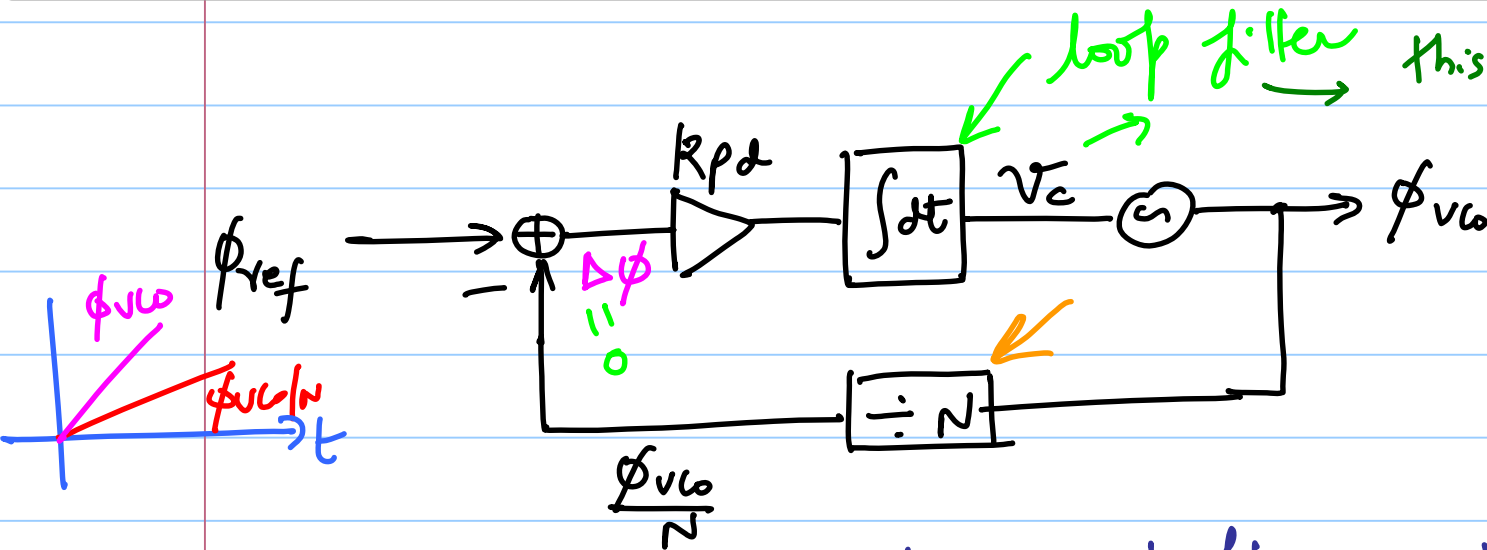


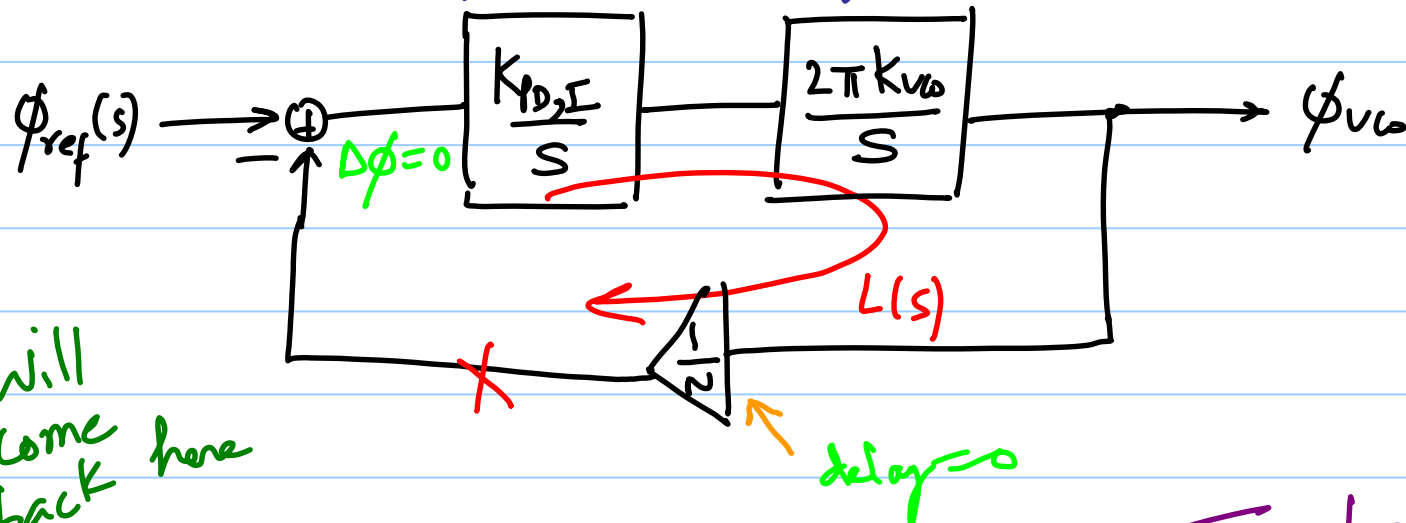
ECE 518 - Lecture 5

Note Title

1/29/2015



Incremental equivalent signal flow graph for the PLL



phase-domain SFG

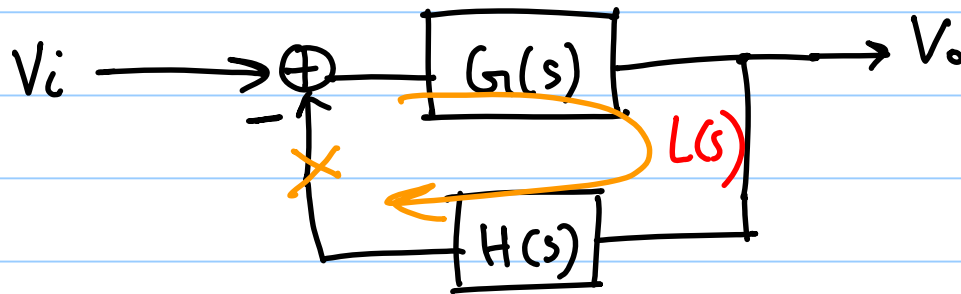
Will come back here

open-loop response:

$$L(s) = \frac{2\pi K_{pd,I} \cdot K_{vco}}{N \cdot s^2}$$

two poles at DC

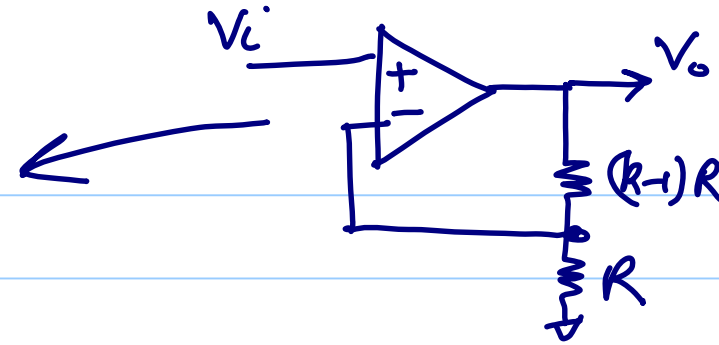
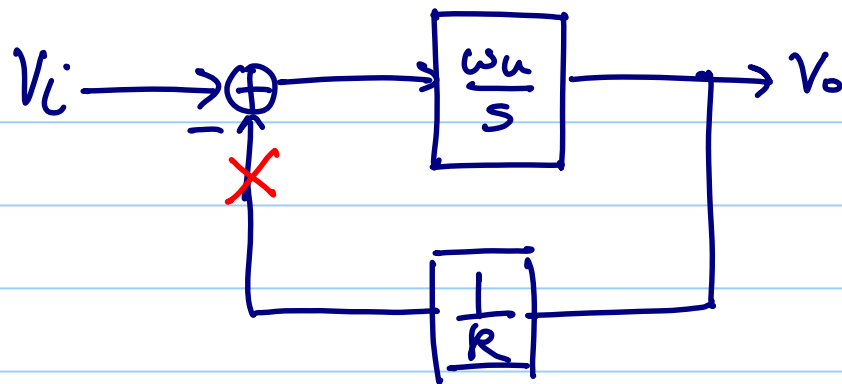
feedback System Review



$$L(s) = G(s)H(s)$$

$$\frac{V_o}{V_i}(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{1}{H(s)} \cdot \frac{1}{1 + \frac{1}{G(s)H(s)}}$$

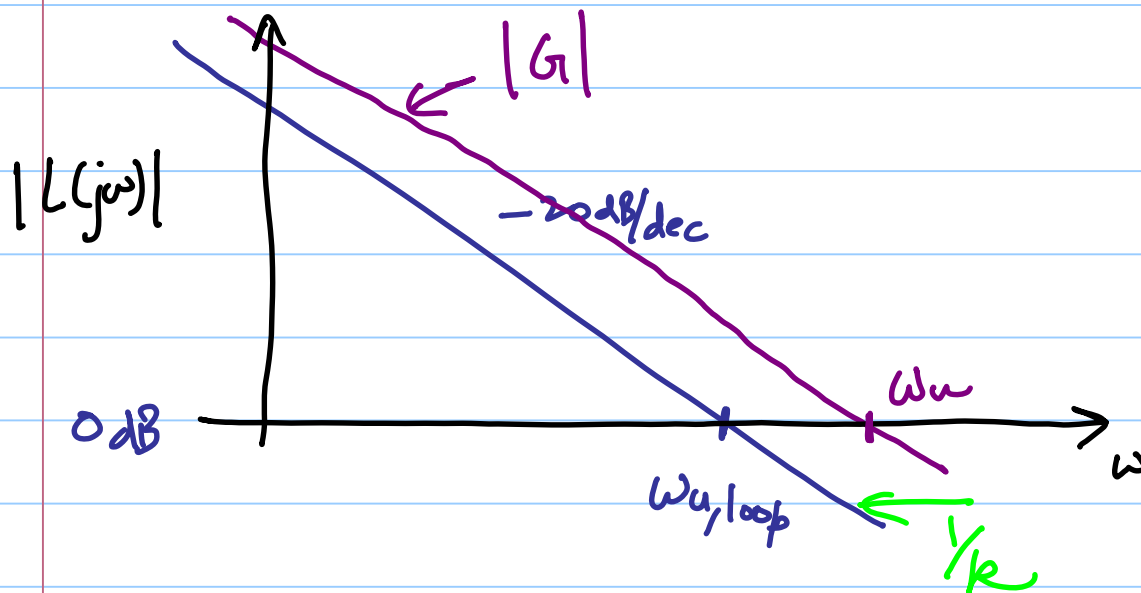
$$\approx \begin{cases} \frac{1}{H(s)} & , \quad |GH| \gg 1 \quad \text{for large } L(s) \\ G(s) & , \quad |GH| \ll 1 \quad \text{small } L(s) \end{cases}$$

