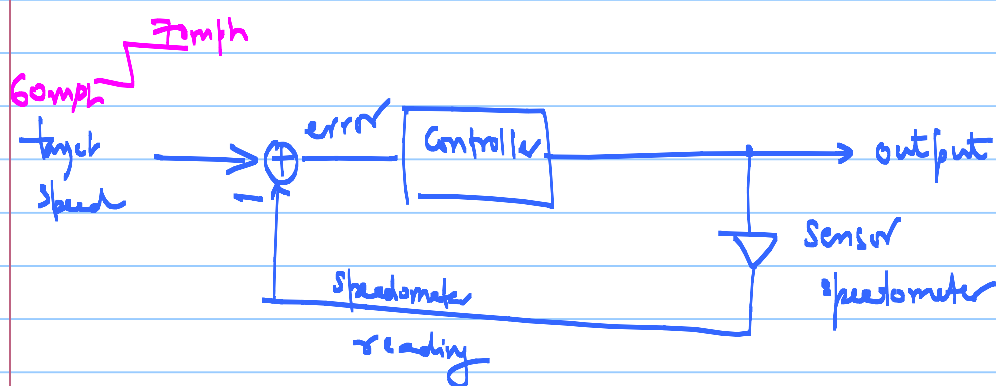
# ECE 511 – Lecture 24

Note Title 4/25/2013

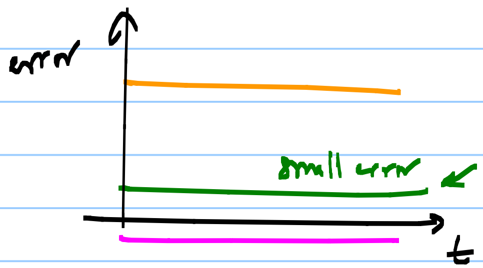
## Negative feedback loops:

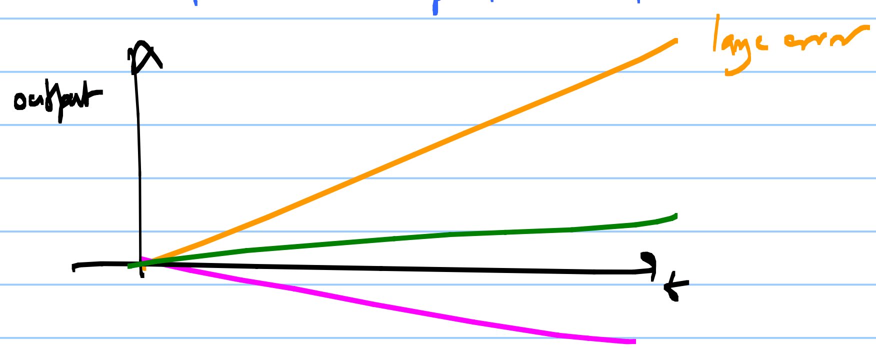



observations: process of continuous sensing and adjustments that drives the output to the correct value.

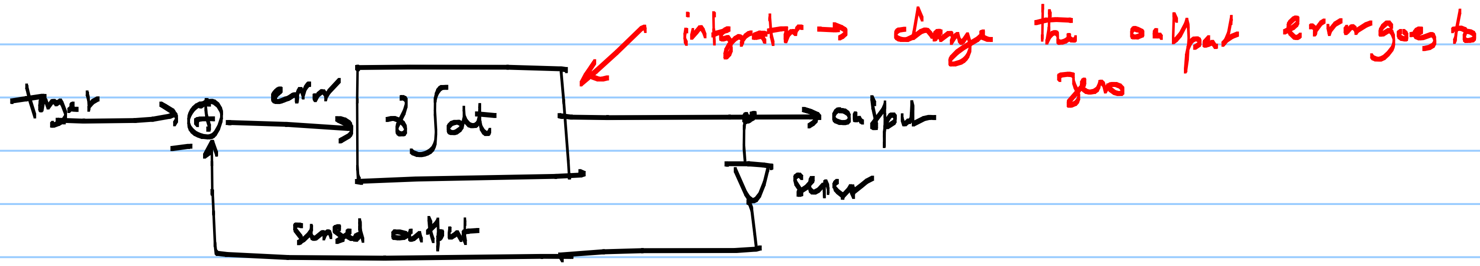
* when the sensed output is very close to the desired value, we drive the output gently so that it changes slowly.

