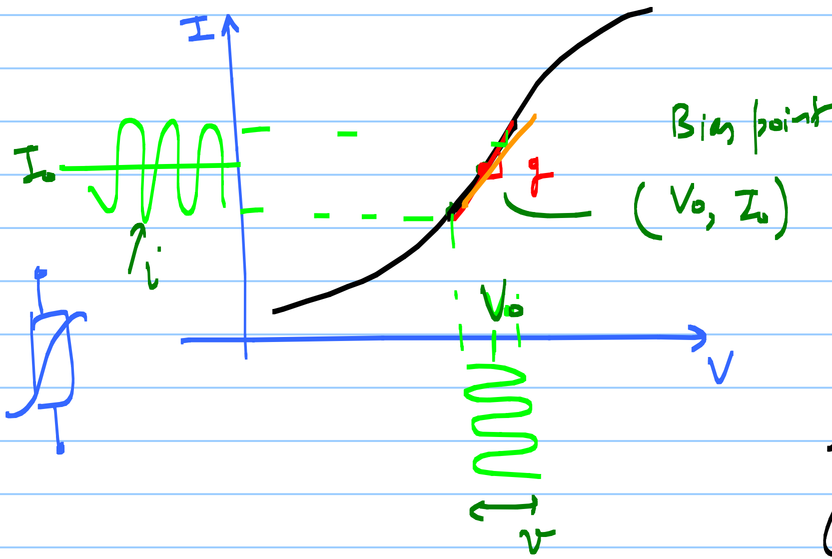
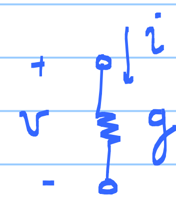
ECE 511 – Lecture 2

Note Title 1/24/2013

$I = f(V)$





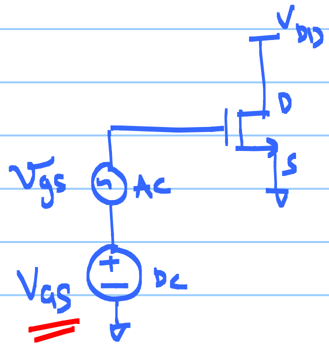
Linearizing about the bias point

Small signal $i = g v$

$$\boxed{g = \underbrace{\frac{dI}{dV}}_{\text{slope}}\Bigg|_{\underbrace{v = V_0}_{\text{bias point}}}}$$

"Small-signal equivalent circuit"

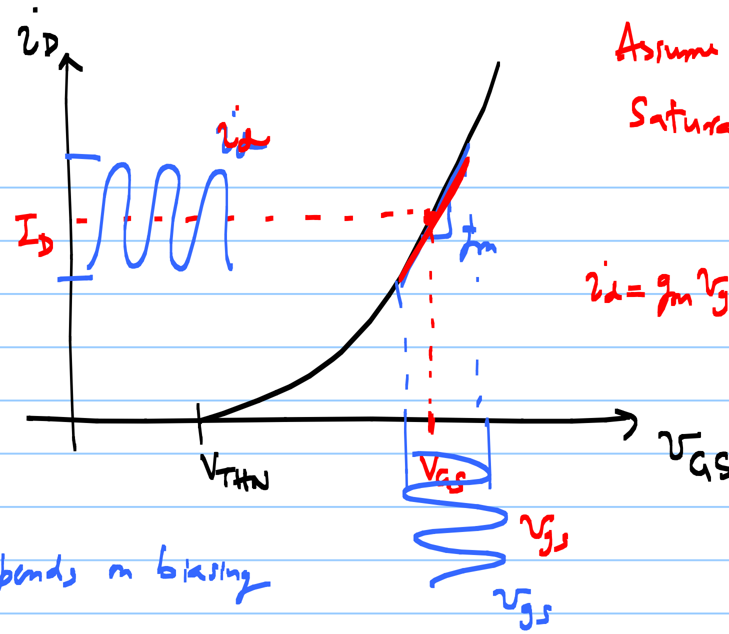
NMOS in Saturation



$$\overbrace{I_D}^{DC} + \overbrace{i_d}^{AC} = i_D$$

$$i_d = \underline{g_m} \cdot v_{gs}$$

g_m → depends on biasing



Assume Saturation

$$i_d = g_m v_{gs}$$

$$i_D = \frac{k'_n}{2}\frac{W}{L}\left(v_{GS} - V_{THN}\right)^2$$

$$\underbrace{I_D}_{DC} + \underbrace{i_d}_{AC} = \frac{k'_n}{2}\frac{W}{L}\left(\underbrace{V_{GS}}_{DC} + \underbrace{v_{gs}}_{AC} - V_{THN}\right)^2$$

$$\boxed{i_d = g_m v_{gs}}$$

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{\underline{v_{GS} = V_{GS}}}$$