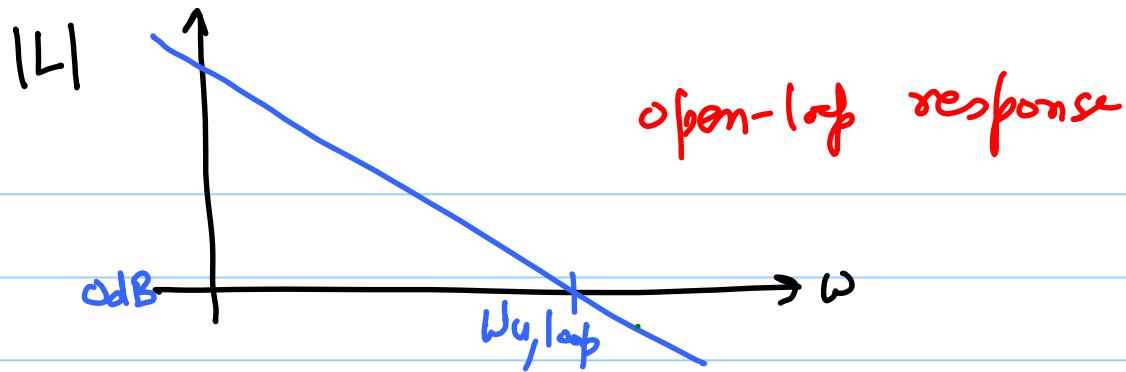


$$L(s) = \frac{\omega_u}{k \cdot s} \quad \text{loop-gain}$$

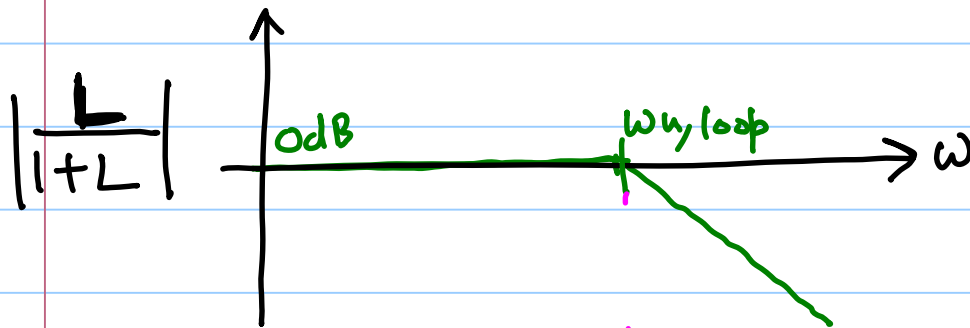
$$\omega_{u, \text{loop}} = \frac{\omega_u}{k}$$

