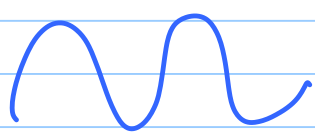
ECE 517 - Lecture 1

Note Title 1/12/2017

Signals Refresher

Fourier Series:



$g(t)$ periodic signal with period $T = 1/f_0$

$$g(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{j2\pi k f_0 t}$$

$$a_k = \frac{1}{T} \int_0^T g(t) \, e^{-j2\pi k f_0 t} \, dt$$

Fourier Transform

$x(t)$

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt \qquad x(t) \xrightarrow{\mathcal{F}} X(f)$$

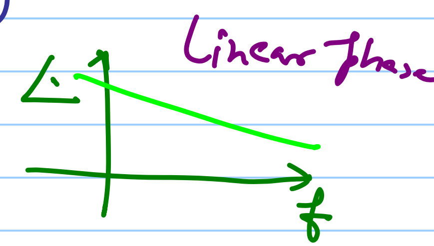
inverse

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df \qquad X(f) \xrightarrow{\mathcal{F}^{-1}} x(t)$$

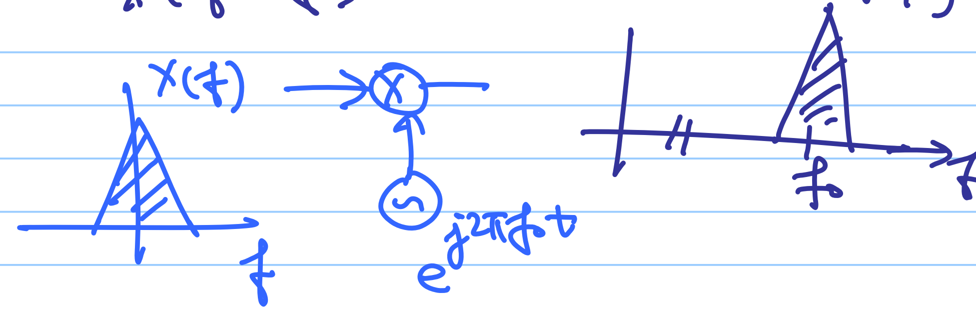
FT properties:

Linearity: $a x(t) + b y(t) \xrightarrow{\mathcal{F}} a X(f) + b Y(f)$

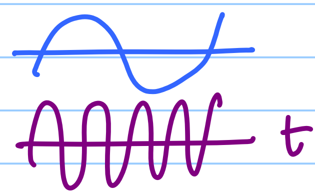
time-delay: $x(t - t_0) \xrightarrow{\mathcal{F}} X(f)\, e^{-(j2\pi f t_0)}$

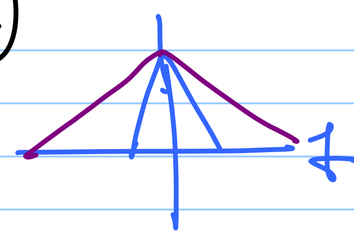


frequency translation: $x(t)\cdot e^{j2\pi f_0 t} \xrightarrow{\mathcal{F}} X(f-f_0)$



Scaling: $x(at) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{|a|} X(f/a)$





Convolution: $x(t) \circledast y(t) \xrightarrow{\mathcal{F}} X(f)\cdot Y(f)$

Multiplication: $x(t) \cdot y(t) \xrightarrow{\mathcal{F}} X(f) \circledast Y(f)$

Duality: $x(t) \xrightarrow{\mathcal{F}} X(f)$

$X(t) \xrightarrow{\mathcal{F}} x(-f)$ } Duality

Parseval's Theorem:

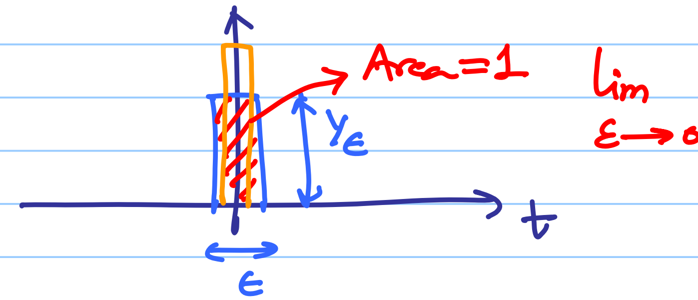
$$\text{Energy} \Rightarrow \int_{-\infty}^{\infty} \underbrace{x(t)\, x^*(t)}_{\text{Power}}\, dt = \int_{-\infty}^{\infty} \underbrace{X(f)\, X^*(f)}_{\text{Power}}\, df$$

Complex Conjugate

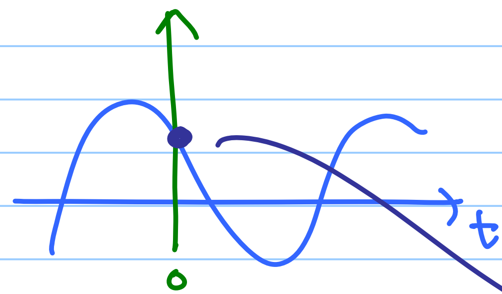
Delta function (?)

