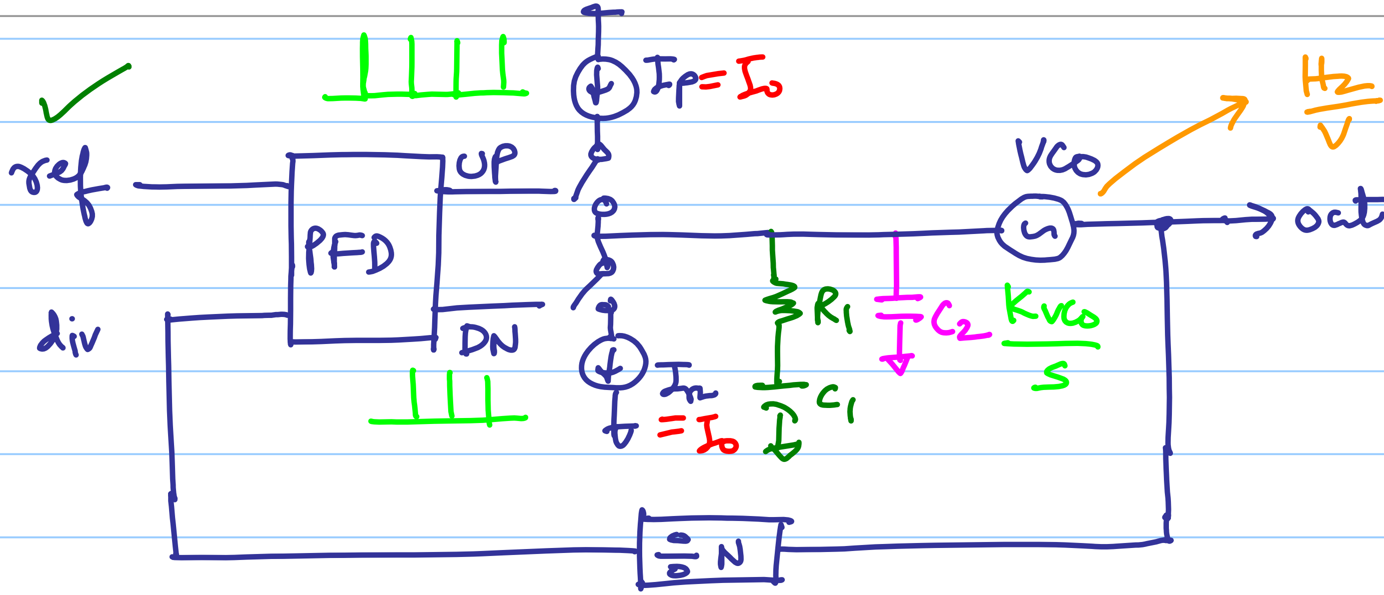
ECE 518 - Lecture 8

Note Title 2/10/2015



Type-II

~~2nd order~~

3rd - order

$$K_{PD,I} = \frac{I_0}{2\pi C_1}$$

$$K_{PD} = \frac{I_0 R_1}{2\pi}$$

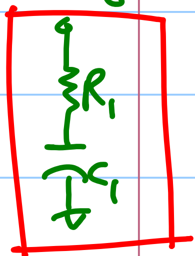
approximation

$$\omega_{u,loop} \simeq \frac{2\pi K_{vco} \cdot K_{PD}}{N} = \frac{K_{vco} \cdot I_0 R}{N}$$

Single-pole approximation

Type-II PLL loop dynamics

2nd-order:



$$L(s) = \frac{2\pi K_{PD,I}\cdot K_{VCO}}{N s^2}\left(1+\frac{sK_{pd}}{K_{pd,I}}\right)$$

open-loop response

$$K_{pd,I} = \frac{I_0}{2\pi C}$$

$$K_{PD} = \frac{I_0 R}{2\pi}$$

$$H_{CL}(s) = \frac{\phi_{out}(s)}{\phi_{ref}(s)} = \frac{G(s)}{1+\underbrace{G(s)H(s)}_{L(s)}} = \frac{1}{\underbrace{H(s)}_{1/N}}\cdot\frac{L(s)}{1+L(s)} = \frac{N\cdot L(s)}{1+L(s)}$$

