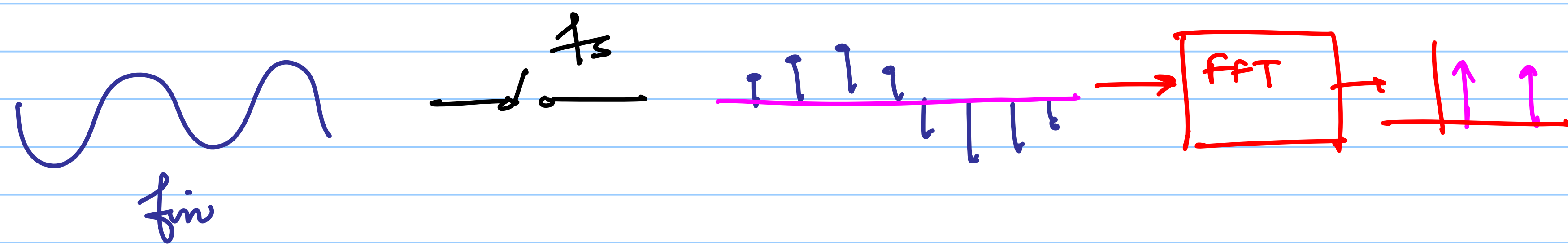
ECE 517 - Lecture 5

Note Title 1/31/2017

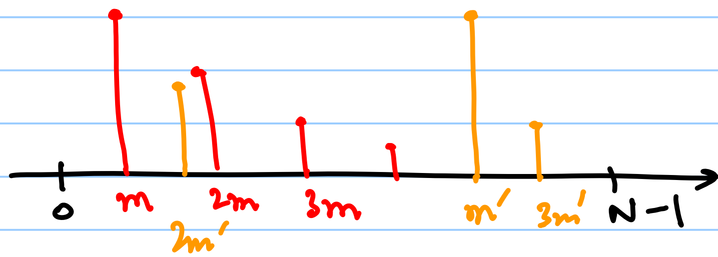


Necessary but not sufficient condition

$$\frac{f_{in}}{f_s} = \frac{m}{N}$$

m, N are mutually prime integers

N is large compared to m



$m \ll N$ so that the significant harmonics don't alias back to the fundamental.

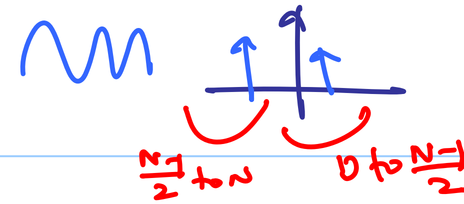
for FFT implementation $N = 2^p$

FFTDemo1.m

input is in the 129^{th} bin

$$f_{in} = \frac{129}{1024} f_s$$

can be normalized to 1 in plots



Absolute value of FFT doesn't matter due to normalization.

