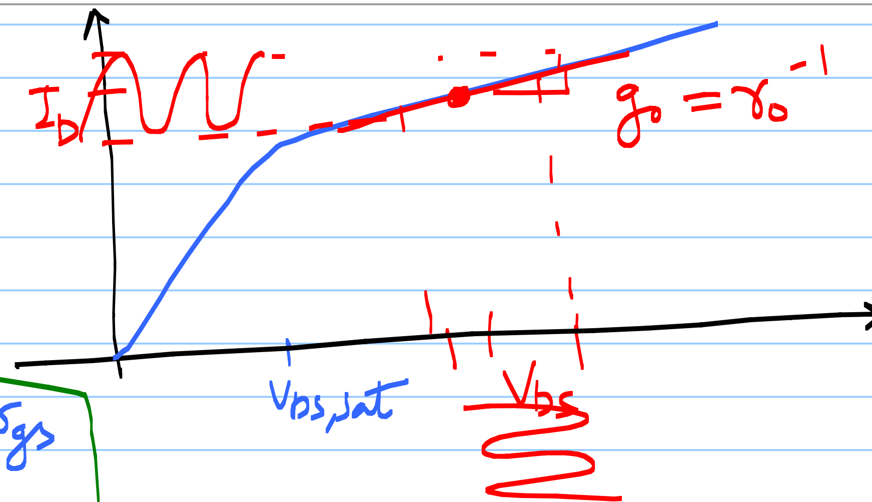
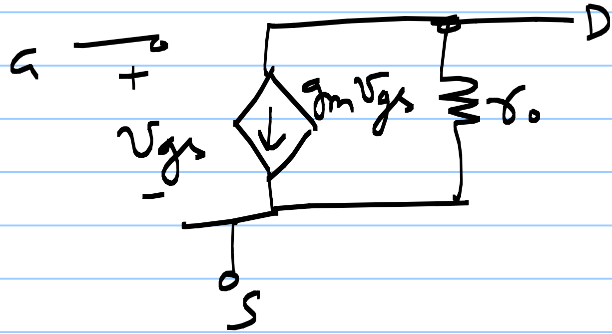
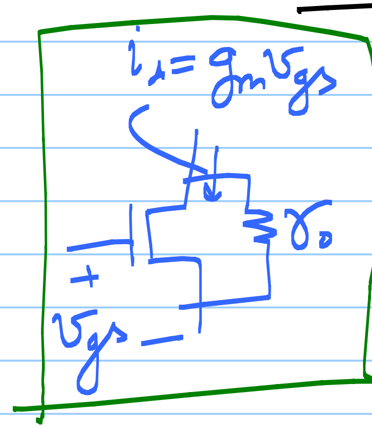
ECE 5411 : Lecture 3.

