ECE518 Memory/Clock Synchronization IC Design

Oscillator Phase Noise

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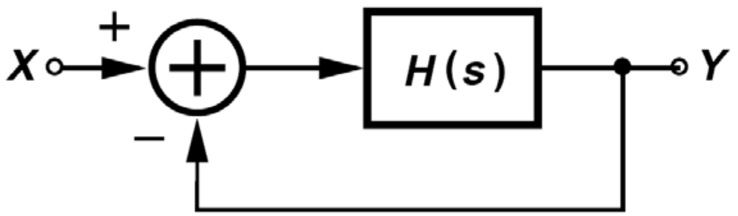
Feedback View of Oscillators

- An oscillator may be viewed as a "badly-designed" negative-feedback amplifier—so badly designed that it has a zero or negative phase margin.

$$\frac{Y}{X}(s) = \frac{H(s)}{1 + H(s)}$$

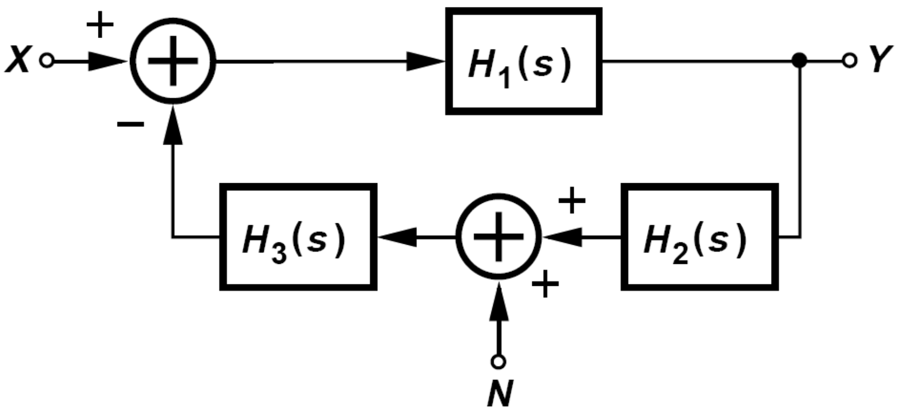


For the above system to oscillate, must the noise at ω_1 appear at the input?

No, the noise can be anywhere in the loop. For example, consider the system shown in figure below, where the noise *N* appears in the feedback path. Here,

$$Y(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)H_3(s)}X(s) + \frac{H_1(s)H_3(s)}{1 + H_1(s)H_2(s)H_3(s)}N(s).$$

Thus, if the loop transmission, $H_1H_2H_3$, approaches -1 at ω_1, *N* is also amplified indefinitely.



Y/X in the Vicinity of $\omega = \omega_1$

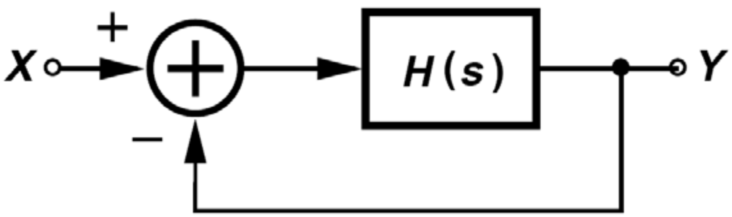
Derive an expression for *Y/X* in figure below in the vicinity of $\omega = \omega_1$ if $H(j\omega_1) = -1$.

We can approximate $H(j\omega)$ by the first two terms in its Taylor series:

$$H[j(\omega_1 + \Delta\omega)] \approx H(j\omega_1) + \Delta\omega \frac{dH(j\omega_1)}{d\omega}$$

Since $H(j\omega_1) = -1$, we have

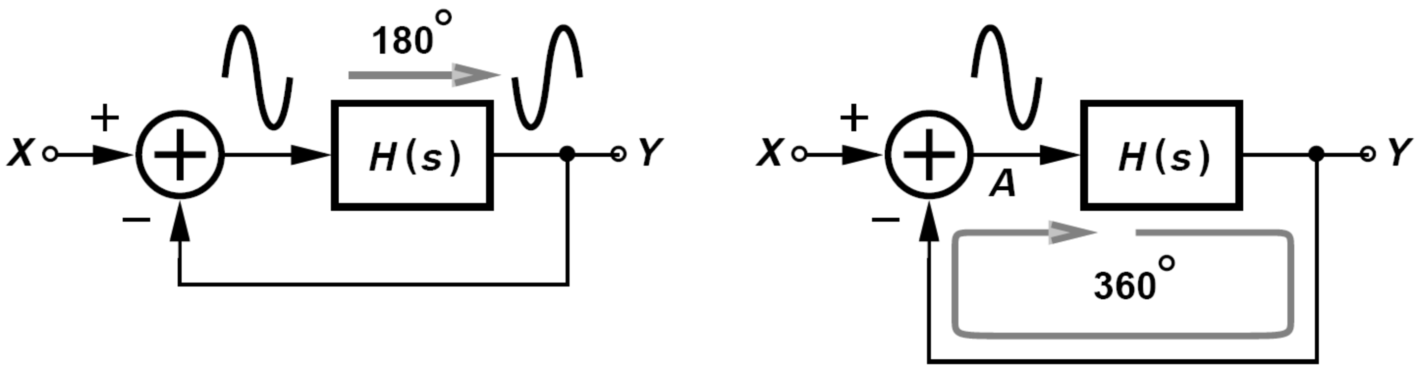
$$\begin{aligned} \frac{Y}{X}[j(\omega_1 + \Delta\omega)] &= \frac{H(j\omega_1) + \Delta\omega \frac{dH(j\omega_1)}{d\omega}}{\Delta\omega \frac{dH(j\omega_1)}{d\omega}} \\ &\approx \frac{H(j\omega_1)}{\Delta\omega \frac{dH(j\omega_1)}{d\omega}} \\ &\approx \frac{-1}{\Delta\omega \frac{dH(j\omega_1)}{d\omega}} \end{aligned}$$



As expected, $Y/X \to \infty$ as $\Delta\omega \to 0$, with a "sharpness" proportional to $dH/d\omega$.

Barkhausen's Criteria

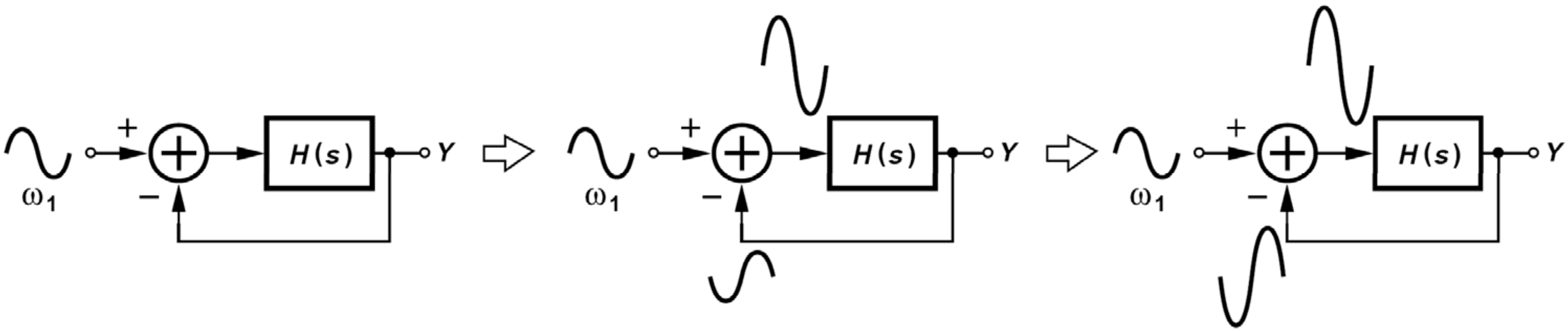
$$|H(s = j\omega_1)| = 1$$
$$\angle H(s = j\omega_1) = 180^\circ$$



- For the circuit to reach steady state, the signal returning to A must exactly coincide with the signal that started at A. We call $\angle H(j\omega_1)$ a "frequency-dependent" phase shift to distinguish it from the 180 ° phase due to negative feedback.
- Even though the system was originally configured to have negative feedback, *H(s)* is so "sluggish" that it contributes an additional phase shift of 180 ° at ω_1, thereby creating positive feedback at this frequency.

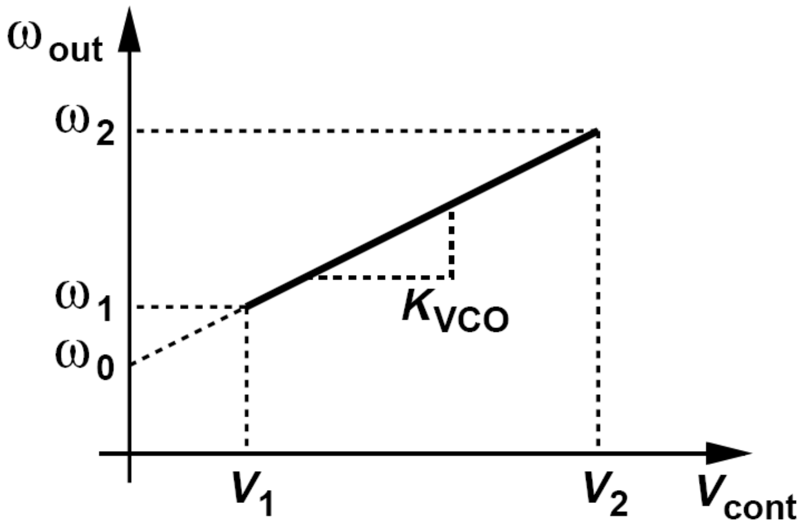
Significance of $|H(jw_1)| = 1$

- For a noise component at ω_1 to "build up" as it circulates around the loop with positive feedback, the loop gain must be at least unity.
- We call $|H(j\omega_1)| = 1$ the "startup" condition.



- What happens if $|H(j\omega_1)| > 1$ and $\angle H(j\omega_1) = 180°$? The growth shown in figure above still occurs but at a faster rate because the returning waveform is amplified by the loop.
- Note that the closed-loop poles now lie in the right half plane.

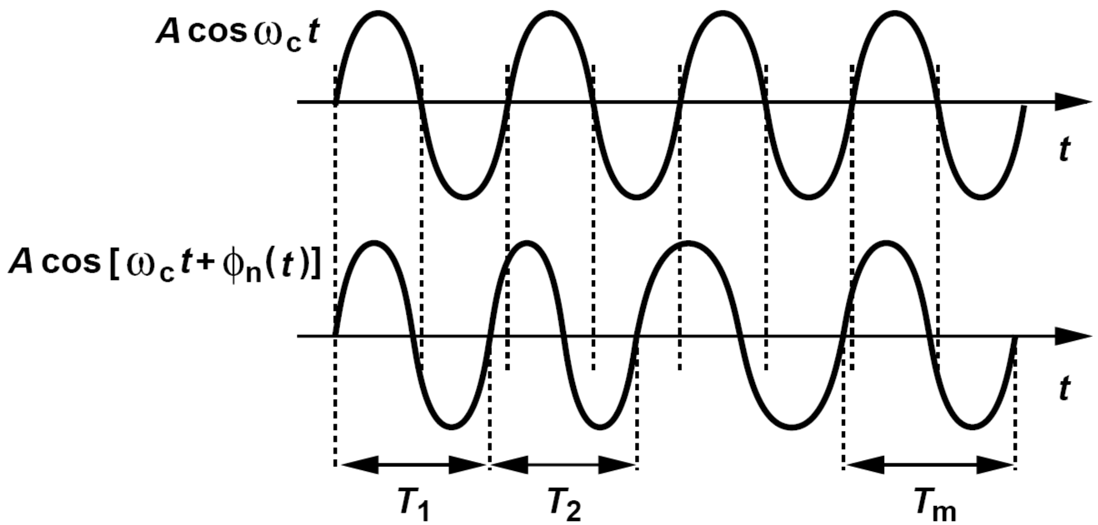
Voltage-Controlled Oscillators: Characteristic



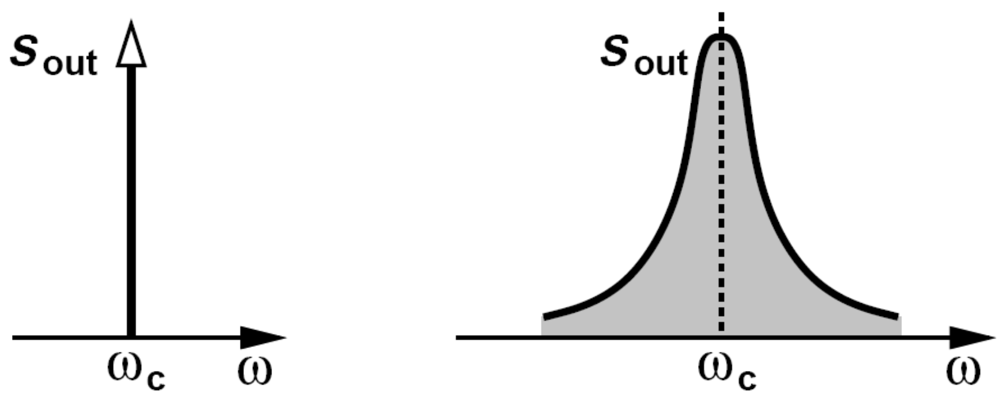
$$\omega_{out} = K_{VCO} V_{cont} + \omega_0$$

- The output frequency varies from ω_1 to ω_2 (the required tuning range) as the control voltage, V_{cont}, goes from V_1 to V_2.
- The slope of the characteristic, K_{VCO}, is called the "gain" or "sensitivity" of the VCO and expressed in rad/Hz/V.