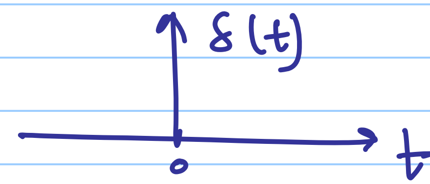
$\delta(t)$

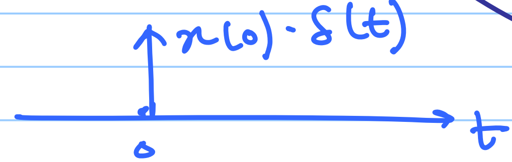
$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$



$x(t) \cdot \delta(t)$



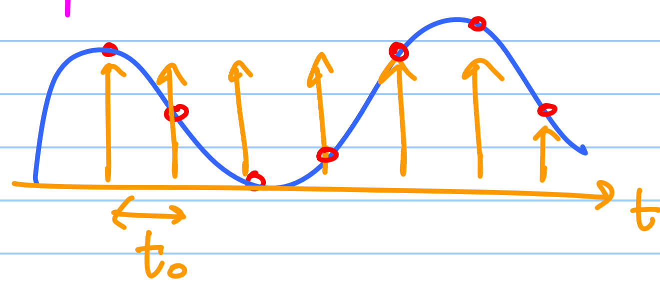




picks the value of the signal at $t=0$

$$x(t) \cdot \delta(t - t_0) = x(t_0) \cdot \delta(t - t_0)$$

$$\underbrace{x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - n t_0)}_{\text{impulse train}} = \sum_{n=-\infty}^{\infty} \underbrace{x(n t_0)}_{\text{Sample}} \cdot \delta(t - n t_0)$$

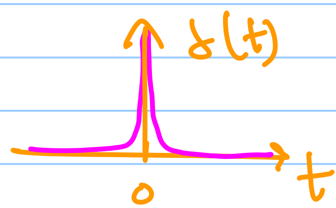


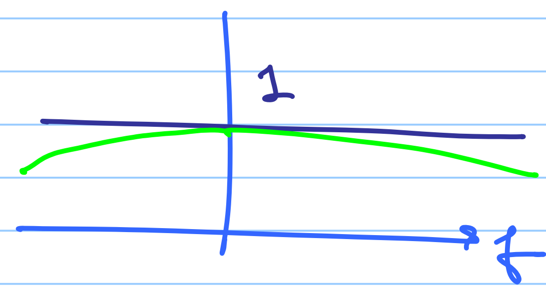
$$x(t) \circledast \delta(t) = x(t)$$

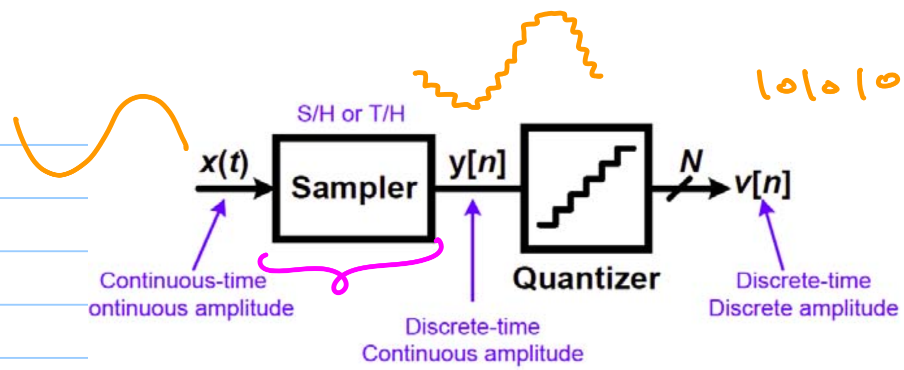
$$\int_{\tau=-\infty}^{\infty} x(\tau)\,\delta(t-\tau) = x(t)$$