Note Title 1/26/2011

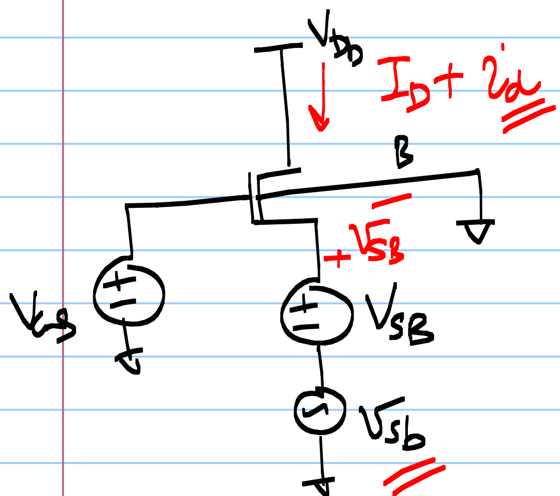
$$r_o = \frac{1}{\lambda I_{D,sat}}$$

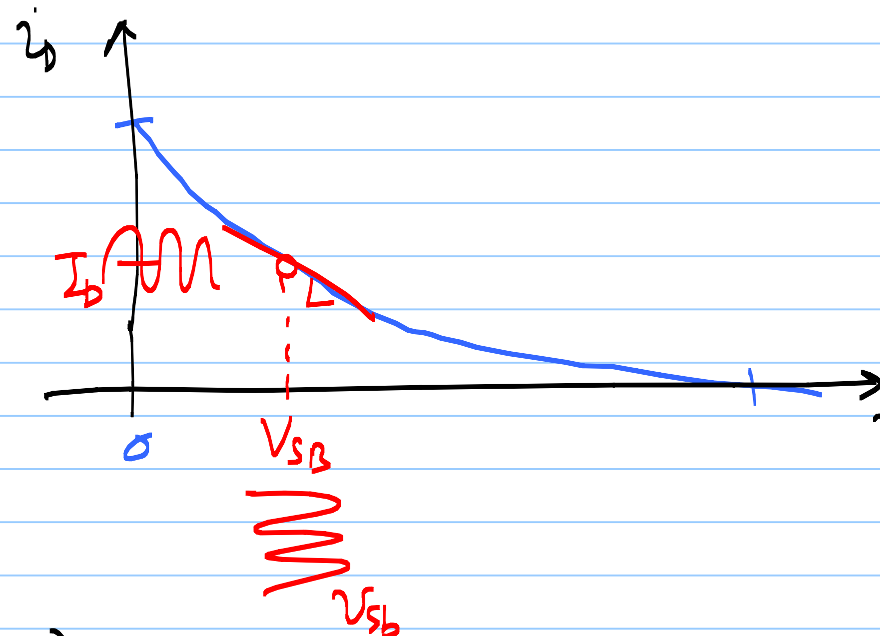






- incremental
- AC
- Small-signal





$$v_{SB} = V_{SB} + v_{sb}$$

$$\underbrace{i_D}_{I_D + i_d} = f(\underbrace{v_{SB}}_{V_{SB} + v_{sb}}) \qquad \left.\begin{matrix} v_{GS} \\ v_{DS} \end{matrix}\right] \text{const}$$

$i_d = g_{mb}\, v_{sb} \longrightarrow$ small-signal parameter (g_{mb})

$$g_m = \left.\frac{\partial i_D}{\partial v_{SB}}\right|_{v_{SB}=V_{SB}}$$

$$= \left.\frac{\partial}{\partial v_{SB}}\left[\frac{K'_n}{2}\,\frac{W}{L}\,(V_{GS}-V_{THN})^2\right]\right|_{v_{SB}=V_{SB}}$$

$$= \left.\underbrace{\frac{K'_n}{\cancel{2}}\cdot\frac{W}{L}\cdot\cancel{2}(V_{GS}-V_{THN})}\cdot\left(\frac{-\partial V_{THN}}{\partial v_{SB}}\right)\right|_{v_{SB}=V_{SB}}$$

$$= g_m \cdot \underbrace{\left(\frac{-\partial V_{THN}}{\partial V_{SB}}\right)},$$

$V_{THN} = f(v_{SB})$

$$= g_m \cdot \underbrace{\left[-\frac{\gamma}{2} \left(|2V_{fp}| + V_{SB} \right)^{-1/2} \right]}_{\eta}$$