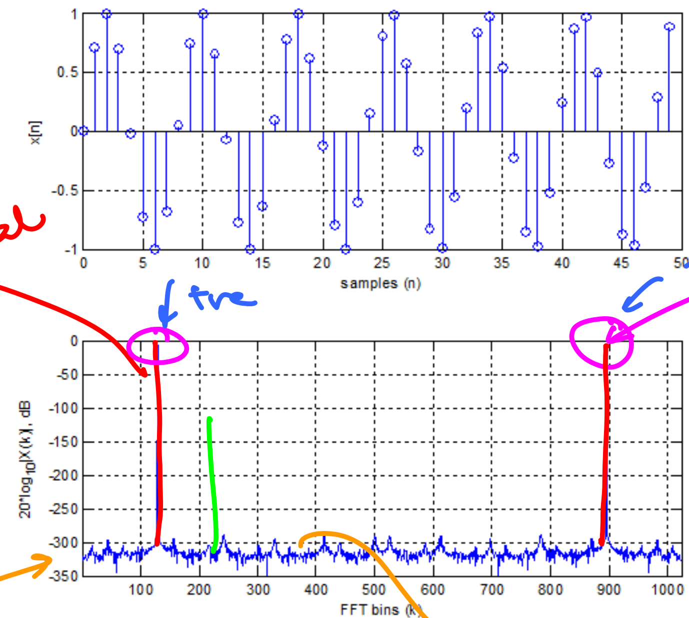
fundamental tone

+ve

-ve

-6 dB

$|X[k]|$



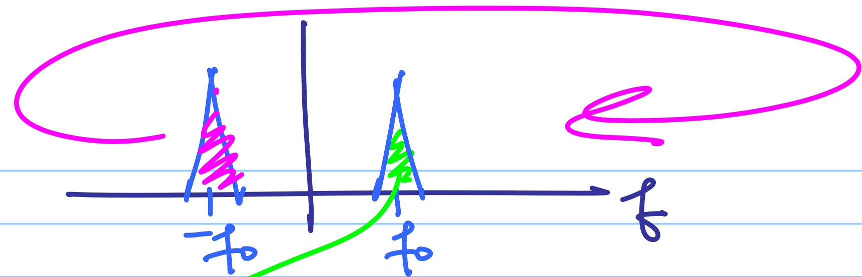
-300 dB

Noise floor

$k \Rightarrow 0$ to 1023

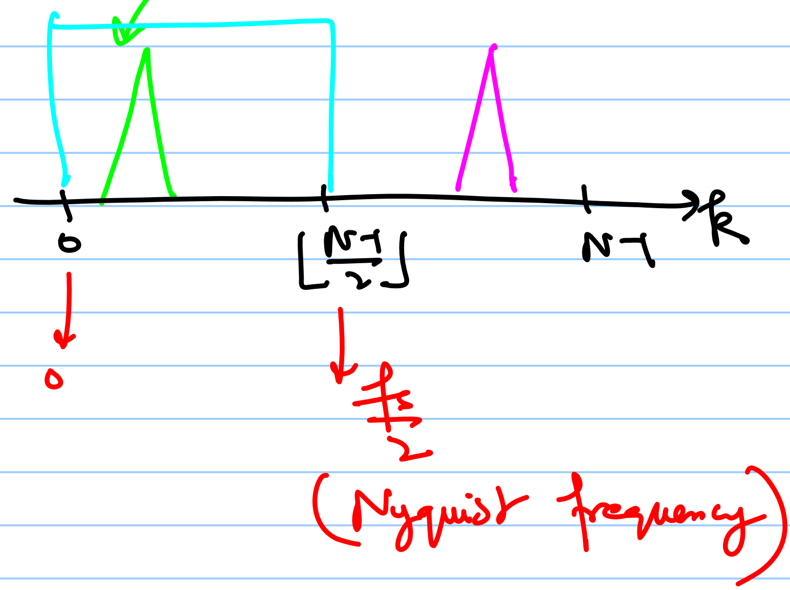
1 to 1024

FT



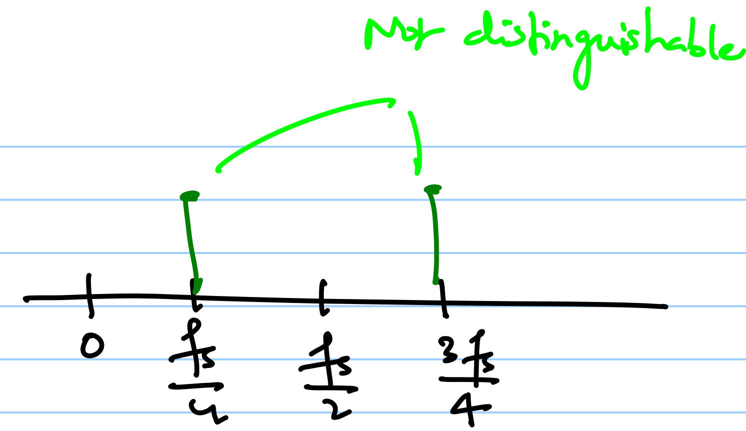
$-f_0$
f_0
f

FFT



0
$\lfloor \frac{N-1}{2} \rfloor$
$N-1$
k
0
$\frac{f_s}{2}$
(Nyquist frequency)

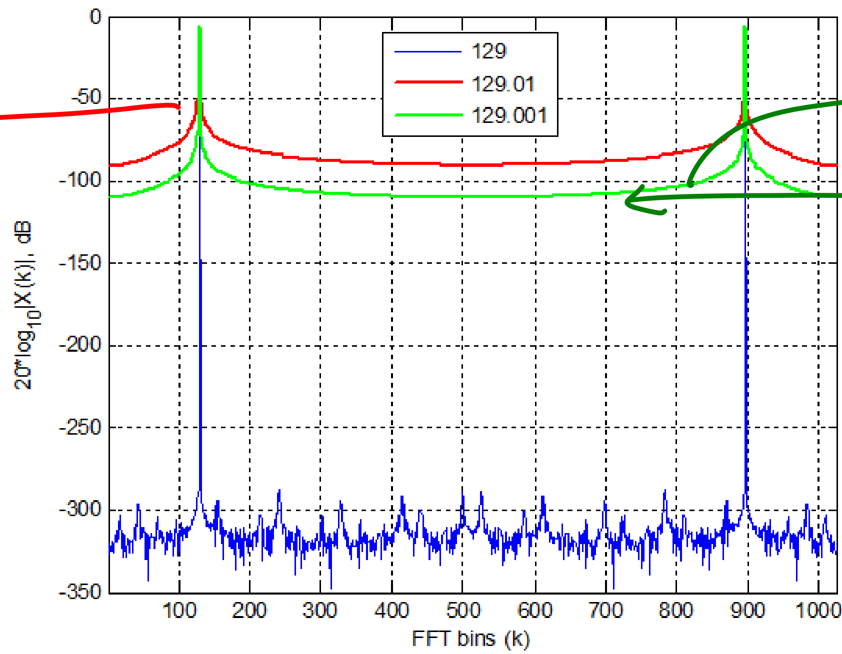
$$\frac{f_{in}}{f_s} = \frac{m}{N} = \frac{2^8}{2^{10}} = \frac{1}{4}$$