* lets say the sensor is stuck
⇒ feedback loop is broken

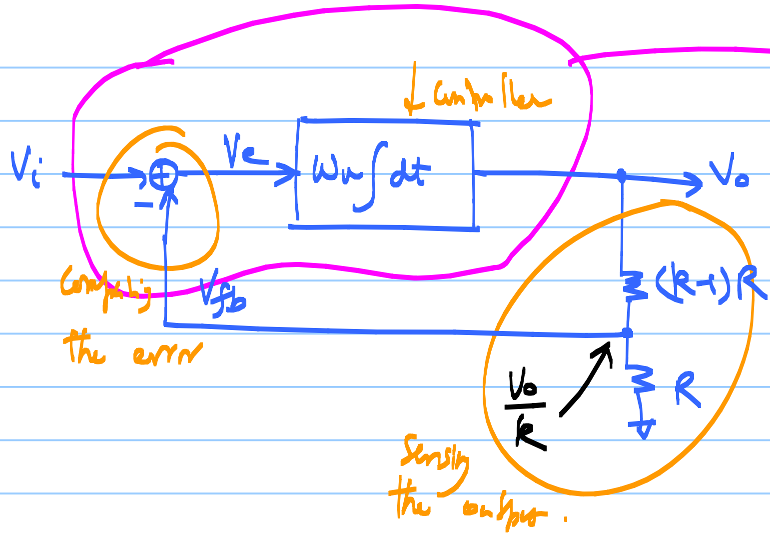


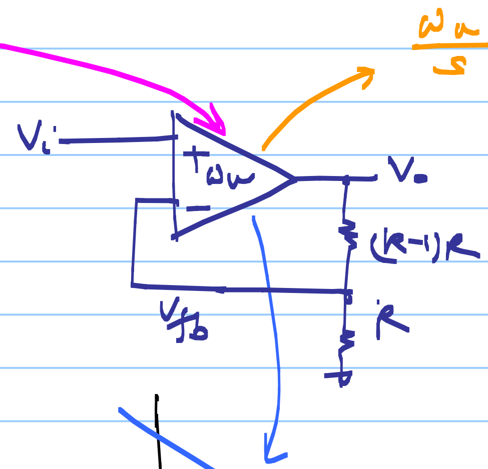



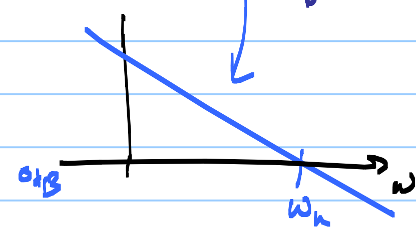
* an integrator is used as the controller



