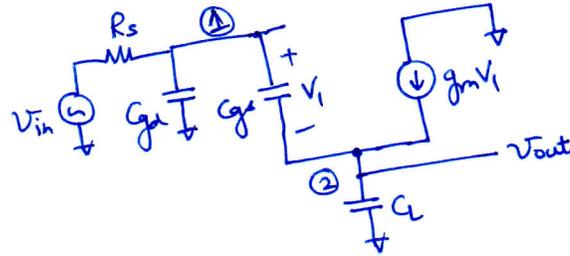
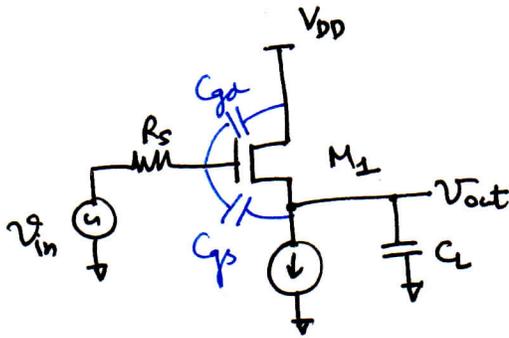


# Source follows frequency response

①



\* strong interaction between nodes ① + ② through  $C_{gs}$ .  
 $\rightarrow$  can't associate a pole with each node

node ②

$$v_1 C_{gs} s + g_m v_1 = v_{out} C_L s$$

$$\Rightarrow v_1 = \frac{C_L s}{g_m + C_{gs} s} v_{out}$$

\* KVL beginning from  $V_{in}$

$$v_{in} = R_s [v_1 C_{gs} s + (v_1 + v_{out}) C_{gd} s] + v_1 + v_{out}$$

substitute  $v_1$

$$\Rightarrow \frac{v_{out}(s)}{v_{in}} = \frac{(g_m + C_{gs} s)}{R_s [C_{gs} C_L + C_{gs} C_{gd} + C_{gd} C_L] s^2 + [g_m R_s C_{gd} + C_L + C_{gs}] s + g_m}$$

$\Rightarrow$  LHP zero

$$\omega_z = -\frac{g_m}{C_{gs}}$$

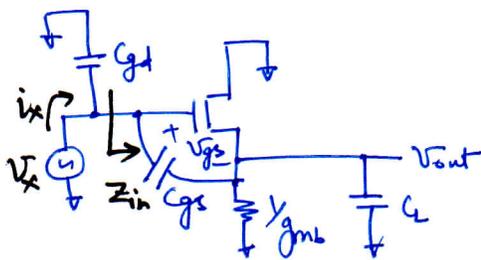
$\Rightarrow$  signal conducted by  $C_{gs}$ , adds to the signal carried by the main transistor with the same polarity.

Assuming  $|\omega_{p1}| \ll |\omega_{p2}|$

Dominant pole is at

$$\begin{aligned}\omega_{p1} &\approx \frac{g_m}{g_m R_s C_{gd} + C_L + C_{gs}} \\ &= \frac{1}{R_s C_{gd} + \frac{C_L + C_{gs}}{g_m}} = \frac{g_m}{C_L + C_{gs}} \text{ iff } R_s = 0.\end{aligned}$$

Input impedance of SF:



\*  $C_{gd}$  shunts the input

$$v_{gs} = \frac{i_x}{sC_{gs}} \Rightarrow i_d = g_m v_{gs} = \frac{g_m i_x}{sC_{gs}}$$

$$\Rightarrow v_x = \frac{i_x}{sC_{gs}} + \left( i_x + \frac{g_m i_x}{sC_{gs}} \right) \left( \frac{1}{g_{mb}} \parallel \frac{1}{sC_L} \right)$$

$$\Rightarrow Z_{in1} = \frac{1}{sC_{gs}} + \left( 1 + \frac{g_m}{sC_{gs}} \right) \frac{1}{g_{mb} + sC_L}$$

@ low-frequencies

$$g_{mb} \gg |sC_L|$$

$$\Rightarrow Z_{in1} \approx \frac{1}{sC_{gs}} \left( 1 + \frac{g_m}{g_{mb}} \right) + \frac{1}{g_{mb}} = \frac{1}{sC_{gs} \left( 1 + \frac{g_m}{g_{mb}} \right)} + \frac{1}{g_{mb}}$$

$\Rightarrow$  Input cap is a fraction of  $C_{gs} + C_{gd}$ .