what if

$$\frac{m}{N} = \frac{129.001}{1024}$$

FFT leakage



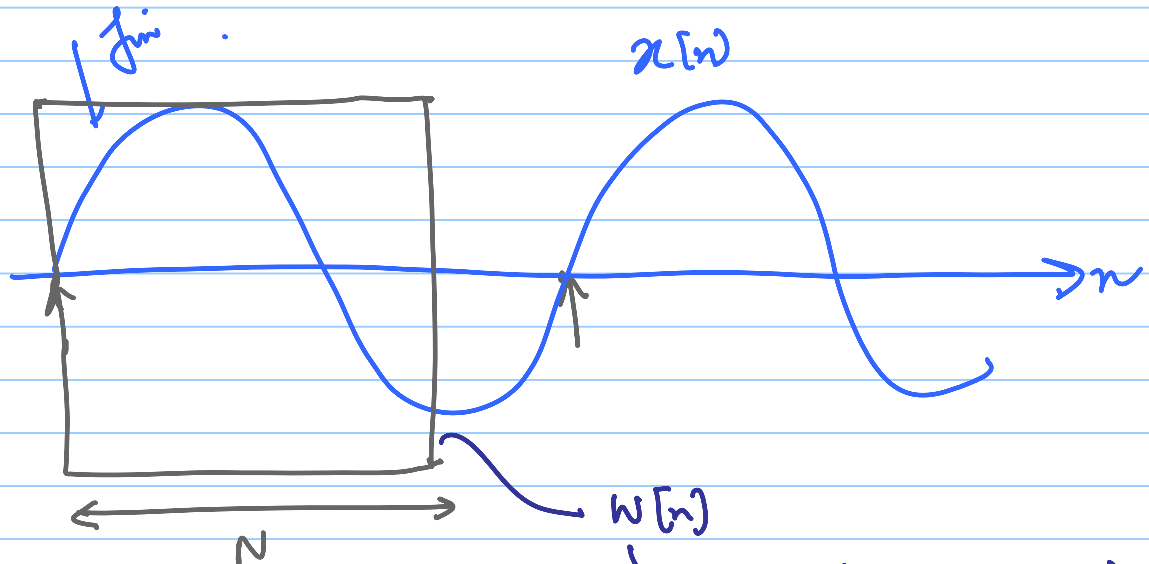
Eiffel tower effect

129.001

noise floor is shifted to ~100 dB

Signal power is leaking to nearby bins.

$$\frac{f_{in}}{f_s} = \frac{m}{N} \Rightarrow \underbrace{N \cdot f_{in} \text{ cycles} = m \text{ cycles of } f_s}$$