Phase Noise: Basic Concepts

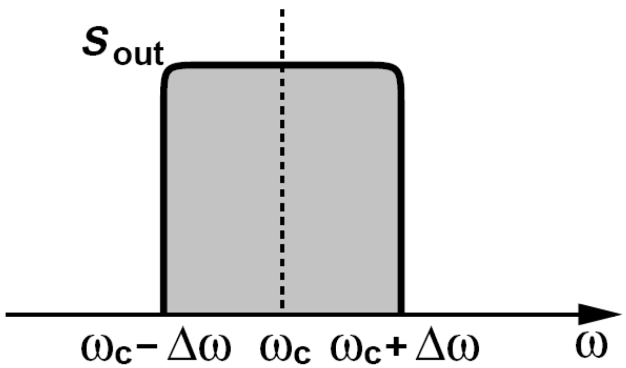


- The noise of the oscillator devices randomly perturbs the zero crossings. To model this perturbation, we write $x(t) = A\cos[\omega_c t + \Phi_n(t)]$, The term $\Phi_n(t)$ is called the "phase noise."
- From another perspective, the frequency experiences random variations, i.e., it departs from ω_c occasionally.



Phase Noise: Declining Phase Noise "Skirts"

Explain why the broadened impulse cannot assume the shape shown below.

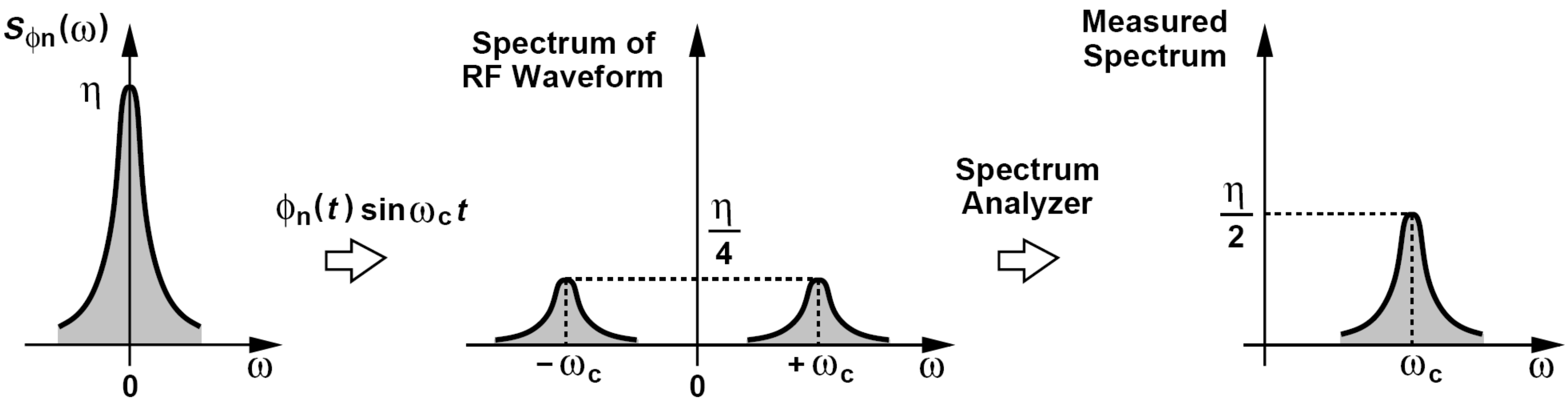


This spectrum occurs if the oscillator frequency has equal probability of appearing anywhere between ω_c - $\Delta\omega$ and ω_c + $\Delta\omega$. However, we intuitively expect that the oscillator prefers ω_c to other frequencies, thus spending lesser time at frequencies that are farther from ω_c. This explains the declining phase noise "skirts".

The spectrum can be related to the time-domain expression.

$$\begin{aligned} x(t) &= A\cos[\omega_c t + \phi_n(t)] \\ &\approx A\cos\omega_c t - A\sin\omega_c t\,\sin[\phi_n(t)] \\ &\approx A\cos\omega_c t - A\phi_n(t)\sin\omega_c t. \end{aligned}$$

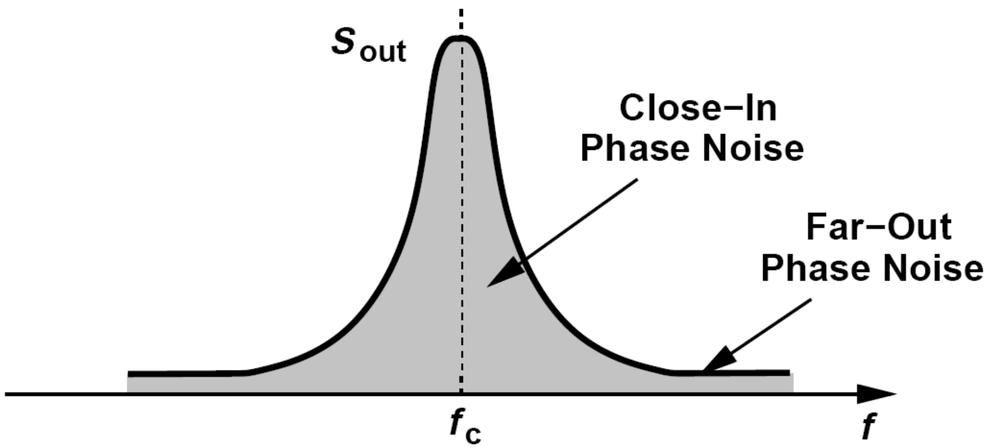
Various Factors of 4 and 2



- **(1) since $\Phi_n(t)$ in equation above is multiplied by sin $\omega_c t$, its power spectral density, $S_{\Phi n}$, is multiplied by 1/4 as it is translated to $\pm\omega_c$;**
- **(2) A spectrum analyzer measuring the resulting spectrum folds the negative frequency spectrum atop the positive-frequency spectrum, raising the spectral density by a factor of 2.**

How is the Phase Noise Quantified?

- Since the phase noise falls at frequencies farther from ω_c, it must be specified at a certain "frequency offset," i.e., a certain difference with respect to ω_c.
- We consider a 1-Hz bandwidth of the spectrum at an offset of Δf, measure the power in this bandwidth, and normalize the result to the "carrier power", called "dB with respect to the carrier".

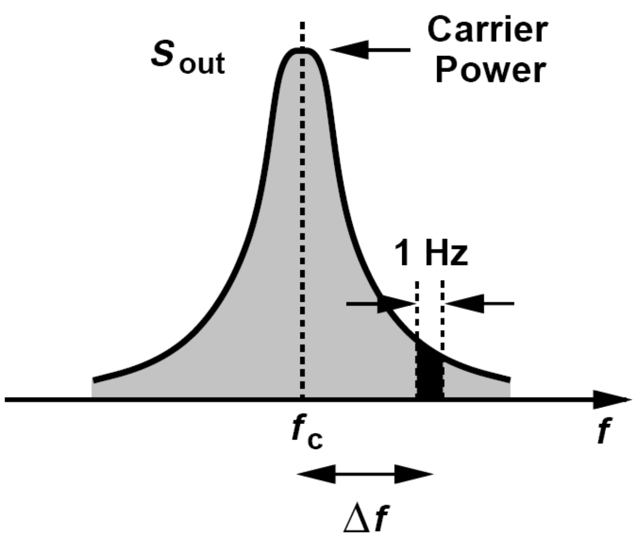


- In practice, the phase noise reaches a constant floor at large frequency offsets (beyond a few megahertz).
- We call the regions near and far from the carrier the "close-in" and the "far-out" phase noise, respectively.

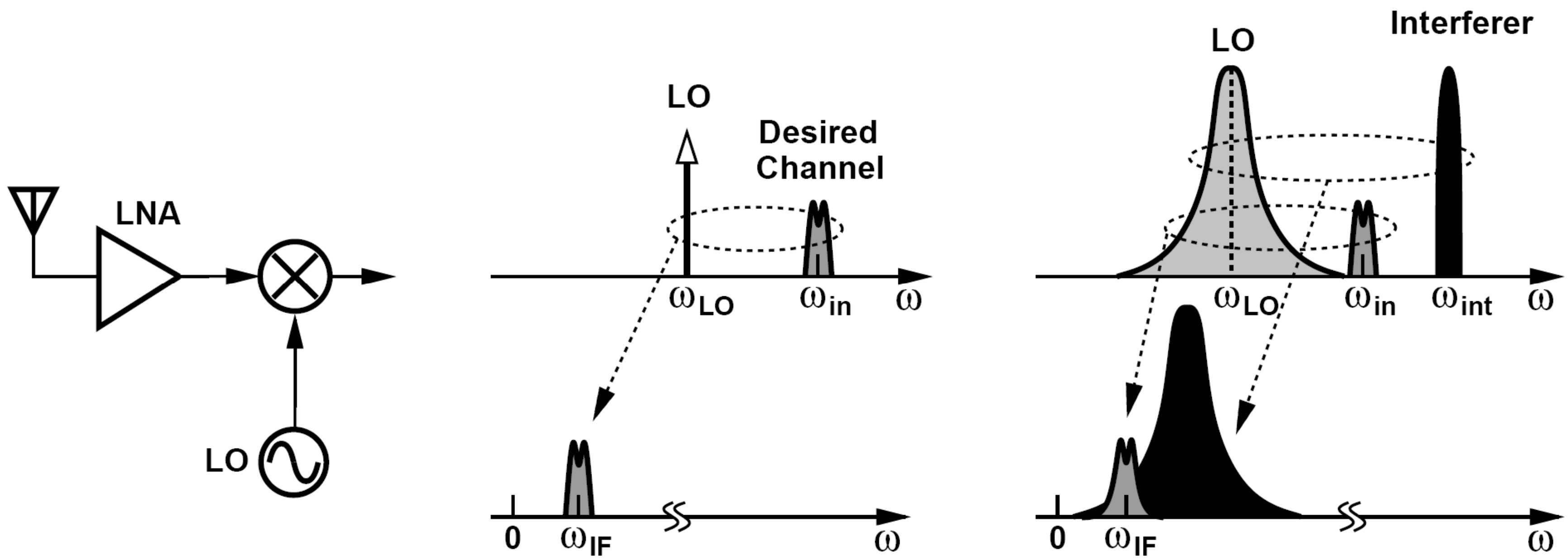
Specification of Phase Noise

At high carrier frequencies, it is difficult to measure the noise power in a 1-Hz bandwidth. Suppose a spectrum analyzer measures a noise power of -70 dBm in a 1-kHz bandwidth at 1-MHz offset. How much is the phase noise at this offset if the average oscillator output power is -2 dBm?

Since a 1-kHz bandwidth carries 10 log(1000 Hz) = 30 dB higher noise than a 1-Hz bandwidth, we conclude that the noise power in 1 Hz is equal to -100 dBm. Normalized to the carrier power, this value translates to a phase noise of -98 dBc/Hz.



Effect of Phase Noise: Reciprocal Mixing



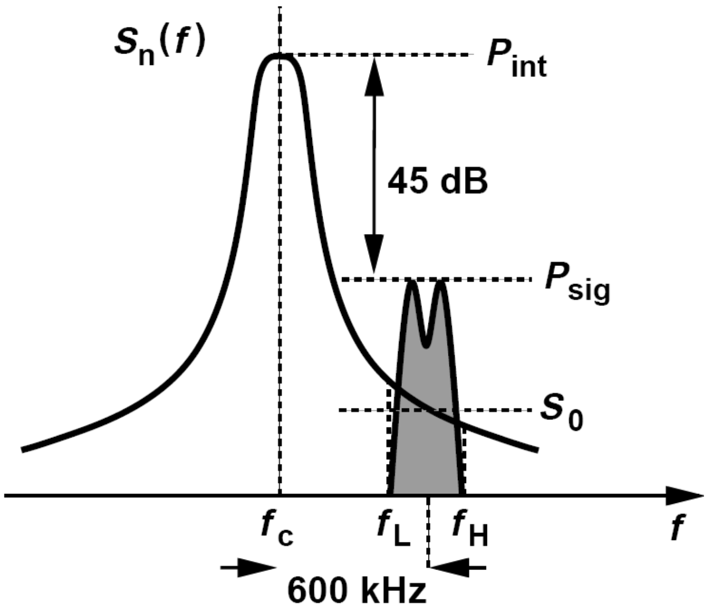
- Referring to the ideal case depicted above (middle), we observe that the desired channel is convolved with the impulse at ω_{LO}, yielding an IF signal at $\omega_{IF} = \omega_{in} - \omega_{LO}$.
- Now, suppose the LO suffers from phase noise and the desired signal is accompanied by a large interferer. The convolution of the desired signal and the interferer with the noisy LO spectrum results in a broadened downconverted interferer whose noise skirt corrupts the desired IF signal.
- This phenomenon is called “reciprocal mixing.”

