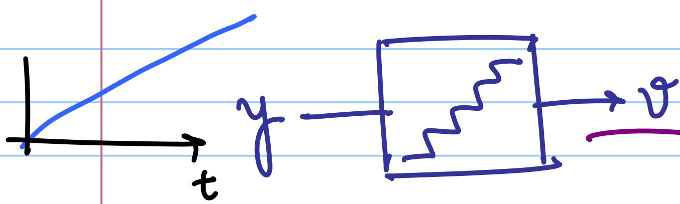
ECE 517 – Lecture 6

Note Title 2/1/2017

Quantizer Noise Modeling

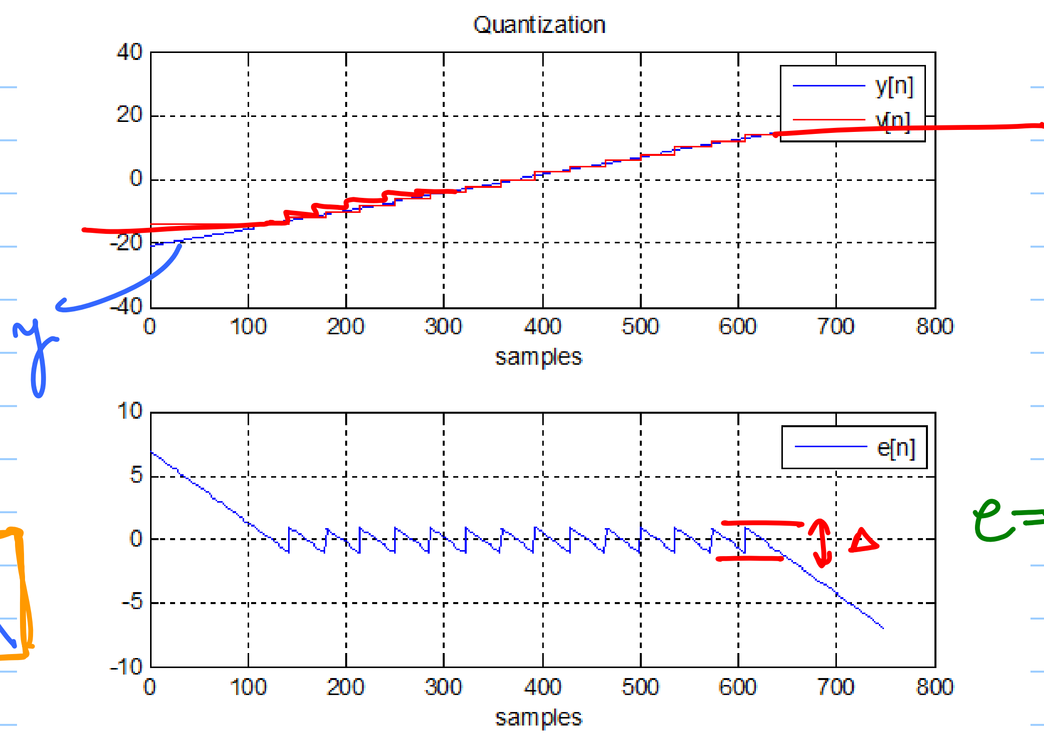


$e: v - y \Rightarrow$ quantization error

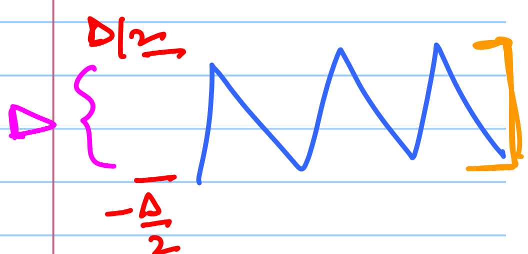
- Memoryless
- hard non-linearity

e ← challenging to analyze

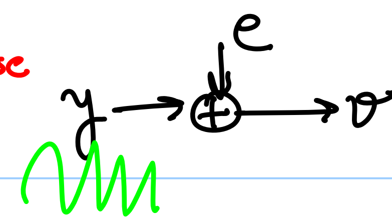
$$y \rightarrow \oplus \rightarrow v, \quad e \downarrow \oplus$$



