Note Title 1/29/2013

ECE 511 - Lecture 3

$\lambda = 0$

AC analysis:

① Draw the DC "picture"
↳ short AC voltage
open AC currents

Solve for the bias points

② Use DC operating point, to find small-signal params
↳ g_m, r_o, g_{mb}, ...

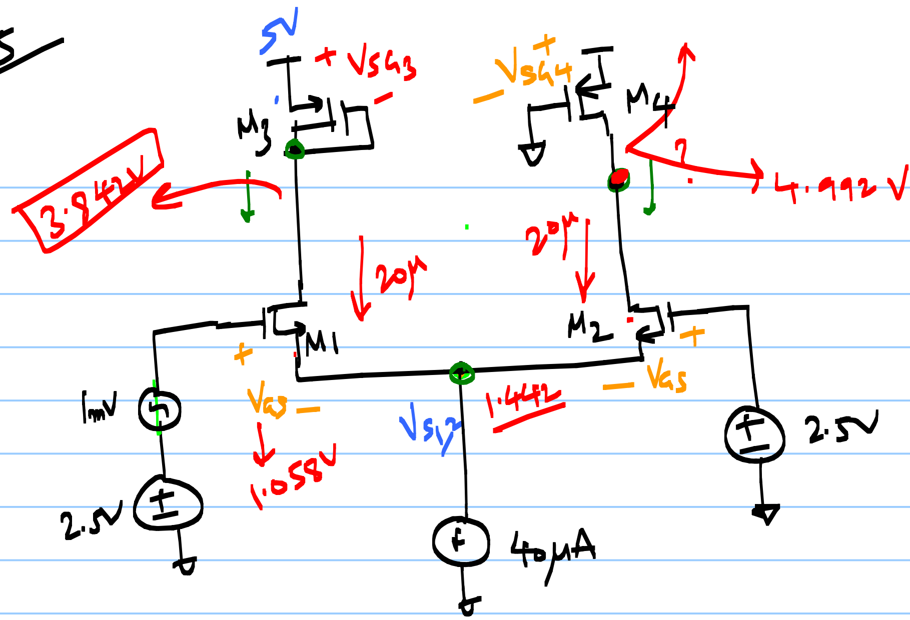
"LINEARIZATION"

③ Draw the AC picture
↳ replace devices by their small-signal models
↳ short DC voltage sources
↳ open DC current sources

Solve for AC values

Final results ⇒ Superposition of DC & AC values

Ex 9.5



All NMOS → $\frac{10}{2}$

All PMOS → $\frac{30}{2}$

Table 6.2

$V_{DD} = 5V$

$V_{THN} = 0.8$

$KP_n = 120 \frac{\mu A}{V^2}$

$V_{THP} = 0.9$

$KP_p = 40 \frac{\mu A}{V^2}$

* find all the voltages & currents

Solve DC picture:

* M_1 & M_2 are in saturation

$$V_{GS_1} = V_{GS_2} = \sqrt{\frac{2 I_D}{KP_n \frac{W}{L}}} + V_{THN} = \sqrt{\frac{2 \times 20\mu}{120\mu \cdot \frac{10}{2}}} + 0.8$$

$$= 1.058V$$

$$V_{S_{1,2}} = 2.5 - 1.058 = \boxed{1.442}$$

~~Since $M_3 \Rightarrow$ SAT~~

$\Rightarrow V_{SG_3} = \sqrt{\frac{2 \cdot 20}{40} \times \frac{2}{30}} + 0.9 = 1.158V$

$V_{D_1} = V_{D_3} = \boxed{3.842V}$

Is M_1 really in SAT? Yes

$V_{DS_1} = 3.842V - 1.442 = 2.4V$

$V_{DS,sat_1} = V_{GS_1} - V_{THN_1} = 1.058 - 0.8 = 258 mV$

* Assume $M_4 \Rightarrow$ Triode (initiall guess)

$V_{SG_4} = 5V$ confirmed

for saturation $\Rightarrow$ $V_{SD} \geq V_{SG} - V_{THP}$

$\cancel{V_S} - V_D \geq \cancel{V_S} - V_G - V_{THP}$

$V_{G_4} \geq V_D - V_{THP}$

$\downarrow 0$

$\Rightarrow \underline{V_D \leq 0.7V}$

$$20\mu = \frac{30}{2} \times 40\frac{\mu A}{V^2} \cdot \left[(5-0.9)V_{SD} - \frac{V_{SD}^2}{2}\right]$$

