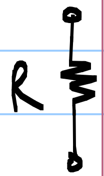
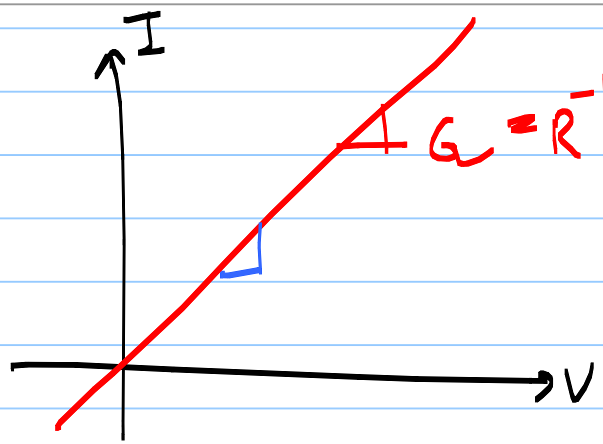
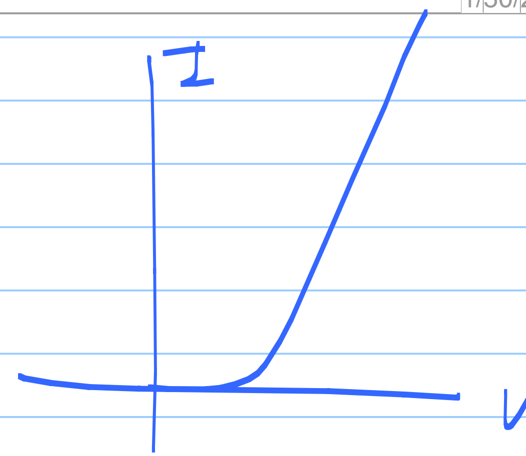
ECE 511 - Lecture 4.

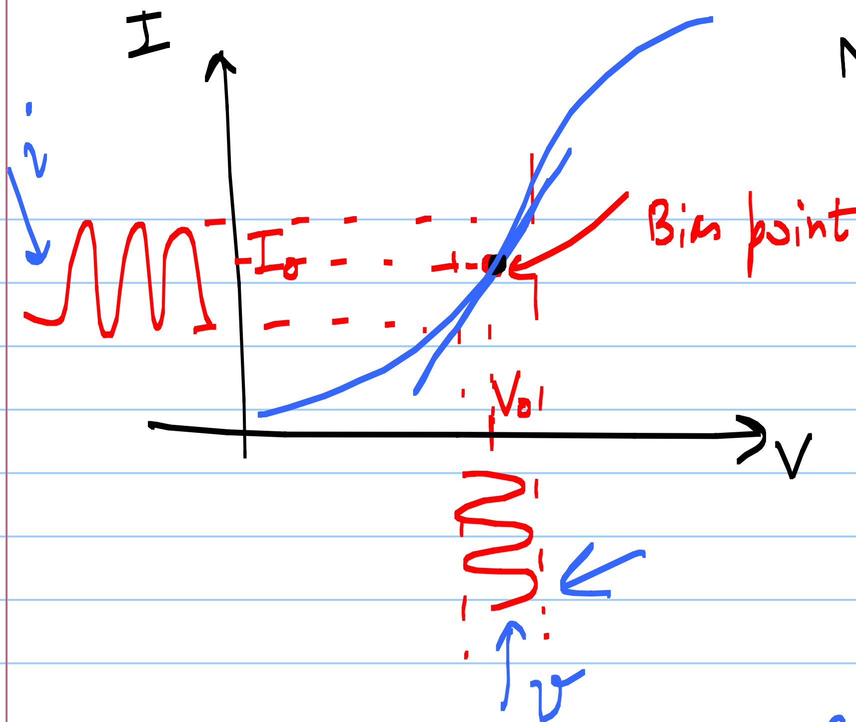
Note Title 1/30/2012







Non-linear resistive device



$$I = f(V)$$

$I = f(V)$ ← Taylor Series expansion around $V = V_0$

$$\Rightarrow I = f(V_0) + f'(V_0)\underbrace{(V-V_0)}_{v} + \frac{f''(V_0)}{2!}\underbrace{(V-V_0)}_{v}{}^2 + \frac{f'''(V_0)}{3!}\overbrace{(V-V_0)^3}^{v^3} + \dots$$