At higher frequencies

$$g_{mb} \ll |sC_c|$$

$$\Rightarrow Z_{in_1} = \frac{1}{sC_{gs}} + \frac{1}{sC_c} + \frac{g_m}{C_{gs}C_c s^2}$$

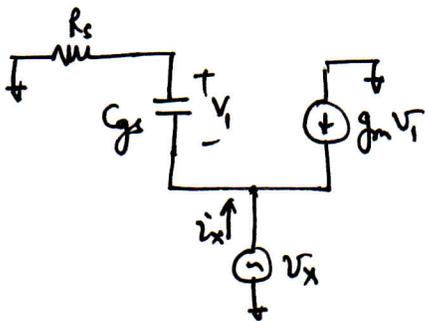
$\Rightarrow$  for same  $s = j\omega$

$$Z_{in_1} = \frac{1}{j\omega} \underbrace{\left[ \frac{1}{C_{gs}} + \frac{1}{C_c} \right]}_{\text{Capacitance}} - \underbrace{\frac{g_m}{C_{gs}C_c \omega^2}}_{\text{negative resistance}}$$

$$Re(Z_{in_1}) = R_{in_1} = \frac{-g_m}{C_{gs}C_c \omega^2} \leftarrow \text{useful in oscillators.}$$

Output impedance:

neglecting  $g_{mb}$  &  $C_{gd}$



$$V_1 C_{gs} s + g_m V_1 = -i_x \rightarrow \textcircled{1}$$

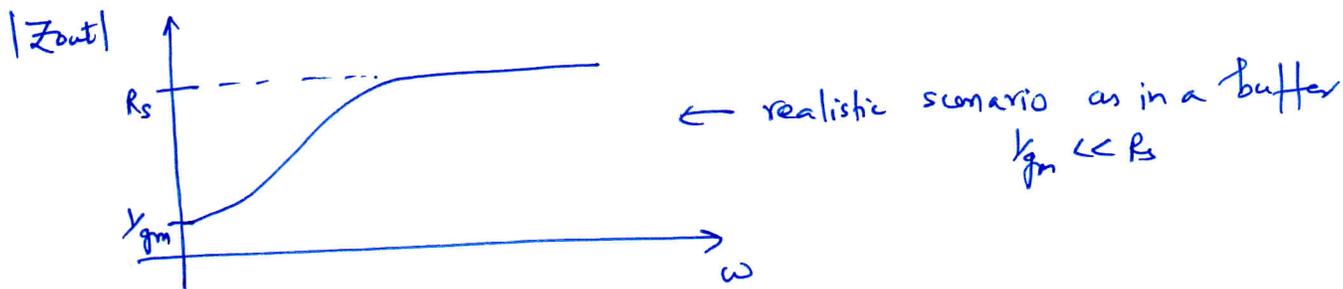
$$\Delta \quad V_1 C_{gs} s R_s + V_1 = -V_x \rightarrow \textcircled{2}$$

$$\frac{\textcircled{2}}{\textcircled{1}} \Rightarrow$$

$$Z_{out} = \frac{V_x}{i_x} = \frac{R_s C_{gs} s + 1}{g_m + s C_{gs}}$$

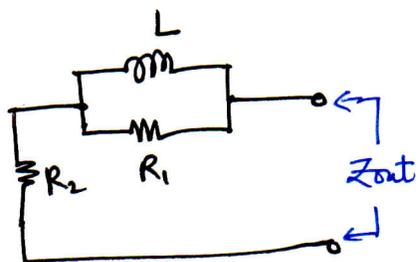
@ low frequencies  $\Rightarrow Z_{out} \approx 1/g_m \rightarrow$  Expected from a SF

@ very high frequencies  $\Rightarrow Z_{out} \approx R_s$  ( $\because C_{gs}$  shorts gate & source)



\* Since the  $Z_{out} \uparrow$  with frequency  
 $\hookrightarrow$  should contain an inductive component!  
 ( $j\omega L$ )

Equivalent impedance can be derived to be !



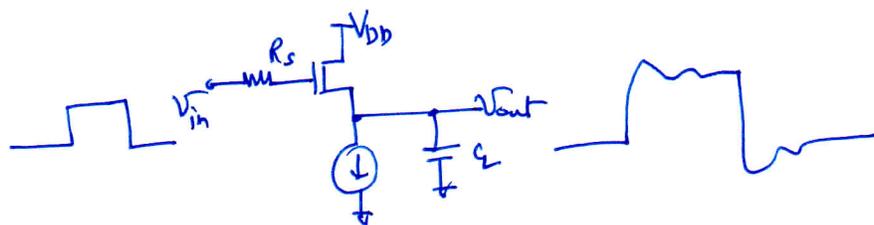
$$R_2 = 1/g_m$$

$$R_1 = R_s - 1/g_m$$

$$L = \frac{C_{gs}}{g_m} (R_s - 1/g_m)$$

$$\therefore L = \frac{C_{gs}}{g_m} (R_s - 1/g_m)$$

$\Rightarrow$  If the SF is driven by a large  $R_s$   
 $\Rightarrow$  then it exhibits substantial inductive behavior  
 $\hookrightarrow$  manifests as ringing