$e = v - y$



Linearized Additive Noise Model



Conditions:

① y stays within the no-overload input range

② $e[n]$ is uncorrelated with the input $y[n]$ (input is "busy")

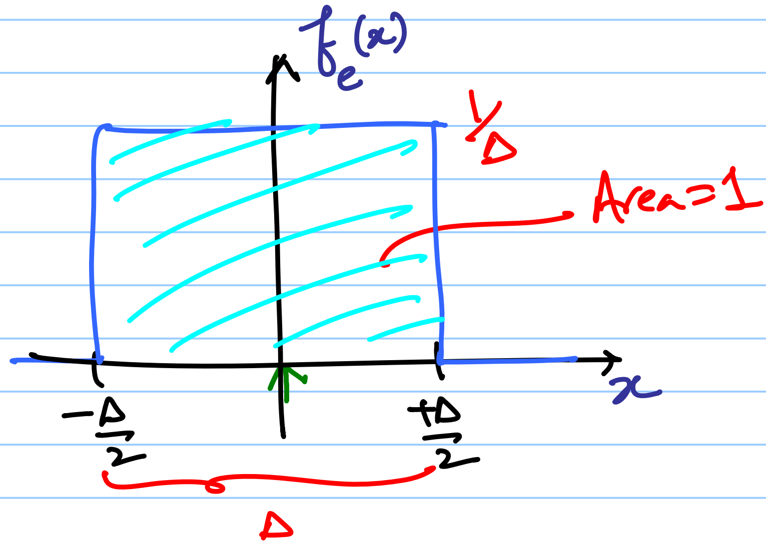
③ spectrum of $e[n]$ is "white" $\Rightarrow$ $e[n]$ is only correlated with itself

$$E\left[e[n]\, e[n+m]\right] = \delta[m] \cdot \sigma^2$$

④ quantization noise, $e[n]$, is uniformly distributed

↳ makes calculations easy

PDF of $e[n]$



Mean of e

$$\mu = E(e) = 0$$

Variance

$$\sigma_e^2 = E[(e-\mu)^2] = E[e^2]$$

$$= \int_{-\infty}^{\infty} x^2 f_e(x)\,dx$$

$$= \frac{1}{\Delta}\int_{-\Delta/2}^{+\Delta/2} x^2\,dx = \frac{\Delta^2}{12}$$

Variance, $\boxed{\sigma_e^2 = \frac{\Delta^2}{12}}$

$\Delta \Rightarrow$ LSB size of the quantizer

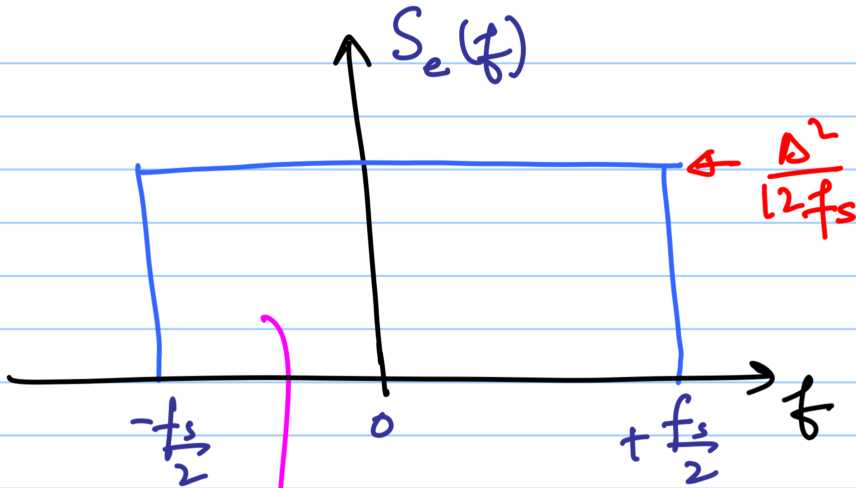
power $\stackrel{\Delta}{=}$ Variance $= \frac{\Delta^2}{12}$