$$G(s) = \frac{\omega_u}{s}$$

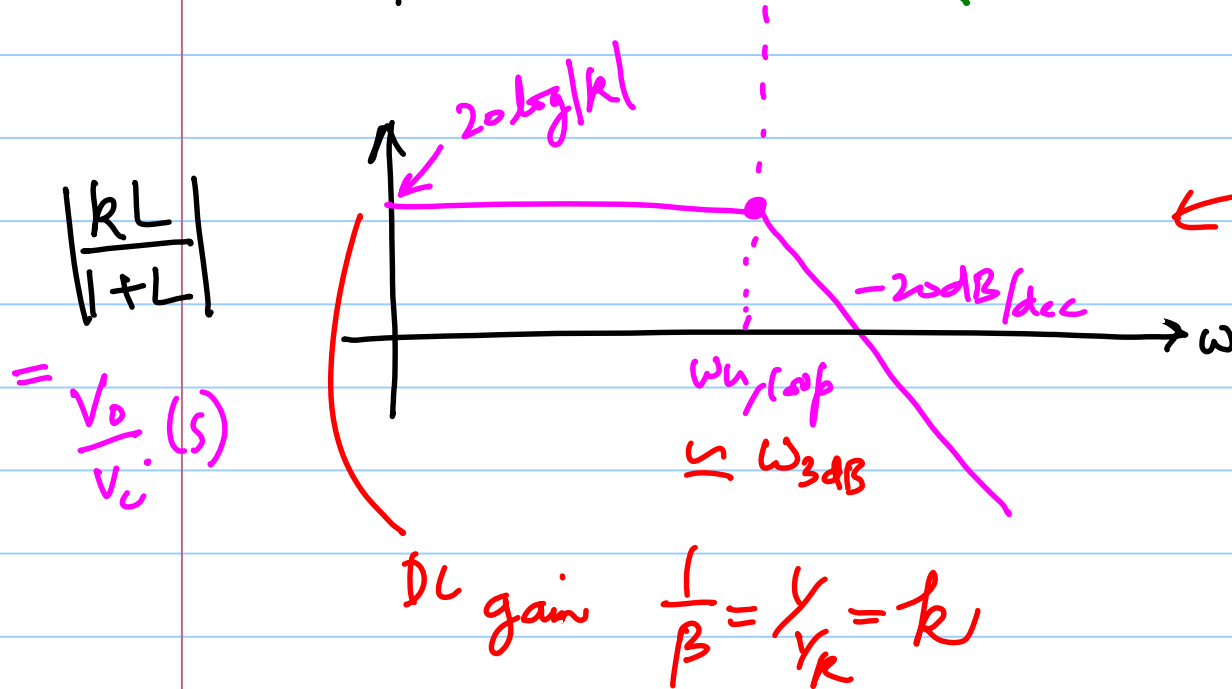




$$|L| \gg 1 \rightarrow 1 \checkmark_{0dB}$$



unity-gain response



$$\left| \frac{G}{1+GH} \right|$$

closed-loop response

$$= \frac{V_o}{V_i}(s)$$

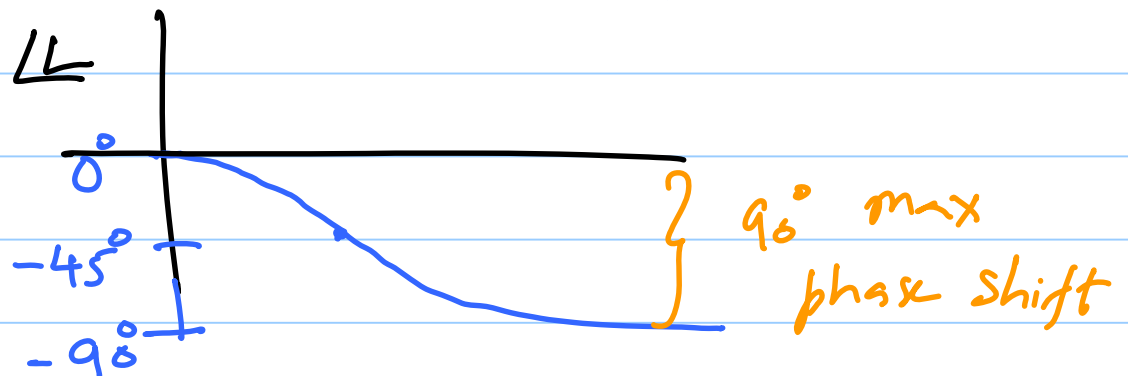
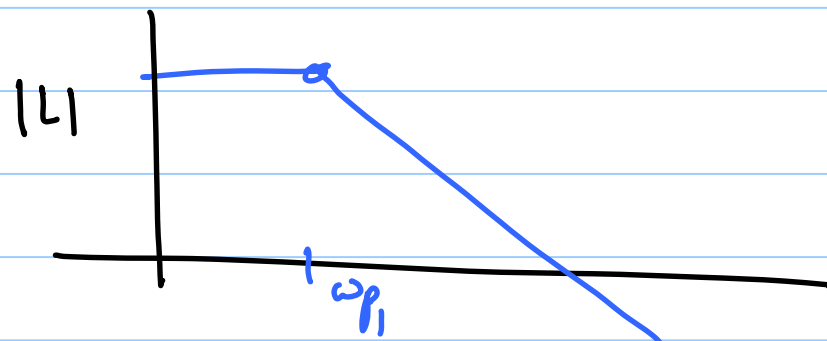
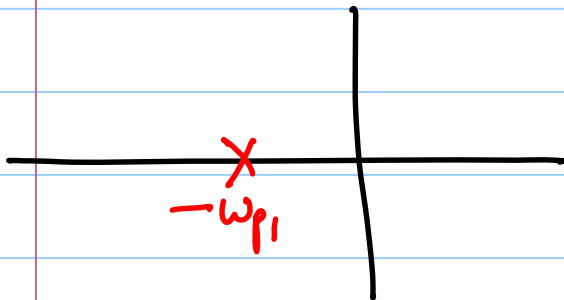
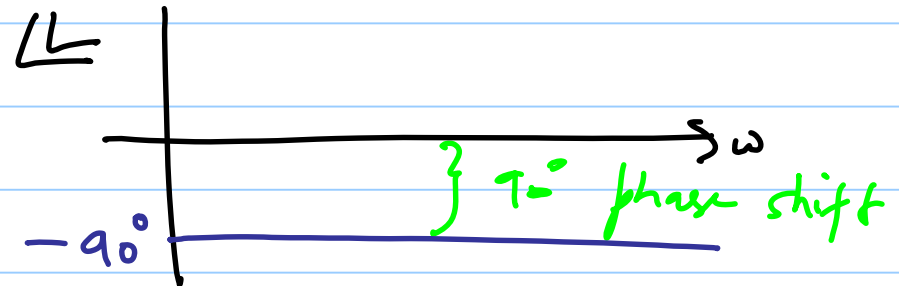
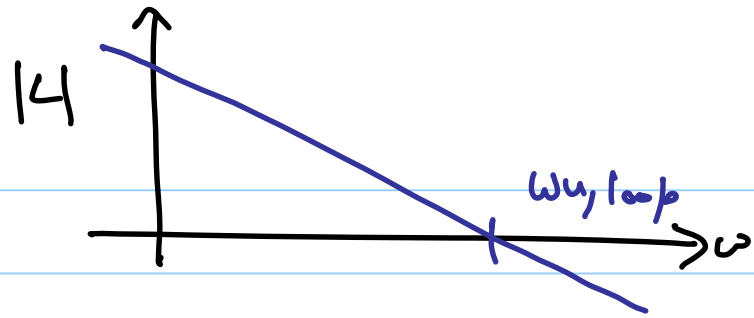
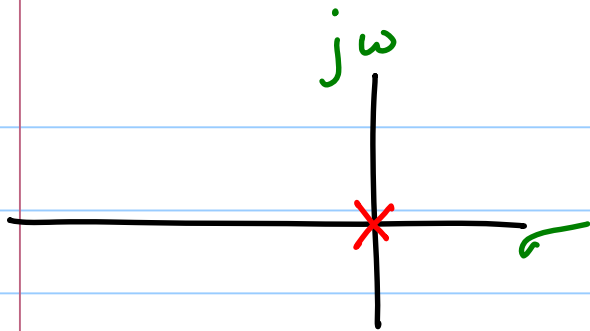
unity loop-gain frequency, $\omega_{u,loop} = \frac{\omega_u}{k}$

