Homework 4

ECE 5/418 – Memory/Clock Synchronization IC Design (Spring 2013)

Due on Tuesday, Feb 26, 2013.

Note: These problems are meant to refresh your signals and circuits concepts. These concepts will be repeatedly used in phase-locked loop design. If you have difficulty with this material please contact the instructor.

Problem 1- Signals and Systems Review:

1. Write the Laplace transforms for the following signals. Here, V_p is a constant voltage, u(t) is the step function, and x(t) is an LTI signal (or system) with its Laplace transform given by X(s). Standard notations apply.

Time domain	Laplace domain
x(t)	X(s)
$V_p u(t)$	
$V_p e^{-\alpha t} u(t)$	
$t \cdot u(t)$	
$t^2 \cdot u(t)$	
$\int_0^t x(\tau) d au$	
$\frac{dx(t)}{dt}$	
$e^{-\alpha t}x(t)$	
$sin(lpha t) \cdot u(t)$	
$cos(lpha t) \cdot u(t)$	
$x_1(t) + x_2(t)$	
$x_1(t) \otimes x_2(t)$	
Initial value: $x(0^+)$	
Final value: $x(\infty)$	

- 2. Sketch Bode plots (magnitude and phase) for the following transfer functions. Verify your plots using Matlab (you can use *bode* function). Also, find the pole and zero locations, and the phase at 0 dB gain.
 - (a) $H(s) = \frac{10(s+1)}{(s+10)(s+100)}$ (b) $H(s) = \frac{10(s+10)}{(s+1)(s+100)}$ (c) $H(s) = \frac{10(s+10)}{s^2(s+100)}$
- 3. A second-order system can be expressed by the transfer function, $H(s) = \frac{k_1 \omega_0^2}{(s^2 + s \frac{\omega_0}{O} + \omega_0^2)}$. Recall that Q is called the quality factor and ω_0 is the natural frequency.
 - (a) Show that the two poles are real if Q < 0.5 and complex conjugate when Q > 0.5.
 - (b) Show that the low frequency magnitude, $|H(j0)| = k_1$ and the magnitude at $\omega = \omega_0$, $|H(j\omega_0)| = k_1 Q.$
 - (c) Derive the peak frequency, ω_{pk} , where the magnitude peaks (i.e. $\frac{d|H(j\omega)|^2}{d\omega}=0$).
 - (d) Find the Q, pole locations, and ω_{pk} for the following transfer functions. Hand sketch and generate Bode plots using Matlab.

i.
$$H(s) = \frac{10^4}{(s^2 + 1001s + 1000)}$$

ii. $H(s) = \frac{100}{(s^2 + 11s + 10)}$

iii.
$$H(s) = \frac{100}{(s^2 + 5s + 100)}$$

Problem 2- RC Circuits Review:



Here $R = 1k\Omega$ and C = 10nF.

- 1. Find Laplace transfer functions $H(s) = \frac{v_{out}(s)}{v_{in}(s)}$ for the above circuits and sketch the Bode plots (mag and phase).
- 2. An input $v_{in} = V_p u(t)$ is applied to the above circuits. Express the output $V_{out}(s)$ in Laplace domain.
- 3. Using the partial fraction and inverse Laplace transform method (refresh from your signals/circuits textbook), find the time-domain output, $v_{out}(t)$. Show the steps.
- 4. An input $v_{in}(t) = sin(10^5 t)$ is applied to the RC circuit in (a). Using Laplace methods, find and plot the transient, steady-state and total outputs.