$$x(t) \circledast \delta(t-t_0) = x(t-t_0)$$

$$\delta(t) \xrightarrow{\mathcal{F}} 1$$







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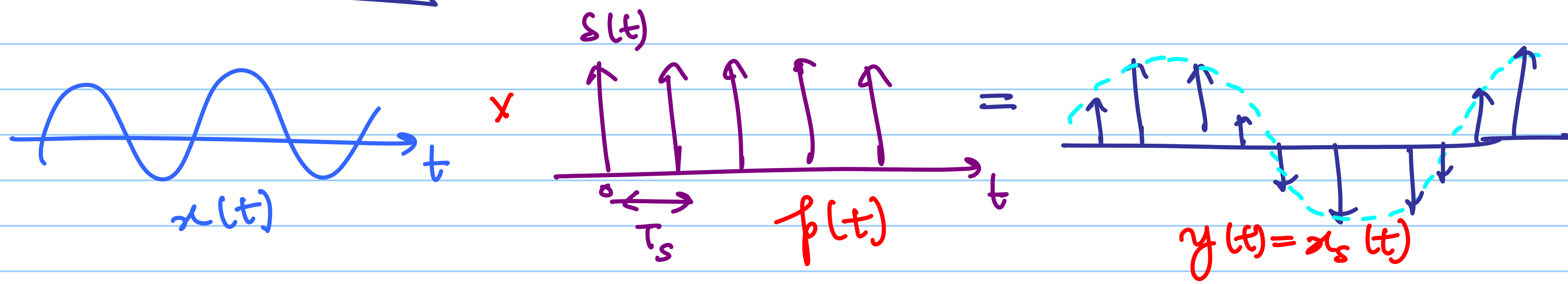
f_s = sample rate = $\frac{1}{T_s}$

$T_s \Rightarrow$ sampling period

Ideal Sampling:



$$y(t) = x(t) \cdot \overbrace{\underbrace{\sum_{n=-\infty}^{\infty} \delta(t - nT_s)}_{p(t)}}^{\text{impulse train}}$$

$$= \underline{x(t)} \cdot p(t)$$

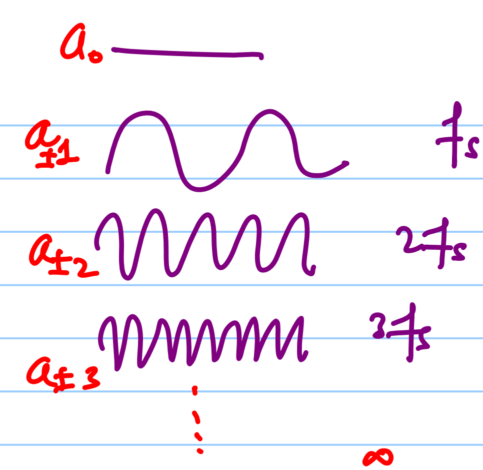
$$\Rightarrow \quad Y(f) = \underline{X(f)} \circledast \underbrace{P(f)}_{??!}$$

$P(f) = ??$

Impulse Train

$p(t) \rightarrow$ periodic function

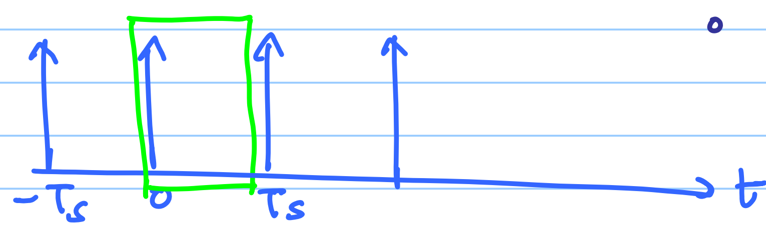
↳ express it as fourier series



Fourier Series representation

$$p(t) = \sum_{k=-\infty}^{\infty} \boxed{a_k}\, e^{j2\pi k f_s t} \longrightarrow (1)$$