↳ $\omega_{u,loop}$ is not always ω_u divided by closed loop-gain 'k'

↳ 1st order system

↳ $\omega_{u,loop}$ is the unity gain frequency of $L(s) = G(s) \cdot H(s)$ not just $G(s)$

1st-order system



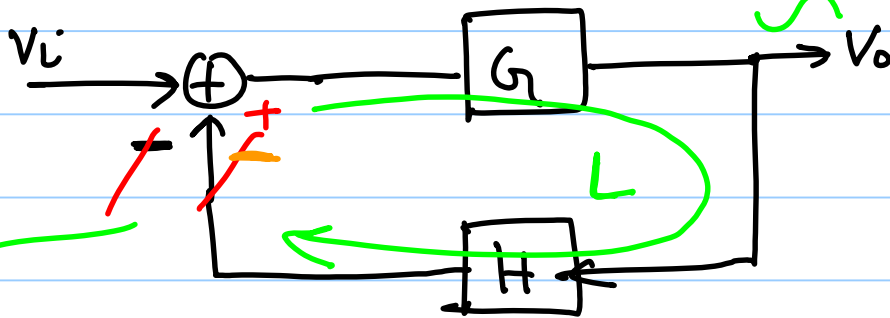
closed-loop
response

$$\frac{V_o}{V_i}(s) = \frac{G(s)}{1 + L(s)}$$

Instability if the loop gain $L = GH = -1$ at
any frequency

$$L = -1$$

at any frequency: $|L| = 1$ and $\angle L = 180^\circ$ "Barkhausen's criterion"



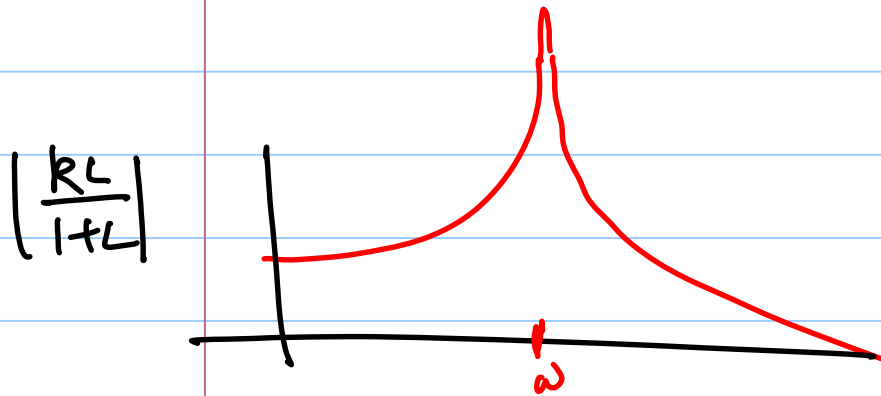
if at any frequency
 $|L| \geq 1$
& $\angle L = 180^\circ$

-ve feedback becomes +ve feedback as the
total phase around the loop is $360^\circ \Rightarrow$ unstable

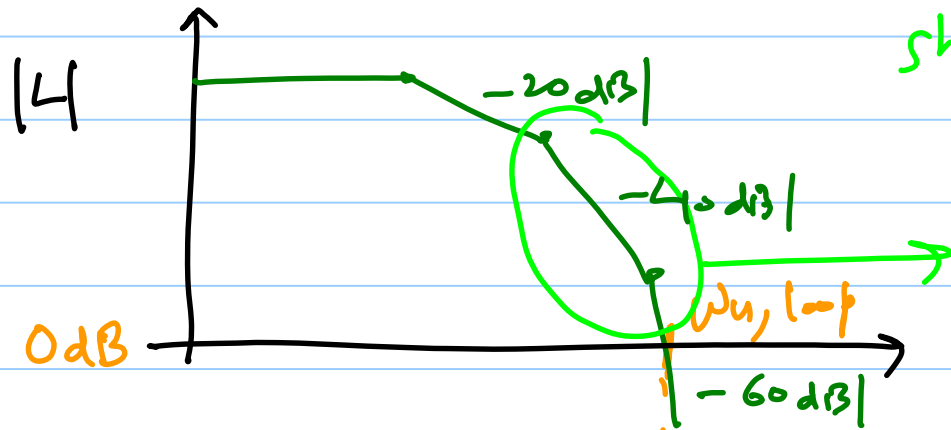
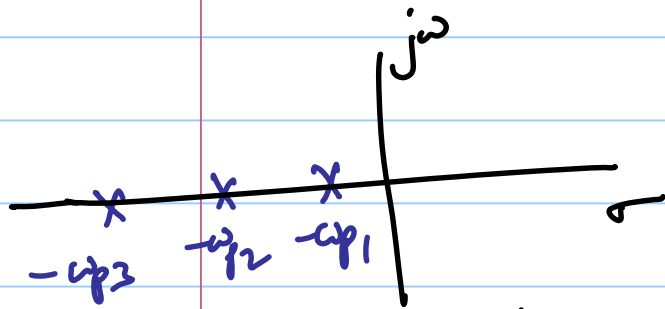
for stability;