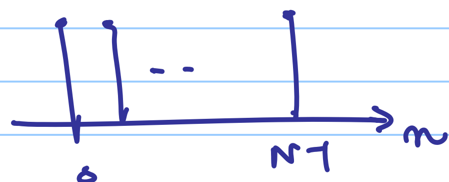


$m = 1$

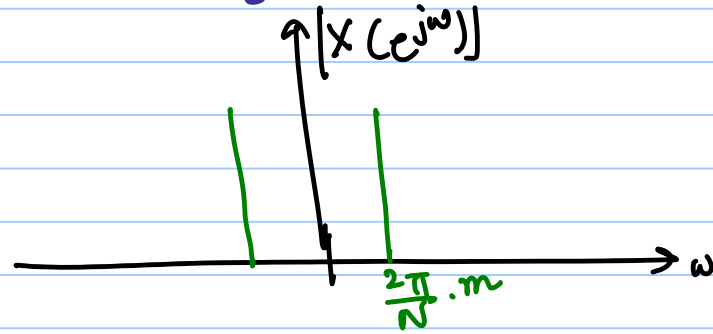
$w[n]$ → rectangular window

$x[n] \cdot w[n]$

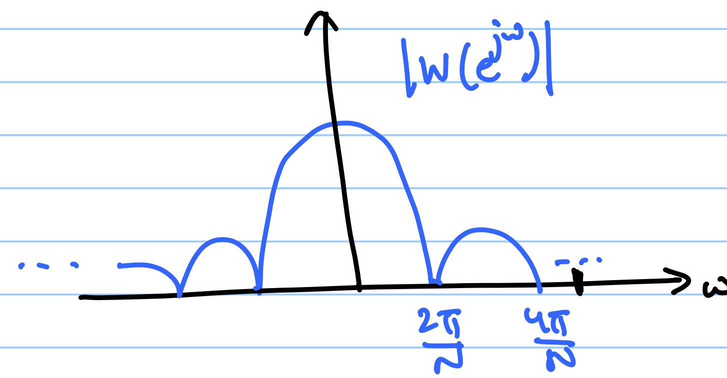
$$p[n] = x[n]\cdot w[n] \xrightarrow{\text{DTFT}} X(e^{j\omega}) \circledast W(e^{j\omega})$$

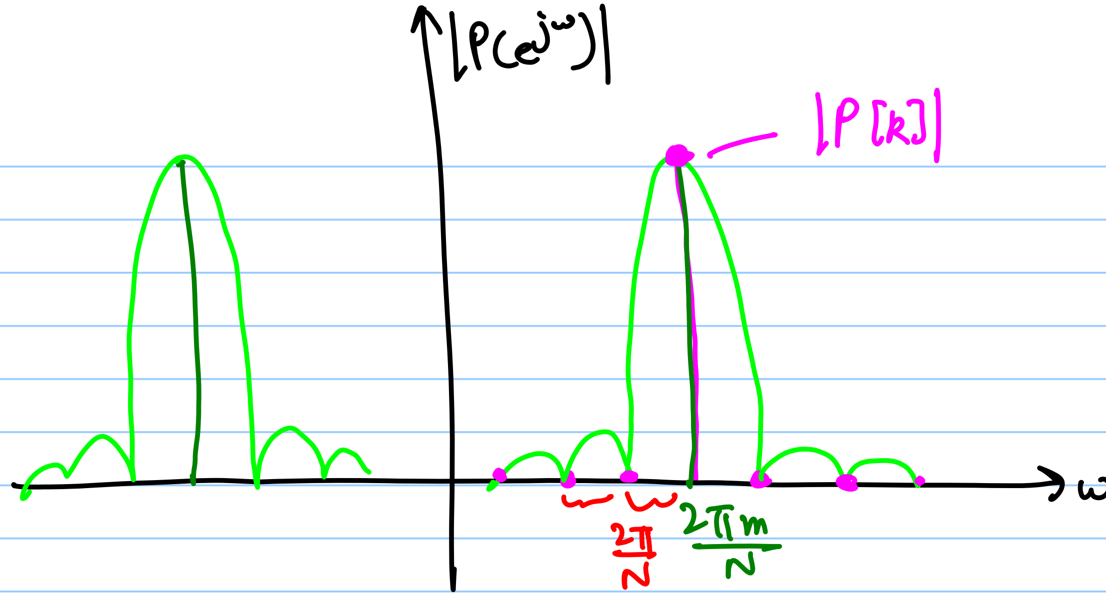


$$\Rightarrow \quad e^{-j\omega\frac{(N-1)}{2}} \, \frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\omega/2\right)}$$



