_{m2}} + \frac{1}{g_{m4}}$$

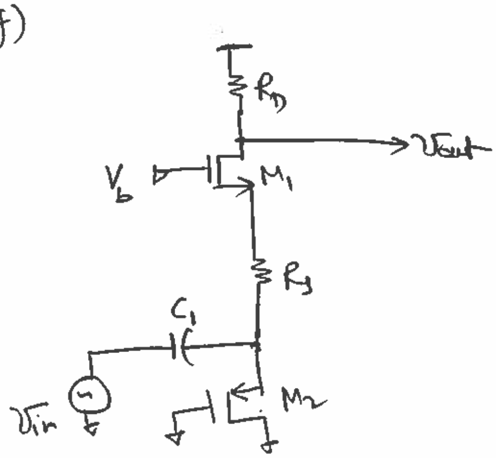
$$\Rightarrow A_v = -G_m R_{out}$$

$$= + \frac{g_{m1}}{1 + g_{m1} R_S} \times \left(\frac{1}{g_{m2}} + \frac{1}{g_{m4}}\right)$$

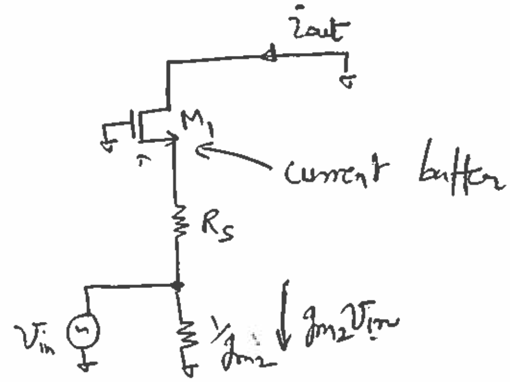
Ans.

Common-Gate Topology

(g)



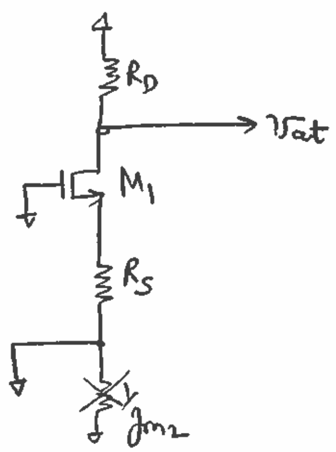
Find G_m



$\Rightarrow \quad i_{out} = g_{m2} v_{in}$

$\Rightarrow \quad G_m = g_{m2}$

Find R_{out}:

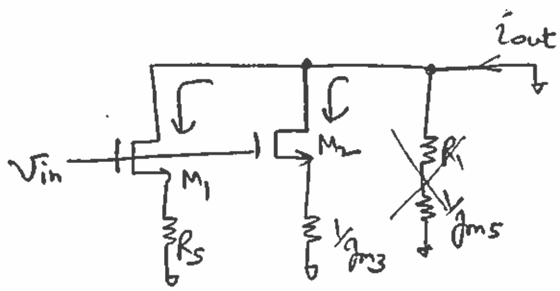


$\Rightarrow \quad R_{out} = R_D \| (g_{m1} r_{o1} R_S)$

$\simeq R_D$

$\Rightarrow \quad A_V = -G_m R_{out} \simeq \underline{-g_{m2} R_D}$

(h) <u>find G_m :</u>



$$\Rightarrow \quad i_{out} = \left(\frac{g_{m1}}{1+g_{m1}R_S} + \frac{g_{m2}}{1+\frac{g_{m2}}{g_{m3}}} \right) V_{in}$$

$$\Rightarrow \quad G_m = \frac{g_{m1}}{1+g_{m1}R_S} + \frac{g_{m2}\, g_{m3}}{g_{m2}+g_{m3}}$$