$V = V_0 + v$ ← AC

$\uparrow$ DC

$$I = \underbrace{f(V_0)}_{I_0} + f'(V_0)v + \frac{f''(V_0)}{\lfloor 2}v^2 + \frac{f'''(V_0)}{\lfloor 3}v^3 + \cdots\cdots$$

$$\Rightarrow \underbrace{I - I_0}_{i} = f'(V_0)\,v + \frac{f''(V_0)}{\lfloor 2}v^2 + \frac{f'''(V_0)}{\lfloor 3}v^3 + \cdots\cdots$$

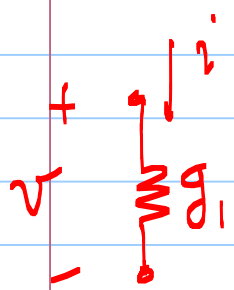
$$i = f'(V_0)\cdot v + \frac{f''(V_0)}{\lfloor 2}v^2 + \frac{f'''(V_0)}{\lfloor 3}v^3 + \cdots$$

$$i = g_1 v + \underline{g_2 v^2} + \underline{g_3 v^3} + \cdots$$

$v \ll V_0$

$\boxed{i = g_1 v}$ $\Leftarrow$ Linear relationship between AC voltage & current

$$g_1 = f'(V_0) = \left.\frac{\partial f}{\partial v}\right|_{v=V_0}$$ ← first derivative at $v = V_0$



