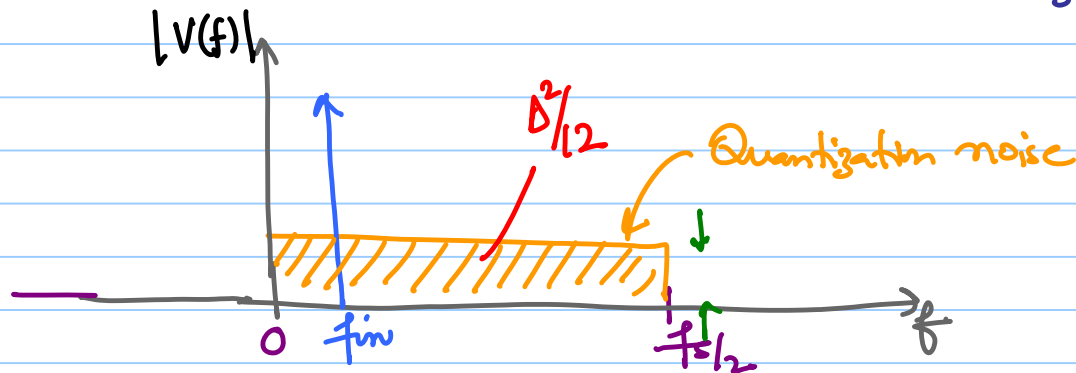
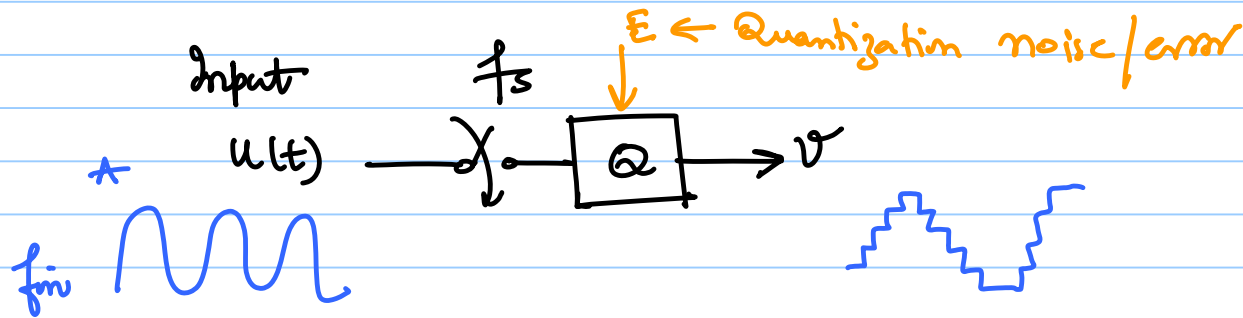


