Pipelined ADC Design
- A Tutorial -

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Outline

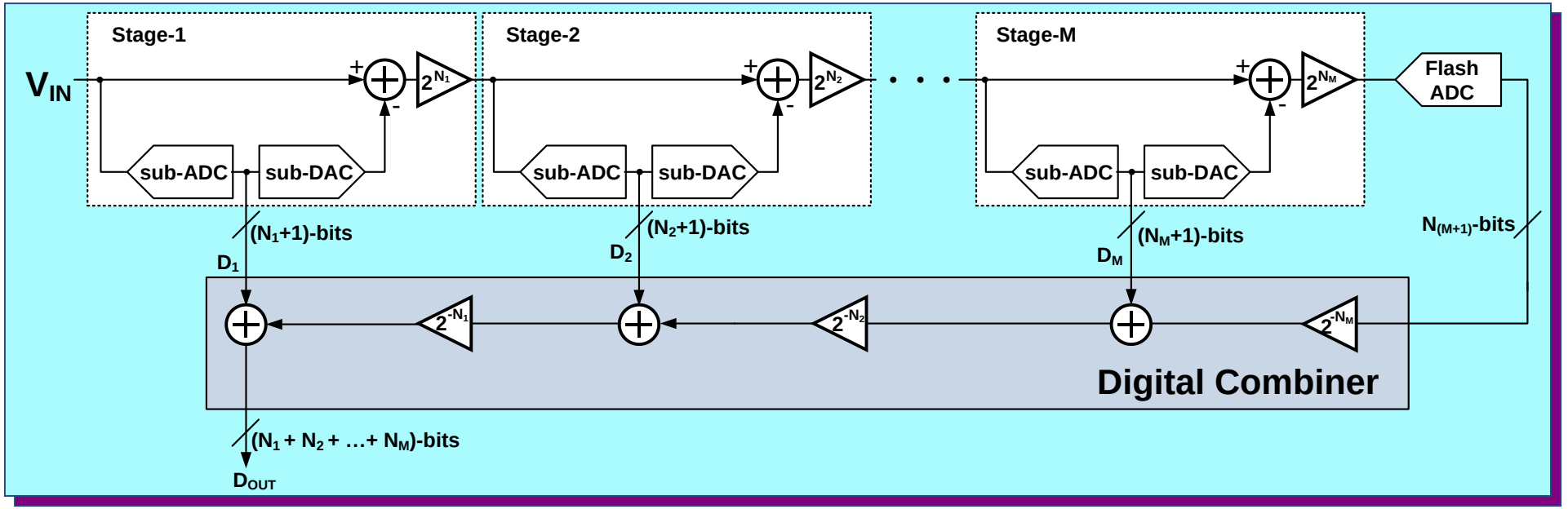
- Review of Pipelined ADCs
- Impact of Scaling on Data Converter Design
- Why Calibration?
- Basics of Digital Calibration Techniques
- Survey of Digital Calibration Techniques
- Conclusion

Introduction

- **Data Converter Design is challenging in Nanoelectronics Era:**
  - **Low intrinsic device gain**
  - **High nonlinearity**
  - **Reduced headroom (reduced dynamic range)**
  - **Large variability and mismatches**
  - **Survival in digital-driven system-on-chip (SOC) environment**
- **Trends in data converter design**
  - **Digitally Assisted Analog Design**
  - **Fuelled by aggressive device scaling**