(AC)
Small-signal parameter

$$= \left.\frac{k'_n}{\cancel{2}} \cdot \frac{W}{L} \cdot \cancel{2}\left(v_{GS} - V_{THN}\right)\right|_{v_{GS} = V_{GS}}$$

$$\boxed{g_m = \underbrace{k'_n \cdot \frac{W}{L}}\left(\underbrace{V_{GS}}_{DC} - V_{THN}\right)}$$

$$g_m = \beta\left(V_{GS} - V_{THN}\right) = \beta V_{OV}$$

$$g_m = \beta(V_{GS} - V_{THN}) \longrightarrow ①$$

$$= \beta \cdot \sqrt{\frac{2I_D}{\beta}} = \sqrt{2\beta I_D} \longrightarrow ②$$

$$V_{GS} - V_{THN} = \sqrt{\frac{2I_D}{\beta}}$$

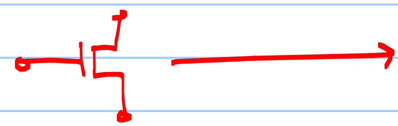
$$= \beta(V_{GS} - V_{THN})$$

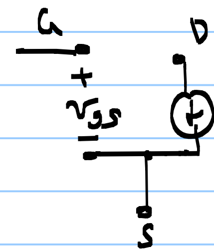
$$= \frac{2I_D}{(V_{GS} - V_{THN})^{\cancel{2}}} \cdot \cancel{(V_{GS} - V_{THN})} = \frac{2I_D}{V_{GS} - V_{THN}} \longrightarrow ③$$

$$\boxed{g_m = \beta V_{OV} = \sqrt{2\beta I_D} = \frac{2I_D}{V_{OV}}}$$

$i_d = g_m v_{gs}$

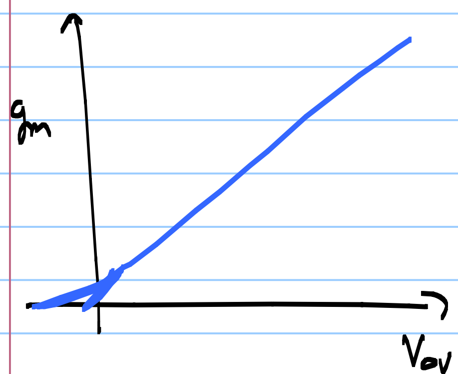
Ac Equivalent Circuit



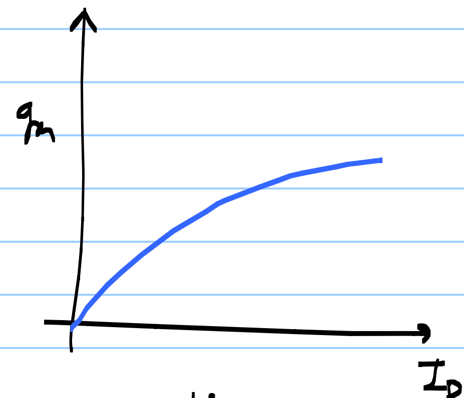


$i_d = g_m v_{gs}$ depends on bias pt.

