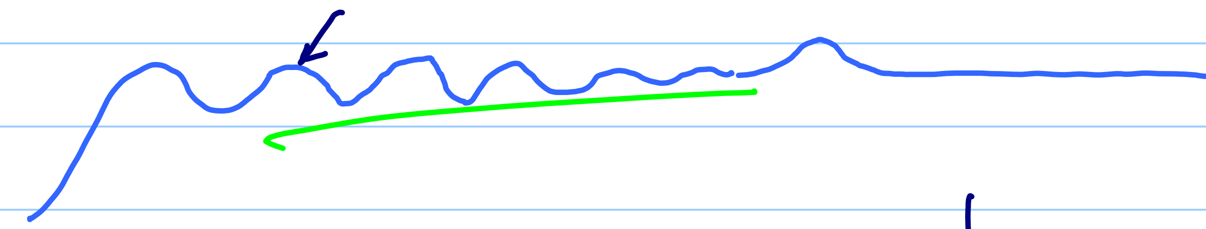
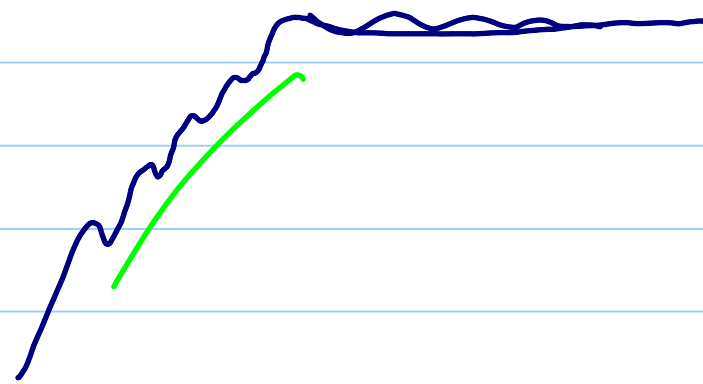
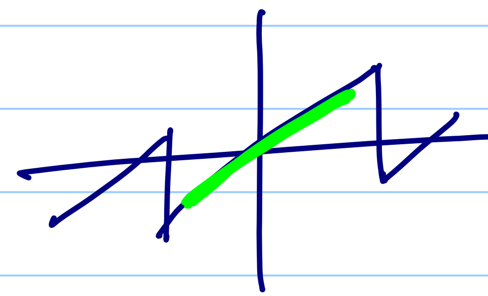
ECE 518 – Lecture 13

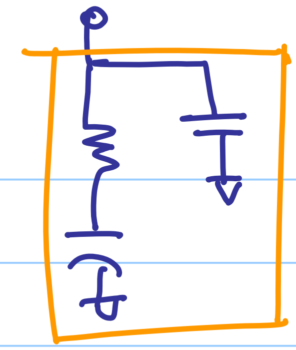
Note Title 2/26/2015







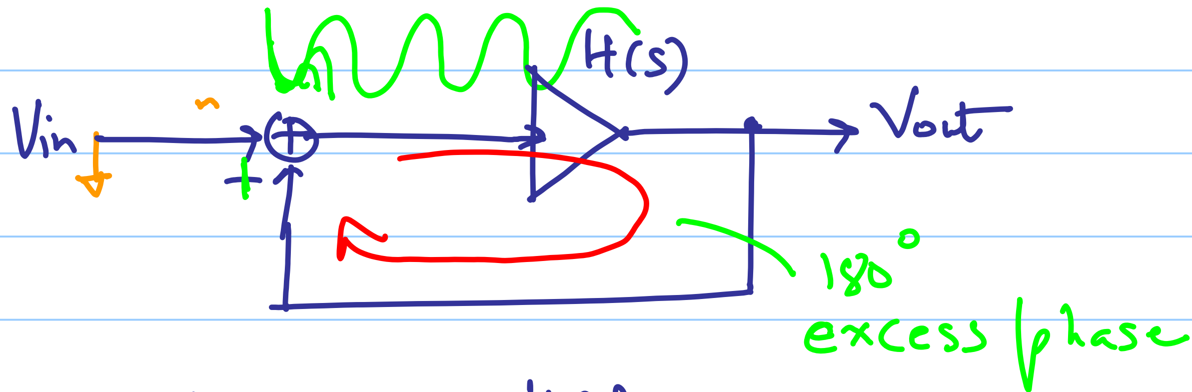
PFD + CP , Loop filter



VCO

Divider

Oscillators Basics



$\beta = 1$

$L(s) = \beta H(s)$

$$H_{cL}(s) = \frac{V_{out}}{V_{in}}(s) = \frac{H(s)}{1 + L(s)}$$

for oscillation at $s = j\omega_0$, we require

$$L(j\omega_0) = -1$$

$$\Rightarrow \quad |L(j\omega_0)| = 1 \quad \& \quad \angle L(j\omega_0) = -180^\circ$$