$\eta < 0$

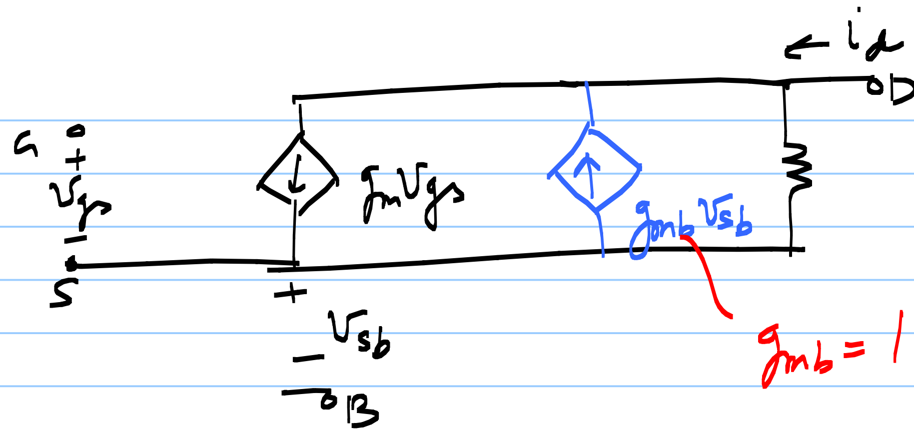
$|\eta| < 1$

$$\boxed{g_{mb} = |\eta| \, g_m}$$

$$\eta = -\frac{\gamma}{2} \frac{1}{\sqrt{|2V_{fp}| + V_{SB}}}$$

* −ve sign in η ⇒ indicates that g_{mb} is opposing 'g_m'

Small signal model for small frequencies

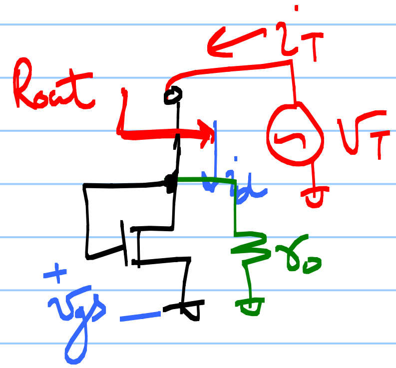


$g_{mb} = |\eta| g_m$

Small-signal parameters

* g_m
* r_o, g_{ds}
* g_{mb}

Diode connected MOSFET



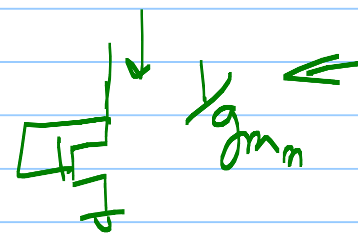
$\frac{1}{g_m} \| r_o \simeq \frac{1}{g_m}$

* Apply a AC test voltage
* find the AC current

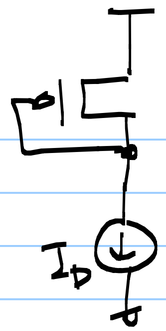
$$\frac{v_T}{i_T} = R_{out}$$

$$R_{out} = \frac{v_T}{i_T} = ?$$

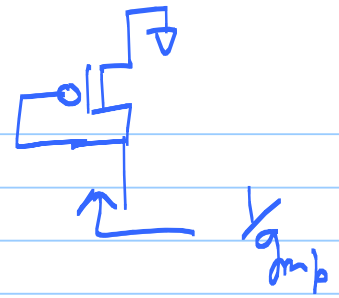
$$= \frac{v_{gs}}{i_d} = \frac{\cancel{v_{gs}}}{g_m \cancel{v_{gs}}} = \frac{1}{g_m}$$



AC picture



I_D



1/g_mp

Steps for small-signal analysis

① Given a circuit, draw the DC picture
↳ Short AC voltage sources
↳ open AC current sources.

Find Bias points for all the devices

② Use the DC operating point from ①,
↳ find small-signal parameters
(g_m, r_o, ...)