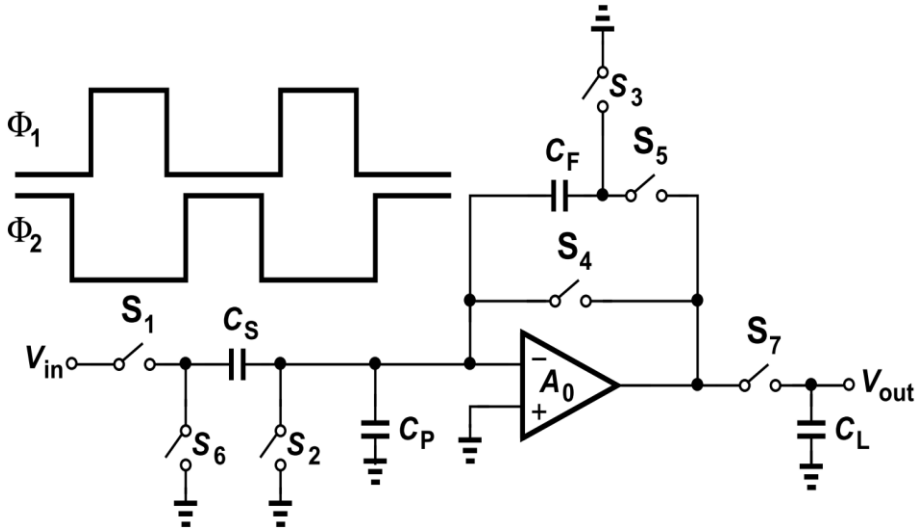
Pipeline ADC
- Review -

Generic Pipelined ADC



- Each stage resolves a small number of bits (i.e. N_1, N_2, …, N_M bits).
- The overall resolution of the ADC is $P = (N_1+N_2+\ldots+N_M+N_{M+1})$.
- Output of stage-i (called "residue" r_i) is digitized to $(P-\Sigma^{i}_{j=1} N_j)$-bits.
- The low resolution ADC digitizing r_i is called the backend of stage-i.

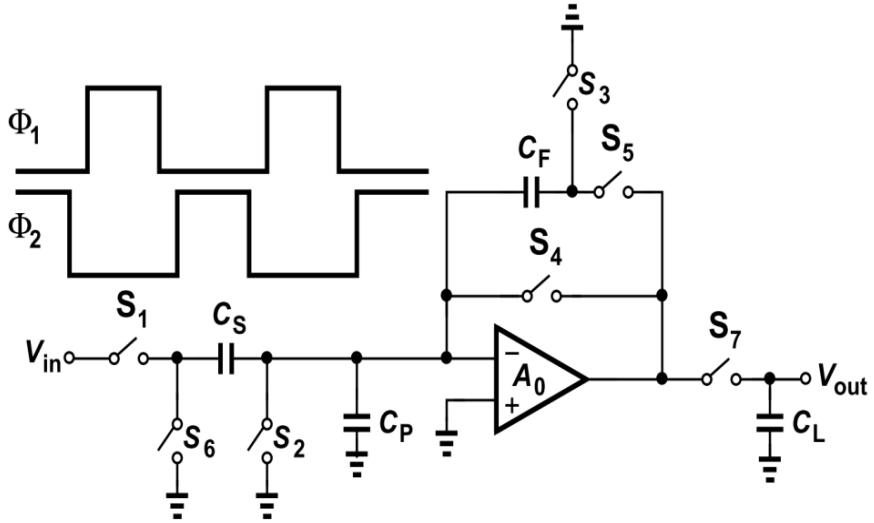
Switched-Capacitor (SC) Circuit – An Overview (1)

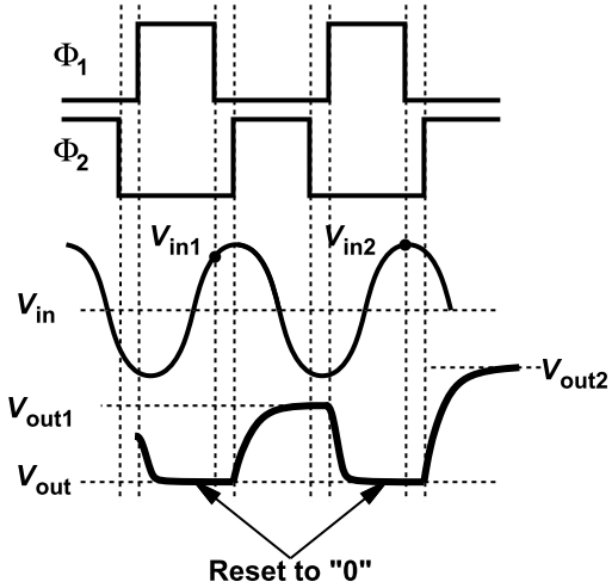


- **Key building blocks:**
  - **Switches**
  - **Capacitors**
  - **Op amp**
  - **Two-phase non-overlapping clock generator**

- **Sample/Reset phase** ➔ Φ_1 is high ➔ Switches S_1 to S_4 are controlled by Φ_1, i.e. S_1 to S_4 close when Φ_1 is high.
  - It is called sample-phase as input signal is sampled.
  - It is also called reset-phase as opamp is reset (more later).
- **Hold/Amplification phase**➔ Φ_2 is high ➔ Switches S_5 to S_7 are controlled by Φ_2, i.e. S_5 to S_7 close when Φ_2 is high.
  - V_{out} is an amplified and held version of the sampled V_{in}.