$$|H_{cL}(j\omega_0)| \longrightarrow \infty$$

*

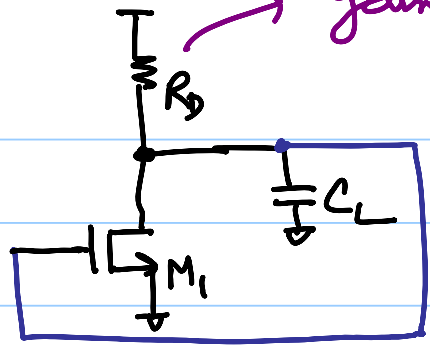
Loop-gain $|L(j\omega_0)| = 1$

& $\angle L(j\omega_0) = -180^\circ$

↳ conditions are necessary but not sufficient

↳ typically we choose the loop-gain to be 2–3

①



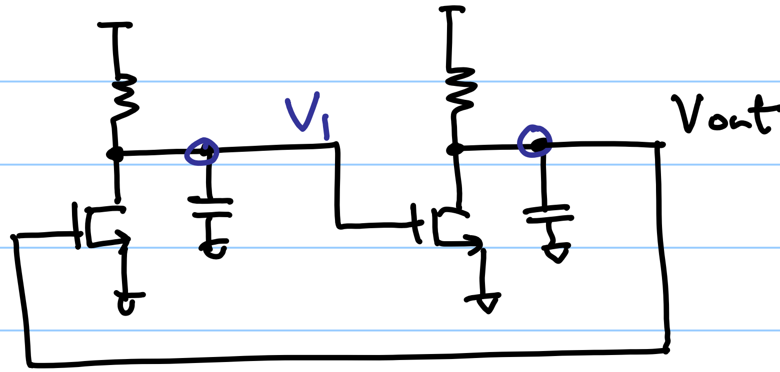
gain $= -g_{m_1} R_D$

single-pole

$$L(s) = \frac{-g_m R_D}{(1 + s R_D C_L)}$$

max phase shift $\Rightarrow 90^\circ$

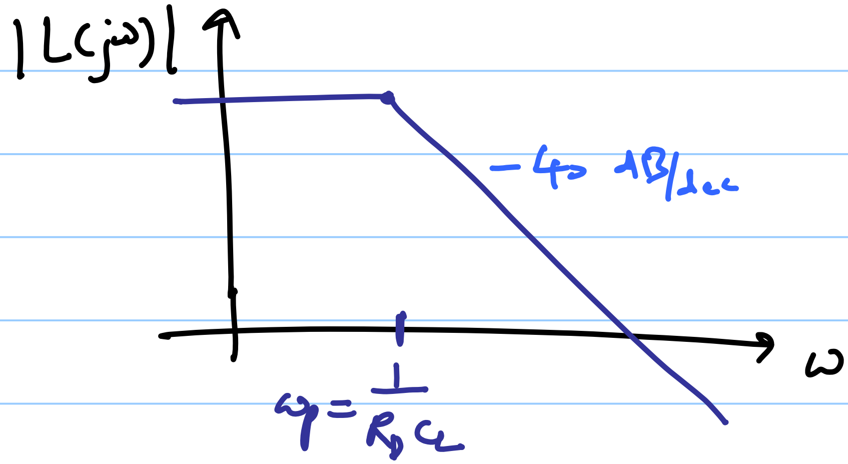
②



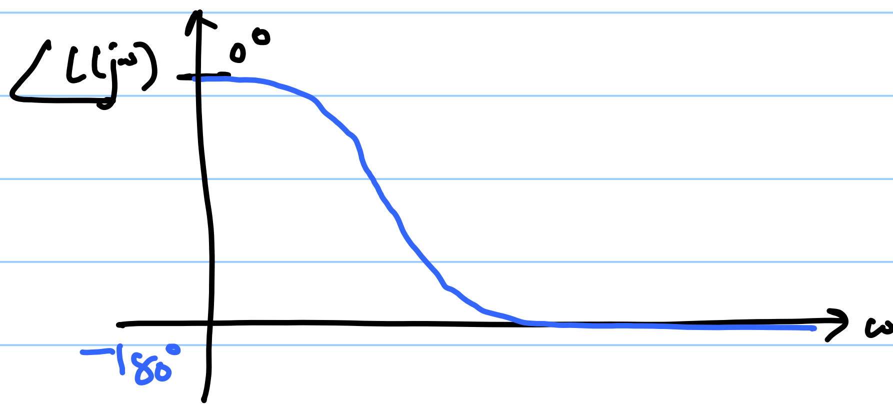
two-poles