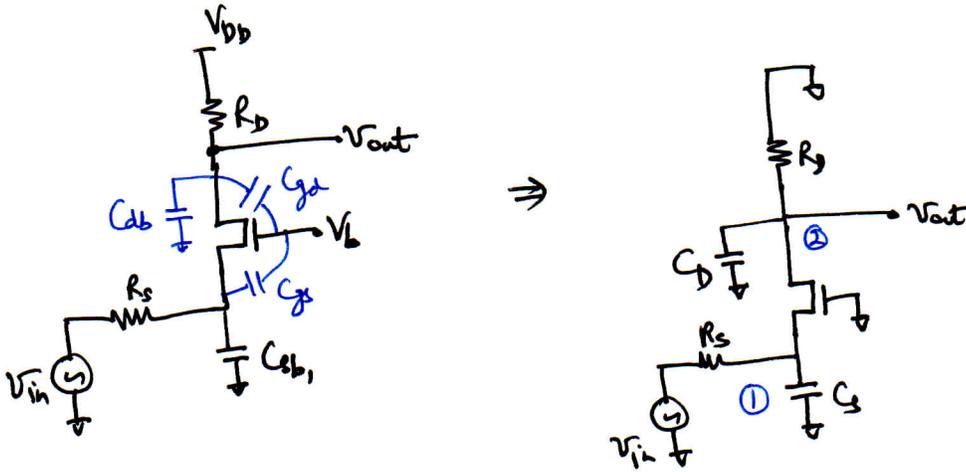


\* Useful as gain peaking loads!

# Common Gate Frequency Response

(5)

In a CG, input and output nodes are 'isolated' if  $\lambda=0$ .



@ node ①

$$C_s = C_{gs1} + C_{sb1}$$

+ Since the nodes are isolated  $\Rightarrow$  no Miller Cap  
 $\Rightarrow$  no zeros in the frequency response

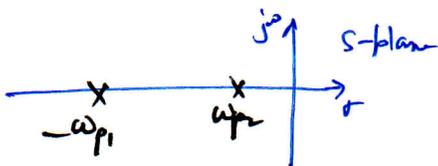
$$\Rightarrow \omega_{p1} = \omega_{in} = \frac{1}{(R_s \parallel \frac{1}{g_m + g_{mb}}) \cdot C_s}$$

@ node ②

$$C_d = C_{gd1} + C_{db1}$$

$$\Rightarrow \omega_{p2} = \omega_{out} = \frac{1}{R_D C_d}$$

$$\Rightarrow \frac{V_{out}}{V_{in}}(\omega) = \frac{(g_m + g_{mb}) R_D}{1 + (g_m + g_{mb}) R_D} \cdot \frac{1}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})}$$



⇒ NB Miller multiplication of Capacitor

↳ potentially wide-band

↳ low impedance input may load the driving stage

↳ good for current input (i.e. transimpedance) amplifiers

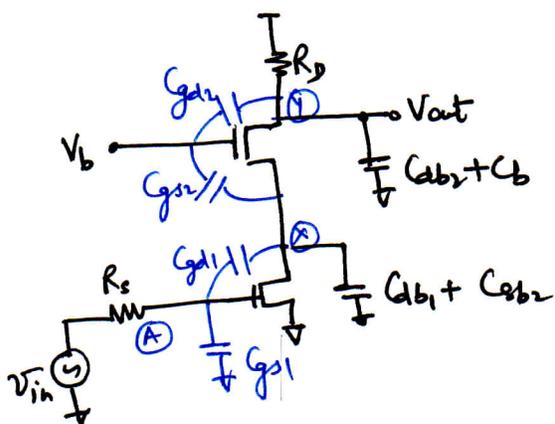
As derived earlier

$$Z_{in} \approx \frac{Z_L}{(g_m + g_{mb})R_o} + \frac{1}{(g_m + g_{mb})}$$

$$\text{where } Z_L = \left( R_o \parallel \frac{1}{sC} \right)$$

# Cascode Amplifier Frequency Response:

⑦



\* Miller effect of  $C_{gd1}$  is determined by the gain from (A) to (X)