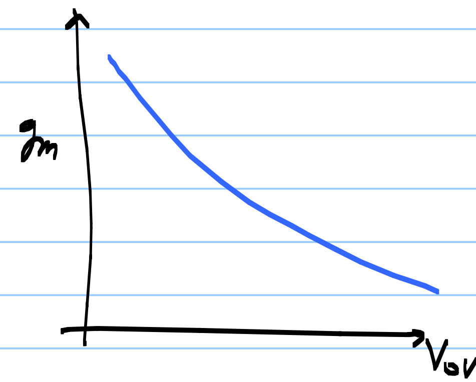
VCCS



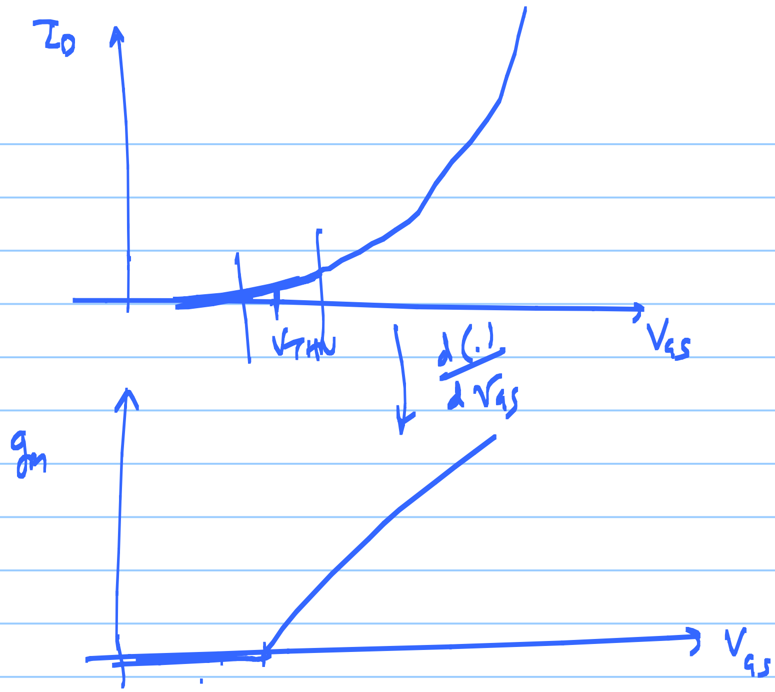
$\frac{W}{L}$ const



$\frac{W}{L}$ const

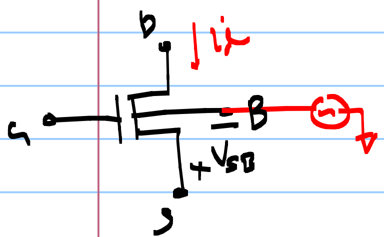


I_D constant

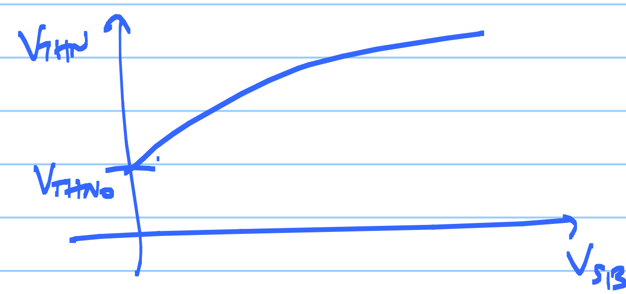


I_D
V_{THN}
V_{GS}
$\frac{d(\cdot)}{dV_{GS}}$
g_m
V_{GS}

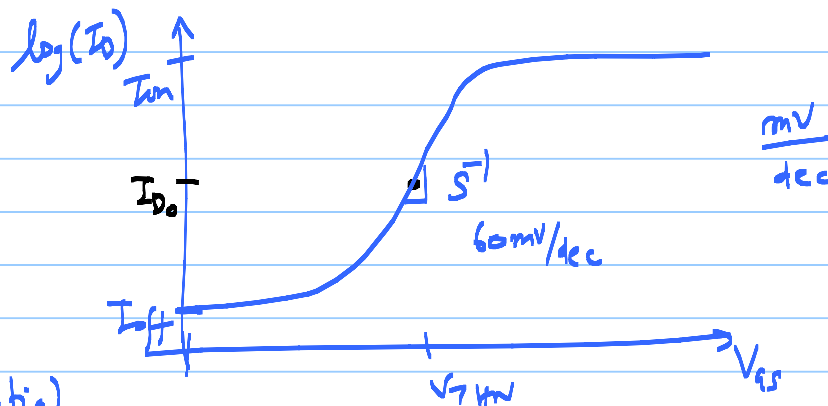
Body Effect



$$V_{THN}(V_{SB}) = V_{THN0} + \gamma_n\left[\sqrt{2|V_{fp}| + V_{SB}} - \sqrt{2|V_{fp}|}\right]$$