<u>Find R_{out} :</u>

$$R_{up} = g_{m4} r_{o4} R_D$$

$$R_{down} = (\underbrace{g_{m1} r_{o1}}_{\text{Large}} R_S) \,\|\, (\underbrace{g_{m2} r_{o2}}_{\text{Large}} \cdot \frac{1}{g_{m3}}) \,\|\, (R_1 + \frac{1}{g_{m5}})$$

$$\simeq R_1 + \frac{1}{g_{m5}}$$

$$\Rightarrow$$

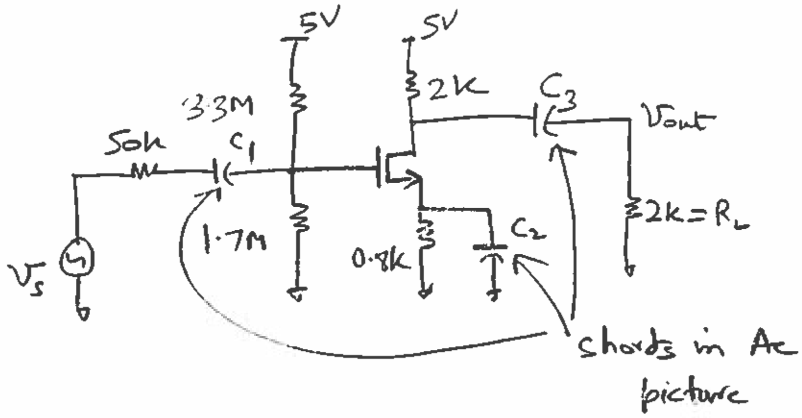
$$A_v = -G_m R_{out} \simeq -\left[\frac{g_{m1}}{1+g_{m1}R_S} + \frac{g_{m2} g_{m3}}{g_{m2}+g_{m3}} \right] \left(R_1 + \frac{1}{g_{m5}} \right)$$

Ans.

Problem 2

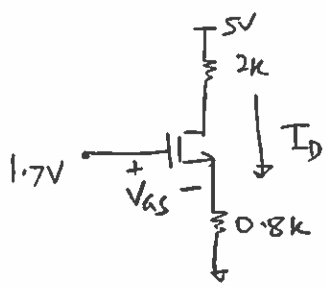
$kP_n = 500\,\frac{\mu A}{V^2}$

$V_{THN} = 0.7V$



(a) DC picture

Given $I_D = 1mA$



$\Rightarrow V_{GS} = 1.7 - 1m \times 0.8k$

$\boxed{V_{GS} = 0.9V}$

$\Rightarrow I_D = 1mA = \frac{kP_n}{2}\left(\frac{W}{L}\right)(V_{GS} - 0.7)^2$

$\Rightarrow \left(\frac{W}{L}\right) = \frac{1m \times 2}{500\,\frac{\mu A}{V^2}} \times \frac{1}{(0.2)^2}$

$= \frac{4}{(0.4)^2} = 100$

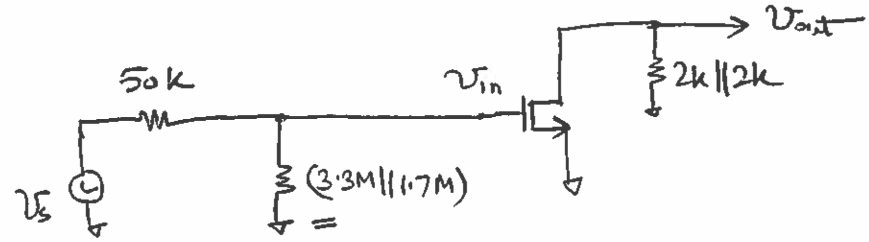
$\Rightarrow \boxed{\frac{W}{L} = 100}$

$V_{DS} = 5 - (2K + 0.8k)\,1mA$

$= 5 - 2.8$

$\Rightarrow \boxed{V_{DS} = 2.2V}$

A2 picture



$$g_m = \sqrt{2\beta I_D} = \frac{2 I_D}{V_{ov}}$$

$$= \frac{2 \times 1\,mA}{0.2\,V}$$

$$\boxed{g_m = 10\,\frac{mA}{V}}$$

(b)