$$L(s) = \frac{(g_{m_1} R_D)^2}{(1 + s R_D C_L)^2}$$

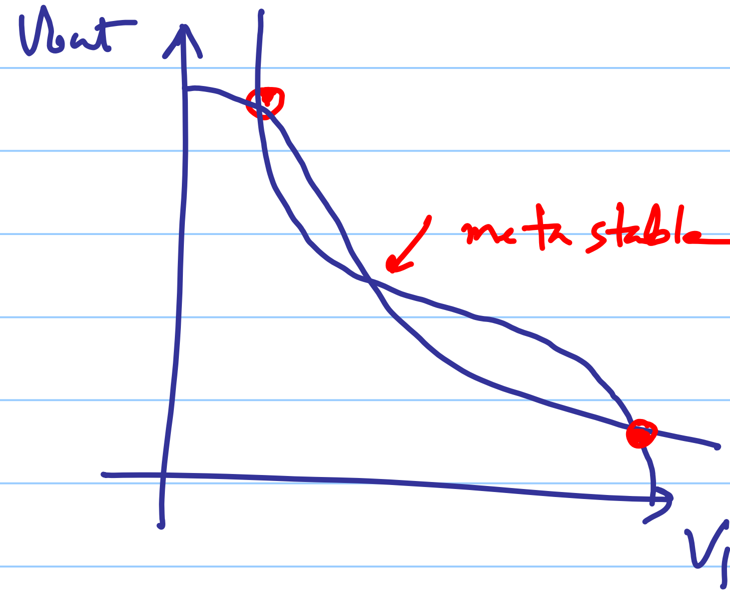
max excess phase shift



$|L(j\omega)|$
-40 dB/dec
ω
$\omega_p = \frac{1}{R_D C_L}$



$\angle L(j\omega)$
0°
-180°
ω



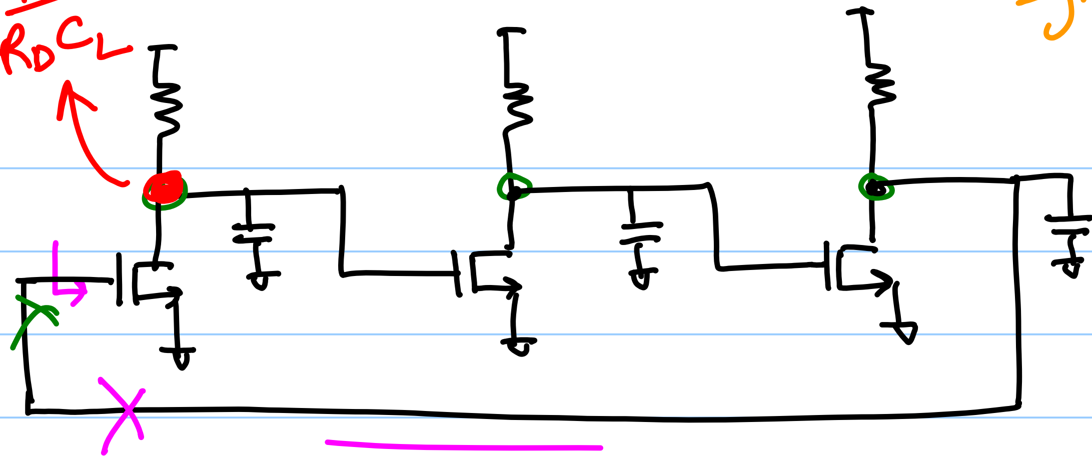
V_{out}
meta stable
V_1

③

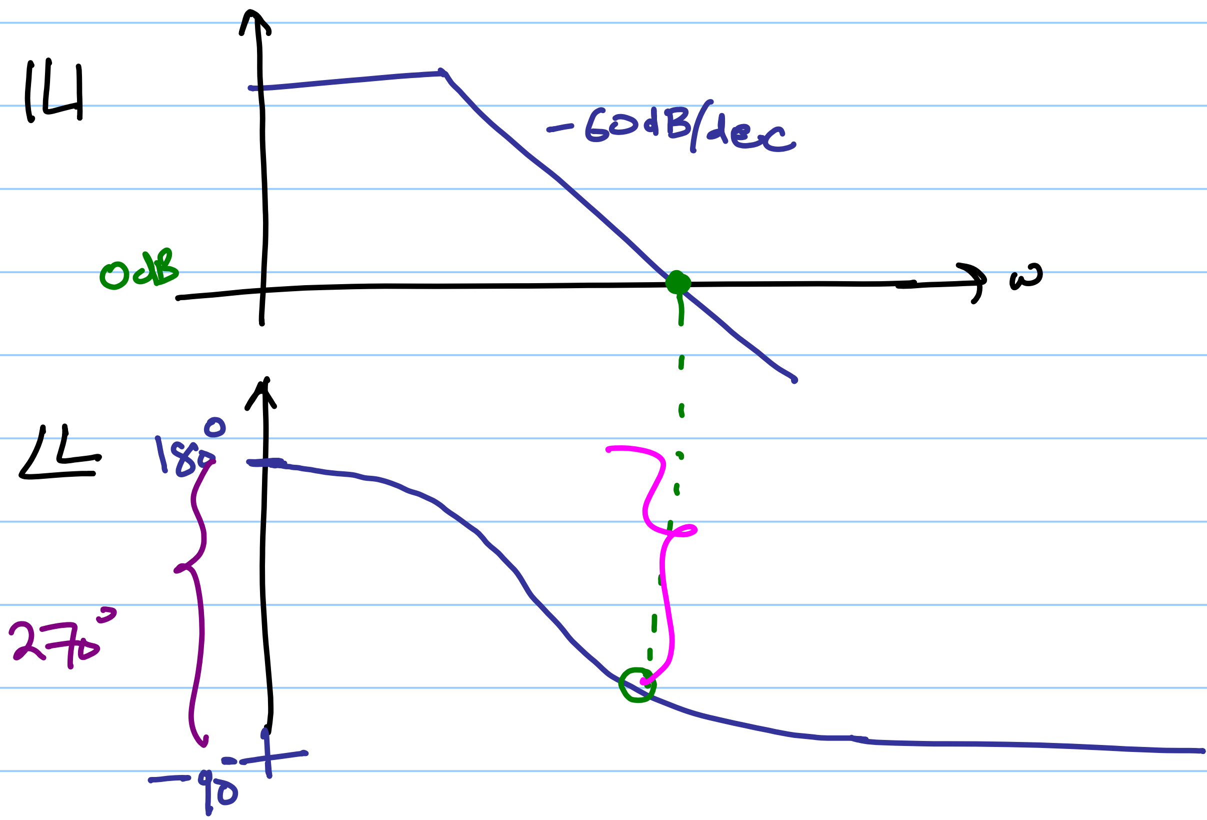
$\omega_0 = \frac{1}{R_D C_L}$

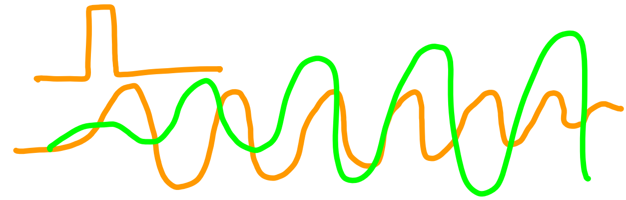
Ignoring Cgs Caps

3-poles



$$L(s) = \frac{-(g_m R_D)^3}{(1 + sR_D C_L)^3}$$





at $\omega = \omega_0$

The circuit oscillates only if the excess phase $= 180^\circ$