$i = g_1 v$

↑ $f'(V_0)$ ← small-signal parameter

↳ bias point

↑ linearized the device about the Bias point

↳ DC operating point

↳ Q-point

$$i = g_1 v + g_2 v^2 + g_3 v^3 + \cdots$$

If v is not small enough

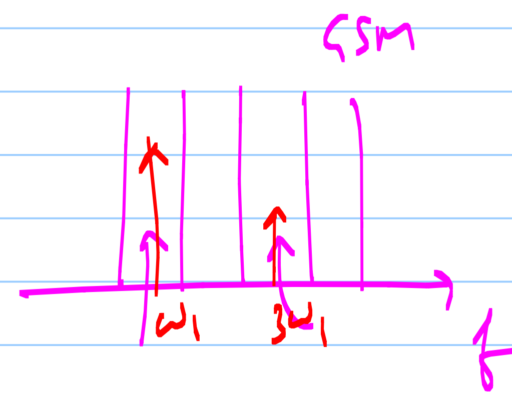
$v = A \sin \omega_1 t$

for small $\underline{\underline{A}}$

$$i = g_1 A \sin \omega_1 t$$

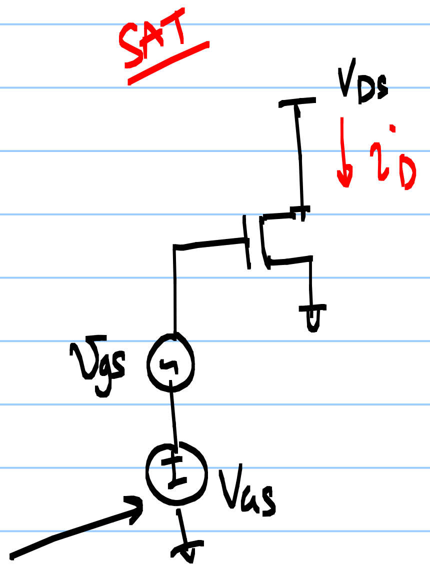
for larger $\underline{\underline{A}}$

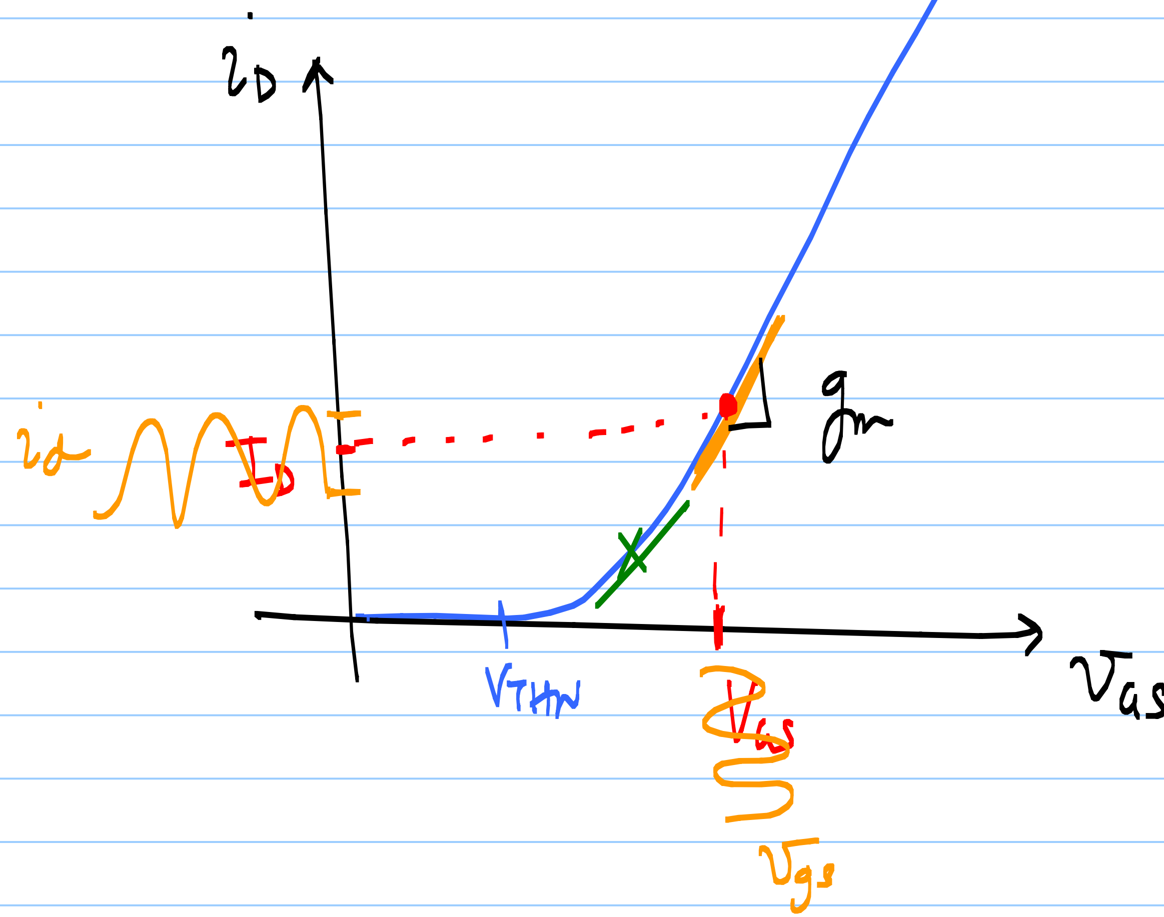
$$i = (\ ) \sin \omega_1 t + (\ ) \sin 2\omega_1 t + (\ ) \sin 3\omega_1 t$$



NMOS in Saturation:

$V_{DS} > V_{DS,sat}$
$= V_{GS} - V_{THN}$





$$i_D = \frac{KP_n}{2}\frac{W}{L}(v_{GS} - V_{THN})^2 \quad \checkmark$$

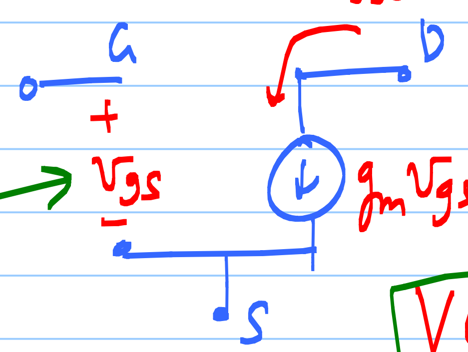
$$\lambda = 0$$

$$i_d = f'(V_0)\, v_{gs}$$

$$\downarrow$$

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{v_{GS}=V_{GS}} \leftarrow \text{Transconductance}$$

$$\boxed{i_d = g_m v_{gs}}$$



$$\boxed{\text{VCCS}}$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}}\Bigg|_{V_{GS}}$$

$$= \frac{KP_n}{\cancel{2}} \cdot \frac{W}{L} \cdot \cancel{2}\,(V_{GS} - V_{THN})\Bigg|_{v_{GS} = V_{GS}}$$

β_n

$$g_m = \underbrace{KP_n \frac{W}{L}}_{\beta_n}(V_{GS} - V_{THN})$$

$$\boxed{g_m = \beta_n\,(\underbrace{V_{GS} - V_{THN}}_{})} \rightarrow (1)$$

$\rightarrow V_{OV} = V_{GS} - V_{THN}$

overdrive voltage

$$I_D = \frac{\beta_n}{2}\left(V_{GS} - V_{THN}\right)^2$$

$$\Rightarrow V_{GS} - V_{THN} = \sqrt{\frac{2I_D}{\beta_n}}$$

$$g_m = \beta_n \left(V_{GS} - V_{THN}\right)$$

$$= \beta_n \cdot \sqrt{\frac{2I_D}{\beta_n}}$$

$$\Rightarrow \boxed{g_m = \sqrt{2\beta_n I_D}} \rightarrow ②$$

$$\beta_n = \frac{2I_D}{\left(V_{GS} - V_{THN}\right)^2}$$

$$g_m = \frac{2I_D}{\left(V_{GS} - V_{THN}\right)^{\cancel{2}}} \times \cancel{\left(V_{GS} - V_{THN}\right)}$$

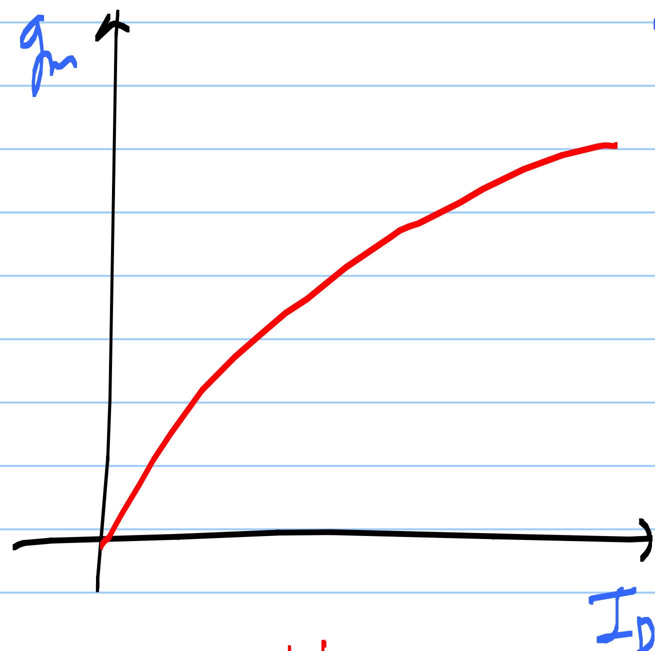
$$\boxed{g_m = \frac{2I_D}{V_{GS} - V_{THN}}} \longrightarrow 3$$