↳ Assuming low value of  $R_D$ , this gain

$$\text{is } -g_{m1} \times \frac{1}{g_{m2} + g_{mb2}} \quad (\text{assuming } \beta \gg 1)$$

$$\approx -\frac{g_{m1}}{g_{m2}} \approx -1$$

\* Assume  $M_1$  &  $M_2$  to be identical

⇒  $C_{gd1}$  is multiplied by 2x, instead of  $|A_v|$

⇒ Miller effect is less significant in cascode amplifiers than in a CS stage. (normal CS)

$$\rightarrow \omega_{pA} = \frac{1}{R_s \left[ C_{gs1} + \left( 1 + \frac{g_{m1}}{g_{m2} + g_{mb2}} \right) C_{gd1} \right]}$$

\* pole due to node (X)

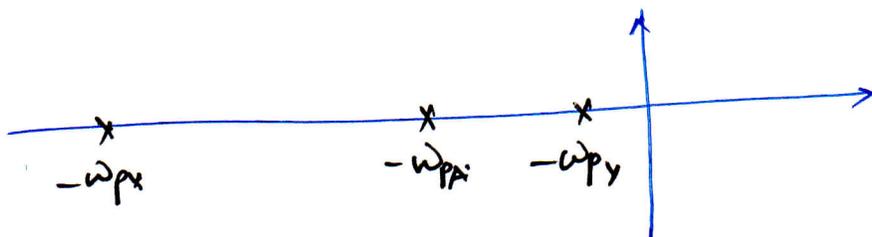
$$C_X = 2C_{gd1} + C_{db1} + C_{cb2} + C_{gs2}$$

$$\rightarrow \omega_{pX} = \frac{g_{m2} + g_{mb2}}{2C_{gd1} + C_{db1} + C_{cb2} + C_{gs2}}$$

\* Output pole:  $\rightarrow$  third pole

$$\omega_{py} = \frac{1}{R_D (C_{db2} + C_L + C_{gd2})}$$

\* Must choose  $\omega_{px}$  to be farther away from the other two  
 $\hookrightarrow$  important for stability



\* What if  $R_D$  is replaced by a cascode current source load?  
 $\hookrightarrow$  higher DC gain  $\rightarrow A_{DC} \approx (g_{m1} r_{o1}^2) \parallel (g_{m2} r_{o2})$ .

$\Rightarrow$  Impedance at node  $\textcircled{x}$  reaches higher value (Derive this) yourself

$\hookrightarrow$  pole at node  $\textcircled{x}$  can be quite lower than

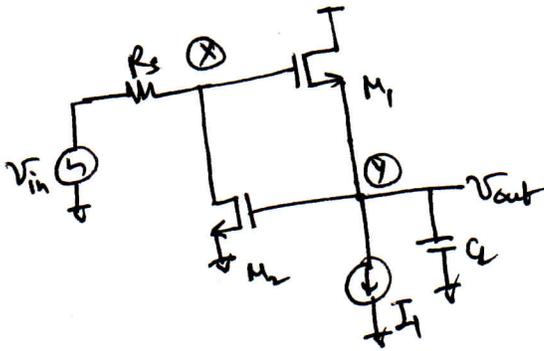
$$\frac{(g_{m2} r_{o2})}{\alpha}$$

$\hookrightarrow$  But in practice this pole doesn't degrade stability much.

$\Rightarrow$  But always be mindful of such scenarios!

Exercise:

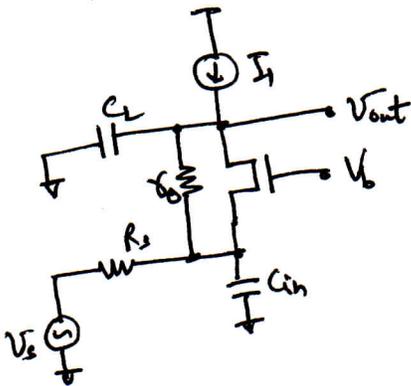
① find the transfer function of the circuit (Assume  $\lambda \rightarrow 0, \gamma \rightarrow 0$ )



Ans: 
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{g_{m1} + C_{xy}s}{R_S g_m s^2 + [C_y + g_{m1} R_c C_x + (1 + g_{m2} R_c) C_{xy}]s + g_{m1} (1 + g_{m2} R_c)}$$

$$C_{xy} = C_{g1} + C_{gd2}$$

② find  $\frac{V_{out}(s)}{V_{in}(s)}$  &  $Z_{in}$  for the circuit ( $\lambda \neq 0$ )



Ans: 
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1 + g_{m2} R_D}{(R_D C_L C_{in} s)^2 + [R_D C_L + C_{in} R_D + (1 + g_{m2} R_D) C_L R_D]s + 1}$$

$$Z_{in} = \frac{1}{g_m + g_{m2}} + \frac{1}{s C_L} \frac{1}{(g_m + g_{m2}) R_D}$$