[linearization around the bias point

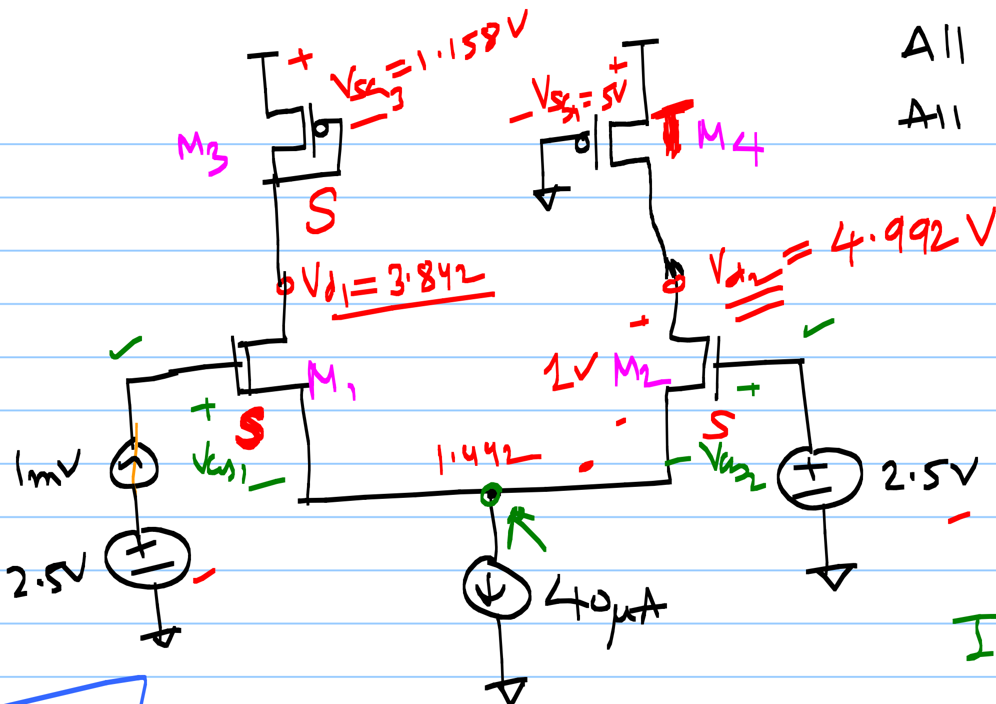
(3) Draw the AC picture for the circuit

↳ replace devices by their small-signal models

↳ short DC voltage sources

↳ open DC current sources

Solve for AC values in this circuit.



All NMOS are 10/2

All PMOS are 30/2

1µm process (Table 6.2)

$V_{DD} = 5V$

$V_{THN} = 0.8$ $\quad V_{THP} = 0.9$

$KP_n = 120 \frac{\mu A}{V^2}$ $\quad KP_p = 40 \frac{\mu A}{V^2}$

$I_{D_1} = I_{D_2} = 20\mu A$

$\boxed{M_1 \& M_2}$

Solve for $V_{GS_1} = V_{GS_2} = \sqrt{\frac{2 I_D \cdot L}{KP_n\ W}} + V_{THN}$

$= \sqrt{\frac{40}{120} \cdot \frac{2}{10}} + 0.8 = 1.058\ V$

$$V_{DS,sat} = V_{GS_1} - V_{THN} = 250\ mV$$

$$V_{S_1} = V_{S_2} = 2.5 - 1.058 = \boxed{1.442V}$$

$\boxed{M_3}$ $V_{SG_3} = \sqrt{\frac{2.20}{40} \cdot \frac{2}{30}} + 0.9 = \boxed{1.158V} \Rightarrow \boxed{V_{SD,sat} = 250mV}$

* M_1 was indeed in Saturation:

$\boxed{M_4}$ $V_{SG} = 5V \Rightarrow$ most likely M_4 is in Triode

for M_4 to be in saturation

$$\Rightarrow V_{SD} \geq V_{SG} - V_{THP}$$

$$V_S - V_D \geq V_S - V_G - V_{THP}$$

$$0 = V_G \geq V_D - V_{THP}$$

$$\Rightarrow \quad \underline{V_{D_4} \le 0.9\,V}$$

Find $V_{D_2} = V_{D_4}$

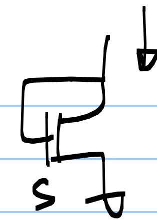
$M_4 \Rightarrow$

$$20\mu = 40\mu \cdot \frac{30}{2}\left[(5-0.9)V_{SD} - \frac{V_{SD}^2}{2}\right]$$

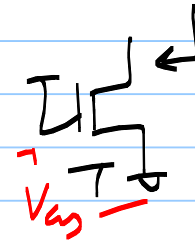
$$\Rightarrow \boxed{V_{SD_4} = 8.13\ mV}$$

$$\Rightarrow \quad V_{D_4} = V_{D_2} = V_{DD} - V_{SD_4} = \underline{4.992\,V}$$

$\therefore$ M_2 is in Saturation indeed.