$$g_m = \beta_n (V_{GS} - V_{THN})$$

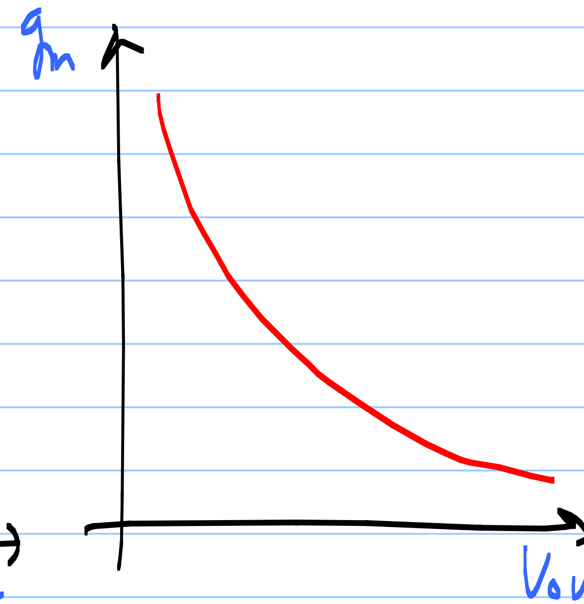
$$g_m = \sqrt{2 \beta_n I_D}$$

$$g_m = \frac{2 I_D}{V_{GS} - V_{THN}}$$

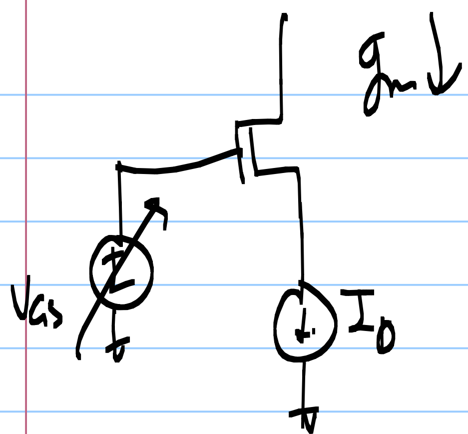
$\frac{W}{L}$ is constant

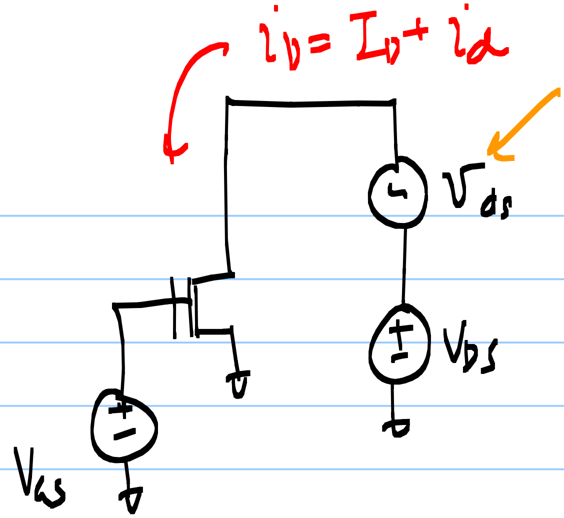


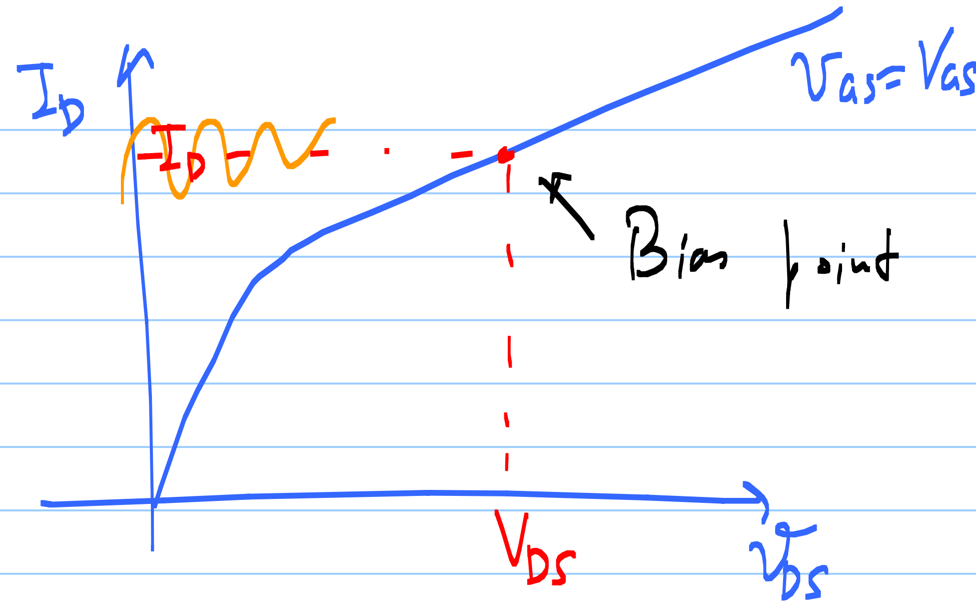
$\frac{W}{L}$ is constant



I_D is constant


g_m
V_{GS}