ECE 517- Lecture 26

Note Title

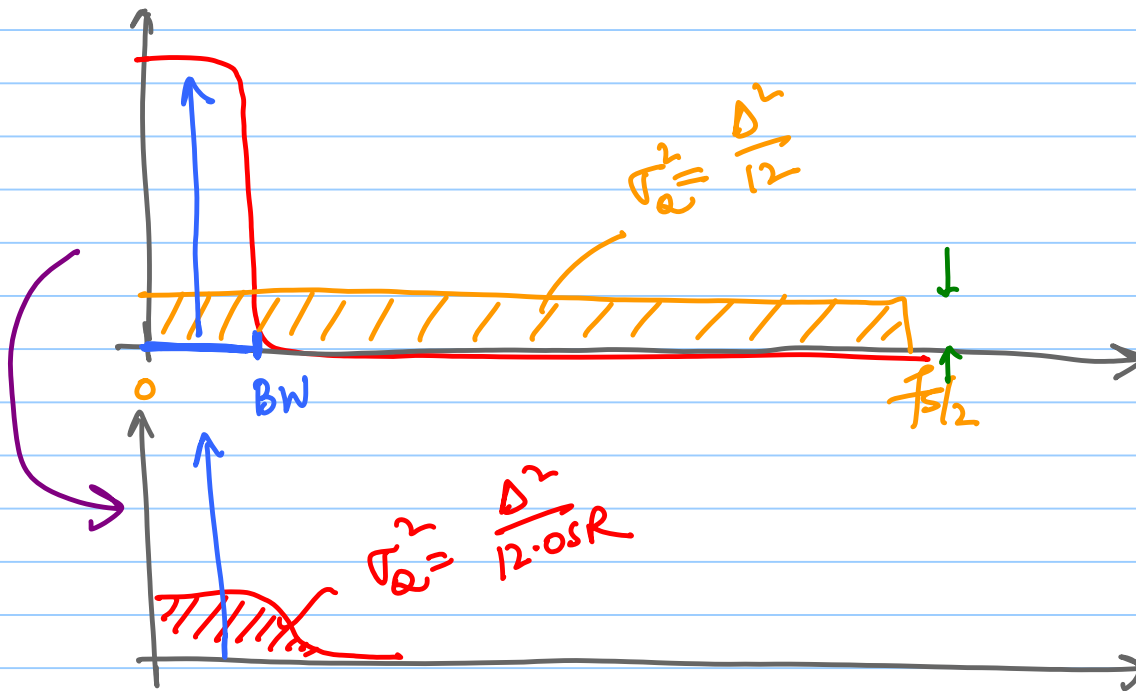
4/20/2017

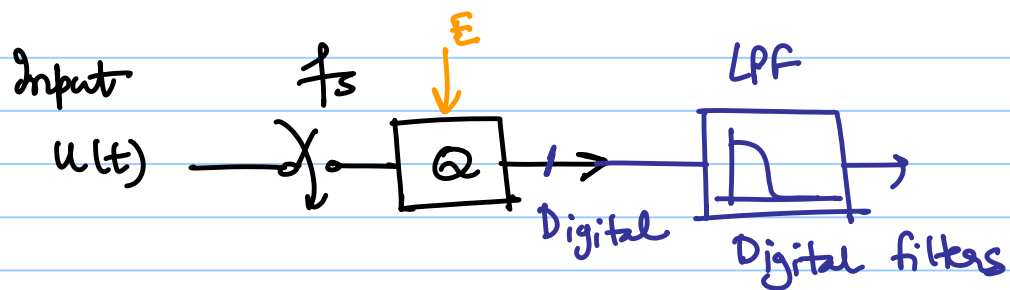


$$SQNR = \frac{P_{\text{signal}}}{P_{\text{noise}}}$$

$$\underline{ENOB} = \frac{SQNR - 1.76}{6.02}$$

Oversampling : $OSR = \frac{f_s/2}{BW}$ ← Nyquist rate of sampling
 ← Bandwidth
 $OSR > 1$





↳ Leverage Digital Signal Processing
 ↳ Oversampling Converters

$$SNR = 10 \log_{10} \left(\frac{\frac{A^2}{2}}{\frac{\Delta^2}{12 \cdot OSR}} \right) = 10 \log_{10} \left(\frac{\frac{(2^{N-1} \Delta)^2}{2}}{\frac{\Delta^2}{12 \cdot OSR}} \right)$$

$$= 6.02N + 1.76 + \underbrace{10 \log_{10}(OSR)}_{\text{Extra term}}$$

$$\text{Increase in ENOB} = \frac{10 \log_{10}(\text{OSR})}{6.02} = \frac{1}{2} \times \log_2(\text{OSR})$$