① for all frequencies where $|L| \geq 1$
 $\angle L \leq 180^\circ$

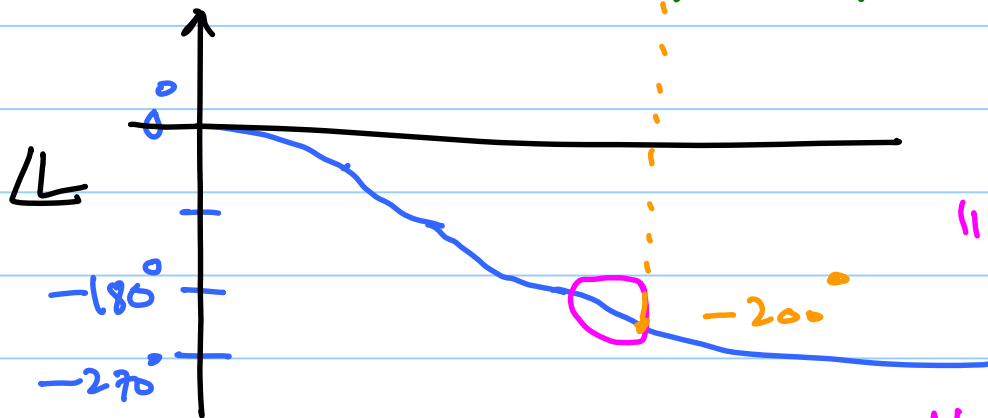
② for all frequencies where $\angle L \geq 180^\circ$
 $|L| < 1$



Multiple poles:



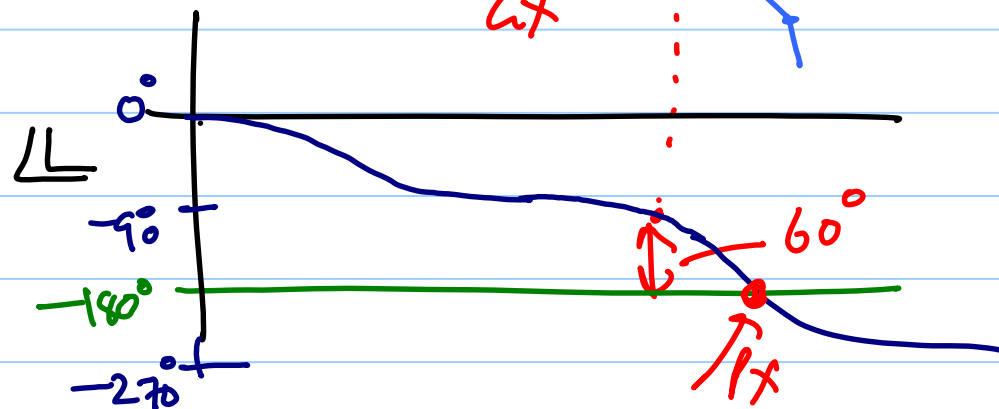
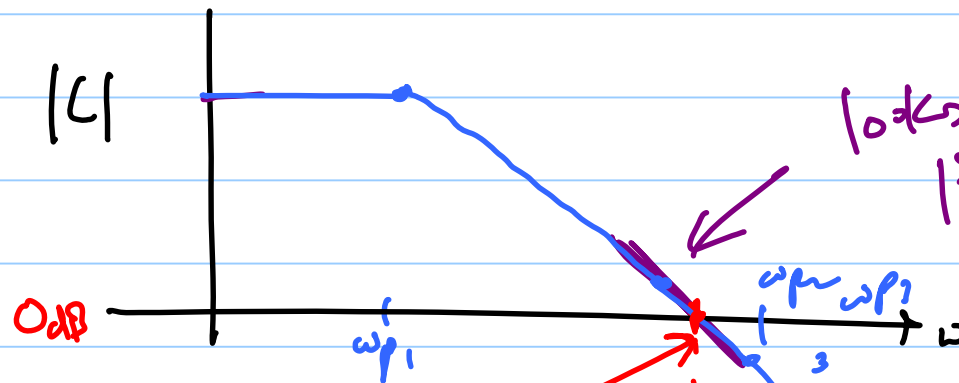
shift the higher poles beyond $\omega_{u, \text{loop}}$



"unstable"

phase margin = -20°

poles far from each other

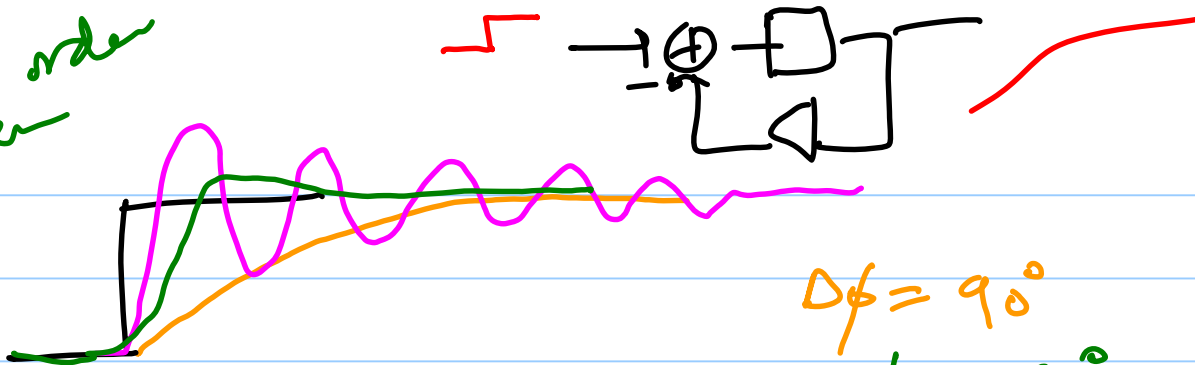


phase margin = 60°

$G_x < P_x$
for stability

$G_x \rightarrow$ gain crossover frequency
 $P_x \rightarrow$ phase "