# Ex. Negative feedback Amplifier

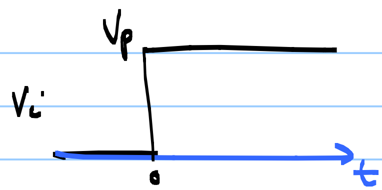


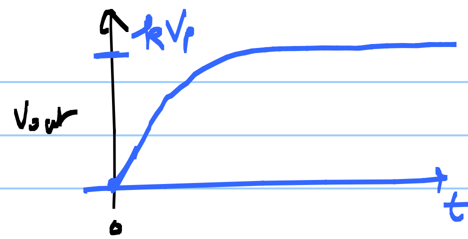



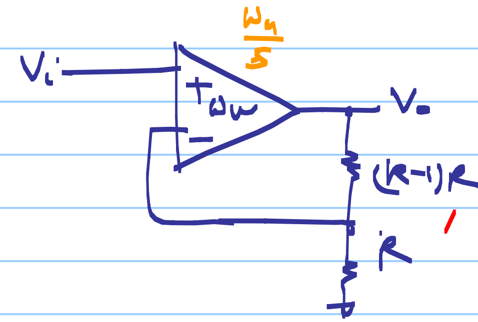

$$e = V_i - \frac{V_o}{k} \to 0 \quad \Rightarrow \quad V_i = \frac{V_o}{k}$$

$$\Rightarrow \boxed{V_o = kV_i}$$ → closed loop gain of $k$




$\cdot\, V_p u(t)$



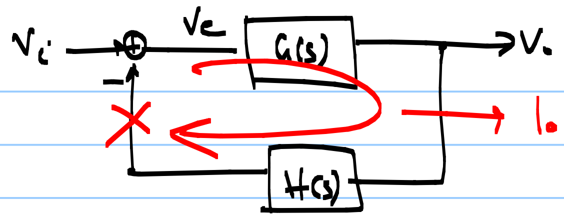

$$V_o(s) = \frac{\omega_u}{s}\left[V_i(s) - \frac{V_o(s)}{k}\right]$$

$$\Rightarrow \quad \frac{V_o}{V_i}(s) = \frac{k}{1+\frac{ks}{\omega_u}} = \frac{k}{(1+s\tau)}$$

$$\tau = \frac{k}{\omega_u}$$

$$V_o(s) = \frac{k}{(1+s\tau)} \cdot \frac{V_p}{s}$$

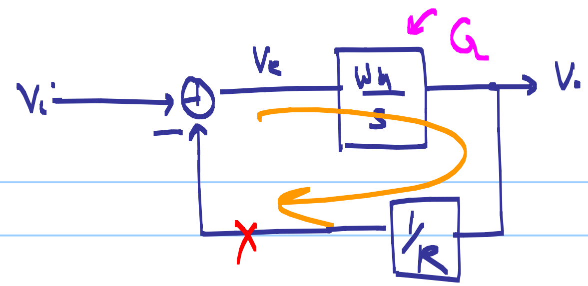
$$v_{out}(t) = kV_p\left(1-e^{-t/\tau}\right) = kV_p\left(1-e^{-\frac{\omega_u t}{k}}\right)$$




loop gain $L(s) = G(s) \cdot H(s)$

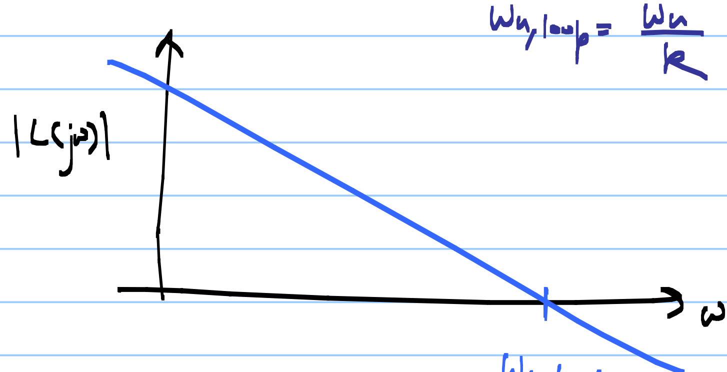
$$\frac{V_o}{V_i}(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{1}{H(s)} \cdot \frac{1}{1 + \frac{1}{G(s)H(s)}}$$

$$\simeq \begin{cases} \frac{1}{H(s)} & |GH| \gg 1 \\ G(s) & |GH| \ll 1 \end{cases}$$