↑ Quadratic

$$\underline{V_{SD} = 8.13\,mV}$$

$$\boxed{V_{D4} = 4.992\,V}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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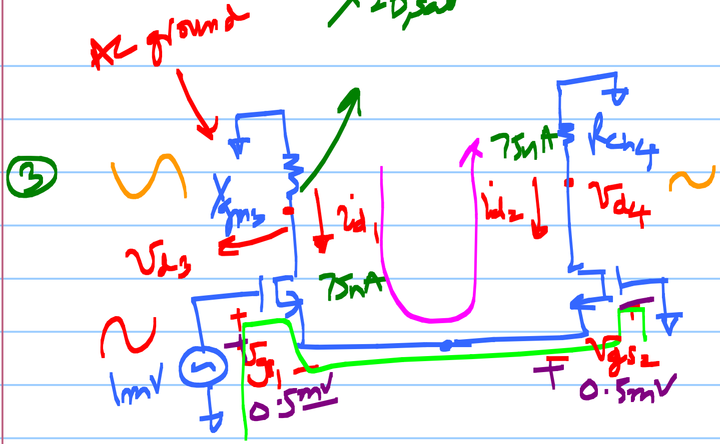
② $\boxed{M_4}$ $\quad R_{ch4} = \dfrac{1}{KP_p \frac{W}{L}(V_{GS} - V_{THN})} = 407\,\Omega$

Deep Triode

$\boxed{M_1, M_2} \Rightarrow \quad g_{m1} = g_{m2} = \sqrt{2\beta_n I_D} = 150\,\frac{\mu A}{V}$ $\quad$ (I_D: 20μA)

$\boxed{M_3} \Rightarrow \quad g_{m3} = \sqrt{2\beta I_D} = 150\,\frac{\mu A}{V} \Rightarrow R = \frac{1}{g_m} = 6.67\,k\Omega$