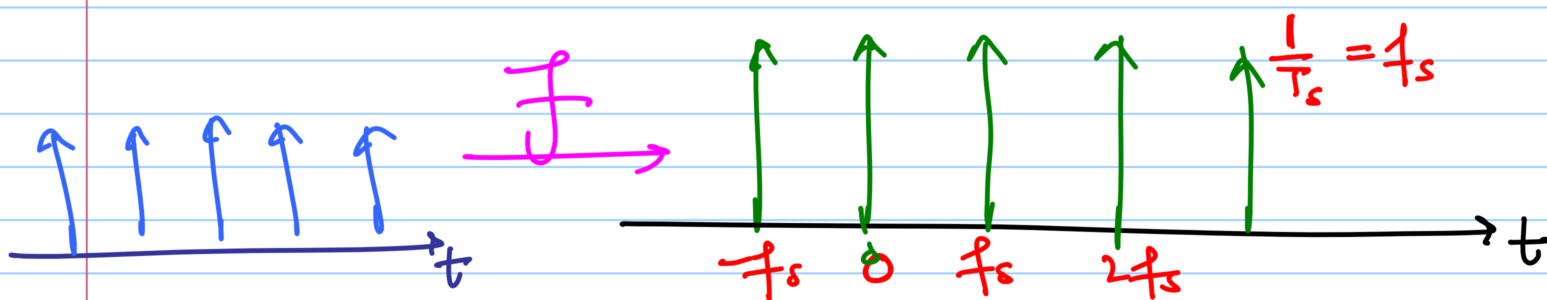
$$a_k = \frac{1}{T_s}\int_{0}^{T_s^-} p(t)\, e^{-j2\pi k f_s t} = \frac{1}{T_s}\int_{0}^{T_s^-} \delta(t)\, e^{-j2\pi k f_s t} = \frac{1}{T_s}\int_{0}^{T_s^-} \delta(t) = \frac{1}{T_s}$$



$$p(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T_s} e^{j2\pi k f_s t} \qquad e^{j\omega_s t}$$

$$\Big\downarrow \mathcal{F}$$

$$P(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta(f - k f_s)$$



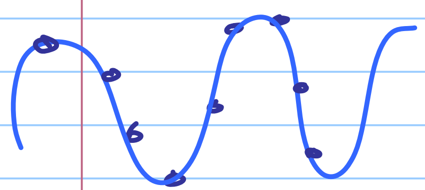
$$x(t)\cdot p(t) \xrightarrow{\mathcal{F}} X(f) \circledast P(f)$$

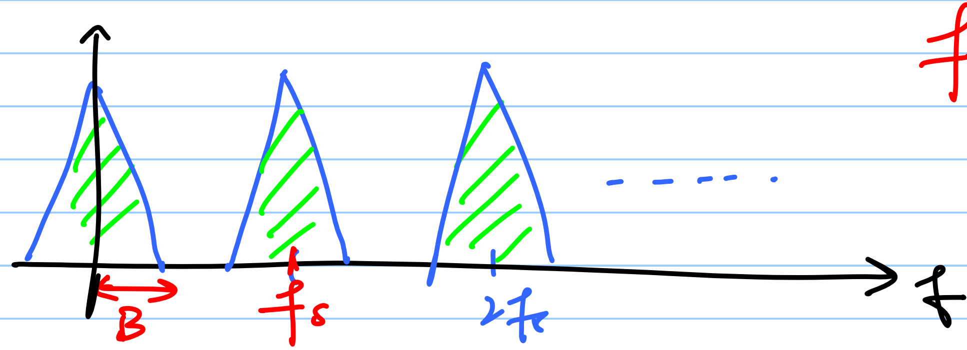
$$= X(f) \circledast \frac{1}{T_s}\sum_{k=-\infty}^{\infty} \delta(f-kf_s)$$

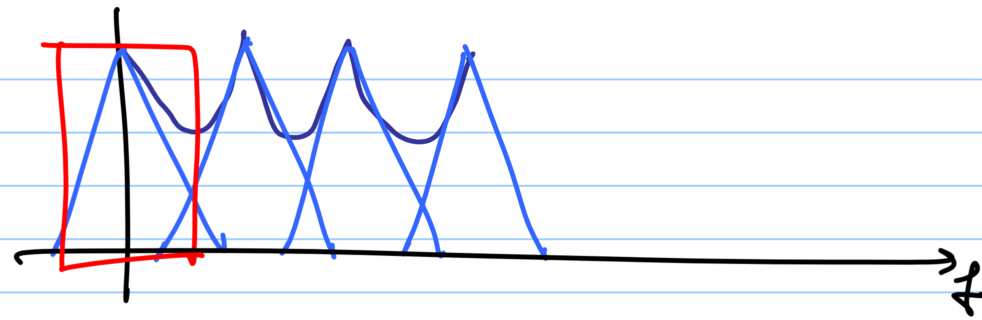
$$= \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(f-kf_s)$$

Nyquist Theorem

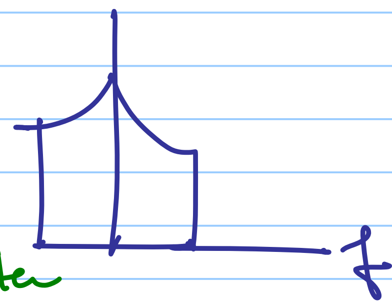
$$f_s \geq 2B$$







"Aliasing"



anti-alias filter