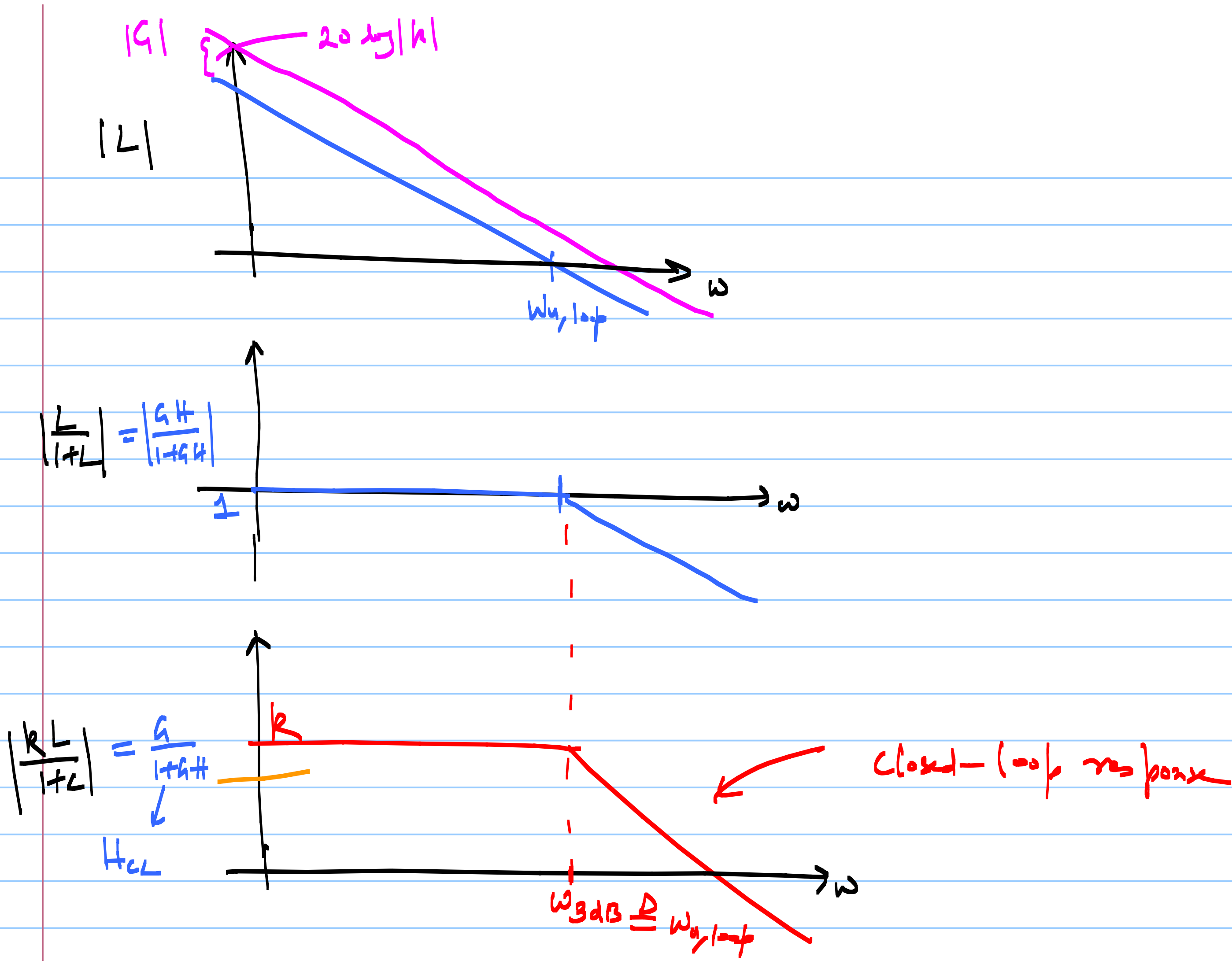

$$L(s) = \frac{\omega_u}{k \cdot s}$$ : (open) loop-gain

$$\omega_{u,loop} = \frac{\omega_u}{k}$$




$$\omega_{u,loop} = \frac{\omega_u}{k}$$

$\omega_{u,loop} \Rightarrow$ unity-gain frequency of the loop



|G|
20 log|h|
|L|
ω
ωu,loop
|L/(1+L)| = |GH/(1+GH)|
1
ω
|kL/(1+L)| = G/(1+GH)
HCL
k
closed-loop response
ω3dB ≜ ωu,loop
ω