$\Rightarrow$ each stage is contributing 60°

$$\boxed{\omega_0 = \frac{1}{R_D C_L}}$$

$$\tan^{-1}\left(\frac{\omega_{osc}}{\omega_0}\right) = 60^\circ$$

$$\frac{\omega_{osc}}{\omega_0} = \tan(60^\circ) = \sqrt{3}$$

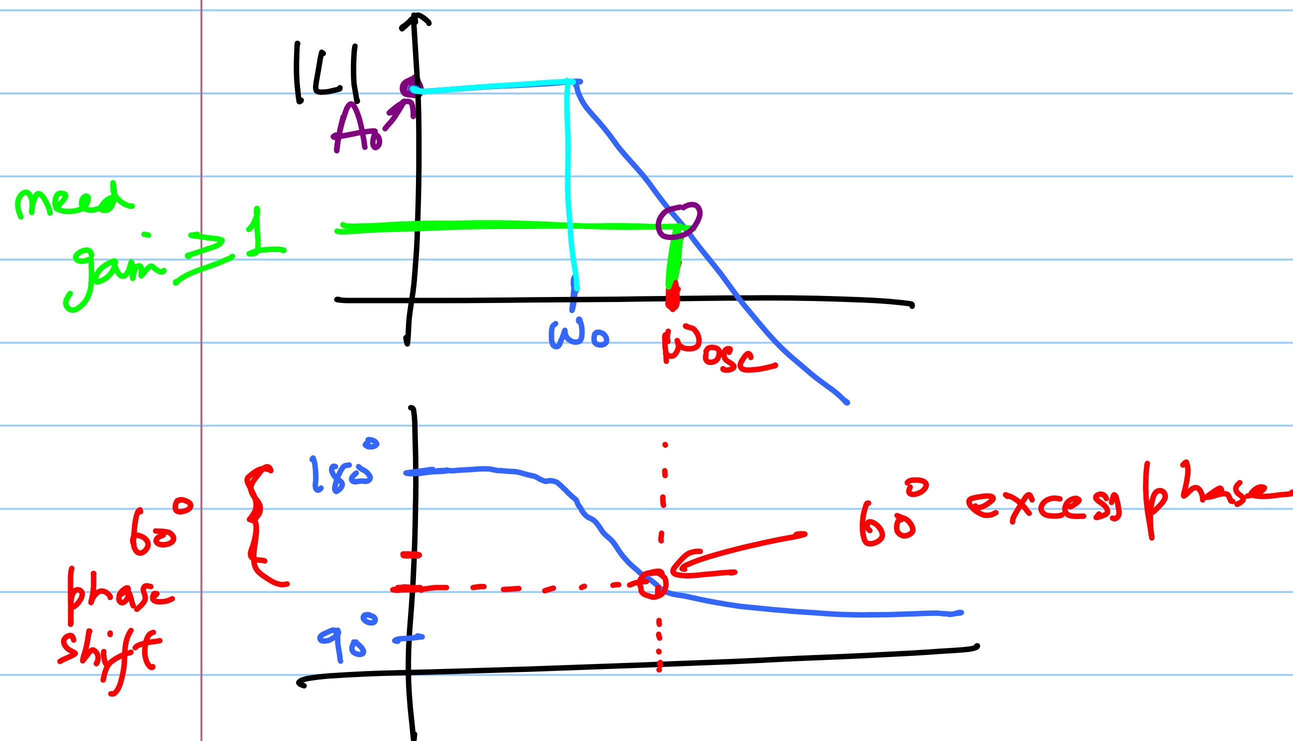
$$\Rightarrow \quad \boxed{\omega_{osc} = \omega_0\sqrt{3}}$$

$\Rightarrow$ Also, the min voltage gain per stage must be such that $\omega_{osc} = \omega_0\sqrt{3}$

Single gain stage

$$L(s) = \frac{-g_m R_D}{(1 + s/\omega_0)}$$

$$\omega_0 = \frac{1}{R_D C_L}$$



$$\angle \tan^{-1}\left(\frac{\omega}{\omega_0}\right) = \angle 60^\circ$$

$$\boxed{\omega_{osc} = \omega_0\sqrt{3}}$$

need a loop-gain of 1 at $\omega_{osc} = \omega_o\sqrt{3}$

$$|L(j\omega_{osc})| = 1$$

$$\Rightarrow \frac{A_o^3}{\left(\sqrt{1+\left(\frac{\omega_{osc}}{\omega_o}\right)^2}\right)^3} = 1$$

$$\Rightarrow \frac{A_o}{\sqrt{1+\left(\frac{\omega_{osc}}{\omega_o}\right)^2}} = 1$$

$$\Rightarrow \frac{A_o}{\sqrt{1+3}} = 1 \quad \Rightarrow \quad A_o = \sqrt{4} = 2 \leftarrow \text{DC gain per stage}$$