Example of Reciprocal Mixing

A GSM receiver must withstand an interferer located three channels away from the desired channel and 45 dB higher. Estimate the maximum tolerable phase noise of the LO if the corruption due to reciprocal mixing must remain 15 dB below the desired signal.

The total noise power introduced by the interferer in the desired channel is equal to

$$P_{n,tot} = \int_{f_L}^{f_H} S_n(f)df$$



For simplicity, we assume $S_n(f)$ is relatively flat in this bandwidth and equal to S_0,

$$\mathrm{SNR} = \frac{P_{sig}}{S_0(f_H - f_L)}$$

which must be at least 15 dB.

$$10\log\frac{S_0}{P_{sig}} = -15\ \mathrm{dB} - 10\log(f_H - f_L)$$

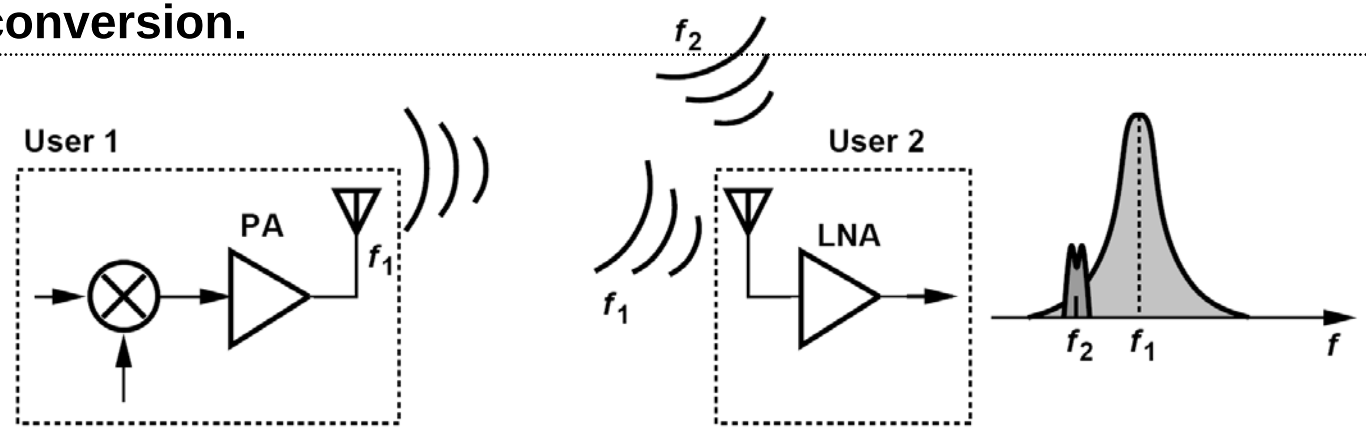
$$10\log\frac{S_0}{P_{int}} = -15\ \mathrm{dB} - 10\log(f_H - f_L) - 45\ \mathrm{dB}$$

If f_H - f_L = 200 kHz, then

$$10\log\frac{S_0}{P_{int}} = -113\ \mathrm{dBc/Hz\ \ at\ \ 600{-}kHz\ \ offset}$$

Received Noise due to Phase Noise of an Unwanted Signal

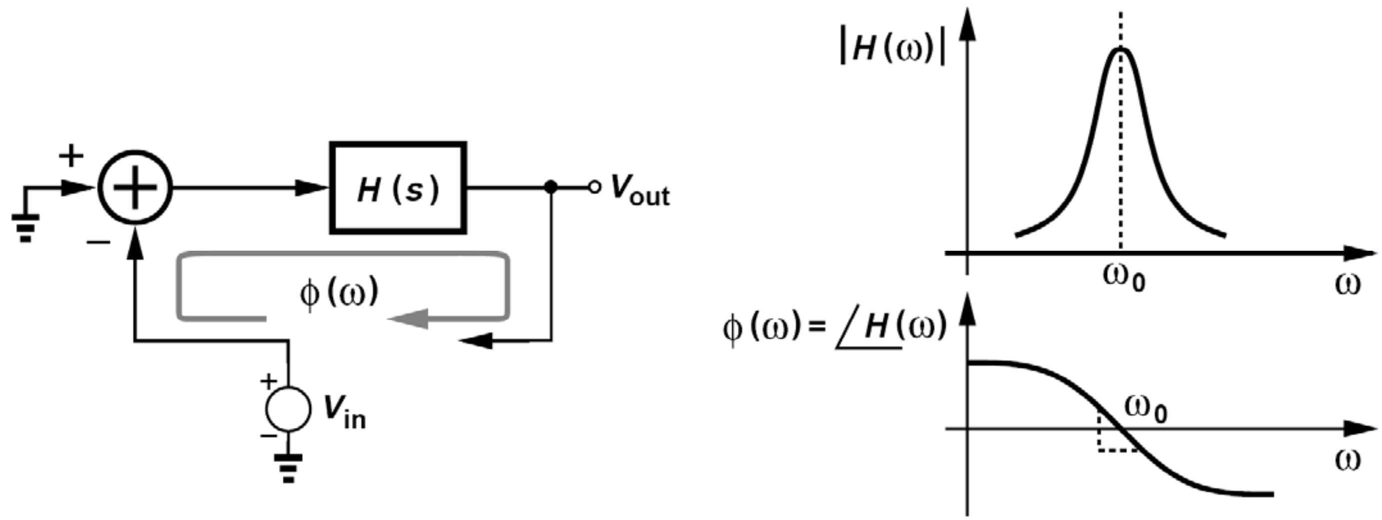
- In figure below, two users are located in close proximity, with user #1 transmitting a high-power signal at f_1 and user #2 receiving this signal and a weak signal at f_2. If f_1 and f_2 are only a few channels apart, the phase noise skirt masking the signal received by user #2 greatly corrupts it even *before* downconversion.



A student reasons that, if the interferer at f_1 above is so large that its phase noise corrupts the reception by user #2, then it also heavily compresses the receiver of user #2. Is this true?

Not necessarily. An interferer, say, 50 dB above the desired signal produces phase noise skirts that are not negligible. For example, the desired signal may have a level of -90 dBm and the interferer, -40 dBm. Since most receivers' 1-dB compression point is well above -40 dBm, user #2's receiver experiences no desensitization, but the phenomenon above is still critical.

Analysis of Phase Noise: Approach I --- Q of an Oscillator

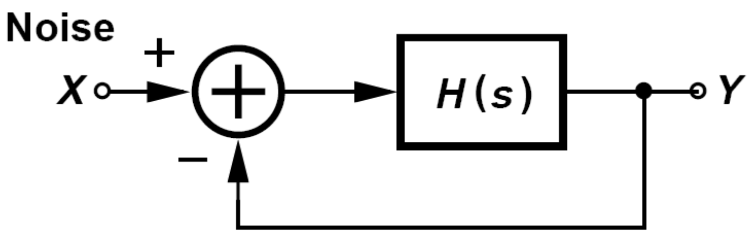


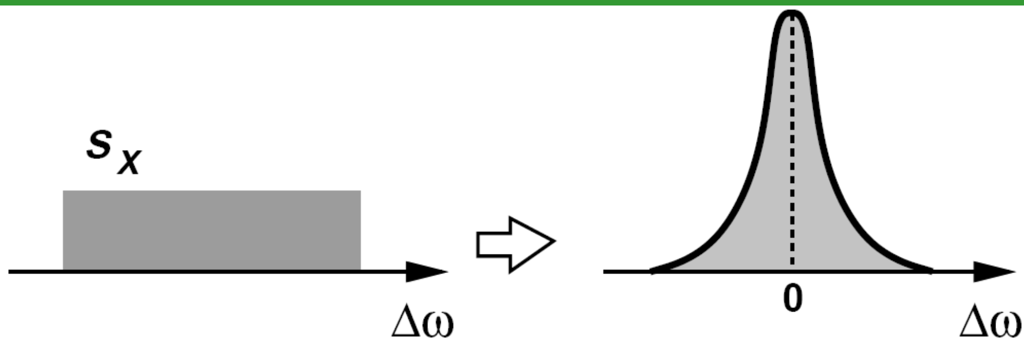
- Another definition of the Q that is especially well-suited to oscillators is shown above, where the circuit is viewed as a feedback system and the phase of the open-loop transfer function, is examined at the resonance frequency.

$$Q = \frac{\omega_0}{2}\left|\frac{d\phi}{d\omega}\right|$$

- Oscillators with a high open-loop Q tend to spend less time at frequencies other than ω_0.

Noise Shaping in Oscillators(I)





$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1+H(s)}$$

In the vicinity of the oscillation frequency, we can approximate *H(jω)* with the first two terms in its Taylor series:

$$H(j\omega) \approx H(j\omega_0) + \Delta\omega\frac{dH}{d\omega}$$

If $H(j\omega_0)$ = -1 and $\Delta\omega dH/d\omega$ << 1,

$$\frac{Y}{X}(j\omega_0 + j\Delta\omega) \approx \frac{-1}{\Delta\omega\frac{dH}{d\omega}}$$

The noise spectrum is "shaped" by

$$|\frac{Y}{X}(j\omega_0 + j\Delta\omega)|^2 = \frac{1}{\Delta\omega^2|\frac{dH}{d\omega}|^2}$$

Noise Shaping in Oscillators (II)

To determine the shape of |*dH/dω*|^2, we write *H(jω)* in polar form, and differentiate with respect to *ω*,

$$\frac{dH}{d\omega} = \left(\frac{d|H|}{d\omega} + j|H|\frac{d\phi}{d\omega}\right)\exp(j\phi)$$

$$\left|\frac{dH}{d\omega}\right|^2 = \left|\frac{d|H|}{d\omega}\right|^2 + \left|\frac{d\phi}{d\omega}\right|^2 |H|^2$$

Note that (a) in an LC oscillator, the term |*d*|*H*|*/dω*|^2 is much less than |*dΦ/dω*|^2 in the vicinity of the resonance frequency, and (b) |*H*| is close to unity for steady oscillations.

$$\left|\frac{Y}{X}(j\omega_0 + j\Delta\omega)\right|^2 = \frac{1}{\frac{\omega_0^2}{4}\left|\frac{d\phi}{d\omega}\right|^2}\frac{\omega_0^2}{4\Delta\omega^2}$$

$$\left|\frac{Y}{X}(j\omega_0 + j\Delta\omega)\right|^2 = \frac{1}{4Q^2}\left(\frac{\omega_0}{\Delta\omega}\right)^2$$

Known as "Leeson's Equation", this result reaffirms our intuition that the open-loop *Q* signifies how much the oscillator rejects the noise.

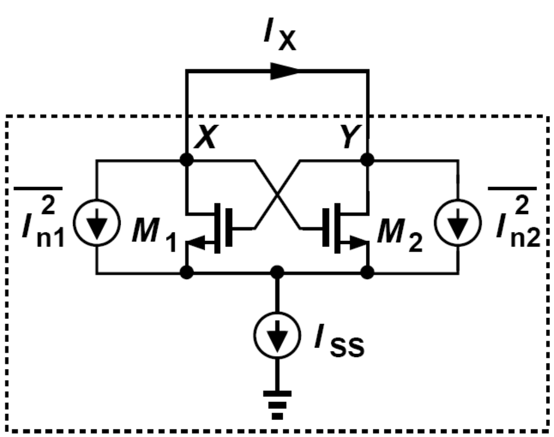
Linear Model (I)

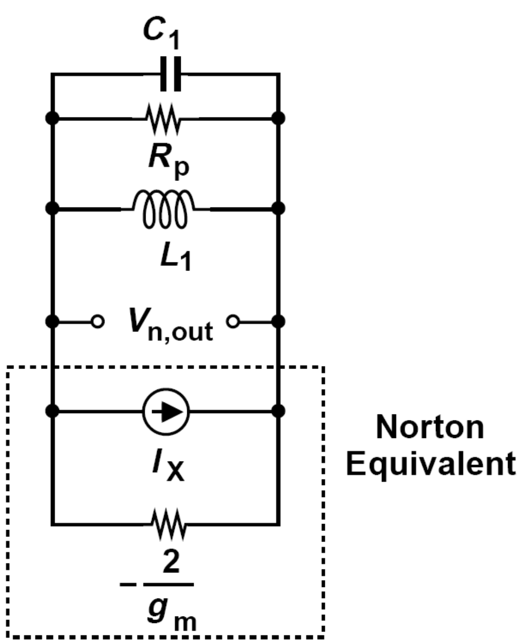
- The small-signal (linear) model may ignore some important effects, e.g., the noise of the tail current source, or face other difficulties.