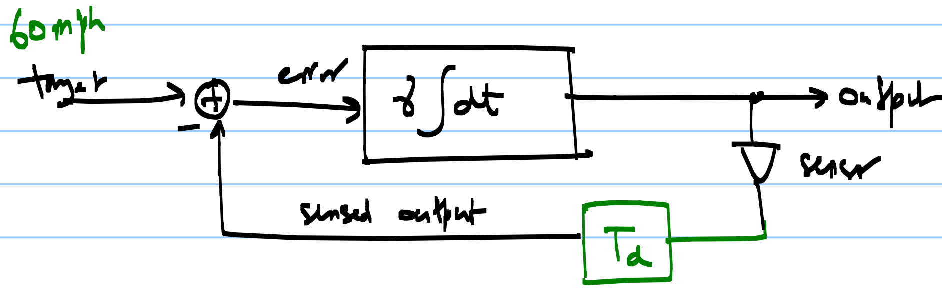
closed-loop DC gain $= \left|\frac{1}{H(s)}\right| = \frac{1}{1/k} =$ "$k$"

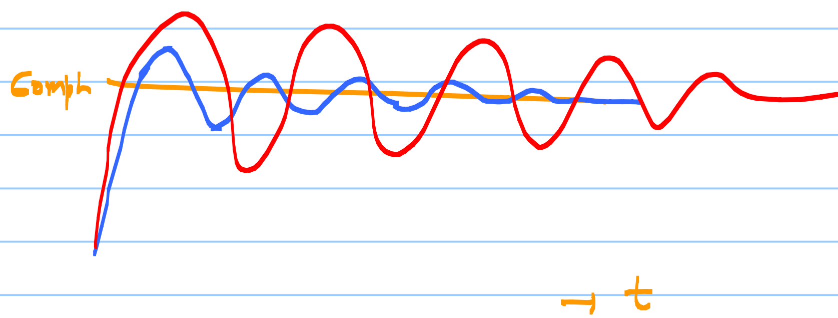
$$H_{CL} = k \frac{L}{1+L} \quad @ \text{ DC}$$

$$= k \cdot \frac{1}{1+\frac{1}{L}}$$

$$= k \text{ for } L \to \infty$$

# Stability Criterion :



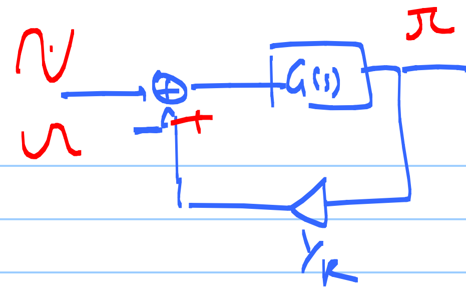




* Excess loop delay can destabilize the system

↳ excess phase

$$\frac{V_o(s)}{V_i(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{G(s)}{1+L(s)}$$