$$= \frac{N\cdot\left(1+\frac{sK_{pd}}{K_{pd,I}}\right)\xrightarrow{} \text{zero}}{\underbrace{s^2\frac{N}{2\pi\cdot K_{pd,I}\cdot K_{VCO}} + \frac{sK_{pd}}{K_{pd,I}}}_{\text{2 poles}} + 1} = \frac{N\left(1+\frac{2\zeta}{\omega_n}s\right)}{\frac{s^2}{\omega_n^2}+\frac{2\zeta s}{\omega_n}+1}$$

Standard form

$$H_{CL}(s) = \frac{N\left(1+\frac{2\zeta}{\omega_n}s\right)}{\frac{s^2}{\omega_n^2}+\frac{2\zeta s}{\omega_n}+1} \Longleftrightarrow \frac{N\left(2\zeta\omega_n s+\omega_n^2\right)}{s^2+2\zeta\omega_n s+\omega_n^2}$$

Standard form

Natural frequency, $\omega_n = \sqrt{\frac{2\pi K_{PD,I} K_{VCO}}{N}} = \sqrt{\frac{I_0 K_{VCO}}{C \cdot N}}$ ✓

Damping factor, $\zeta = \frac{K_{pd}}{2}\sqrt{\frac{2\pi K_{VCO}}{N K_{PD,I}}} = \frac{R}{2}\sqrt{\frac{I_0 K_{VCO} C}{N}}$ ✓

$K_{VCO} \rightarrow \frac{Hz}{V}$

$\downarrow$

2π discrepancy

Quality factor $Q \triangleq \frac{1}{2\zeta} = \frac{1}{R}\sqrt{\frac{NC}{I_0 \cdot K_{VCO}}}$

ω_z

zero; $\omega_z = \dfrac{K_{PD,I}}{K_{PD}} = \dfrac{1}{RC}$ "LHP zero"

closed loop

poles:

$$= \frac{s^2}{\omega_n^2} + \frac{2\xi s}{\omega_n} + 1$$

roots of $D(s) = s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$$\omega_{p1,2} = \omega_n\left[-\xi \pm \sqrt{\xi^2 - 1}\right]$$

$$\omega_z = \frac{\omega_n}{2\xi}$$

* for well separated poles $\xi >> 1$

$$D(s) \triangleq \left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right) = \frac{s^2}{\omega_{p1}\omega_{p2}} + s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + 1$$

$\omega_n = \sqrt{\omega_{p1}\omega_{p2}}$ ⟸ geometric mean

$$\frac{2\xi}{\omega_n} = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}$$ ← harmonic mean