$$A_o^3 = 2^3 = 8 \leftarrow \text{DC gain of the loop}$$

$\Rightarrow$ We need a DC gain of 2 per stage in a 3-stage oscillator

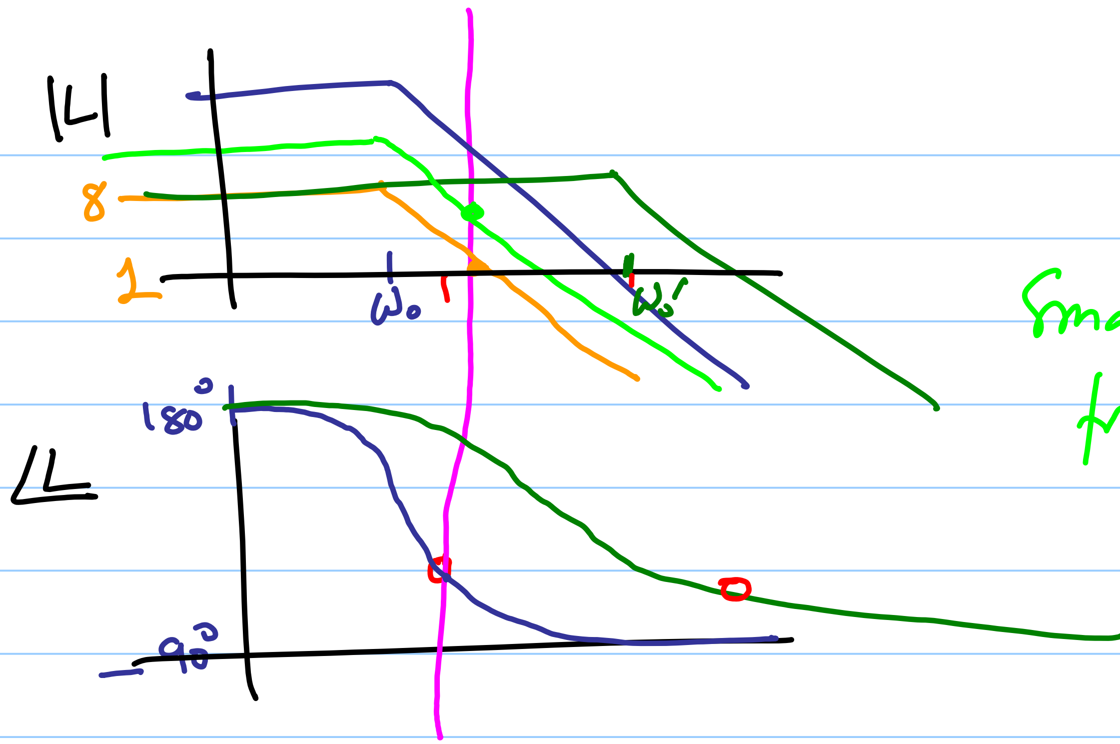
$$\frac{A_0}{\left(1+\frac{s}{\omega_0}\right)} \leftarrow |g_m R_D| \geq 2$$

Two conditions

$$A_0 \geq 2$$

$$\omega_{osc} = \omega_0 \sqrt{3}$$

$$L(s) = \frac{-A_0^3}{\left(1+\frac{s}{\omega_0}\right)^3}$$



Small signal transfer functions

* what happens when $A_0 \neq 2$

- for $A_0 < 2$, the circuit fails to oscillate
- for $A_0 > 2$??

$$L(s) = \frac{-A_0^3}{(1+s/\omega_0)^3}$$

$$H_{CL}(s) = \frac{V_{out}}{V_{in}}(s) = \frac{\frac{-A_0^3}{(1+s/\omega_0)^3}}{1+\frac{A_0^3}{(1+s/\omega_0)^3}}$$

$$= \frac{-A_0^3}{\left(1+\frac{s}{\omega_0}\right)^3 + A_0^3} \leftarrow D(s) = 0$$

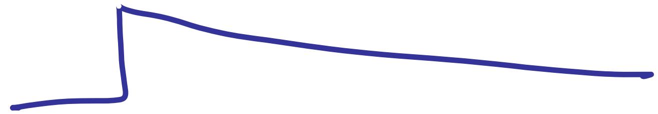
closed-loop poles?

$$\left(1+\frac{s}{\omega_0}\right)^3 + A_0^3 = 0$$

3 roots of the polynomial

$$s = \begin{cases} -(A_0+1)\omega_0 \\ \left(\frac{A_0(1 \pm j\sqrt{3})}{2} - 1\right)\omega_0 \end{cases}$$

* $e^{-(A_o+1)\omega_o t}$



exponentially decaying term.