Compute the total noise injected to the differential output of the cross-coupled oscillator when the transistors are in equilibrium. Note that the two-sided spectral density of the drain current noise is equal to $\overline{I_n^2} = 2kT\gamma g_m$.

The output noise is obtained as

$$\overline{V_{n,out}^2} = \left(\overline{I_X^2} + \frac{2kT}{R_p}\right) \frac{R^2 L_1^2 \omega^2}{R^2(1 - L_1 C_1 \omega^2)^2 + L_1^2 \omega^2}$$

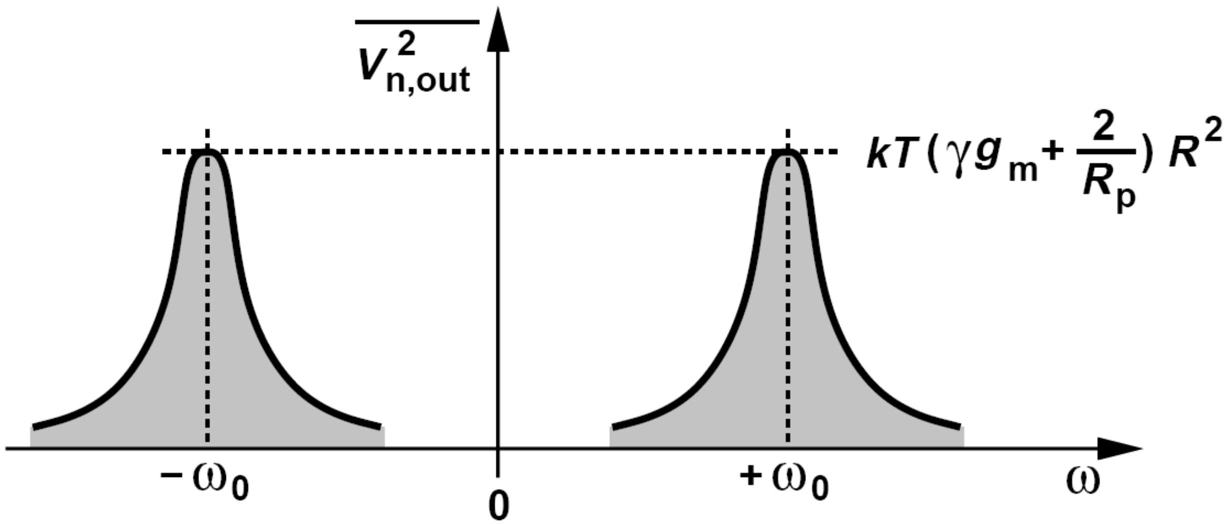




Linear Model (II)

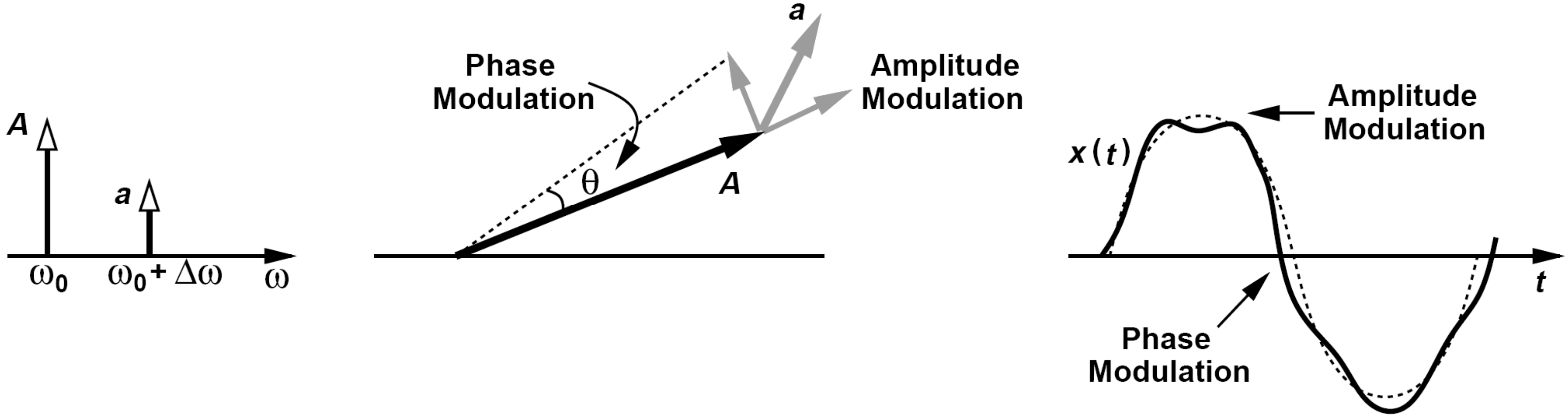
Since I_{n1} and I_{n2} are uncorrelated

$$\overline{V_{n,out}^2} = \left(kT\gamma g_m + \frac{2kT}{R_p}\right)\frac{R^2 L_1^2\omega^2}{R^2(1-L_1C_1\omega^2)^2 + L_1^2\omega^2}$$



Unfortunately, this result contradicts Leeson's equation. g_m is typically quite higher than $2/R_p$ and hence $R \neq \infty$.

Conversion of Additive Noise to Phase Noise



- At any point in time, the small phasor can be expressed as the sum of two other phasors, one aligned with *A* and the other perpendicular to it. The former modulates the amplitude and the latter, the phase.

The output of the limiter can be written as

$$
\begin{aligned}
x_{lim}(t) &= \frac{A}{2}\cos\omega_0 t - \frac{a}{2}\cos(\omega_0+\Delta\omega)t + \frac{a}{2}\cos(\omega_0-\Delta\omega)t \\
&\approx \frac{A}{2}\cos\left(\omega_0 t - \frac{2a}{A}\sin\Delta\omega t\right).
\end{aligned}
$$

We expect that narrowband random additive noise in the vicinity of ω_0 results in a phase whose spectrum has the same shape as that of the additive noise but translated by ω_0 and normalized to *A*/2.

Conversion of Additive Noise to Phase Noise: Analytically Proof of the Previous Conjecture

We write $x(t) = A\cos\omega_0 t + n(t)$. It can be proved that narrowband noise in the vicinity of ω_0 can be expressed in terms of its quadrature components

$$n(t) = n_I(t)\cos\omega_0 t - n_Q(t)\sin\omega_0 t$$

$$x(t) = [A + n_I(t)]\cos\omega_0 t - n_Q(t)\sin\omega_0 t$$

In polar form, $$x(t) = \sqrt{[A + n_I(t)]^2 + n_Q^2(t)}\cos\left[\omega_0 t + \tan^{-1}\frac{n_Q(t)}{A + n_I(t)}\right]$$

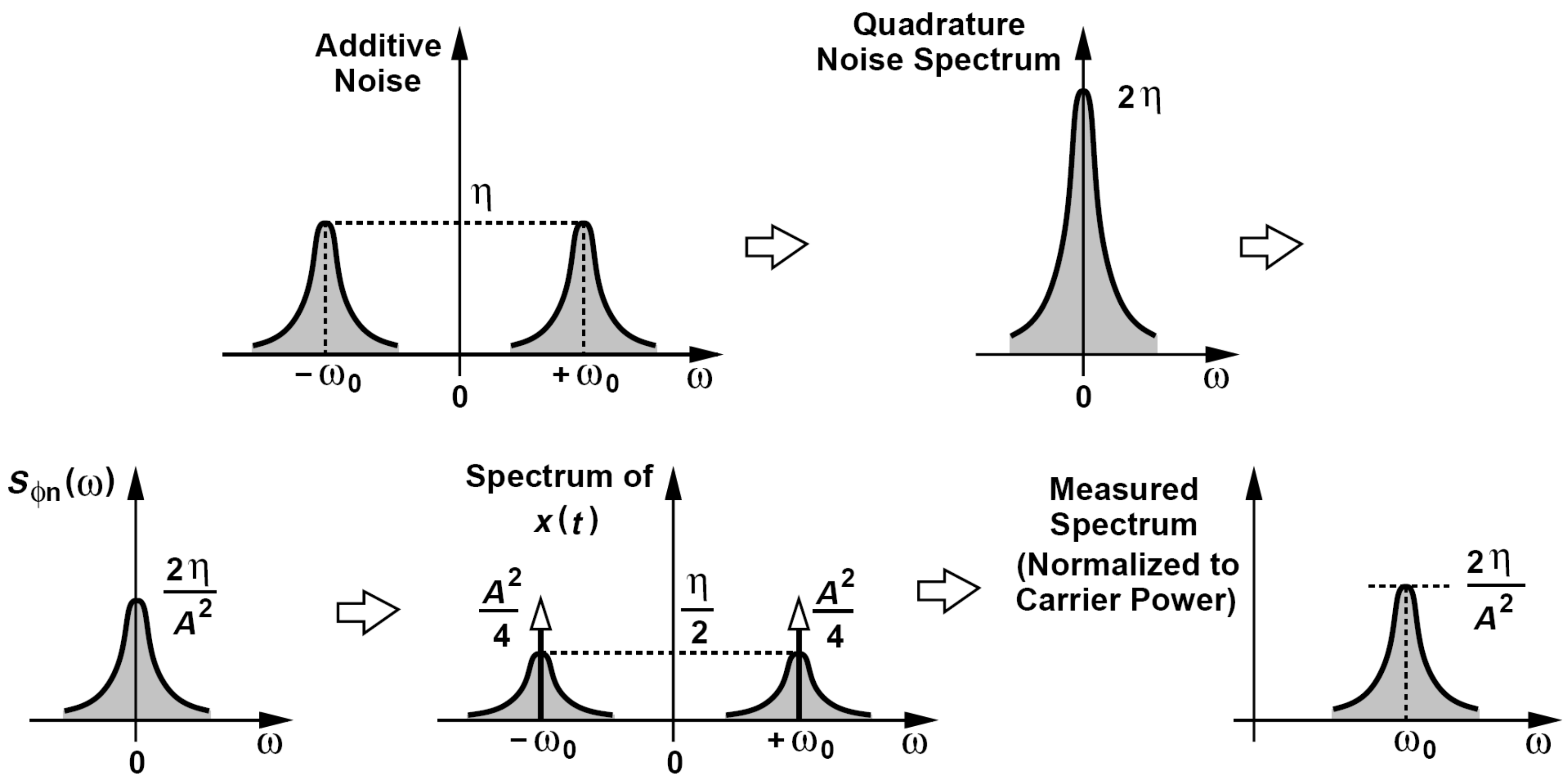
The phase component is equal to: $$\phi_n(t) \approx \frac{n_Q(t)}{A}$$

$$S_{\phi n}(\omega) = \frac{S_{nQ}(\omega)}{A^2}$$

We are ultimately interested in the spectrum of the RF waveform, *x(t)*, but excluding its AM noise.

$$\begin{aligned} x(t) &\approx A\cos\left[\omega_0 t + \frac{n_Q(t)}{A}\right] \\ &\approx A\cos\omega_0 t - n_Q(t)\sin\omega_0 t \end{aligned}$$

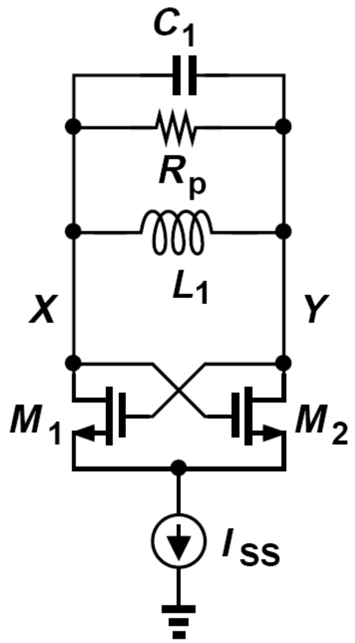
Conversion of Additive Noise to Phase Noise: Summarization

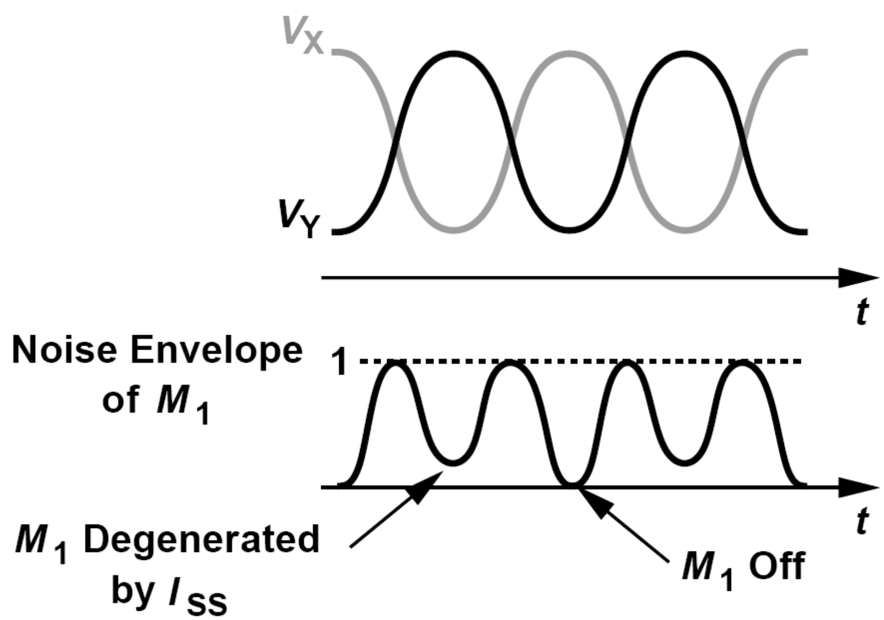


- Additive noise around $\pm\,\omega_0$ having a two-sided spectral density with a peak of η results in a phase noise spectrum around ω_0 having a normalized one-sided spectral density with a peak of $2\eta/A^2$

Cyclostationary Noise

- Since oscillators perform this noise modulation periodically, we say such noise sources are "cyclostationary," i.e., their spectrum varies periodically.