I_D





$$i_D = \frac{K'_n}{2}\frac{W}{L}(V_{GS}-V_{TH,n})^2\left[1+\lambda(v_{DS}-V_{DS,sat})\right]$$

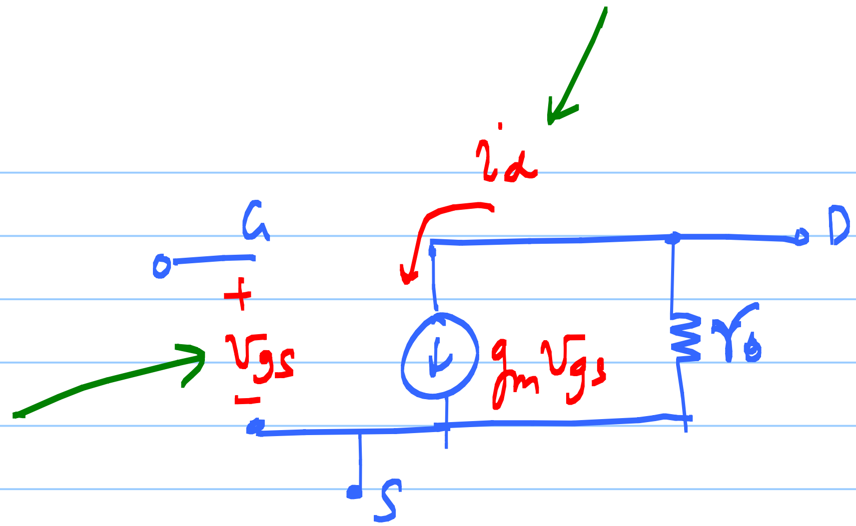
$$i_d = \underbrace{g_o}_{\text{output conductance } g_o = g_{DS}} v_{ds} = (r_o^{-1})\, v_{ds}$$

$$g_o = r_o^{-1} = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_{\substack{v_{DS} = V_{DS} \\ V_{GS} = const}}$$