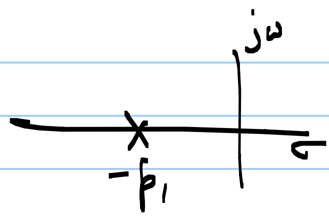



Instability if loop gain $L(j\omega) = -1$

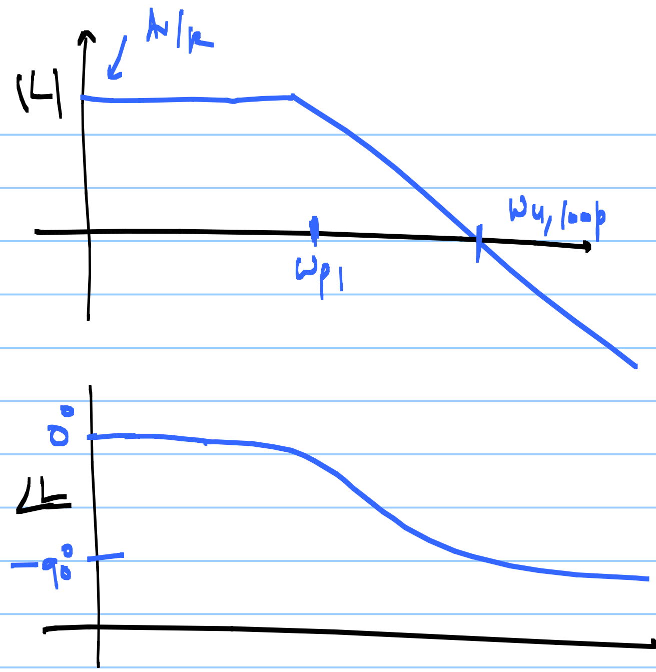
$|L(j\omega)| = 1$ and $\angle L(j\omega) = -\pi$