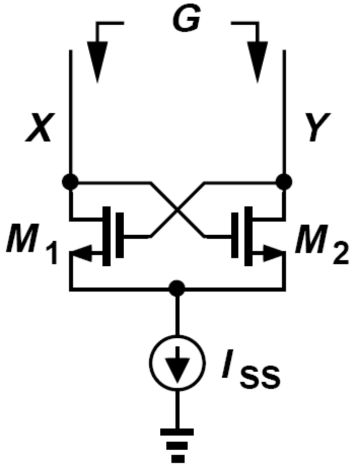


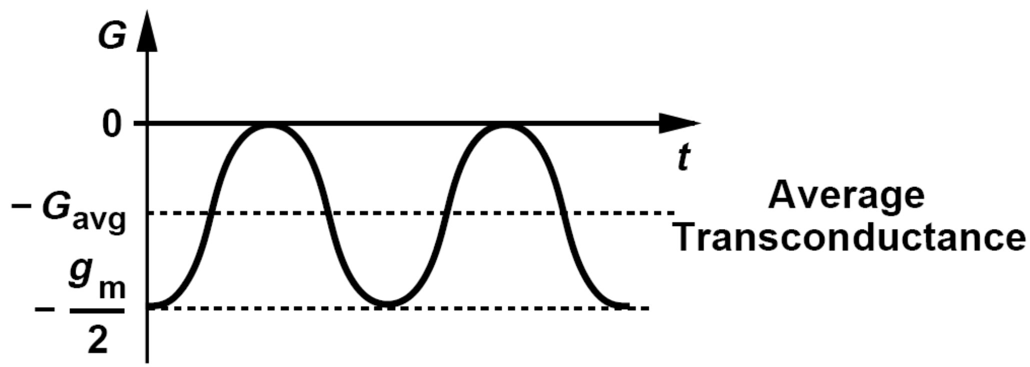


- The total noise current experiences an envelope having ***twice the oscillation frequency*** and swinging between zero and unity.
- Let us approximate the envelope by a sinusoid, 0.5 cos2$\omega_0 t$ + 0.5. White noise multiplied by such an envelope results in white noise with three-eighth the spectral density.

Time-Varying Resistance

- In addition to cyclostationary noise, the time variation of the resistance presented by the cross-coupled pair also complicates the analysis. We may consider a *time average* of the resistance as well.

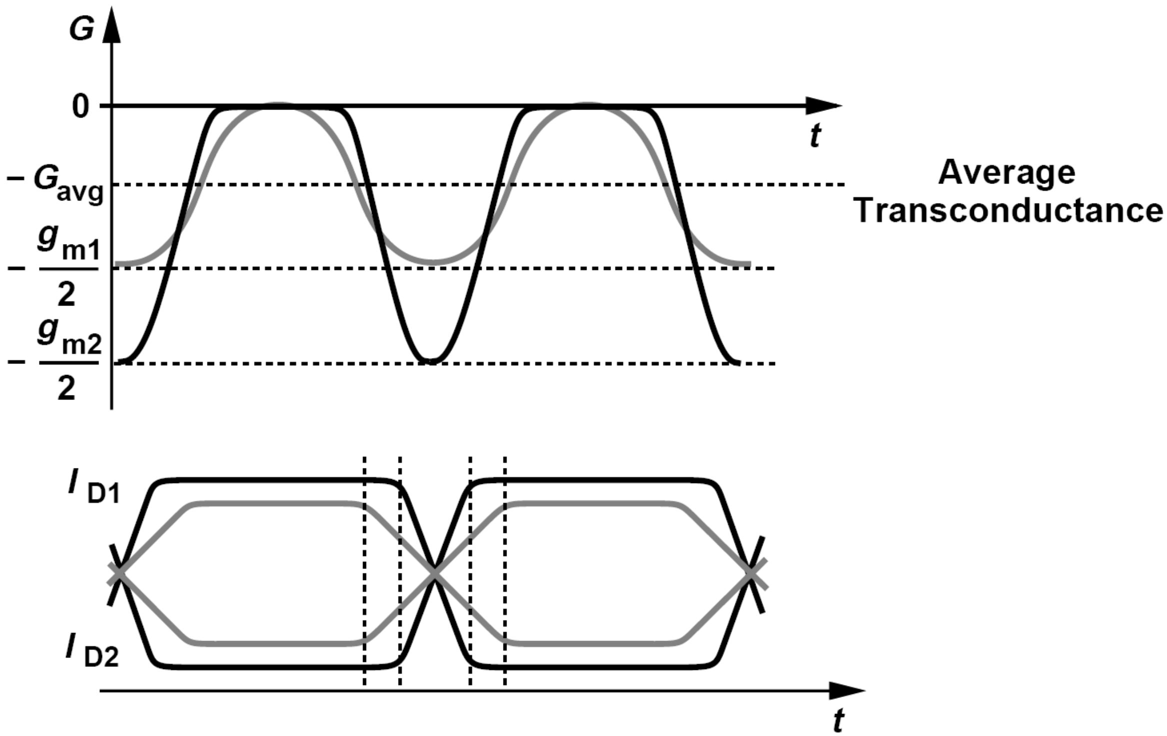




- The resistance seen between the drains of M_1 and M_2 periodically varies from $-2/g_m$ to nearly infinity. The corresponding conductance, G, thus swings between $-g_m/2$ and nearly zero, exhibiting a certain average, $-G_{avg}$.
- If $-G_{avg}$ is not sufficient to compensate for the loss of the tank, R_p, then the oscillation decays. Conversely, if $-G_{avg}$ is more than enough, then the oscillation amplitude grows. In the steady state, therefore, $G_{avg} = 1/R_p$.

Time-Varying Resistance: Effect of Increasing Tail Current

What happens to the conductance waveform and G_{avg} if the tail current is increased?



Since G_{avg} must remain equal to $1/R_p$, the waveform changes shape such that it has greater excursions but still the same average value. A larger tail current leads to a greater peak transconductance, $-g_{m2}/2$, while increasing the time that the transconductance spends near zero so that the average is constant. That is, the transistors are at equilibrium for a shorter amount of time.

Phase Noise Computation (I)

We now consolidate our formulations of (a) conversion of additive noise to phase noise, (b) cyclostationary noise, and (c) time-varying resistance.

- **1. We compute the average spectral density of the noise current injected by the cross-coupled pair.**

If a sinusoidal envelope is assumed, the two-sided spectral density amounts to $kT\gamma g_m \times (3/8)$

- **2. To this we add the noise current of R_p.**

$(3/8)kT\gamma g_m + 2kT/R_p$ is obtained.

- **3. We multiply the above spectral density by the squared magnitude of the net impedance seen between the output nodes.**

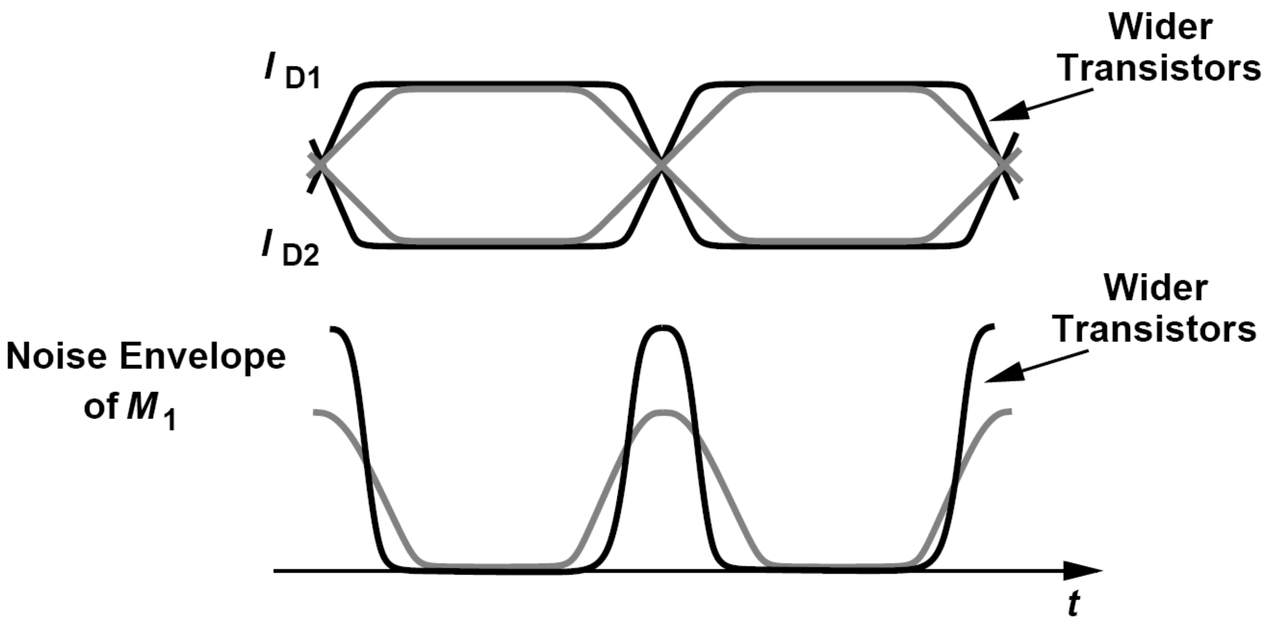
$$\overline{V_{n,out}^2} = kT\left(\frac{3}{8}\gamma g_m + \frac{2}{R_p}\right)\frac{L_1^2\omega^2}{(1-L_1C_1\omega^2)^2} \Rightarrow \overline{V_{n,out}^2} = kT\left(\frac{3}{8}\gamma g_m + \frac{2}{R_p}\right)\frac{1}{4C_1^2\Delta\omega^2}$$

- **4. We divide this result by $A^2/2$ to obtain the one-sided phase noise spectrum around ω_0.**

$$S(\Delta\omega) = \frac{\pi^2}{2}\frac{kT}{I_{SS}^2}\left(\frac{3}{8}\gamma g_m + \frac{2}{R_p}\right)\frac{\omega_0^2}{4Q^2\Delta\omega^2}$$

Phase Noise Computation (II)

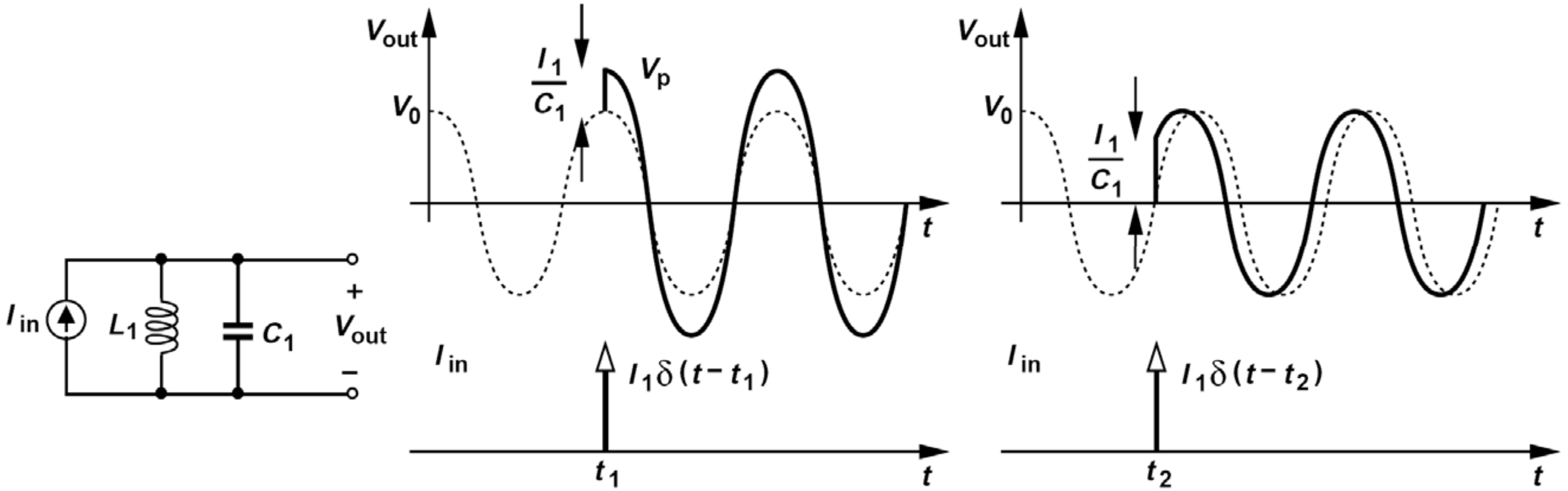
- A closer examination of the cross-coupled oscillator reveals that the phase noise is in fact independent of the transconductance of the transistors.