$\Rightarrow$ $\boxed{\omega_{p1} \approx \frac{\omega_n}{2\xi}}$

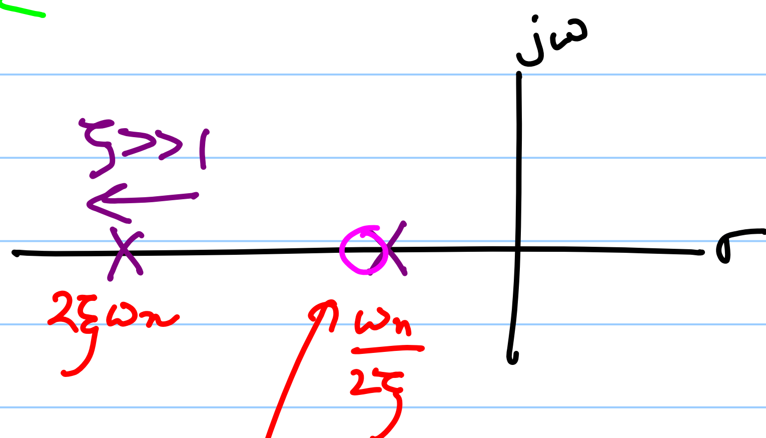
$\omega_{p2} \approx 2\xi\omega_n$

$\omega_z = \frac{\omega_n}{2\xi}$

$AM \Rightarrow \frac{x_1 + x_2}{2}$

$GM = \sqrt{x_1 x_2}$

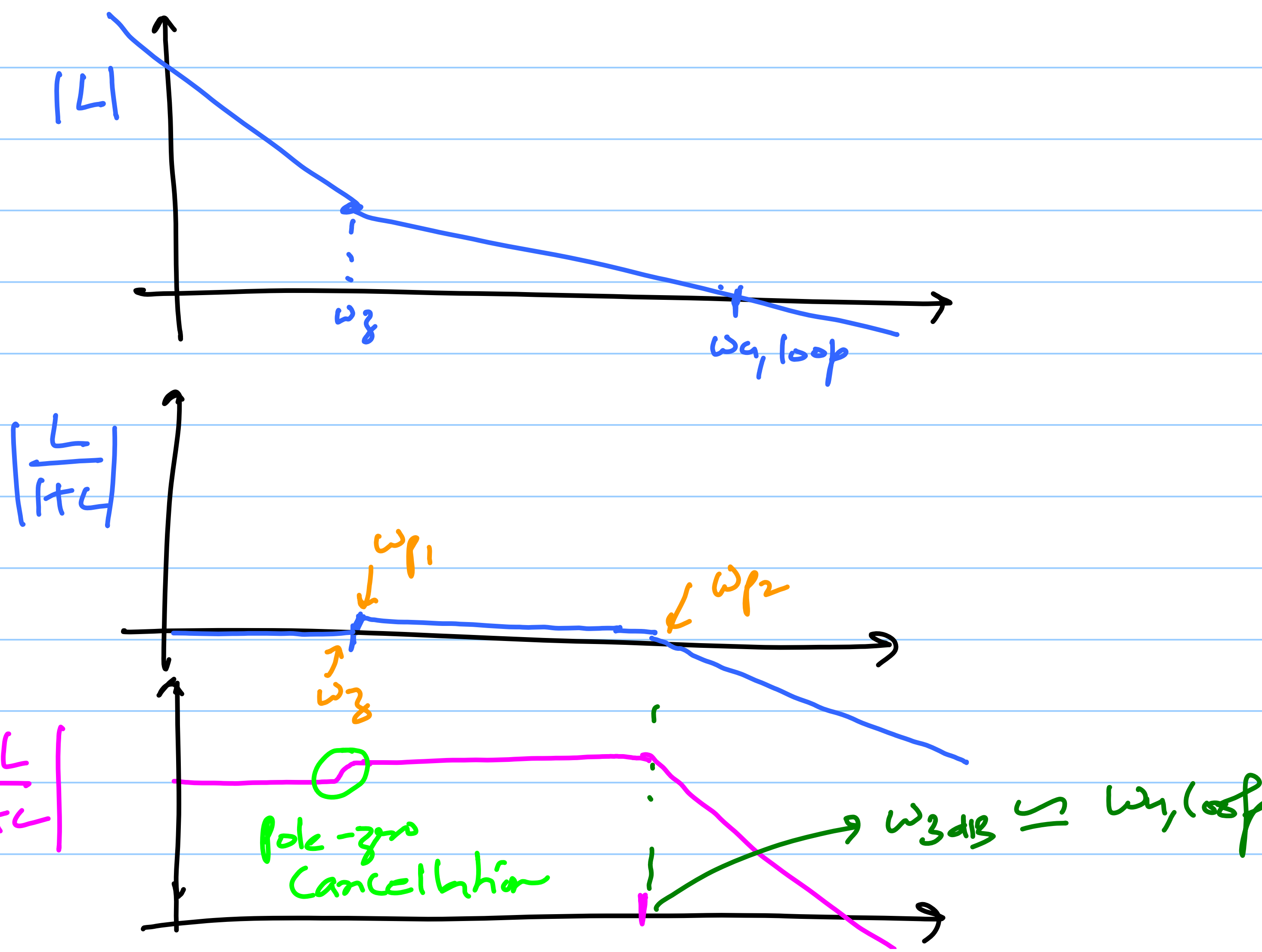
$\frac{1}{HM} = \frac{1}{x_1} + \frac{1}{x_2}$