$$r_o = \frac{1}{\lambda I_{D,sat}} = \infty$$



KCL @ $V_{s_{1,2}}$

$$i_{d_1} + i_{d_2} = 0$$

$$\boxed{i_{d_2} = -i_{d_1}}$$

$$\boxed{v_{gs_2} = -v_{gs_1}}$$

$$i_{d_1} + i_{d_2} = 0$$

$$\cancel{g_{m_1}} \cdot v_{gs_1} + \cancel{g_{m_2}} v_{gs_2} = 0$$

$$\Rightarrow \boxed{v_{gs_2} = -v_{gs_1}}$$

$$-1mV + v_{gs_1} - v_{gs_2} = 0$$

$$\boxed{v_{gs_1} = v_{gs_2} = \frac{1mV}{2} = 0.5mV}$$

$$i_{d_1} = g_m v_{gs_1} = 0.5mV \times 150\mu = \underline{75nA}$$

$$i_{d_2} = -75nA$$

$$v_{d_1} = v_{d_3} = -i_{d_1} \cdot \frac{1}{g_{m3}} = 0.5\,mV$$

$$v_{d_2} = v_{d_4} = -i_{d_4} \cdot R_{ch4} = 0.03\,mV$$

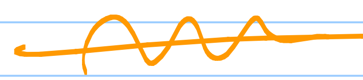
$$v_{D1,3} = V_{D1,3} + v_{d1,3}$$

$$= 3.942 - \underbrace{0.5 \times 10^{-3} \sin(\omega t)}_{Ac}$$

$$v_{D2,4} = 4.992 + \underbrace{0.03 \times 10^{-3} \sin(\omega t)}_{Ac}$$

$$i_{D_1} = I_{D_1} + i_{d_1}$$

$$i_{D_2} = I_{D_2} + i_{d_2}$$



.AC

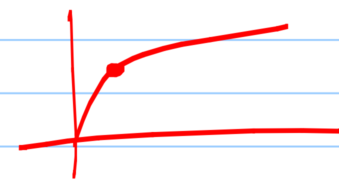
.tran

Long-channel g_m, r_o

from Chap 6 $\Rightarrow$ $\lambda \propto \frac{1}{L}$

$$I_{D,sat} = \frac{k'_n}{2} \cdot \frac{W}{L} \cdot V_{DS,sat}^2$$

$V_{ov} = V_{GS} - V_{THN}$



$$r_o = \frac{1}{\lambda I_{Dsat}} \propto \frac{L^2}{V_{DS,sat}^2}$$

$$\boxed{r_o \propto \frac{L^2}{V_{ov}^2}}$$

$L \uparrow \Rightarrow r_o \uparrow$

$V_{ov} \downarrow \Rightarrow r_o \uparrow$

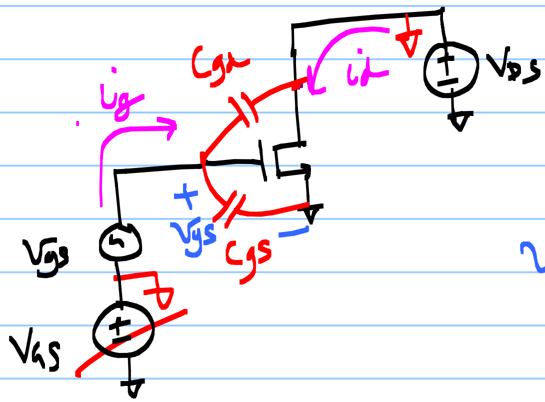
$$g_m = \beta V_{ov} \propto \frac{V_{ov}}{L}$$

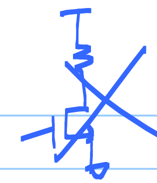
$\boxed{g_m r_o}$ $\uparrow$ ← max open loop gain from a transistor

MOSFET Transition frequency (f_T)

SAT

AC picture





Current gain

@ $f = f_T = \left|\frac{i_d}{i_g}\right| = 1$

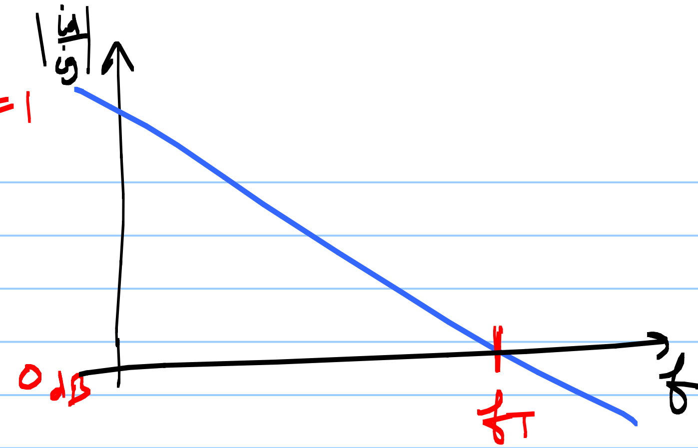
$$v_{gs} = \frac{i_g}{s(C_{gd}+C_{gs})} \longrightarrow (1)$$

$$i_d = g_m v_{gs}$$

$$\Rightarrow i_d = \frac{g_m i_g}{s(C_{gd}+C_{gs})}$$

$$\frac{i_d}{i_g} = \frac{g_m}{\underline{\underline{s}}(C_{gd}+C_{gs})}$$

$$\left|\frac{i_d}{i_g}\right| = \frac{g_m}{2\pi f\,(C_{gd}+C_{gs})} = 1$$



$$\Rightarrow \boxed{f_T = \frac{g_m}{2\pi\,(C_{gs}+C_{gd})}}$$

ignore junction caps

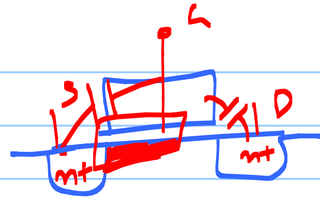
⇒ more intuition on f_T

g_m sat

$$\boxed{C_{gs} = \frac{2}{3} W L C_{ox}'}$$

$C_{gd} = CGDO \cdot W$

$C_{gs} >> C_{gd}$



RF

Aside

$f_{max} > f_T$

$$f_T = \frac{g_m}{2\pi C_{gs}} = \frac{3KP_n(V_{GS} - V_{THN})}{4\pi L^2 C_{ox}'} = \frac{3\mu_n}{4\pi} \cdot \frac{V_{ov}}{L^2}$$

Long channel CMOS process

$$\boxed{f_T \propto \frac{V_{ov}}{L^2}}$$

Fastest operation

$L = L_{min}$

$V_{ov} \uparrow$

moderate speed $V_{ov} \simeq 2-5\% \; V_{DD}$

high speed $V_{ov} > 5\%$ of V_{DD}