The decrease in the width and the increase in the height of the noise envelope pulses cancel each other and gm can be simply replaced with 2/R_p in the above equation

$$S(\Delta\omega) = \frac{\pi^2}{R_p}\frac{kT}{I_{SS}^2}\left(\frac{3}{8}\gamma + 1\right)\frac{\omega_0^2}{4Q^2\Delta\omega^2}$$

Analysis of Phase Noise: Approach II



Suppose an impulse of current is injected into the oscillating tank at the peak of the output voltage producing a voltage step across C_1. If

$$I_{in}(t) = I_1\delta(t - t_1)$$

Then the additional energy gives rise to a larger oscillation amplitude

$$V_p = V_0 + \frac{I_1}{C_1}$$

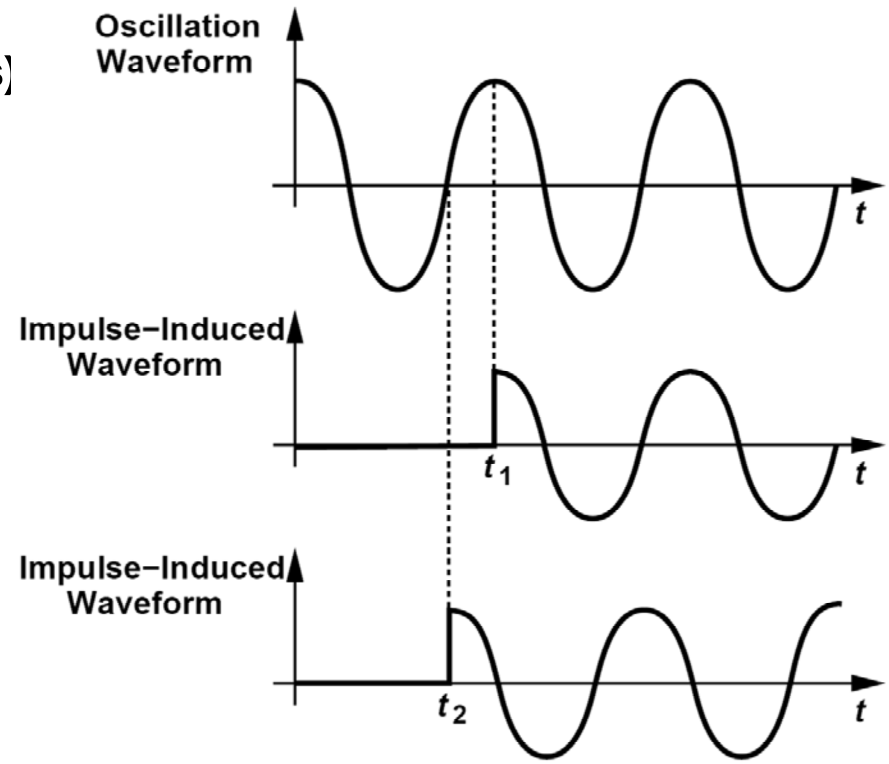
The injection at the peak does not disturb the phase of the oscillation.

- **Noise creates only amplitude modulation if injected at the peaks and only phase modulation if injected at the zero crossings.**

Computation of Impulse Response Using Superposition

Explain how the effect of the current impulse can be determined analytically.

The linearity of the tank allows the use of superposition for the injected currents (the inputs) and the voltage waveforms (the outputs). The output waveform consists of two sinusoidal components, one due to the initial condition (the oscillation waveform) and another due to the impulse. Figure on the right illustrates these components for two cases: if injected at t_1, the impulse leads to a sinusoid exactly in phase with the original component, and if injected at t_2, the impulse produces a sinusoid 90 ° out of phase with respect to the original component. In the former case, the peaks are unaffected, and in the latter, the zero crossings.

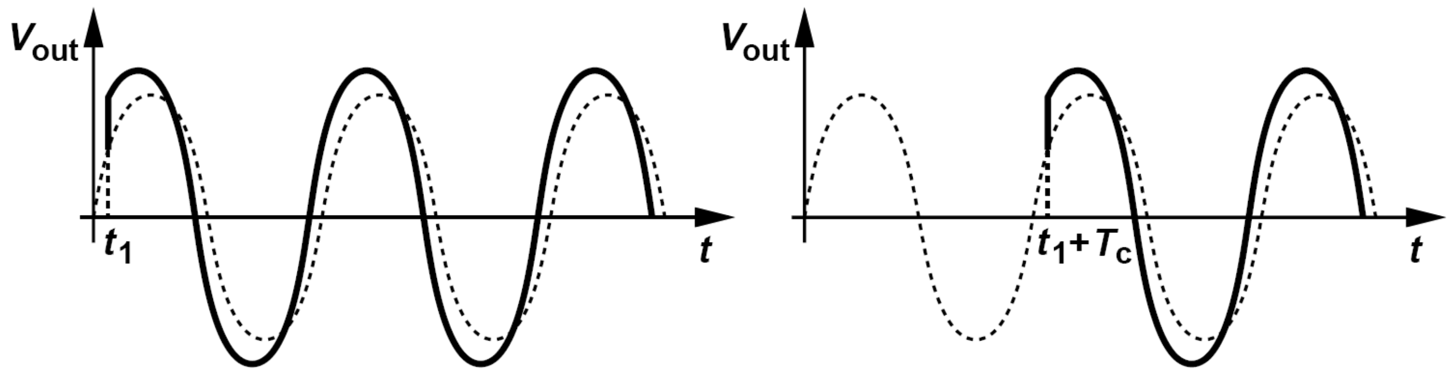


Quantifying Noise Hitting the Output Waveform: Impulse Sensitivity Function

We define a linear, time-variant system from each noise source to the output phase. The output phase in response to a noise *n(t)* is given by

$$\phi(t) = h(t, \tau) * n(t)$$

- **In an oscillator, $h(t, \tau)$ varies periodically: a noise impulse injected at $t = t_1$ or integer multiples of the period thereafter produces the same phase change.**
- **The impulse response, $h(t, \tau)$, is called the "impulse sensitivity function" (ISF).**



Explain how the LC tank has a time-variant behavior even though the inductor and the capacitor values remain constant.

The time variance arises from the finite initial condition (e.g., the initial voltage across C_1). With a zero initial condition, the circuit begins with a zero output, exhibiting a time-invariant response to the input.

Computation of Phase Impulse Response of a Tank

Compute the phase impulse response for the lossless LC tank

The overall output voltage can be expressed as

$$V_{out}(t) = V_0 \cos\omega_0 t + \Delta V[\cos\omega_0(t-t_1)]u(t-t_1)$$

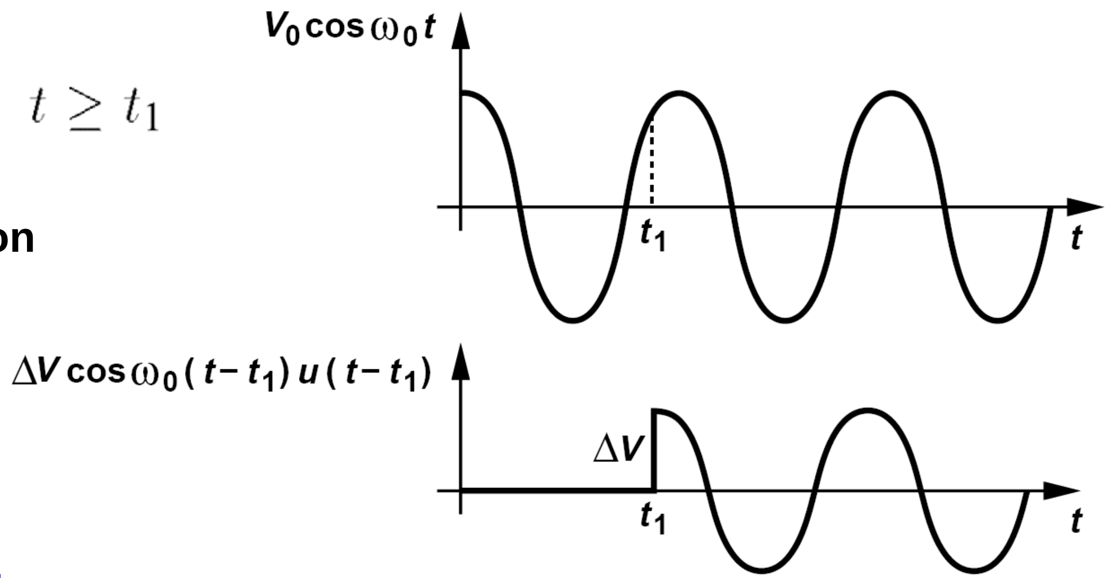
For $t \geq t_1$, V_{out} is equal to the sum of two sinusoids:

$$V_{out}(t) = (V_0 + \Delta V \cos\omega_0 t_1)\cos\omega_0 t + \Delta V \sin\omega_0 t_1 \sin\omega_0 t \quad t \geq t_1$$

$$V_{out}(t) = V_0 \cos\omega_0 t + \Delta V \cos\omega_0(t-t_1) \quad t \geq t_1$$

The phase of the output is therefore equal to

$$\phi_{out} = \tan^{-1}\frac{\Delta V \sin\omega_0 t_1}{V_0 + \Delta V \cos\omega_0 t_1} \quad t \geq t_1$$



Interestingly, Φ_{out} is not a linear function of ΔV in general. But, if $\Delta V << V_0$, then

$$\phi_{out} \approx \frac{\Delta V}{V_0}\sin\omega_0 t_1 \quad t \geq t_1$$

$$h(t, t_1) = \frac{1}{C_1 V_0}\sin\omega_0 t_1 u(t-t_1)$$

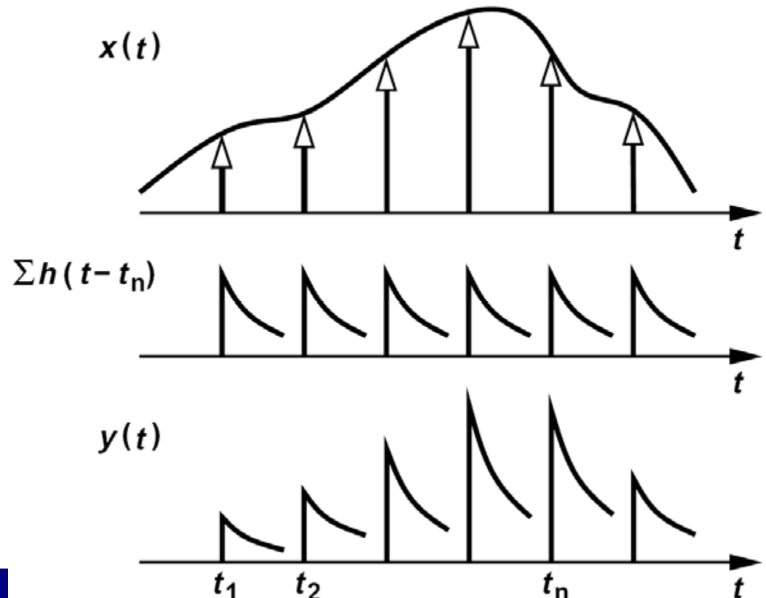
Convolution in Time-Invariant and Time-Variant

linear, time-invariant system

$$x(t) \approx \sum_{n=-\infty}^{+\infty} x(t_n)\delta(t-t_n)$$

$$y(t) \approx \sum_{n=-\infty}^{+\infty} x(t_n)h(t-t_n)$$

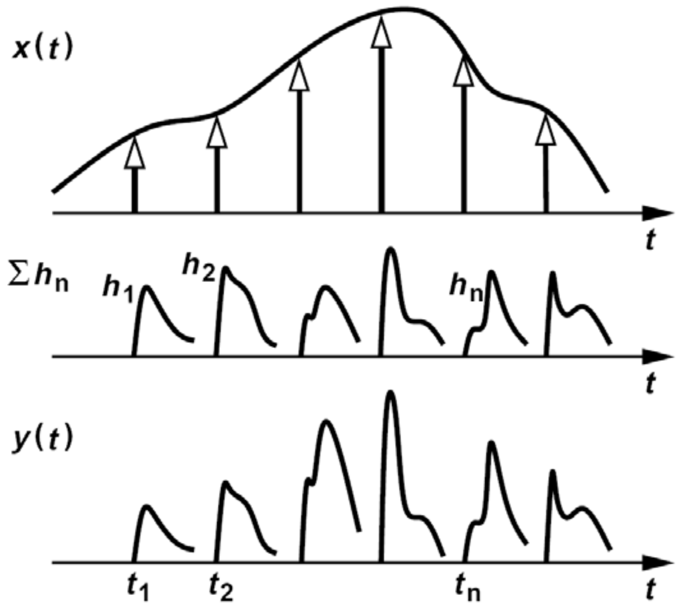
$$= \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$$



time-variant linear system

$$y(t) \approx \sum_{n=-\infty}^{+\infty} x(t_n)h_n(t)$$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t,\tau)d\tau$$



Example of Phase Noise Calculation

Determine the phase noise resulting from a current, $i_n(t)$, having a white spectrum, $S_i(f)$, that is injected into the tank.