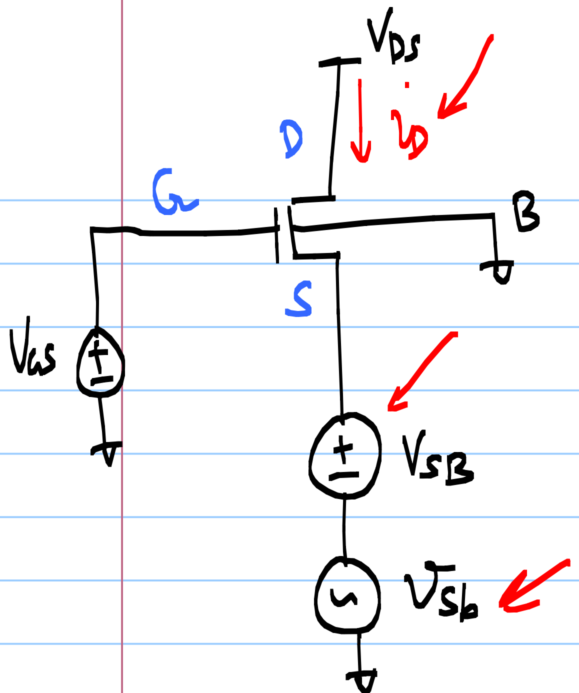
$$= \underbrace{\frac{K'_n}{2} \frac{W}{L} (V_{GS} - V_{THN})^2}_{I_{D,sat}} \cdot \lambda$$

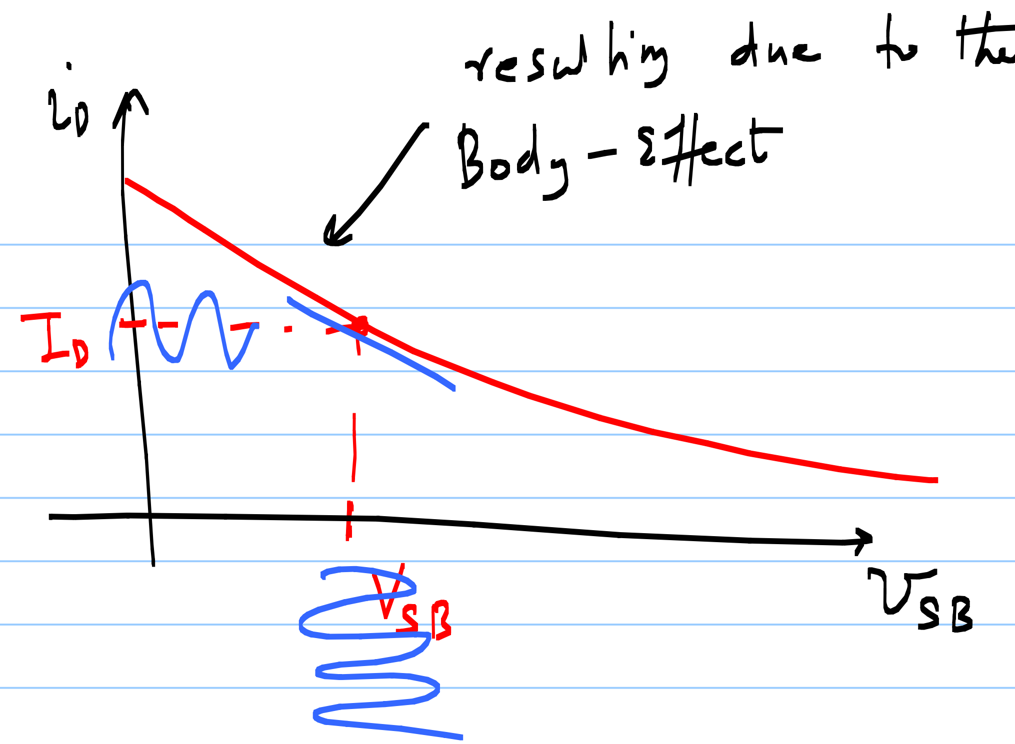
$$= \lambda \cdot I_{Dsat}$$

$$\Rightarrow \quad \boxed{r_o = \frac{1}{\lambda \cdot I_{Dsat}}}$$



$\boxed{\text{VCCS}}$





$$i_D = f(v_{SB})$$

$$i_d = g_{mb} \cdot v_{sb}$$

Body g_m

$$g_{mb} = \frac{\partial i_D}{\partial v_{SB}}\Bigg|_{v_{SB}=V_{SB}}$$

$$= \frac{\partial}{\partial v_{SB}}\left[\frac{KP_n}{2}\cdot\frac{W}{L}\left(V_{GS}-V_{THN}\right)^2\right]\Bigg|_{v_{SB}=V_{SB}}$$