$\circledast$



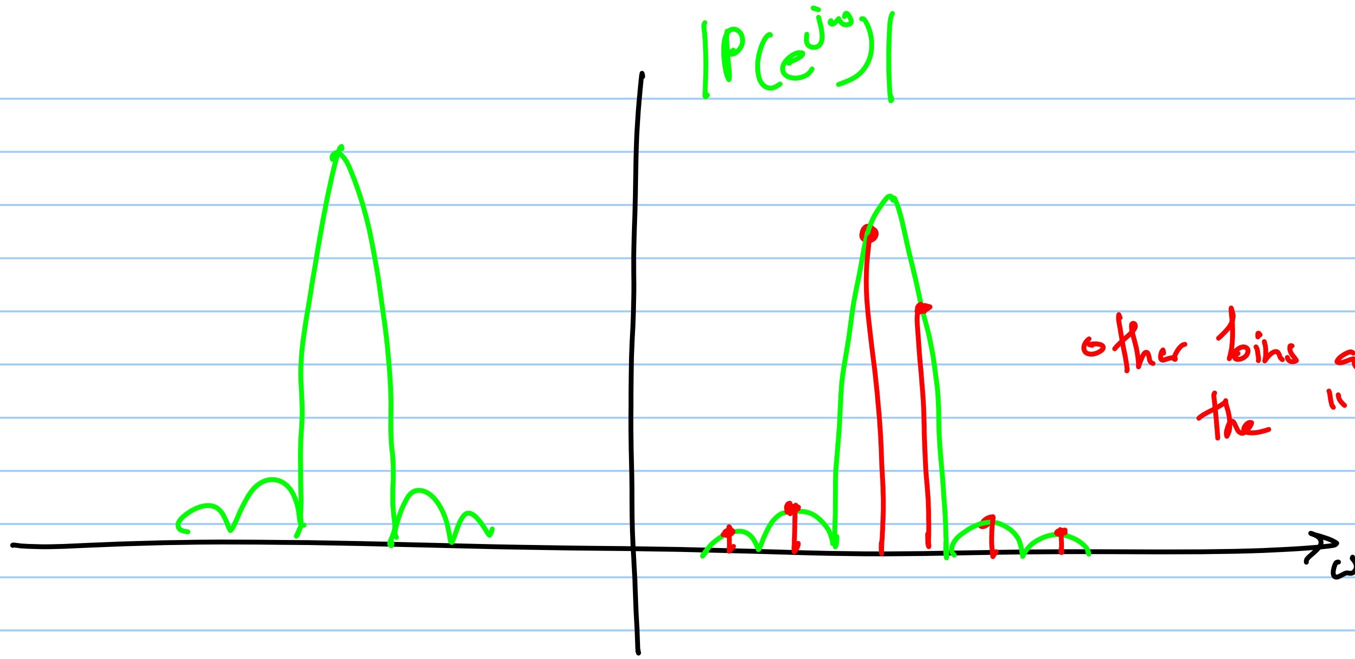


$$\frac{f_{in}}{f_s} = \frac{m}{N}$$

all non-signal bins are 0.