Q has ∞ resolution

$\Delta \to 0$

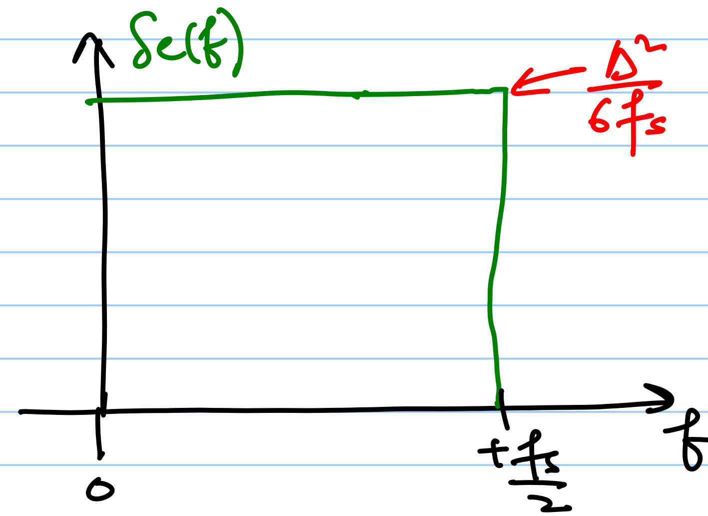
$\sigma_e^2 \to 0$

Quantization Noise Power Spectral Density (PSD)

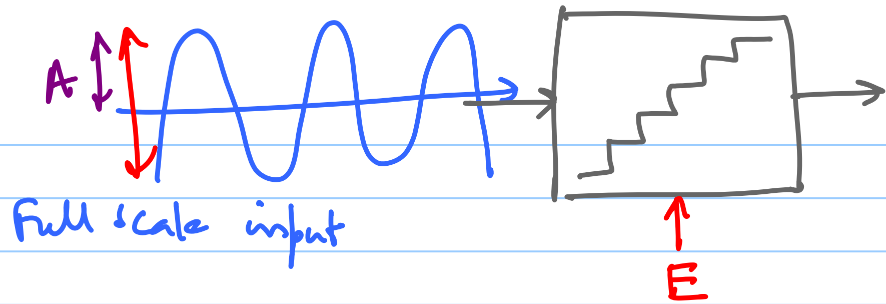


total power $= \frac{\Delta^2}{12}$

OR



one-sided spectrum



Signal to quantization noise ratio
(SQNR)

$$= \frac{P_s}{\Delta^2/12}$$

for an <u>N-bit</u> ADC/Quantizer :

Full scale range $= 2^N \cdot \Delta$

maximum sine amplitude $= 2^{N-1} \cdot \Delta$

maximum signal power $= \frac{A^2}{2} = \frac{(2^{N-1}\Delta)^2}{2}$

$$\text{Peak SQNR} = \frac{P_s}{\sigma_e^2} = \frac{(2^{N-1}\Delta)^2}{2 \cdot \frac{\Delta^2}{12}} = \frac{2^{2N-2}}{2} \cdot 12 = \frac{3}{2} \cdot 2^{2N}$$

$$SQNR_{dB} = 10 \log_{10}\left(\frac{3}{2} \cdot 2^{2N}\right)$$

Power

$$P_{dB} = 10 \log_{10}(P)$$

$$= 20 \log_{10}(V)$$

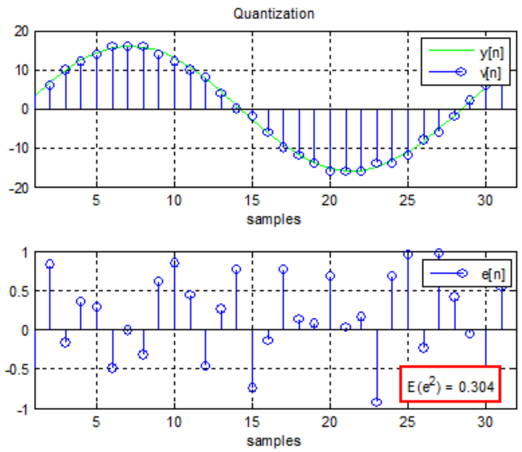
$$\boxed{SQNR = 6.02\,N + 1.76\ dB}$$

→ for a FS sine input

$$SQNR_{dB} \simeq 6 \cdot N$$

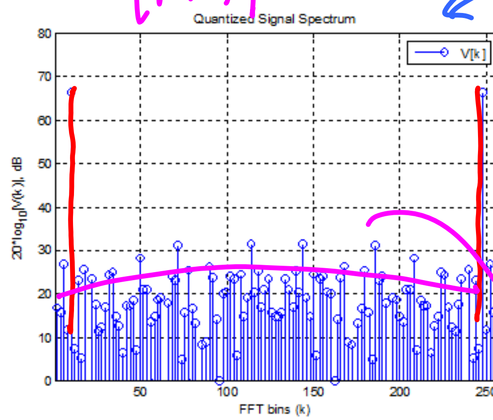
1 bit increase in resolution ⇒ 6-dB increase in SQNR