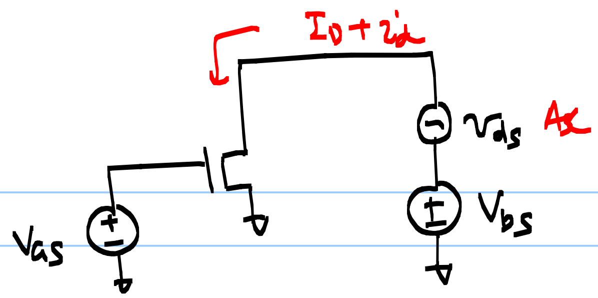


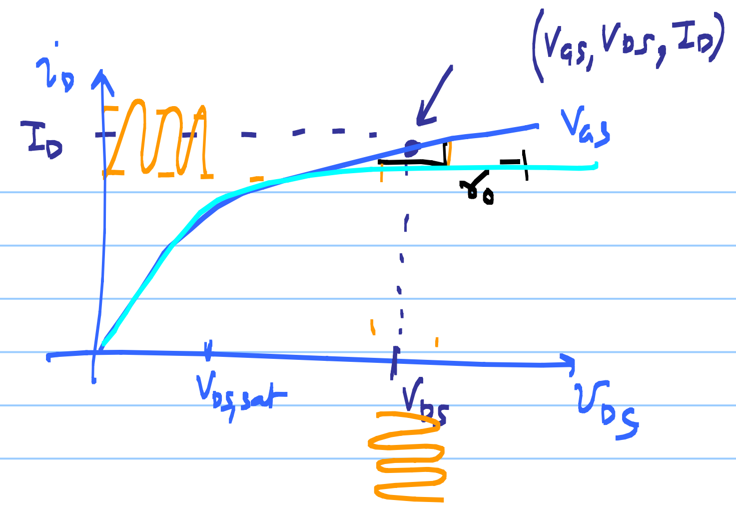
Subthreshold regime



$$I_D = I_{D0}\cdot\frac{W}{L}\cdot e^{\frac{V_{GS}-V_{THN}}{nV_T}}$$

$$g_m = \frac{I_D}{nV_T} \quad \left(\text{good } \frac{g_m}{I_D} \text{ ratio}\right)$$

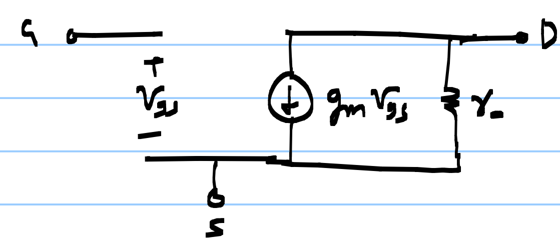


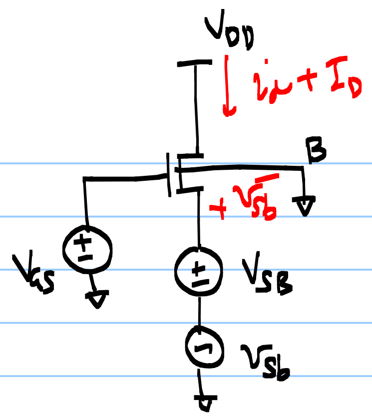


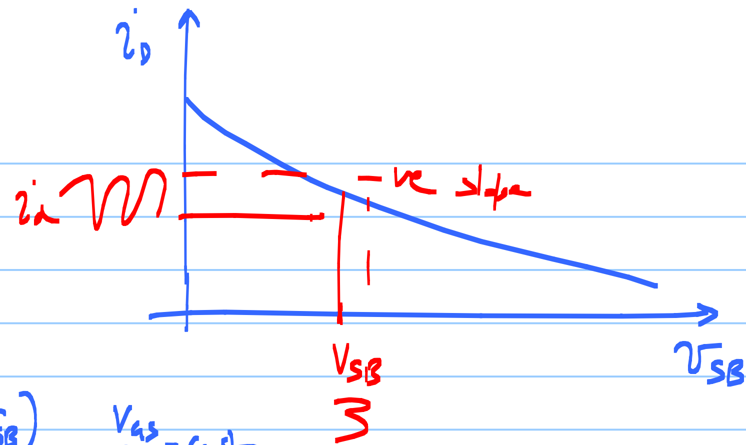
$$i_d = g_{DS} \cdot v_{ds} = \frac{v_{ds}}{r_o}$$

$$r_o = \left(\left. \frac{\partial i_D}{\partial v_{DS}} \right|_{\substack{v_{DS} = V_{DS} \\ V_{GS} = \text{const}}} \right)^{-1}$$

$$\boxed{r_o = \frac{1}{\lambda I_{D,sat}}}$$







$$i_D = f(v_{SB}) \qquad \begin{matrix} V_{GS} \\ V_{DS} \end{matrix} = \text{const.}$$

$$i_d = g_{mb} \cdot v_{sb}$$

$f(v_{SB})$

$$g_{mb} = \left.\frac{\partial i_D}{\partial v_{SB}}\right|_{V_{SB}} = \frac{\partial}{\partial v_{SB}}\left.\left[\frac{KP_n}{2}\frac{W}{L}\left(V_{GS}-V_{THN}\right)^2\right]\right|_{V_{SB}}$$

$$= \underbrace{KP_n \cdot \frac{W}{L}\left(V_{GS}-V_{THN}\right)}\left.\underbrace{\left[-\frac{\partial V_{THN}}{\partial v_{SB}}\right]}\right|_{V_{SB}}$$