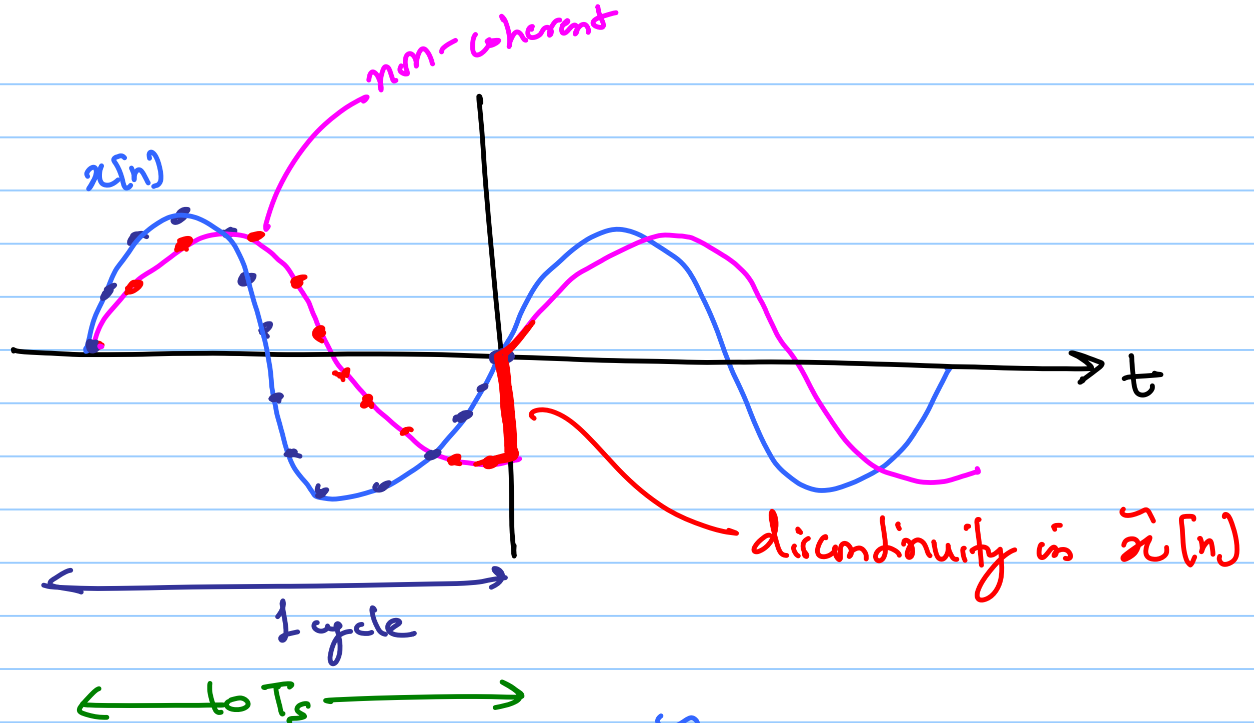
Synchronous sampling

or coherent sampling



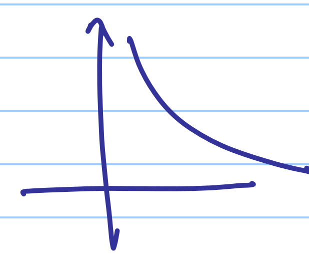
$$\frac{m}{N} \neq \frac{f_{in}}{f_s}$$

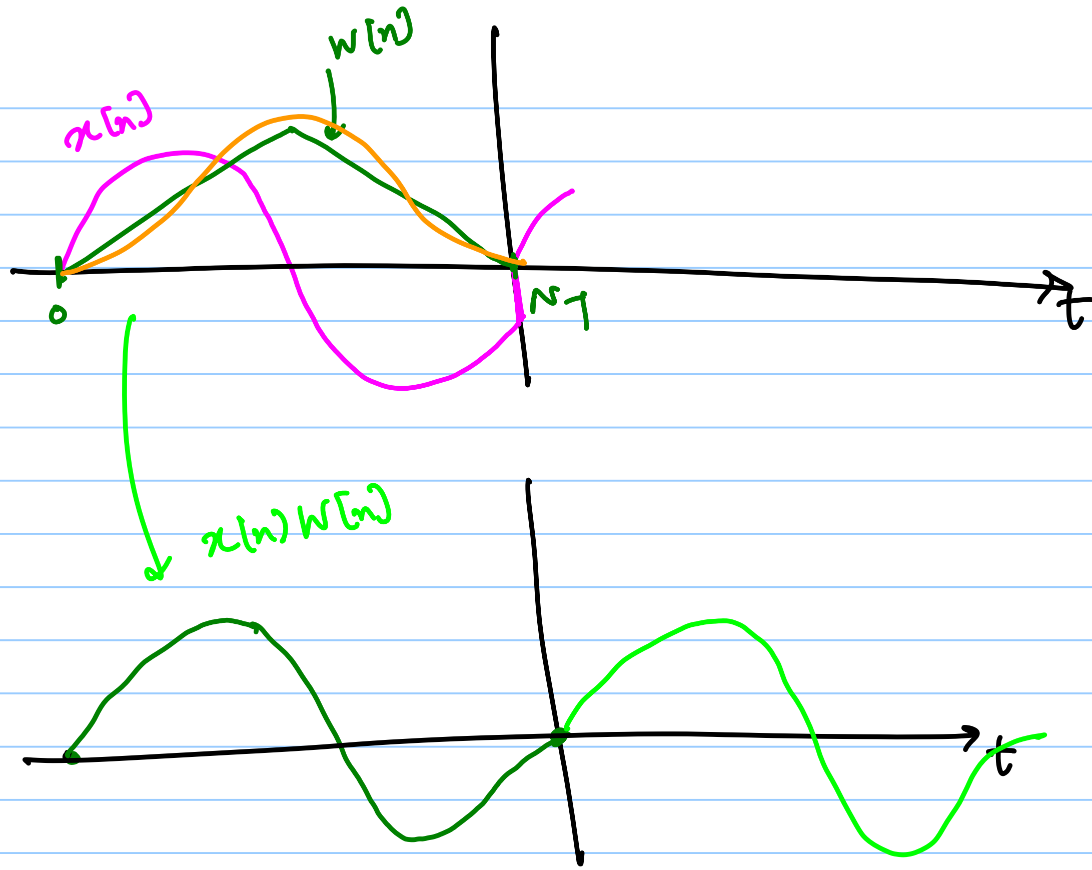
other bins are filled up with the "leaked" sine wave.