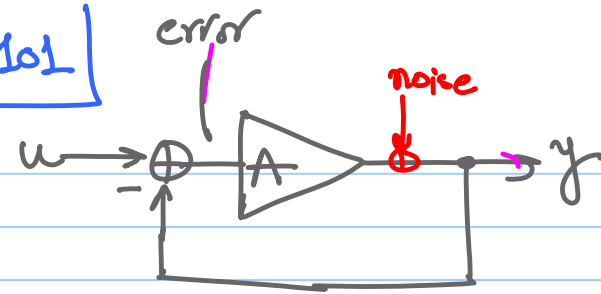
$$\text{OSR} = 2$$

$$\frac{10 \log_{10}(2)}{6} = 0.5$$

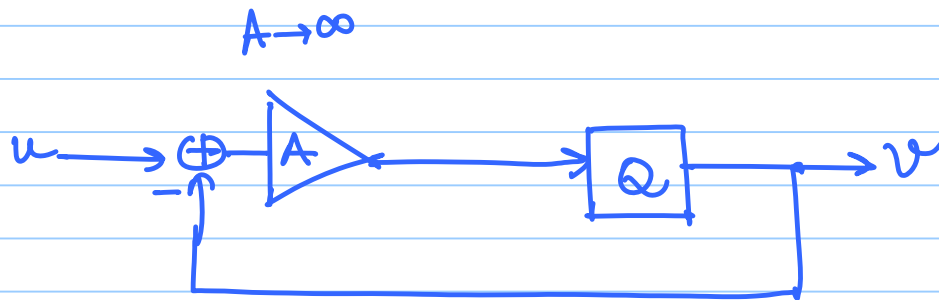
* $\frac{1}{2}$ bit extra resolution per doubling in OSR!

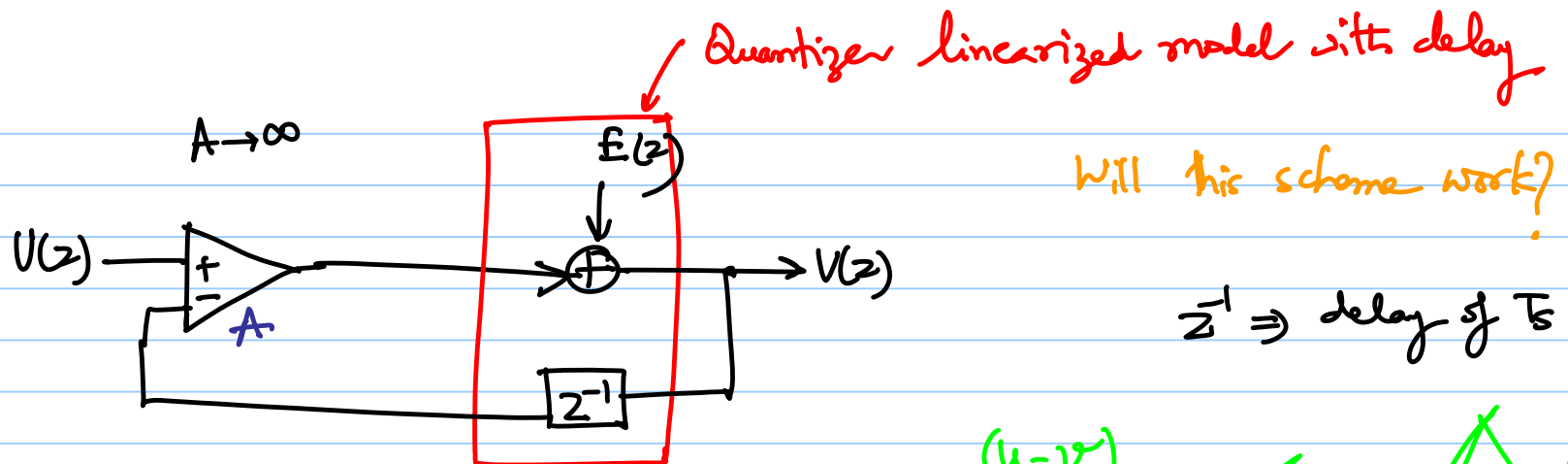
$$\left. \begin{array}{l} N_0 = 3\text{-bit Quantizer} \\ \text{OSR} = 64 \end{array} \right\} N = 3 + \frac{1}{2} \cdot \log_2(64) = 6\text{bits}$$

feedback Control 101



$A \rightarrow \infty$
 $\text{error} \rightarrow 0$
 $y \approx u$
 $y \rightarrow \text{tracks } u$





$$(u(z) - z^{-1}V(z))A + E(z) = V(z)$$

$$\Rightarrow V(z) = \left(\frac{A}{1 + Az^{-1}} \right) U(z) + \underbrace{\left(\frac{1}{1 + Az^{-1}} \right) E(z)}$$

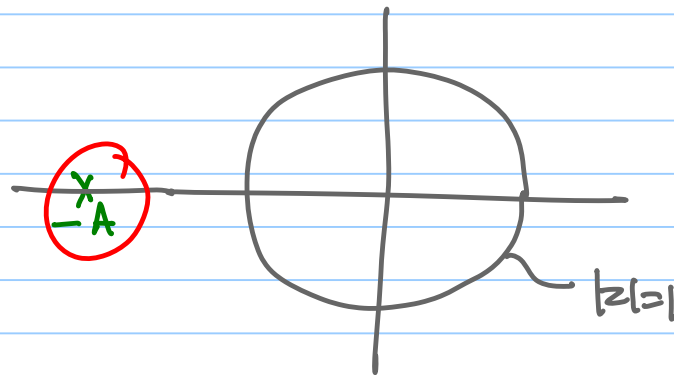
(u-v)

blows up!