$$\frac{v_{out}}{v_{in}} = -g_m (r_o \| R_D \| R_L) = -g_m \times 1k$$

$$= -\ 10\,\frac{m\cancel{A}}{\cancel{V}} \times \cancel{1}k\Omega$$

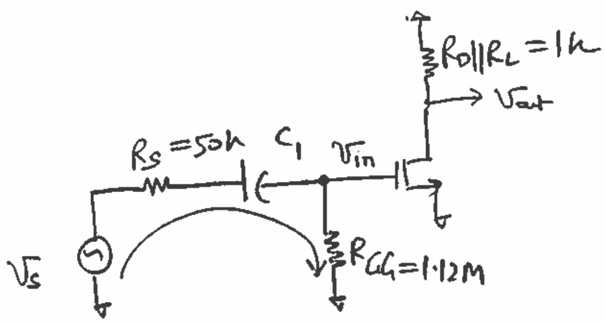
$$= -10$$

$\Rightarrow$

$$\frac{v_{out}}{v_s} = -\frac{1.122M}{50k + 1.122M} \times 10$$

$$\boxed{A_v = -9.573}$$

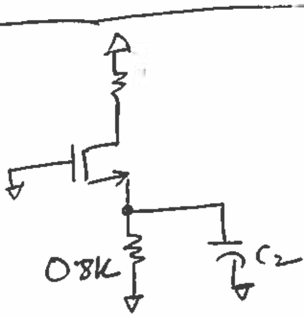
(c)



Make suitable Approximations here

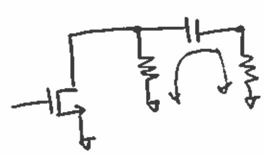
$$\omega_{P_1} = \frac{1}{(R_s + R_{GG})\, C_1} \leq \frac{\omega_{in}}{10} = 10$$

$$\Rightarrow \quad C_1 \geq \frac{1}{10 \times (50k + 1.122M)} = 8.53 \times 10^{-8} F = \underline{85.3\,nF}$$



$$\frac{1}{0.8k\, C_2} \leq 10$$

$$\Rightarrow C_2 \geq \underline{125\mu F}$$



$$\frac{1}{(2k+2k)\, C_3} \leq 10$$

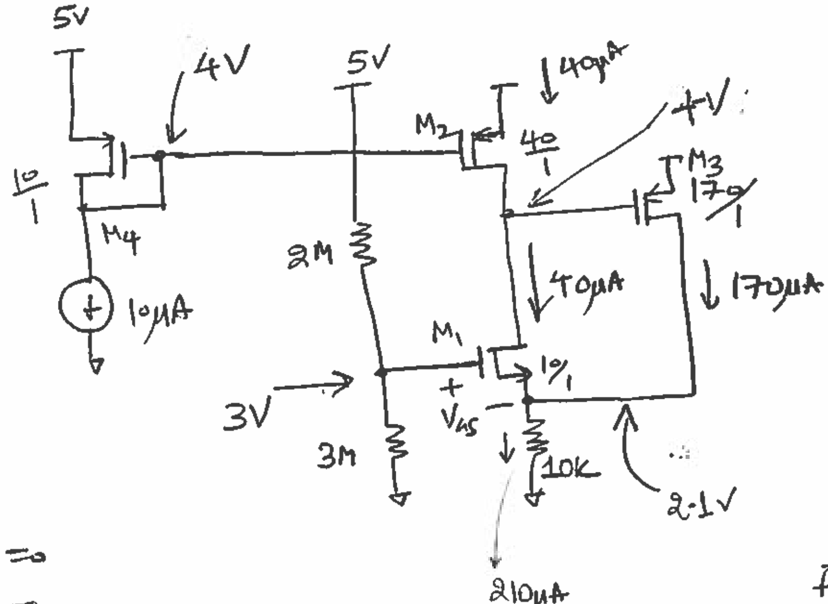
$$\Rightarrow C_3 \geq \underline{25\mu F}$$

Problem 3

$KP_n = 250 \frac{\mu A}{V^2}$, $V_{THN} = 0.7$

$KP_p = 50 \frac{\mu A}{V^2}$, $V_{THP} = 0.8$

(a) DC picture:



Here $\lambda = 0$

M1:
for $I_D = 40\mu A$, $\left(\frac{W}{L}\right) = 10$

$\Rightarrow V_{GS_1} = \sqrt{\frac{2I_D}{\beta}} + V_{THN}$

$= 0.9V$

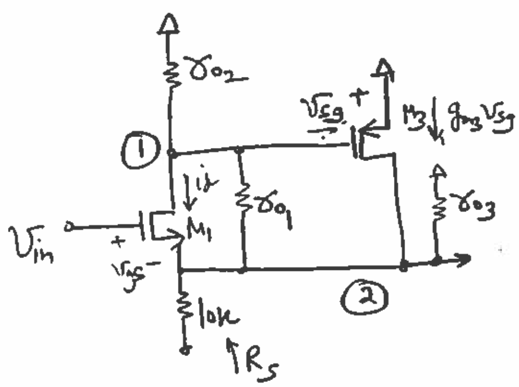
$\Rightarrow \quad V_{GS} = 3 - 2.1V = \boxed{0.9V}$

$V_{DS} = 4.0V - 2.1V$

$\Rightarrow \boxed{V_{DS} = 1.9V}$

feedback stabilizes gate of M3 at 4V such that it sinks 170μA to set 40μA in M1

(b)



$$\frac{v_1}{r_{o2}} + \frac{(v_1 - v_2)}{r_{o1}} + g_{m_1}(v_{in} - v_2) = 0$$

$$\Rightarrow \frac{v_1}{r_{o1}||r_{o2}} - v_2\left(g_{m_1} + \frac{1}{r_{o1}}\right) = -g_{m_1} v_{in} \longrightarrow ①$$

$$\frac{v_2}{r_{o3}} + \frac{v_2}{R_s} + \frac{(v_2 - v_1)}{r_{o1}} - g_{m_3} \underset{-v_1}{v_{sg}} - g_{m_1}(v_{in} - v_2) = 0$$

$$\Rightarrow \frac{v_2}{r_{o3}||R_s||r_{o1}} - g_{m_1} v_2 + \left(g_{m_3} - \frac{1}{r_{o1}}\right) v_1 = g_{m_1} v_{in} \longrightarrow ②$$

$$v_1 = v_2\left(g_{m1} + \frac{1}{r_{o1}}\right) r_{o1}\|r_{o2} - g_{m1} r_{o1}\|r_{o2} \cdot v_{in}$$

plug into ②

$$v_2\left[\frac{1}{r_{o1}\|r_{o3}\|R_s} - g_{m2}\right] + \left(g_{m3} - \frac{1}{r_{o1}}\right)\left[v_2 \times \left(g_{m1} + \frac{1}{r_{o1}}\right) r_{o1}\|r_{o2} - g_{m1} r_{o1}\|r_{o2}\, v_{in}\right] = g_{m1} v_{in}$$

$$\Rightarrow \quad v_2\left[\frac{1}{r_{o1}\|r_{o3}\|R_s} - g_{m2} + \left(g_{m3} - \frac{1}{r_{o1}}\right)\left(g_{m1} + \frac{1}{r_{o1}}\right) r_{o1}\|r_{o2}\right] = \left[g_{m1} + g_{m1} r_{o1}\|r_{o2}\left(g_{m3} - \frac{1}{r_{o1}}\right)\right] v_{in}$$

$$\Rightarrow \quad \frac{v_{out}}{v_{in}} = \frac{v_2}{v_{in}} = \frac{g_{m1}\left[1 + \left(g_{m3} - \frac{1}{r_{o1}}\right) r_{o1}\|r_{o2}\right]}{\left(\frac{1}{r_{o1}\|r_{o3}\|R_s} - g_{m2}\right) + \left(g_{m3} - \frac{1}{r_{o1}}\right)\left(g_{m1} + \frac{1}{r_{o1}}\right) r_{o1}\|r_{o2}} \longrightarrow ③$$

Calculations:

M_1: $g_{m1} = 0.4 \frac{mA}{V} \Rightarrow r_{o1} = 250\,k\Omega$

M_2: $g_{m} = 0.4 \frac{mA}{V} \Rightarrow r_{o2} = 250\,k\Omega$

M_3: $g_{m3} = 1.7 \frac{mA}{V} \Rightarrow r_{o3} = 58.82\,k\Omega$

plug these values into ③