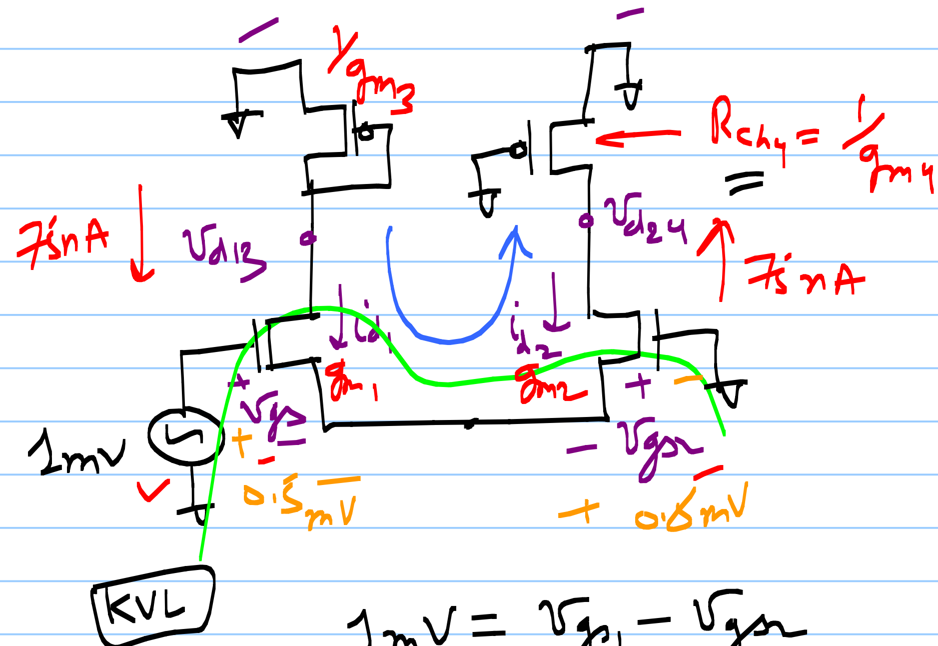
$1/g_m$



$$R_{ch} = \frac{1}{KP_n \frac{W}{L} (V_{GS} - V_{THN})}$$

$$= \frac{1}{g_m}$$

Very small



$$1mV = v_{gs1} - v_{gs2}$$

$\boxed{M_4}$ $R_{ch4} = 407\,\Omega$

$\boxed{M_1 \& M_2}$

$$g_{m1} = g_{m2} = \sqrt{2\beta_n I_D} = 150 \frac{\mu A}{V}$$

$\boxed{M_3}$ $\frac{1}{g_{m3}} = 6.67\,k\Omega$

$$i_{d1} + i_{d2} = 0$$

$$i_{d2} = -i_{d1}$$

$$v_{gs2} = g_{m2}\, i_{d2} = -g_{m1}\, i_{d1} = -v_{gs1}$$

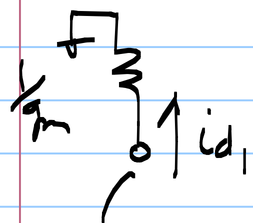
$$\boxed{v_{gs2} = -v_{gs1}} = 0.5\,mV$$

$$i_{d1} = -i_{d2} = g_{m1} v_{gs1} = \boxed{75\,nA}$$

$$i_{D1} = \overset{DC}{I_{D1}} + \overset{AC}{i_{d1}} = \left[20 + 0.075 \cdot \sin(\omega_0 t)\right] \mu A$$

$$i_{D2} = I_{D2} + i_{d2} = \left[20 - 0.075 \cdot \sin(\omega_0 t)\right] \mu A$$

$$i_{D} + i_{D2} = 40\,\mu A$$



$$v_{d_1} = v_{d_3} = -i_{d_1} \cdot \frac{1}{g_{m_3}} = -0.5\,mV$$

(neglected all r_o's)

$$v_{d_2} = v_{d_4} = -i_{d_4} \cdot R_{ch_4} = 0.03\,mV$$

$$v_{D_{1,3}} = \left[3.842 - 0.5 \times 10^{-3} \sin(\omega t)\right] V$$

$$V_{D_{2,4}} = \left[4.792\,V + 0.03 \times 10^{-3} \sin(\omega t)\right] V$$