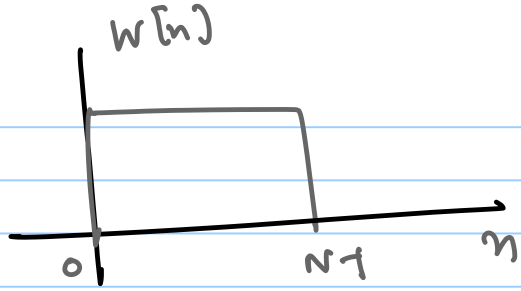
$$\frac{m}{N} = \frac{1}{10}$$

$\tilde{x}[n]$

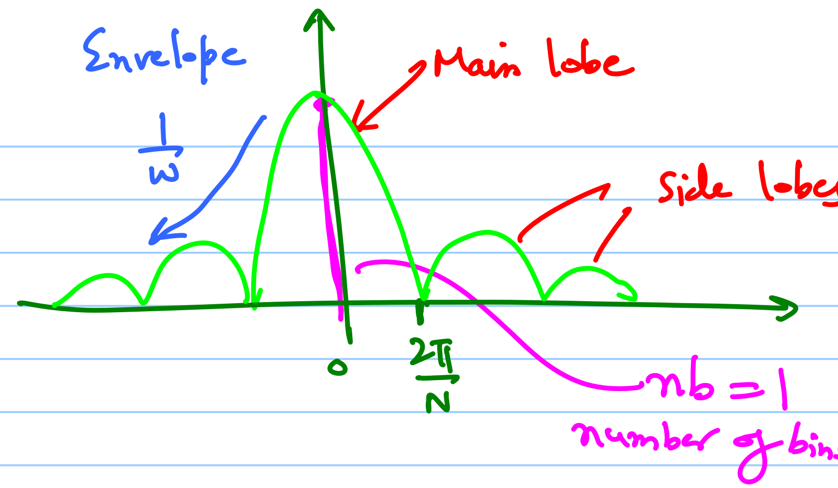




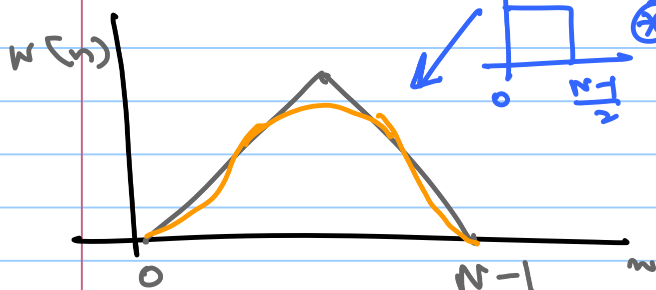
less emphasis on the ends & more importance to the signal in the middle.

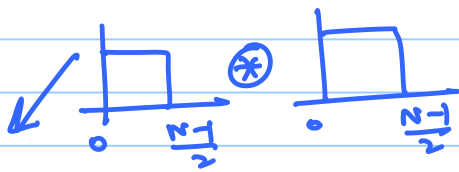


$$\sim \frac{\sin\left(\frac{N\omega}{2}\right)}{\sin\left(\omega/2\right)}$$



Triangular window

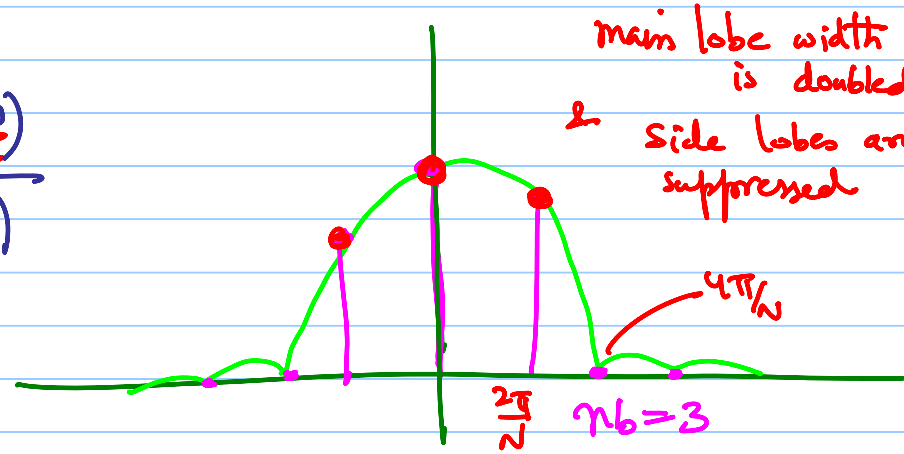




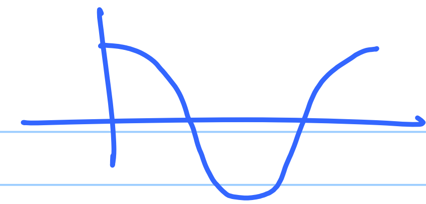
$$\propto \frac{\sin^2\left(\frac{N\omega}{4}\right)}{\sin^2\left(\frac{\omega}{2}\right)}$$