pole-zero doublet

One-pole approximation

$\omega_{3dB} = \underbrace{2\zeta\omega_n}$ ← closed-loop BW



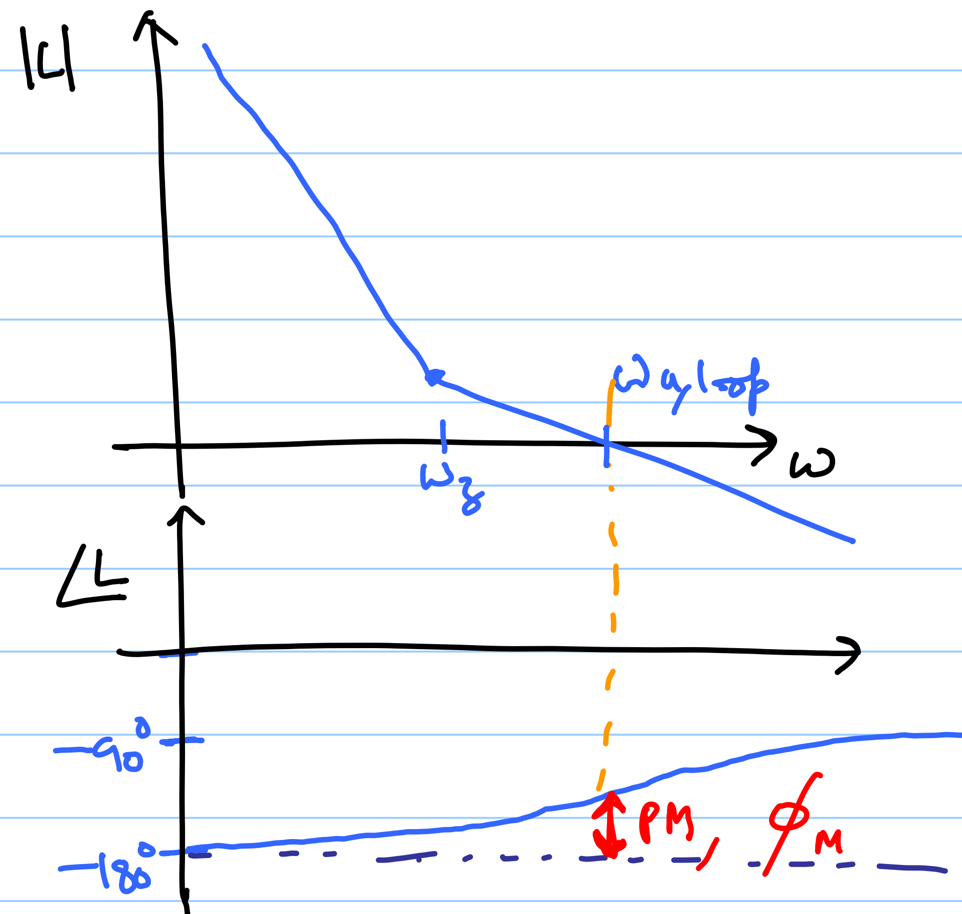
$$\frac{\phi_{out}}{\phi_{ref}}(s) = \left|\frac{NL}{1+L}\right|$$

Phase Margin of a Type-II PLL

a) 2^{nd}- order PLL

$$L(s) = \frac{K_{vco} \cdot I_o R_1}{2\pi} \frac{(1 + sR_1C_1)}{Ns^2}$$

$$\Rightarrow \quad \omega_{u,loop} \simeq \frac{K_{vco} \cdot I_o R_1}{N}$$



$$\phi = -180^0 + \underbrace{\tan^{-1}\left(\frac{\omega}{\omega_z}\right)}_{\text{LHP zero}}$$

Phase Margin $= \tan^{-1}\left(\frac{\omega_{u,loop}}{\omega_z}\right) \longleftarrow$ phase added by the LHP zero at $\omega = \omega_{u,loop}$

$$= \tan^{-1}\left(\frac{K_{vco} I_o R_1^2 C_1}{N}\right)$$