2nd-order
system

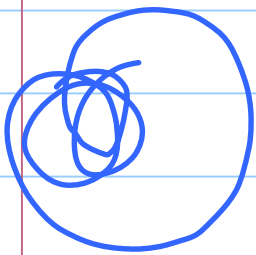
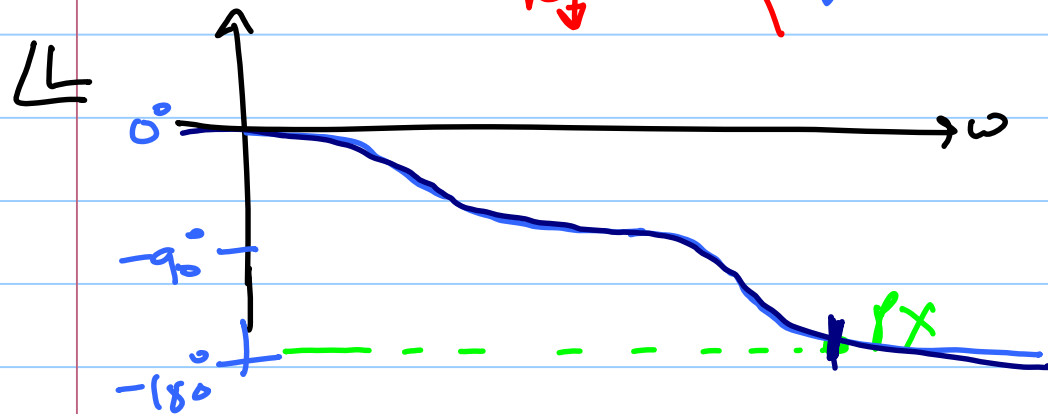
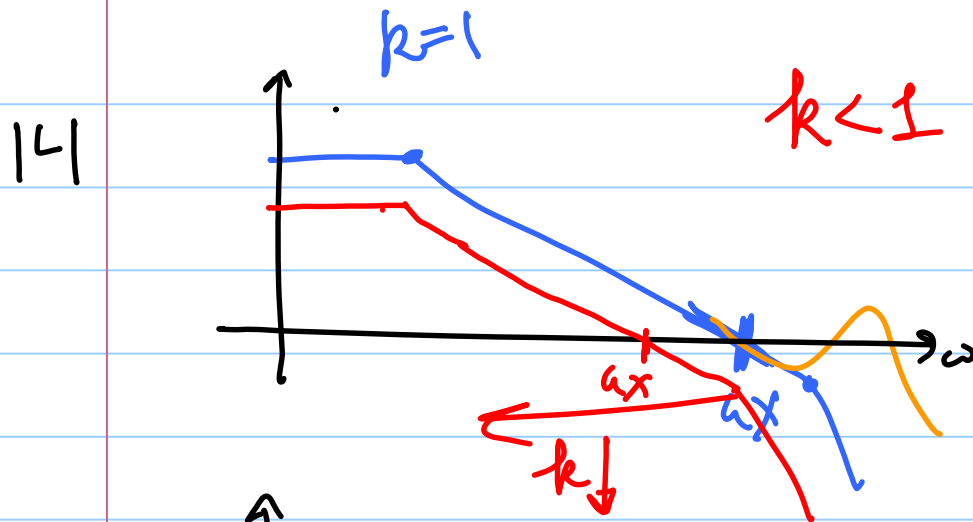
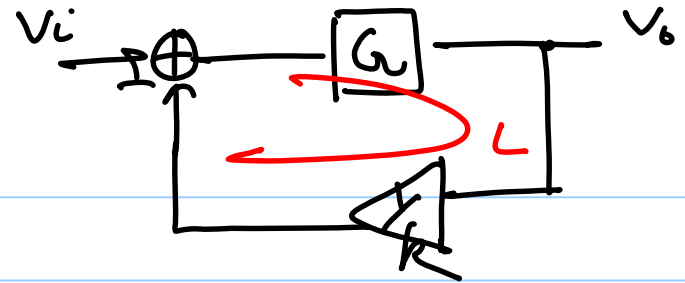


$$\Delta\phi = 90^\circ$$

$$\Delta\phi \leq 60^\circ$$

$$\Delta\phi = 20^\circ$$

critically damped



Worst-case stability condition for $k=1$

if stable for $k=1$
 $\Rightarrow$ stable for all $k < 1$

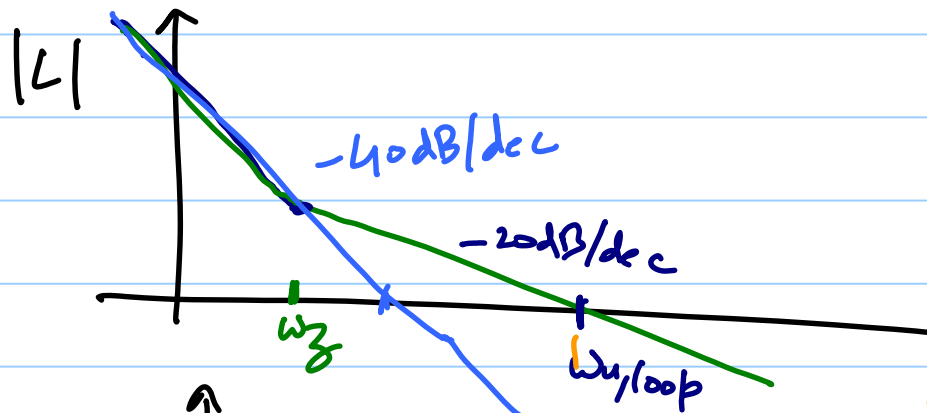
$\hookrightarrow$ there should only be one ux