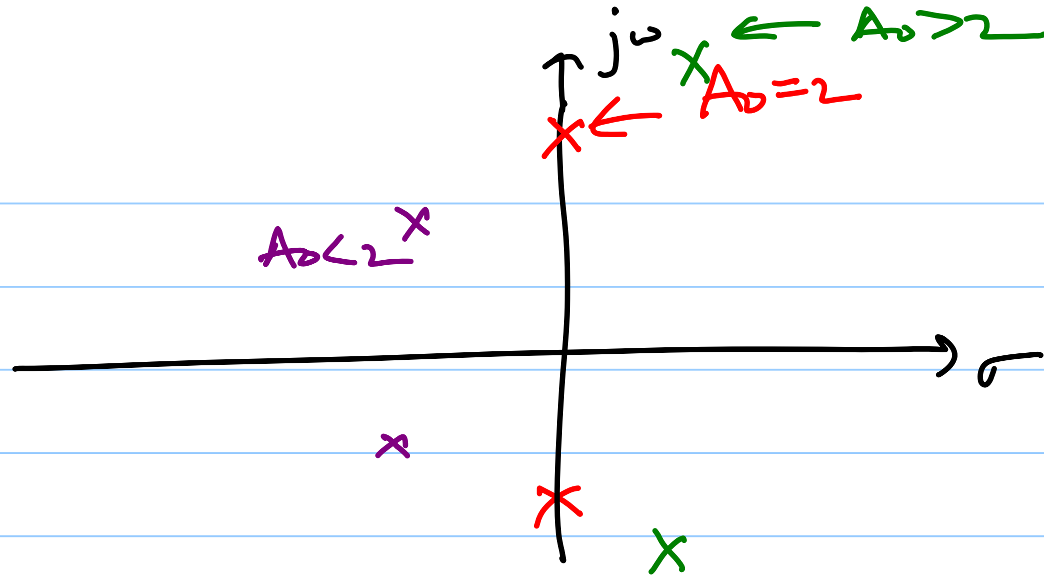
* The complex conjugate poles.

$$\omega_{p_{2,3}} = \omega_o\left[\frac{A_o(1 \pm j\sqrt{3})}{2} - 1\right]$$

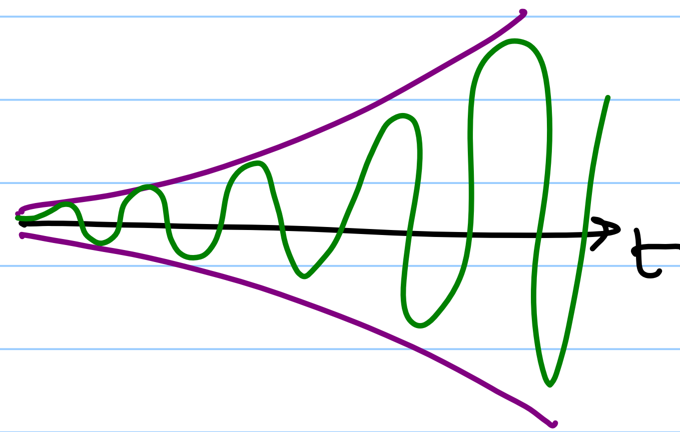
steady-state response

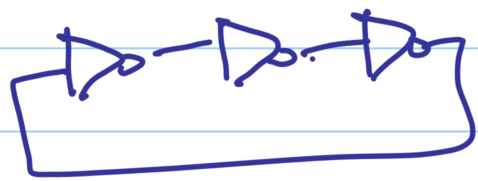
for $A_o = 2$

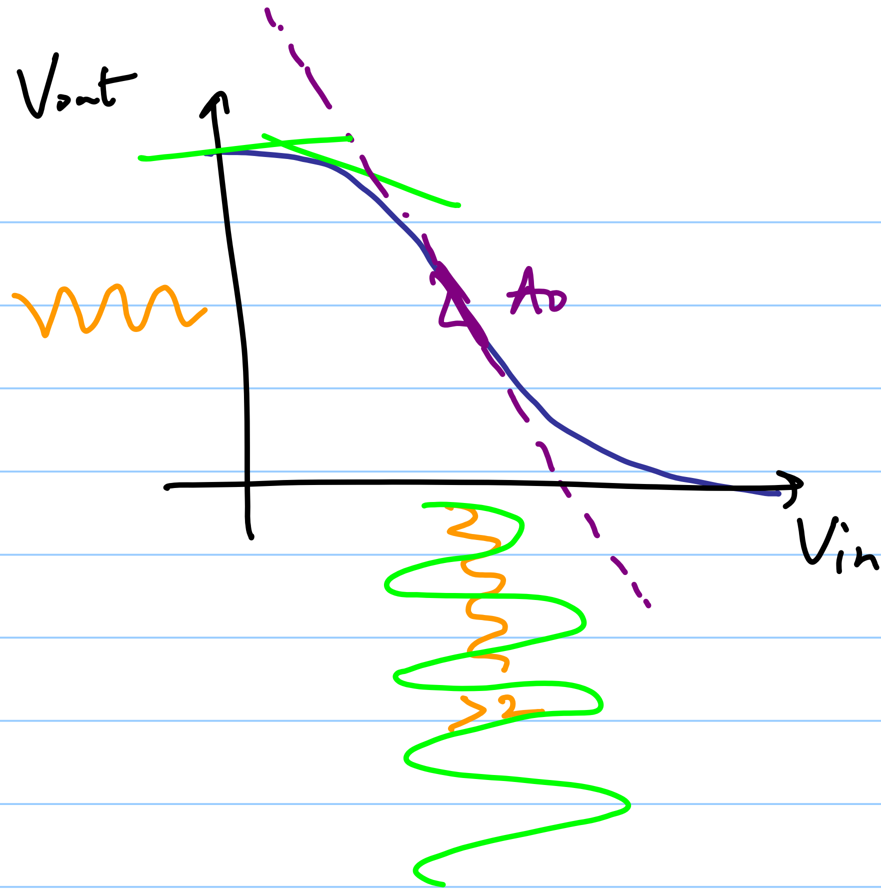
$$\omega_{p_{2,3}} = \omega_o\left[1 \pm j\sqrt{3} - 1\right] = \pm j\omega_o\sqrt{3}$$