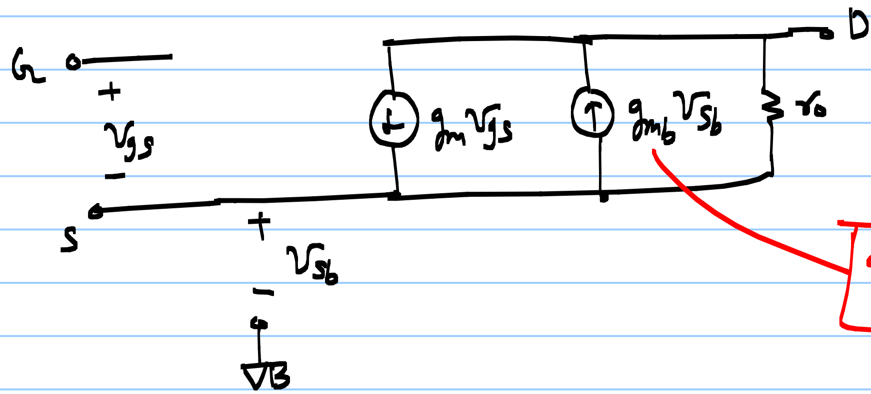
$$\boxed{g_{mb} = g_m \cdot \eta}$$

body coefficient

$$\eta = -\frac{\gamma}{2}\left(\left|2V_{fp} + V_{SB}\right|^{-1/2}\right)$$

g_{mb} opposes the g_m

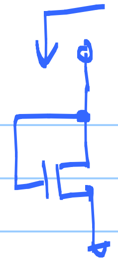
AC equivalent circuit



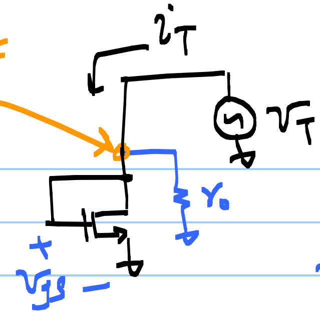
$$\boxed{g_{mb} = |\eta|\, g_m}$$

$\eta < 1$

$R_{out} = ?$



Low-Z node



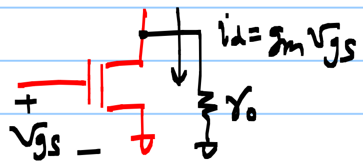
$$R_{out} = \frac{v_T}{i_T}$$

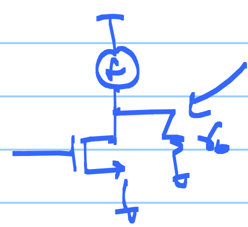
$$i_T = g_m v_T$$

$$R_{out} = \frac{v_T}{i_T} = \frac{1}{g_m} \| r_o$$

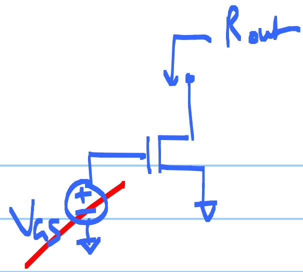
$$\cong \frac{1}{g_m} \leq 1k\Omega$$

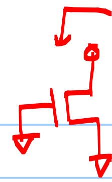
A_c





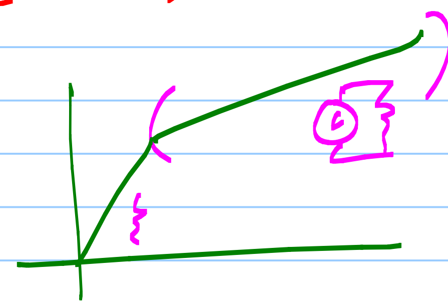
moderate impedance node





$$R_{ch} = \frac{1}{kP_n \frac{W}{L}(V_{GS}-V_{THN})}$$

$$R_{ch} < \frac{1}{g_m}\Big|_{sat}$$



① DC picture