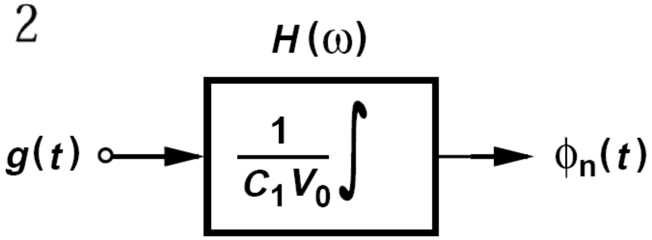
$$\begin{aligned}\phi_n(t) &= \int_{-\infty}^{+\infty} i_n(\tau)\frac{1}{C_1V_0}\sin\omega_0\tau u(t-\tau)d\tau \\ &= \frac{1}{C_1V_0}\int_{-\infty}^{t} i_n(\tau)\sin\omega_0\tau d\tau.\end{aligned}$$

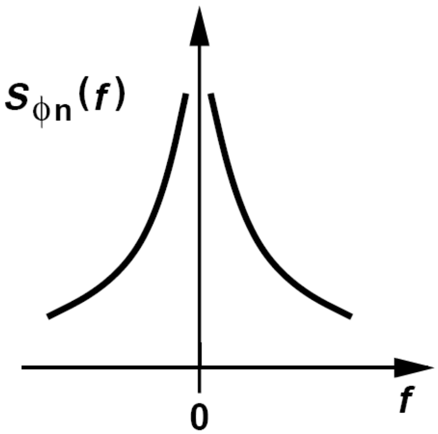
with half the spectral density of in(t):

$$S_g(f) = \frac{1}{2}S_i(f)$$

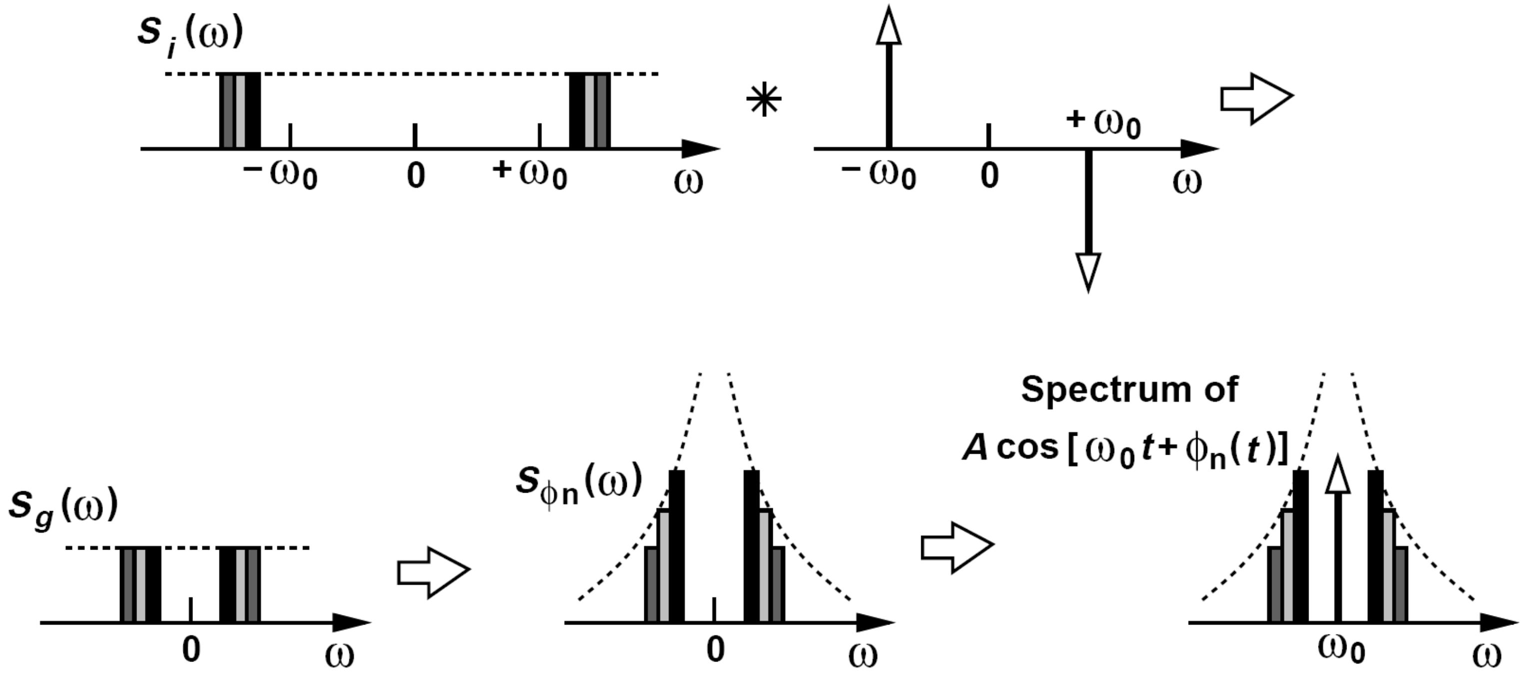
We note that (1) the impulse response of this system is simply equal to $(C_1V_0)^{-1}\ u(t)$, and (2) the Fourier transform of u(t) is given by $(j\omega)^{-1} + \pi\delta(\omega)$.

$$\begin{aligned}S_{\phi n}(f) &= |H(j\omega)|^2 S_g(f) \\ &= \frac{1}{C_1^2V_0^2}\frac{1}{(2\pi f)^2}\frac{S_i(f)}{2}\end{aligned}$$





Summary of Conversion of Injected Noise to Phase Noise around the Carrier



Which frequency components in $i_n(t)$ in the above example contribute significant phase noise?

Since $i_n(t)$ is multiplied by sin $\omega_0 t$, noise components around ω_0 are translated to the vicinity of zero frequency and subsequently appear in equation above. Thus, for a sinusoidal phase impulse response (ISF), only noise frequencies near ω_0 contribute significant phase noise.

Effect of Flicker Noise

Due to its periodic nature, the impulse response of oscillators can be expressed as a Fourier series:

$$h(t,\tau) = [a_0 + a_1\cos(\omega_0 t + \phi_1) + a_2\cos(2\omega_0 t + \phi_2) + \cdots]u(t-\tau)$$

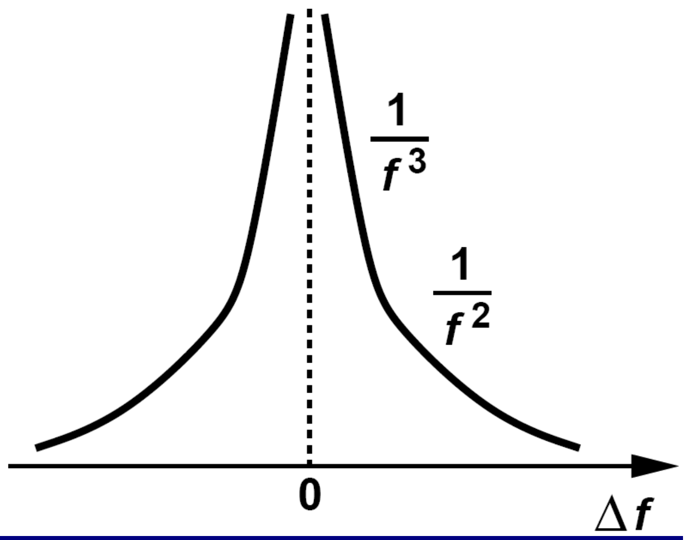
In particular, suppose $a_0 \neq 0$. Then, the corresponding phase noise in response to an injected noise $i_n(t)$ is equal to:

$$\phi_{n,a0} = \int_{-\infty}^{t} a_0 i_n(\tau)d\tau$$

$$\Rightarrow \quad S_{\phi n,a0}(f) = \frac{a_0^2}{\omega^2}S_i(f)$$

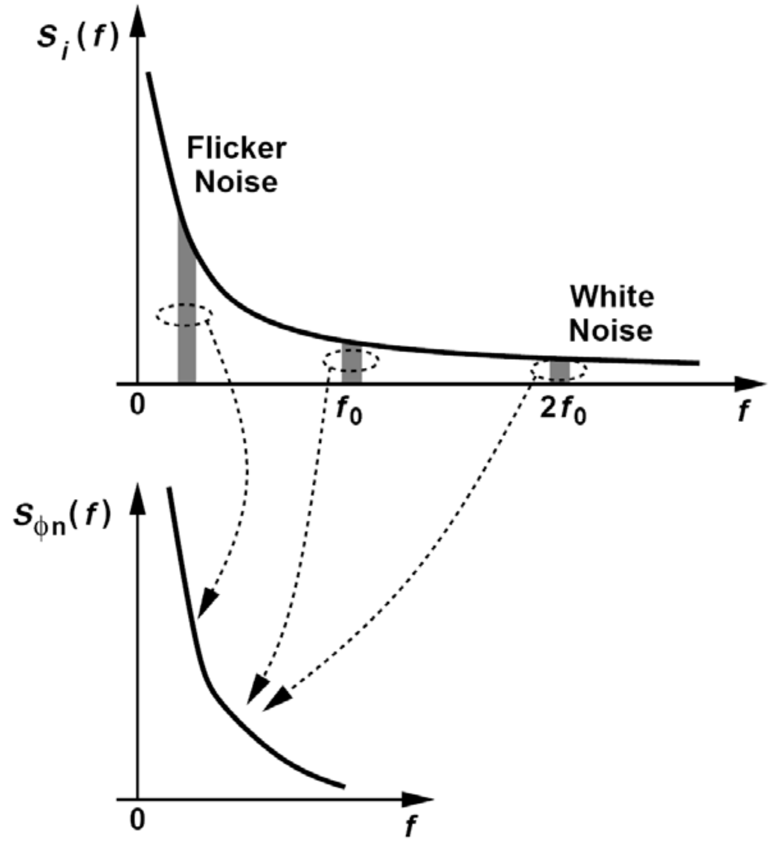
- **If the "dc" value of *h(t, τ)* is nonzero, then the flicker noise of the MOS transistors in the oscillator generates phase noise.**

$$S_{\phi n,a0}(f) = \frac{a_0^2}{4\pi^2}\frac{K}{WLC_{ox}}\frac{1}{f^3}$$



Noise around Higher Harmonics / Cyclostationary Noise

$a_1 \cos(\omega_0 t + \Phi_1)$ translates noise frequencies around ω_0 to the vicinity of zero and into phase noise. By the same token, $a_m \cos(m\omega_0 t + \Phi_j)$ converts noise components around $m\omega_0$ in $i_n(t)$ to phase noise.