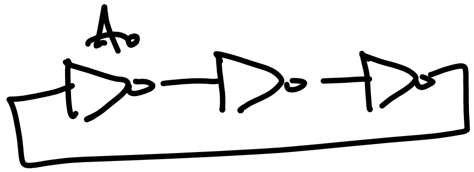
for $A_0 > 2$



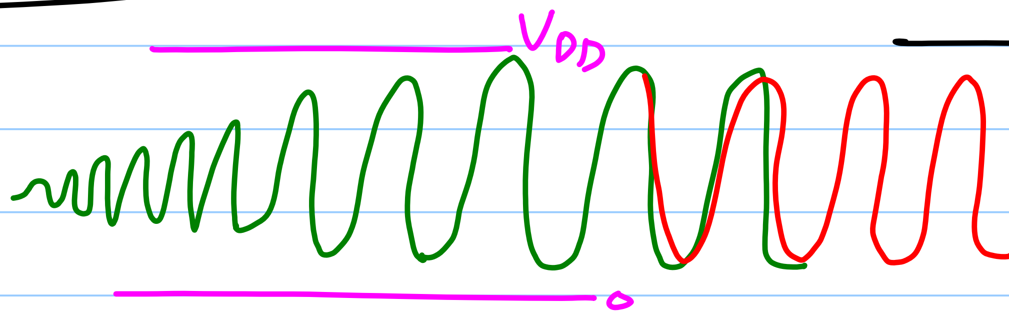


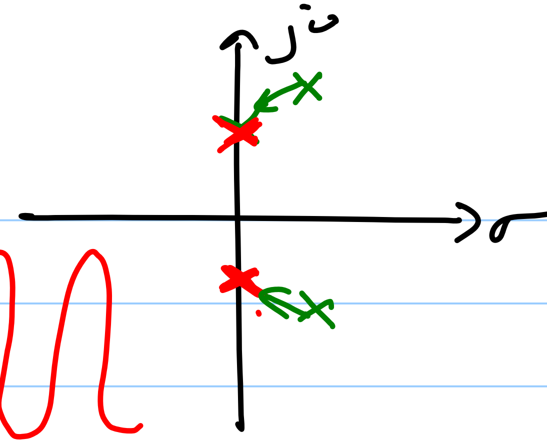

Vout
A0
Vin

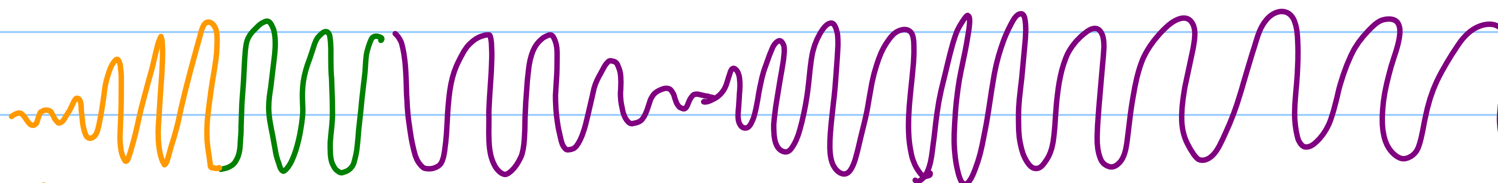
* In practice, as the oscillation amplitude increases
  - ↳ gain stages experience nonlinearity
    - ↳ limits the maximum amplitude
      - ↳ "average loop gain" drops
        - ↳ eventually the conjugate poles move to the $j\omega$ axis



$A_0 > 2$







$A_0 > 2$ $A_0 = 2$ $A_0 < 2$