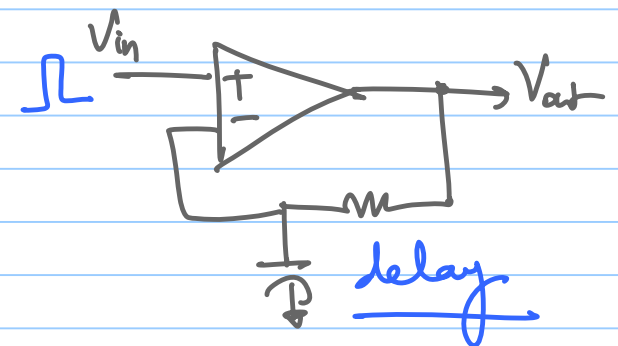
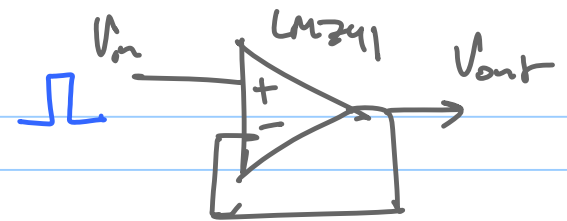
$$\frac{1}{1 + Az^{-1}}$$

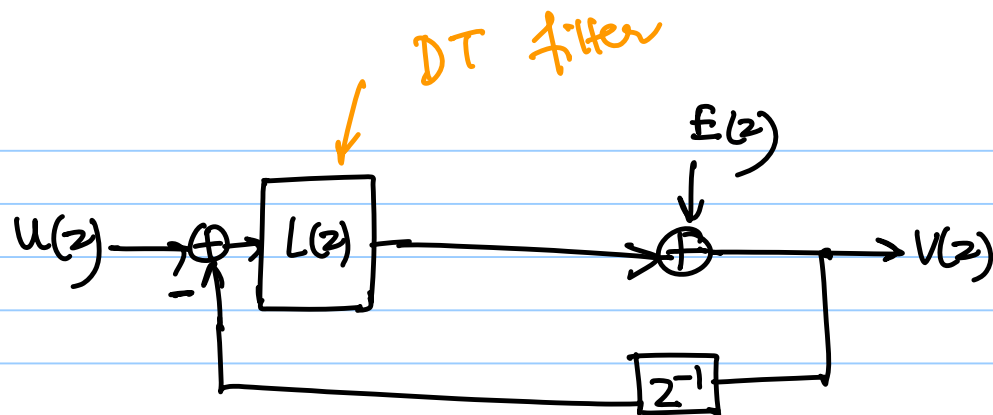
$$\Rightarrow \text{pole!} \quad D^r = 0 \Rightarrow 1 + Az^{-1} = 0$$

$$\Rightarrow z = -A$$



unstable system!





choose

$$L(z) = \frac{1}{1-z^{-1}}$$

$$(U(z) - V(z))L(z) + E(z) = V(z)$$

$$\Rightarrow V(z) = \frac{L(z)}{1 + z^{-1}L(z)} U(z) + \frac{1}{1 + z^{-1}L(z)} E(z)$$

signal transfer function

STF

NTF

noise transfer function



$$e^{j\alpha} - e^{-j\alpha} = 2j \sin \alpha$$

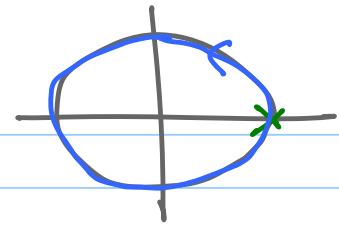
$$|L(e^{j\omega})| = \frac{1}{2|\sin(\omega/2)|}$$