$$= \underbrace{KP_n\cdot\frac{W}{L}\left(V_{GS}-V_{THN}\right)}_{g_m}\cdot\left[-\frac{\partial V_{THN}}{\partial v_{SB}}\right]_{v_{SB}=V_{SB}}$$

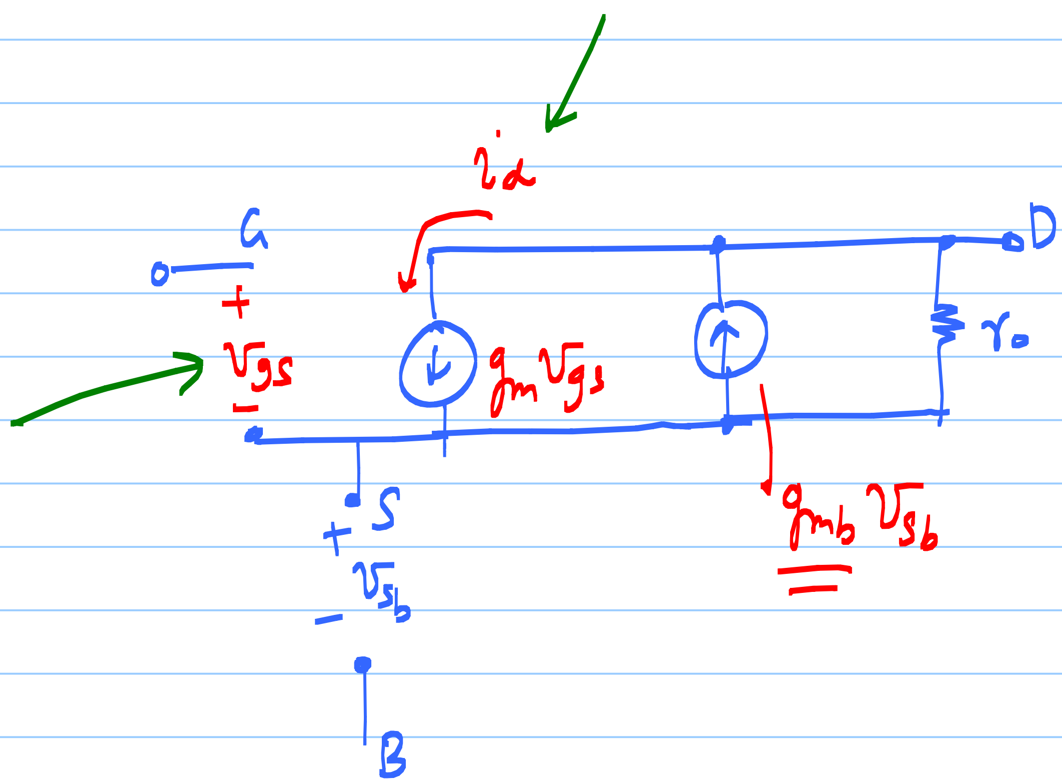
$$= g_m\cdot\underbrace{\left(-\frac{\gamma}{2}\left(|2V_{fp}|+V_{SB}\right)^{-1/2}\right)}_{\eta}$$

$\eta \leftarrow$ body factor

$$\boxed{g_{mb} = \eta \cdot g_m}$$

$$|\eta| < 1$$

$$\eta = \frac{-\gamma}{2\sqrt{2|V_{fb}| + V_{SB}}}$$



G
D
i_d
v_{gs}
$g_m v_{gs}$
r_o
S
v_{sb}
B
$g_{mb} v_{sb}$