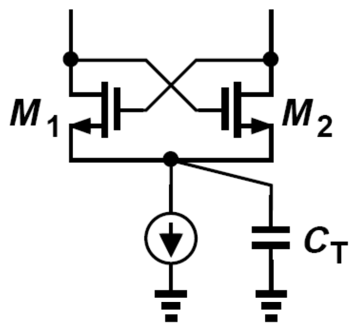
Cyclostationary noise can be viewed as stationary noise, *n(t)*, multiplied by a periodic envelope, *e(t).*

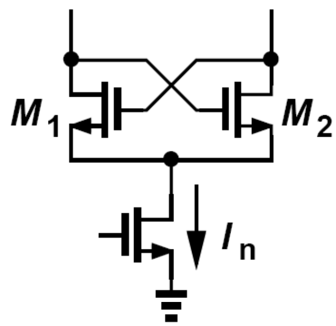
$$y(t) = \int_{-\infty}^{+\infty} n(\tau)e(\tau)h(t,\tau)d\tau$$

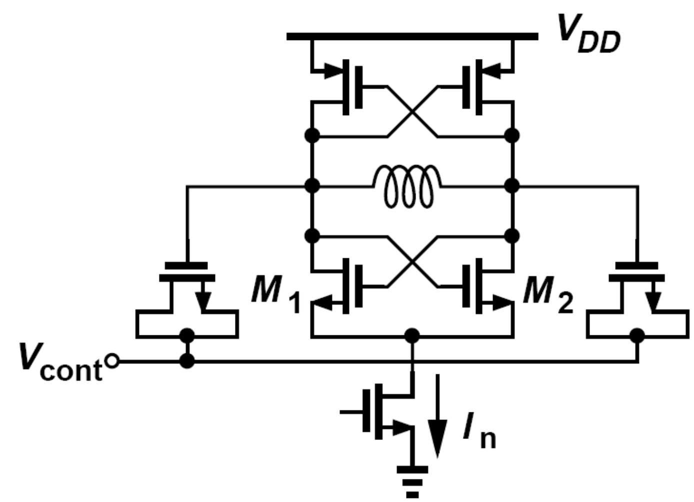
The effect of *n(t)* on phase noise ultimately depends on the product of the cyclostationary noise envelope and *h(t, τ).*

Noise of Bias Current Source: Tail Noise Mechanisms in Cross-Coupled Oscillator

- Oscillators typically employ a bias current source so as to minimize sensitivity to the supply voltage and noise therein.







Cross−coupled transistors enter triode region and	Tail current source has noise at $2\omega_0$.	Tail current source has flicker noise and
Tail has large capacitance.		Output CM level is modulated by flicker noise or
		Varactors have even−order voltage dependence.

More on Bias Current Source Noise

- **To obtain the phase noise in the output voltage, (1) the current sidebands computed in the above example must be multiplied by the impedance of the tank at a frequency offset of ±*Δω*, and (2) the result must be normalized to the oscillation amplitude.**

The relative phase noise can be expressed as :

$$S(\Delta\omega) = \frac{\frac{16\overline{I_n^2}}{9\pi^2}\left(\frac{1}{2C\Delta\omega}\right)^2}{\frac{4}{\pi^2}I_{SS}^2R_p^2} = \frac{4\overline{I_n^2}}{9I_{SS}^2}\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2$$

- **The thermal noise near higher even harmonics of ω_0 plays a similar role, producing FM sidebands around ω_0.**

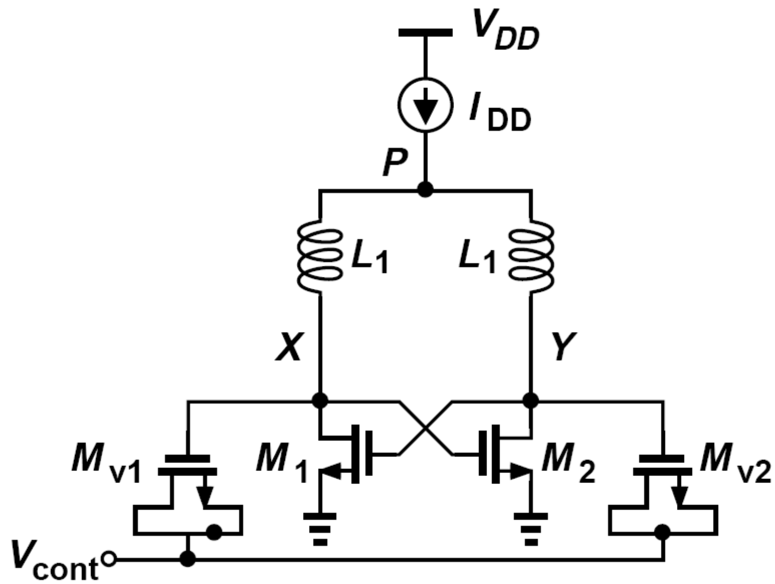
The summation of all of the sideband powers results in the following phase noise expression due to the tail current source

$$S(\Delta\omega) = \frac{\pi^2\overline{I_n^2}}{16I_{SS}^2}\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2$$

Top Bias Current Source Phase Noise

Suppose I_{DD} contains a noise current $i_n(t)$, producing a common-mode voltage change of

$$\Delta V = \frac{1}{g_m}\frac{i_n(t)}{2}$$



The output waveform can be expressed as

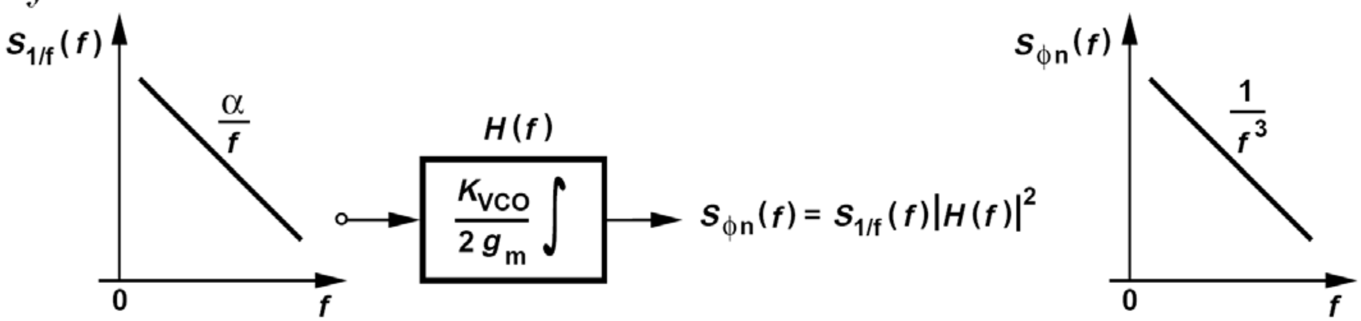
$$V_{out}(t) = V_0 \cos\left[\omega_0 t + \int K_{VCO}\frac{i_n(t)}{2g_m}dt\right]$$

If $\Phi_n(t)$ << 1 rad, then

$$V_{out}(t) \approx V_0 \cos\omega_0 t - V_0\frac{K_{VCO}}{2g_m}\left[\int i_n(t)dt\right]\sin\omega_0 t$$

We recognize that low-frequency components in $i_n(t)$ are upconverted to the vicinity of ω_0.

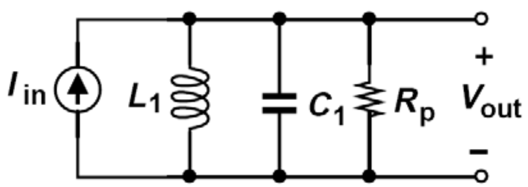
$$S_{\phi n}(f) = \left(\frac{K_{VCO}}{2g_m}\frac{1}{2\pi f}\right)^2\frac{\alpha}{f}$$

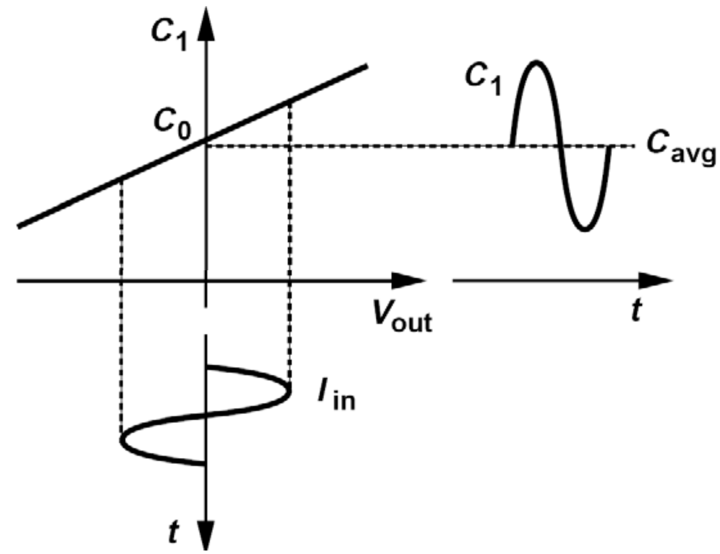


AM/PM Conversion (I)

- The amplitude modulation resulting from the bias current noise does translate to phase noise in the presence of nonlinear capacitances in the tanks.

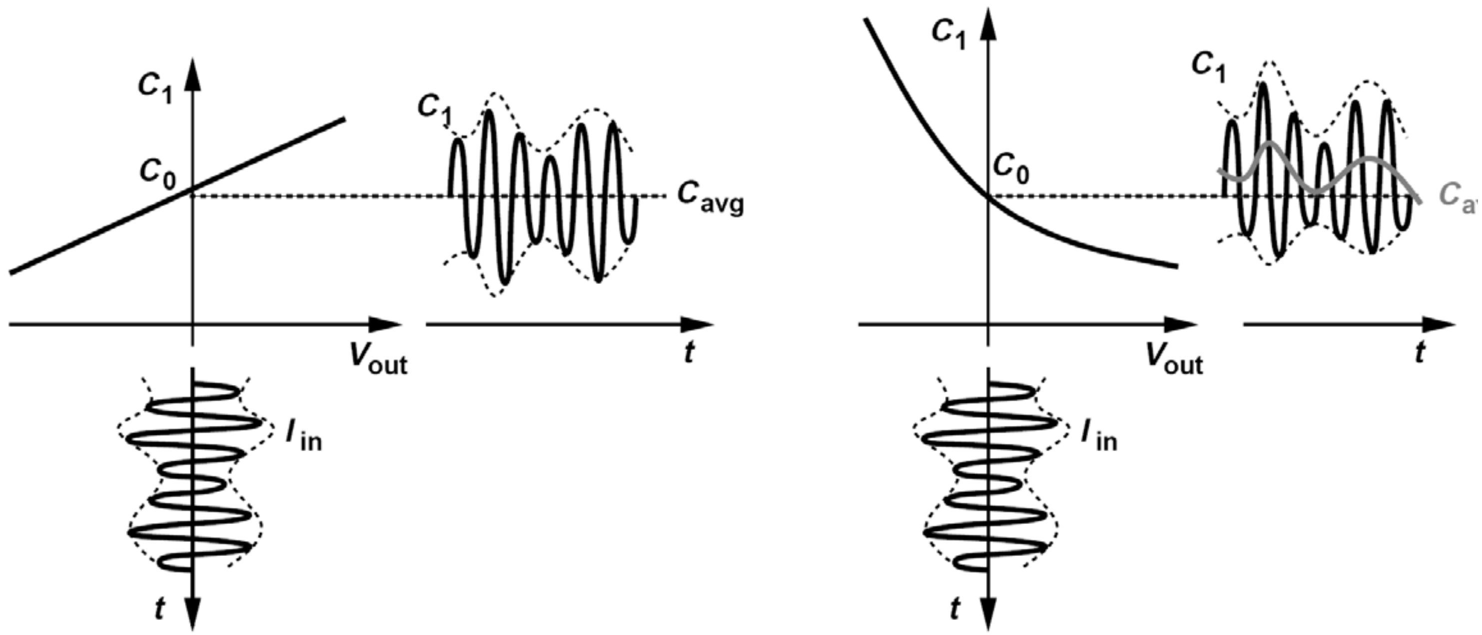
$$C_{var} = C_{avg} + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$





- First assume that the voltage dependence of C_1 is odd-symmetric around the vertical axis. C_{avg} is independent of the signal amplitude.
- The average tank resonance frequency is thus constant and no phase modulation occurs.

AM/PM Conversion (II)



- The above results change if C_1 exhibits even-order voltage dependence, e.g., $C_1 = C_0(1 + \alpha_1 V + \alpha_2 V^2)$. Now, the capacitance changes more sharply for negative or positive voltages, yielding an average that depends on the current amplitude.
- The tail current introduces phase noise via three distinct mechanisms:

  (1) its flicker noise modulates the output CM level and hence the varactors;

  (2) its flicker noise produces AM at the output and hence phase noise;

  (3) its thermal noise at $2\omega_0$ gives rise to phase noise.

Figures of Merit of VCOs

- **Our studies in this chapter point to direct trade-offs among the phase noise, power dissipation, and tuning range of VCOs.**

A figure of merit (FOM) that encapsulates some of these trade-offs is defined as

$$\mathrm{FOM}_1 = \frac{(\text{Oscillation Frequency})^2}{\text{Power Dissipation} \times \text{Phase Noise} \times (\text{Offset Frequency})^2}$$

Another FOM that additionally represents the trade-offs with the tuning range is

$$\mathrm{FOM}_2 = \frac{(\text{Oscillation Frequency})^2}{\text{Power Dissipation} \times \text{Phase Noise} \times (\text{Offset Frequency})^2} \times \left(\frac{\text{Tuning Range}}{\text{Oscillation Frequency}}\right)^2$$

- **In general, the phase noise in the above expressions refers to the worst-case value, typically at the highest oscillation frequency.**
- **Also, note that these FOMs do not account for the load driven by the VCO.**

References

1. B. Razavi, "RF Microelectronics," 2nd Ed., Prentice Hall, 2012.