Aside

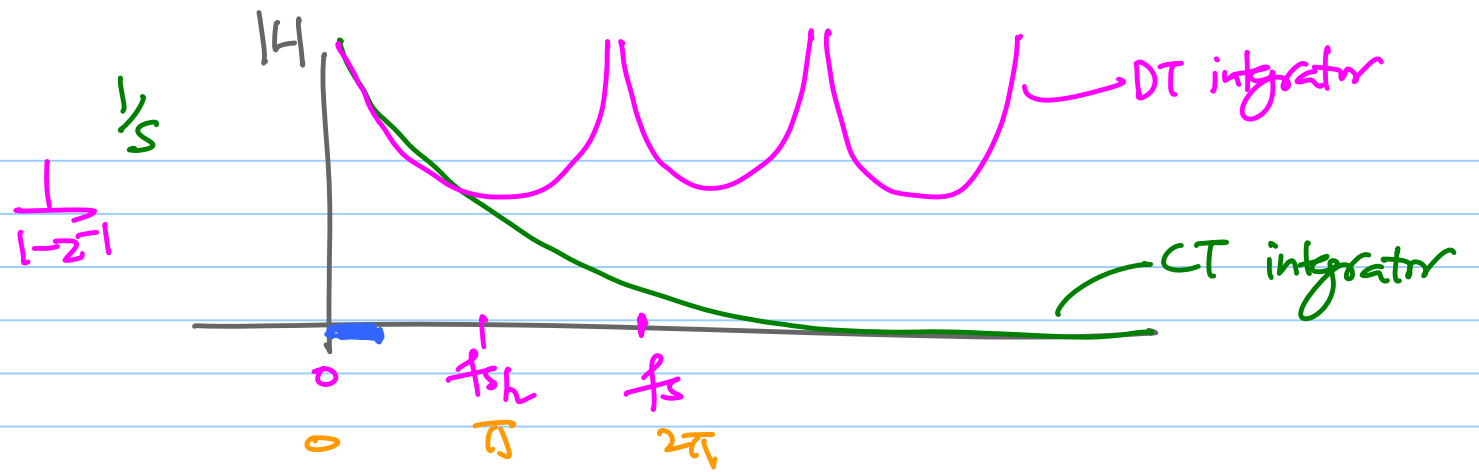
$$L(z) = \frac{1}{1-\bar{z}}$$

$$z \rightarrow e^{j\omega}$$

DTFT



$$\begin{aligned} L(e^{j\omega}) &= \frac{1}{1-e^{-j\omega}} = \frac{1}{e^{j\omega/2} (e^{j\omega/2} - e^{-j\omega/2})} \\ &= \frac{e^{j\omega/2}}{2j \sin(\omega/2)} \end{aligned}$$