if this happens then need to use Nyquist stability

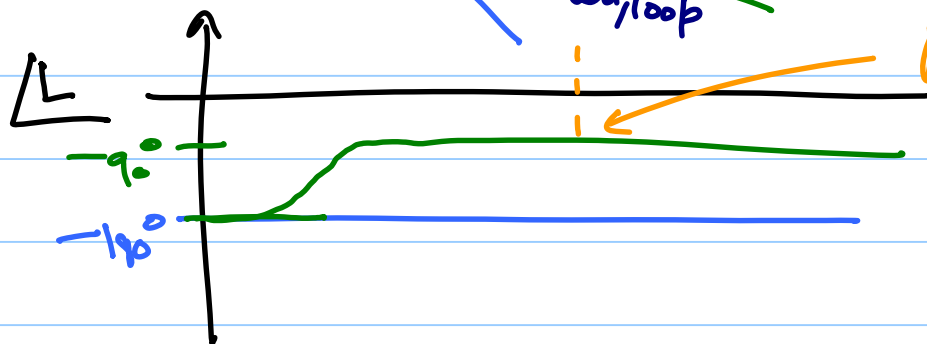
In Summary:

- * for stability, the mag plot must have a slope of -20dB/dec around the unity gain frequency
- * There can be any number of poles to the right as long as they are sufficiently far away from the c.c.
- * To stabilize a higher order system, it must be made to "look-like" a first-order system at and around the $\omega_{u, \text{lof}}$.

$$L(s) = \frac{2\pi k_{p,I} \cdot K_{v10}}{N \cdot s^2}$$



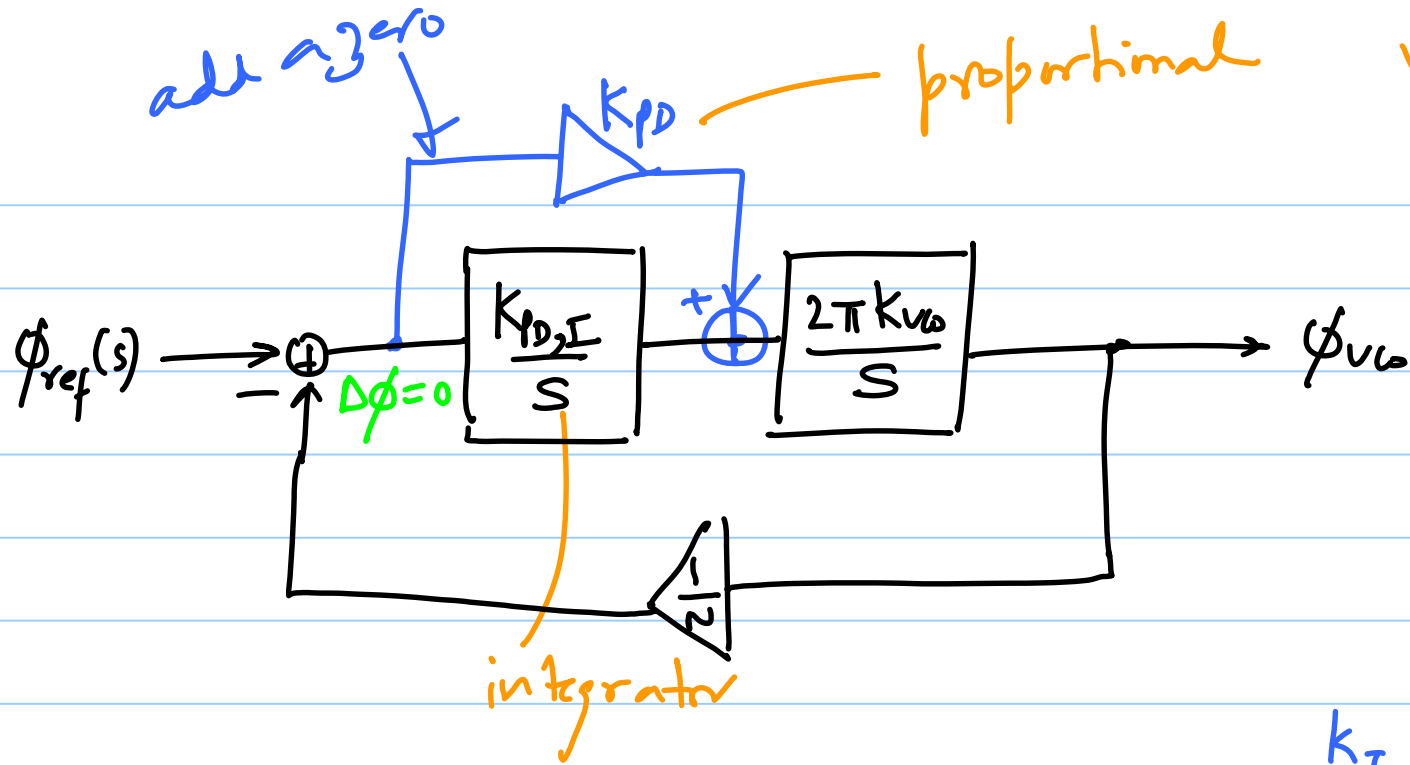
not stable



$PM \approx 90^\circ$

Add a zero

★



"PI-controller"

Type-II system
 $\hookrightarrow$ # of poles $\geq$ DC

$$\begin{aligned} & \frac{k_I}{s} + k_p \\ &= \frac{(k_I + s k_p)}{s} \quad \text{zero} \\ &= \frac{(1 + s/\omega_z)}{s} \end{aligned}$$