e



FFT

Spectrum of V[n]



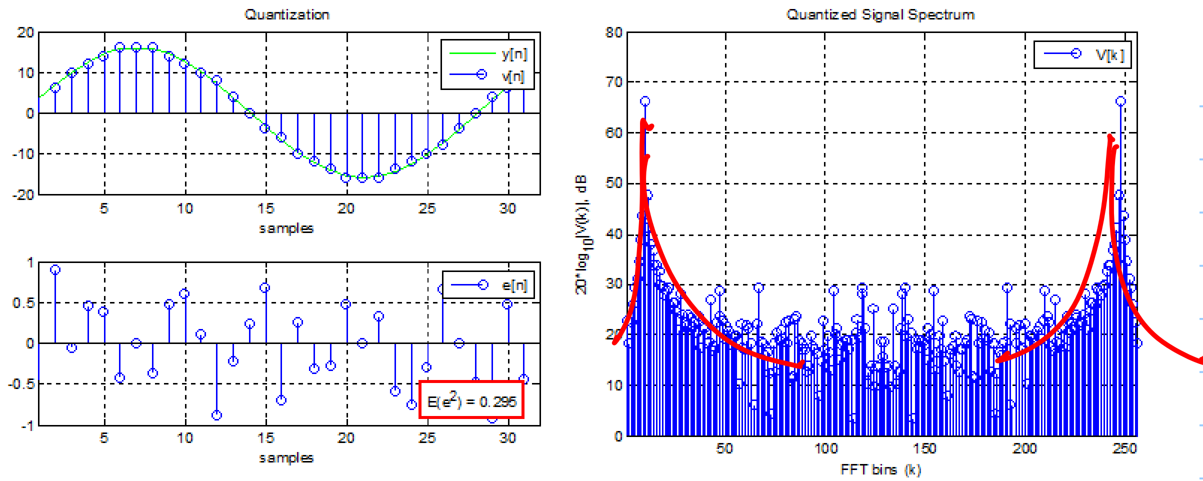
Matlab Spectre

SQNR

nLev=17, Δ=2, $f_{in}/f_s = 9/256$:

•$E(e^2) = \underline{0.304} \approx \Delta^2/12$ $= 1/3$

file:Quantization_Noise1.m

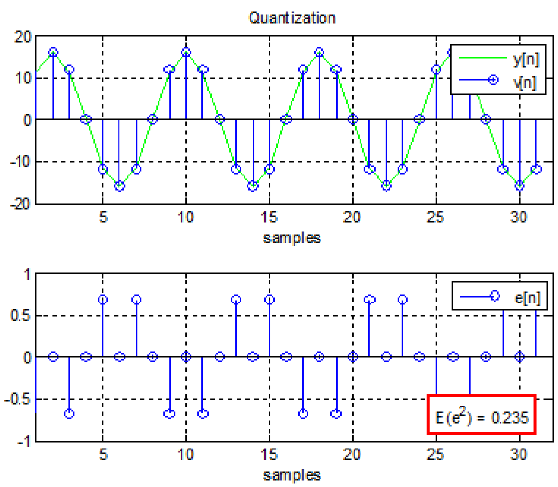


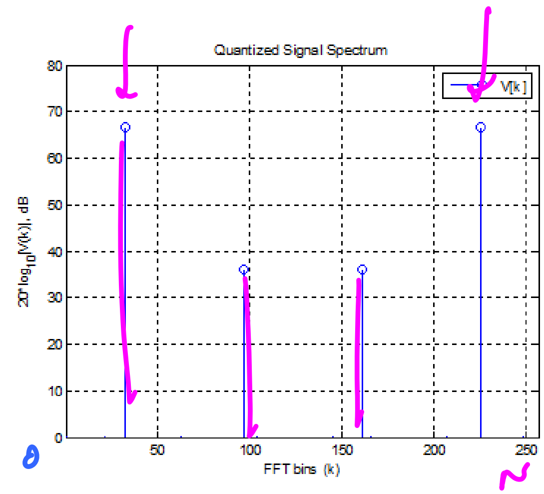
nLev=17, Δ=2, $f_{in}/f_s = 9.1/256$:

•$E(e^2) = 0.295 \approx \Delta^2/12$

•Notice the FFT leakage.

file:Quantization_Noise1.m



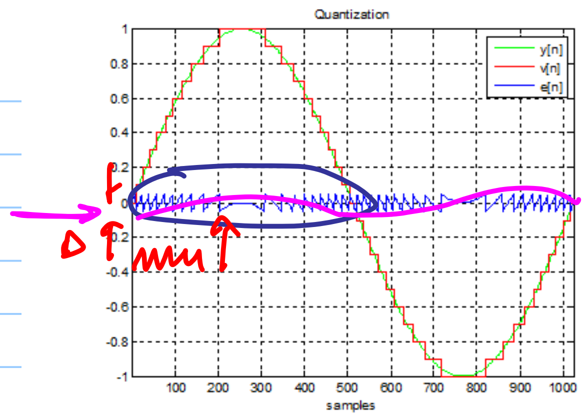


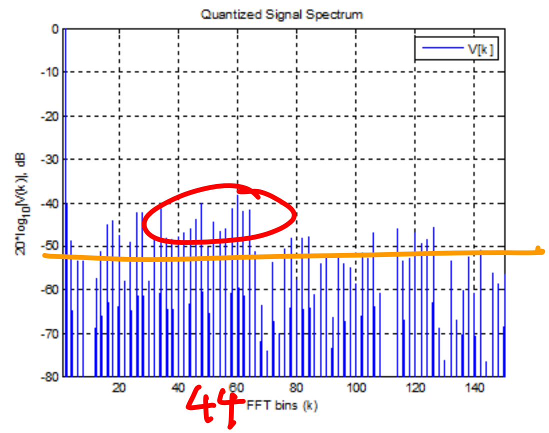
nLev=17, Δ=2, $f_{in}/f_s = 32/256 = 1/8$:

•$E(e^2) = 0.235 < \Delta^2/12$

•Quantization *noise* approximation not valid

file:Quantization_Noise1.m

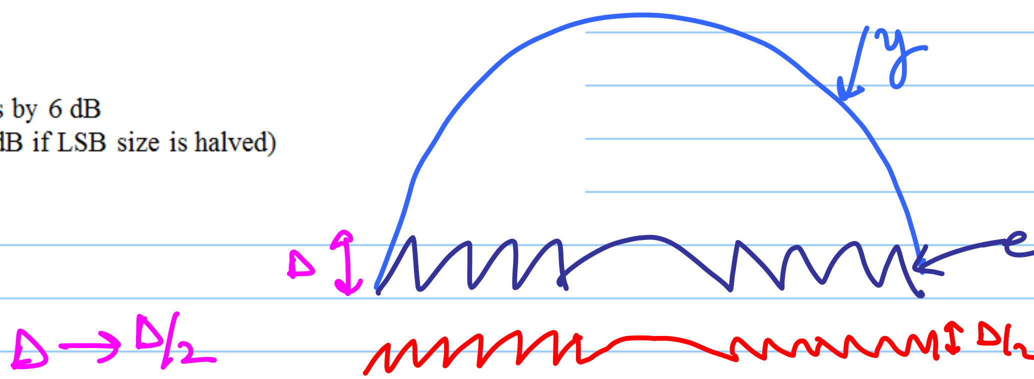


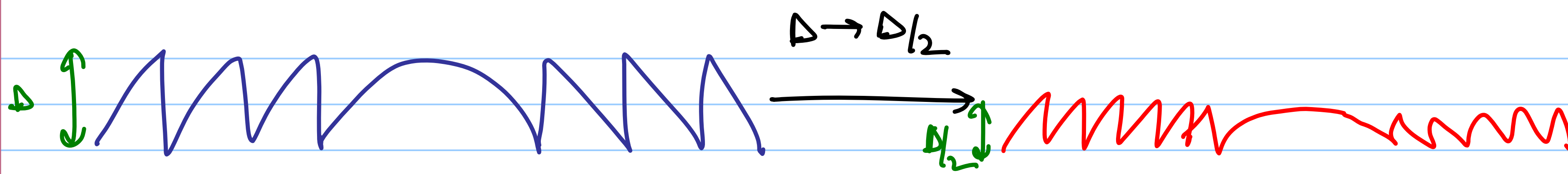


A=1, Δ=0.1, $f_{in}/f_s = 1/1024$:

- Most of the tones around the 44^{th} bin
- Average quantization noise floor lowers by 6 dB
- SFDR = -39 dB (SFDR increases by 9 dB if LSB size is halved)

file:Quantization_Noise2a.m





$e(t)$

$\frac{1}{2}e(2t)$

tones are spread out by 2

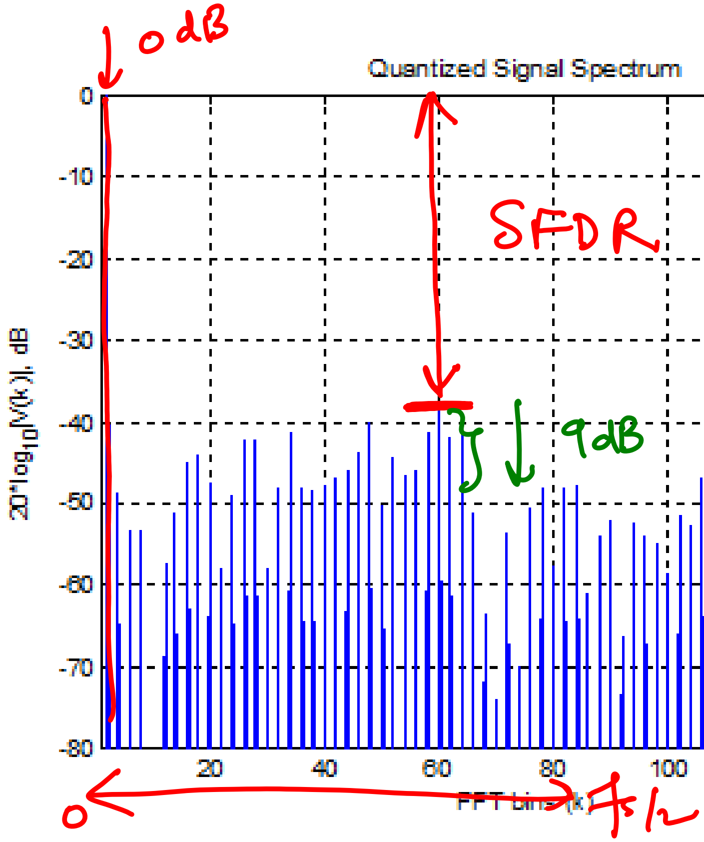
tone
$\frac{1}{2} \to$ power drops by 6dB

3dB reduction

$\Rightarrow$ 9 dB reduction in tone power

$\Delta \to \frac{\Delta}{2}$

SFDR improves by 9 dB



Spurious tones (spur)

SFDR ⇒ Spurious free dynamic range

Static

↳ DNL / INL

$$SNDR = 10 \log_{10} \left(\frac{P_s}{P_N + P_{distortion}} \right)$$

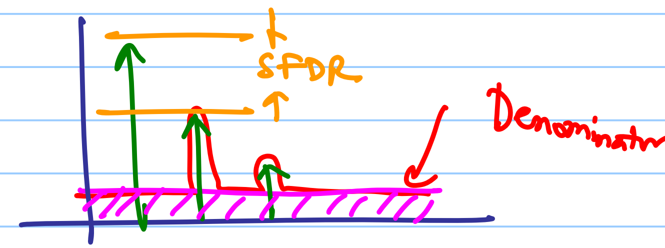
Effective number of bits

$$ENOB \triangleq \frac{SNDR - 1.76}{6.02}$$

$$SFDR_{dB} = 10 \log_{10} \left(\frac{\text{signal power}}{\text{largest spurious power}} \right)$$