Barkhausen Criterion

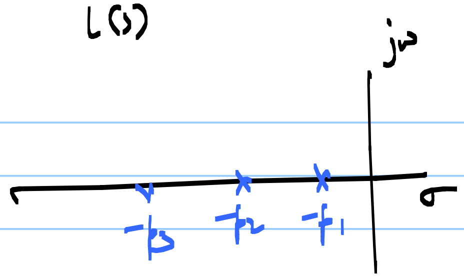
Single real pole

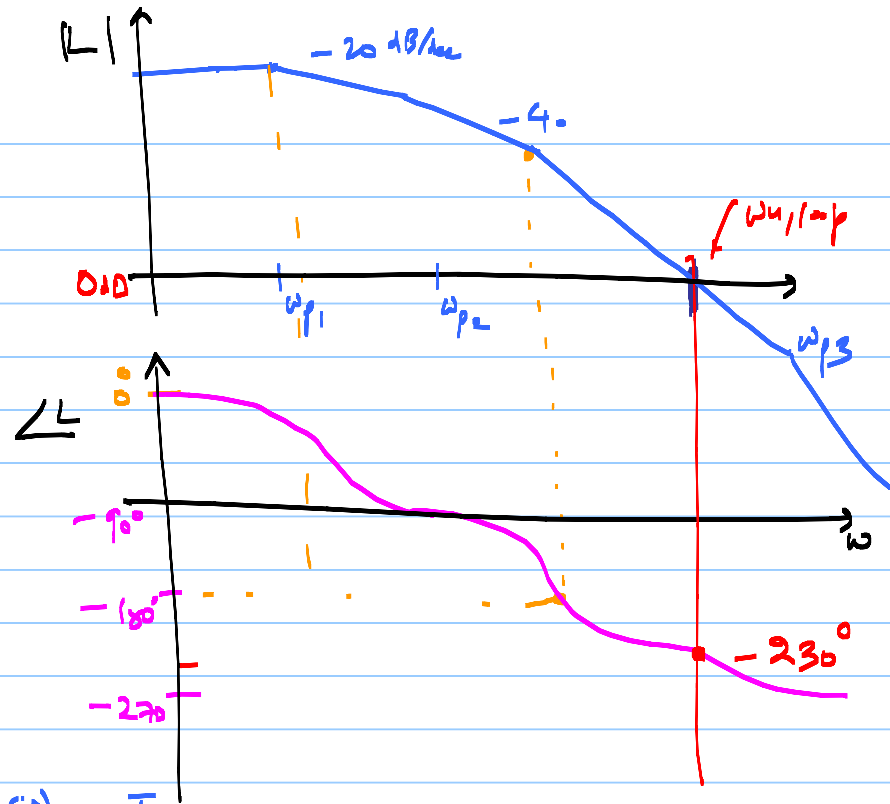



$$L(s) = \frac{A_v}{1 + s/\omega_{p1}} \cdot \frac{1}{R}$$




→ 1st order system is unconditionally stable

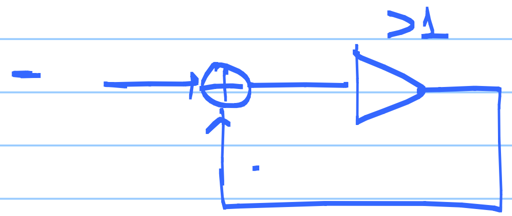



if for any frequency

$|L(j\omega)| \geq 1$ and $\angle L(j\omega) = -\pi$

$\rightarrow$ instability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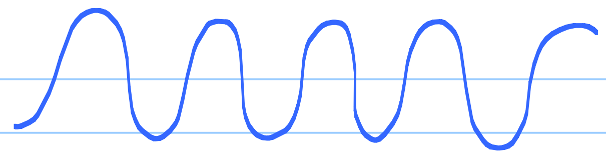
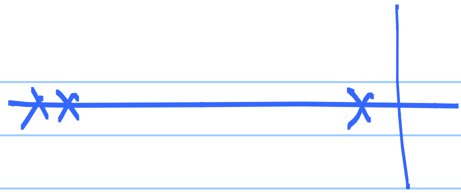
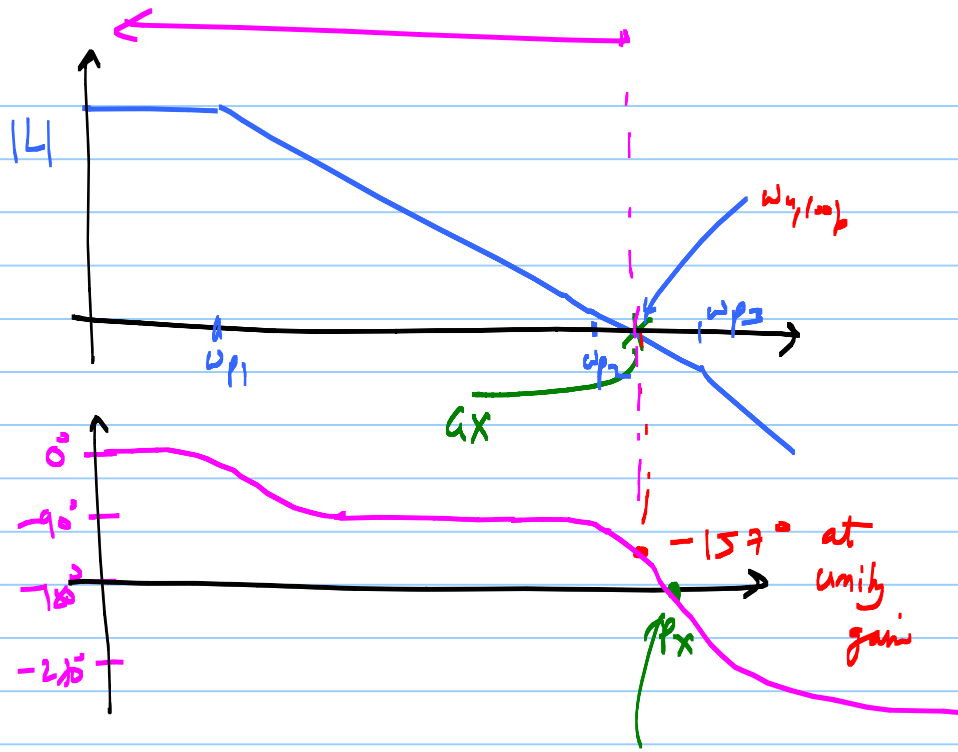




GX: gain crossover frequency
(0 dB)

PX: phase crossover frequency
($\angle L = -180^\circ$)




GX < PX