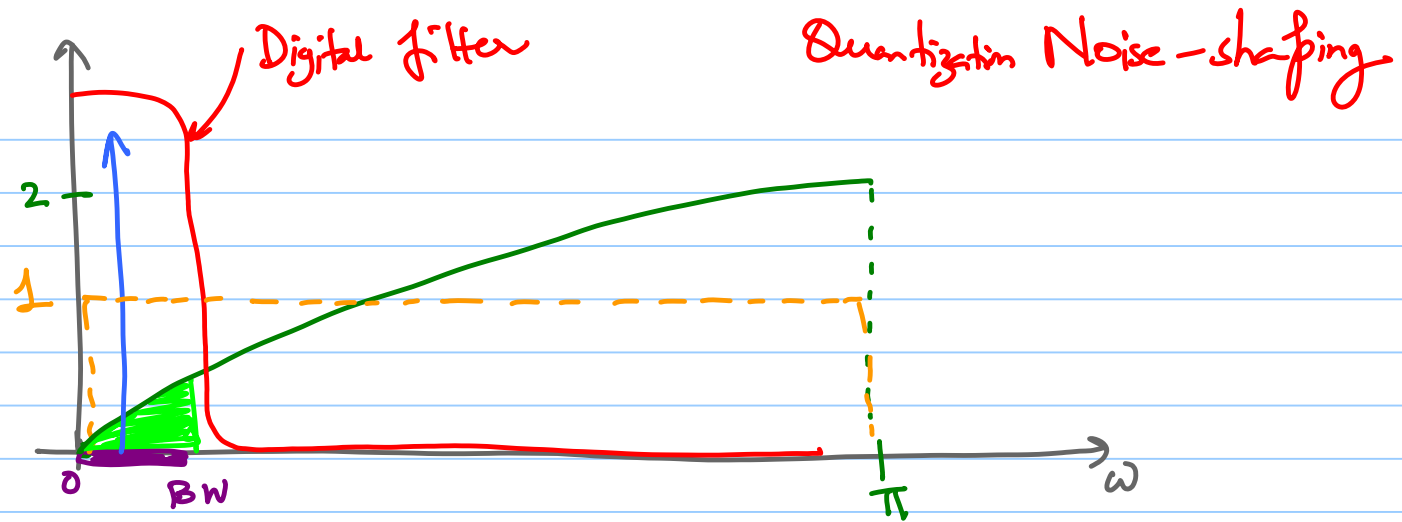


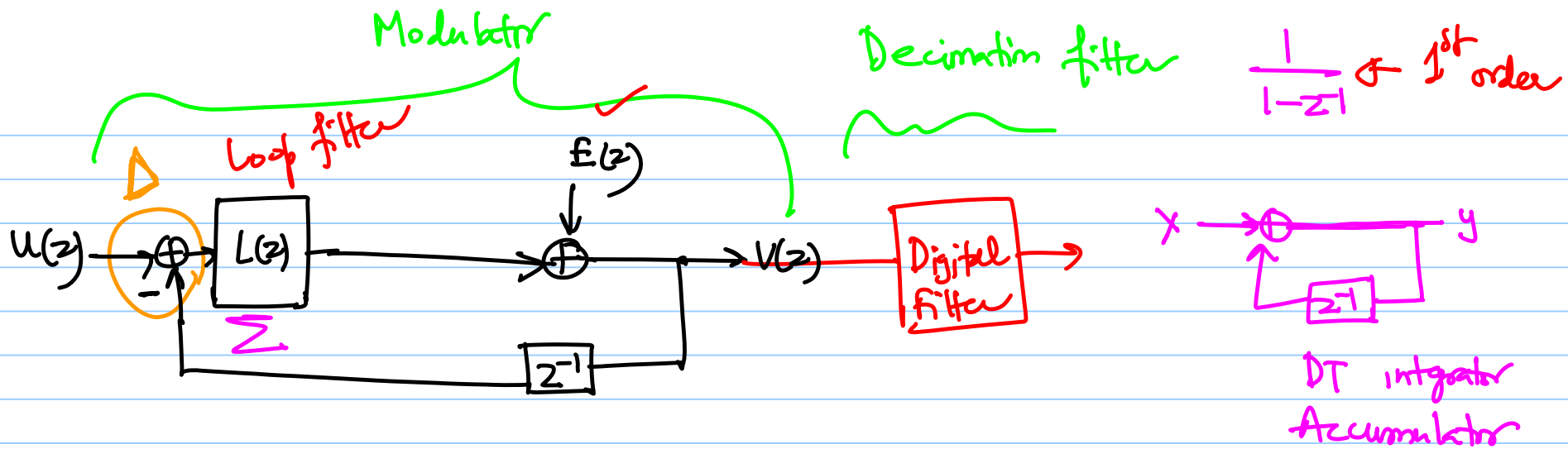
$$NTF(z) = \frac{1}{1 + z^{-1} L(z)} = \frac{1}{1 + \frac{z^{-1}}{1 - z^{-1}}} = \frac{(1 - z^{-1})}{\cancel{1 + z^{-1}} - \cancel{z^{-1}}} = (1 - z^{-1})$$

$$STF(z) = \frac{L(z)}{1 + z^{-1} L(z)} = 1$$

$$NTF = 1 - z^{-1} \Rightarrow NTF(e^{j\omega}) = 1 - e^{-j\omega} = e^{-j\omega/2} [e^{j\omega/2} - e^{-j\omega/2}] = 2j e^{-j\omega/2} \sin(\omega/2)$$

$$|NTF| = 2 |\sin(\omega/2)|$$





$\Delta \Sigma$ ADC
 $\Sigma \Delta$ ADCs

order $\Rightarrow M$
 $(M + \frac{1}{2})$ bits per 2x in OSR