Dynamic

frequency domain characteristics



ENOB < N

THD : Total harmonic distortion

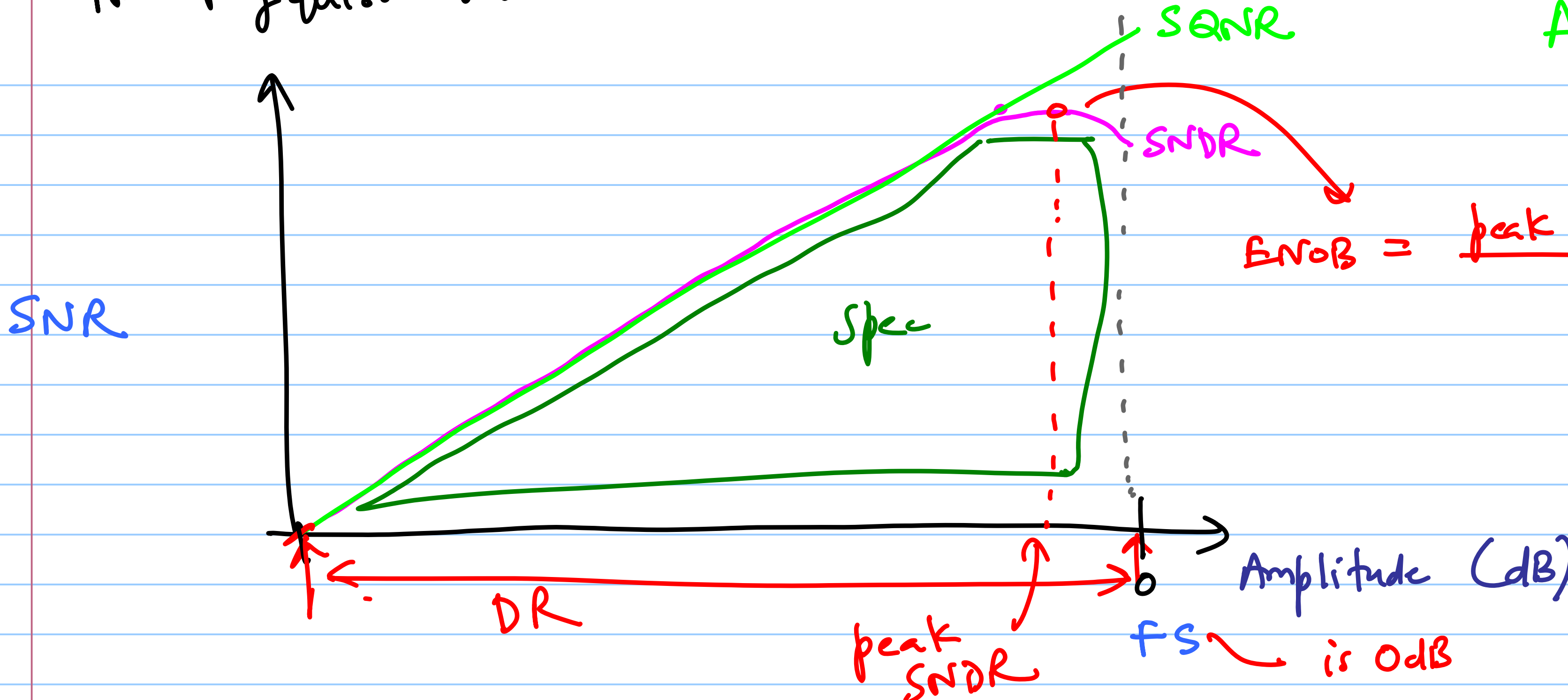
$$THD = 10 \log_{10} \left(\frac{\sum_{k=2}^{\infty} X_k^2}{X_1^2} \right)$$

Dynamic Range

$$DR = 10 \log_{10} \left(\frac{\text{Maximum signal power detected}}{\text{Smallest signal power detected}} \right)$$

The range from the full scale (fs) to the smallest detectable signal is the dynamic range for Nyquist rate ADC

$A \uparrow$ $P_s \uparrow$



$$ENOB = \frac{\text{peak SNDR} - 1.76}{6.02}$$