main lobe width is doubled

& Side lobes are suppressed



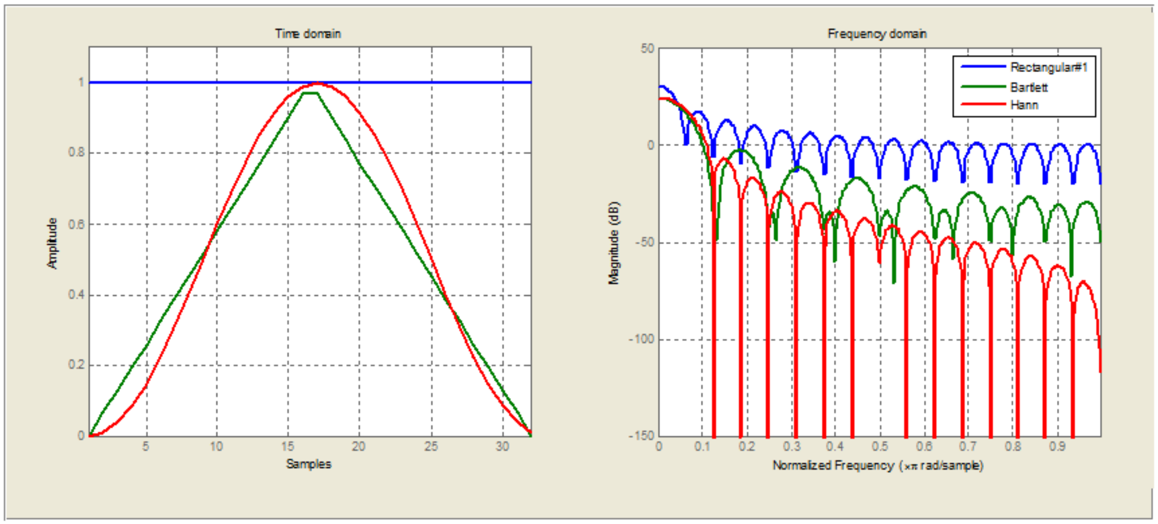
Raised Cosine (Hann, Hanning) Windows



$$w[n] = \frac{1}{2}\left[\underbrace{1}_{1} - \underbrace{\cos\left(\frac{2\pi n}{N}\right)}_{2}\right], \qquad 0 \le n \le N-1$$

Signal bins, $n_b = 3$

More sidelobe suppression




```
% Compare Rect, Bartlett and Hann windows
L = 32;
wvtool(rectwin(L), bartlett(L), ds_hann(L));
```

Hann window is defacto used in simulations.

Blackmann-Harris window

$$w(n) = a_0 + a_1 \cos\left(\frac{2\pi}{N} n\right) + a_2 \cos\left(\frac{2\pi}{N} 2n\right) + a_3 \cos\left(\frac{2\pi}{N} 3n\right)$$

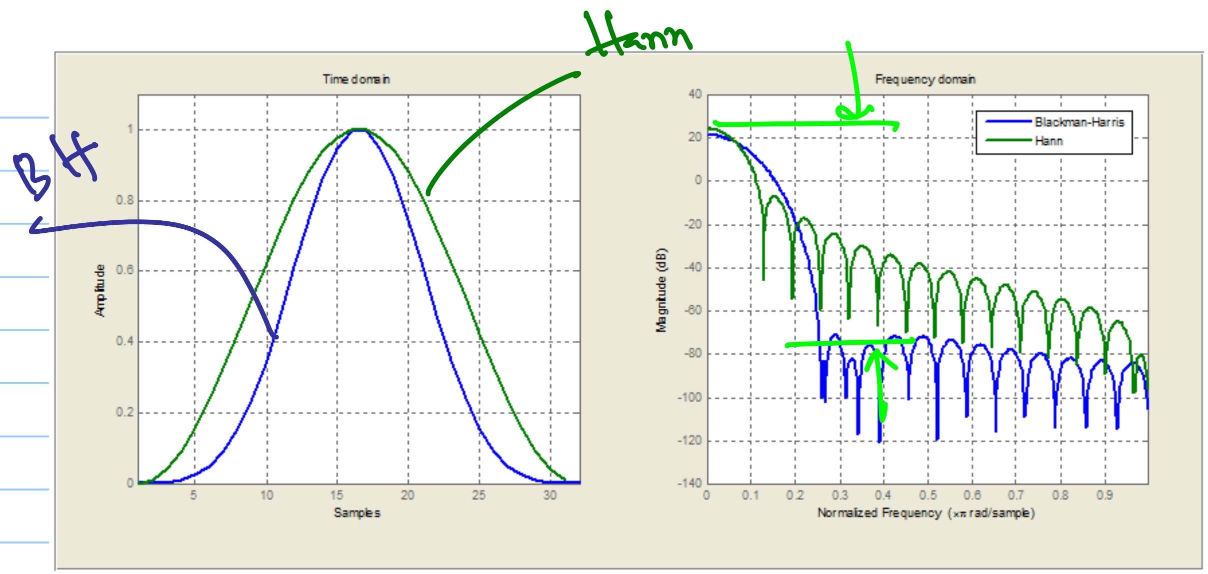
$a_0 = 0.35875$

$a_1 = 0.48829$

$a_2 = 0.14128$

$a_3 = 0.01168$

$m_b = 7$




```
% Compare Blackman-Harris and Hann windows
L = 32;
wvtool(blackmanharris(L), ds_hann(L));
```

Experimental measurement
Collect ~ million samples.