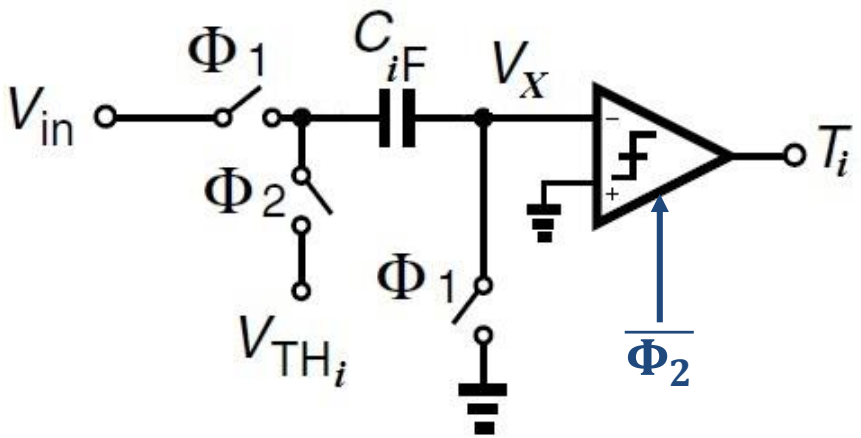
Switched-Capacitor (SC) Circuit – An Overview (2)

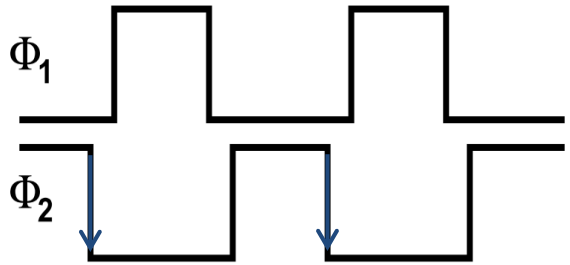




- Falling edge of Φ_1 samples the input, V_{in1} and V_{in2}.
- In Φ_2 the sampled values get amplified to give V_{out1} & V_{out2}, respectively.
- During Φ_1 the sampled charge is $Q_S = C_S V_{in}$.
- During Φ_2 the charge is $Q_H = C_F V_{out}$.
- Since charge is conserved, $Q_S = Q_H \Rightarrow V_{out} = \frac{C_S}{C_F} V_{in}$.
- The gain of the circuit is the ratio of the sampling capacitor (C_S) to feedback capacitor (C_F).

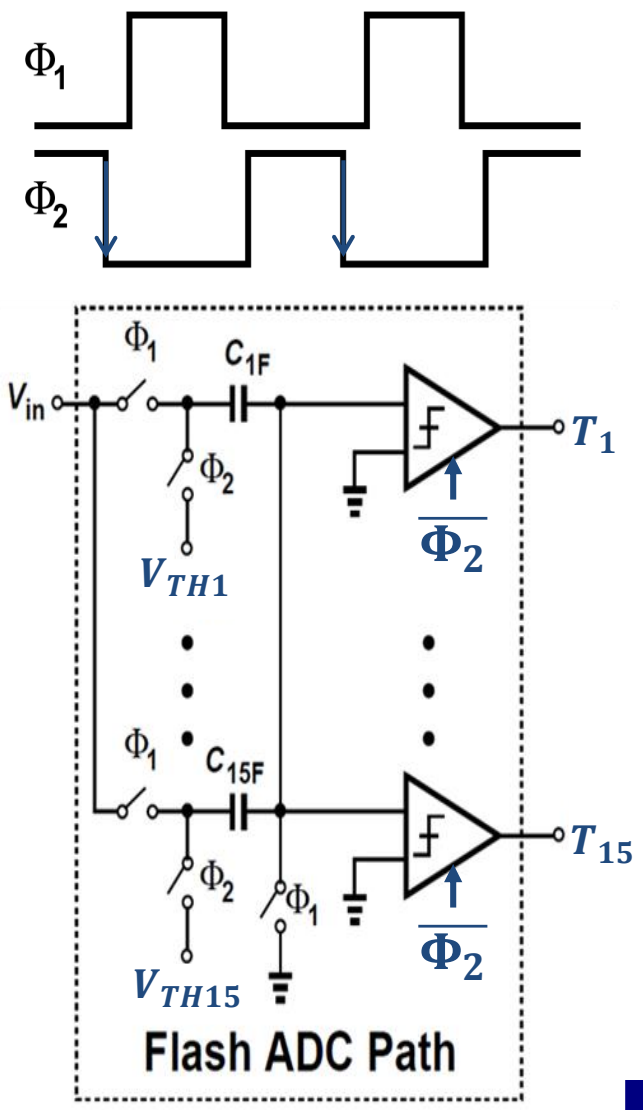
Switched-Capacitor Comparator





- During Φ_1 the sampled charge $Q_S = C_{iF}V_{in}$.
- Since charge is conserved the charge during Φ_2 is given by $Q_H = C_{iF}(V_{THi} - V_X)$ resulting in $V_X = V_{in} - V_{THi}$.
- When the comparator is clocked with the falling edge of Φ_2 it makes a decision based on whether $V_{in} > V_{THi}$ or $V_{in} < V_{THi}$.

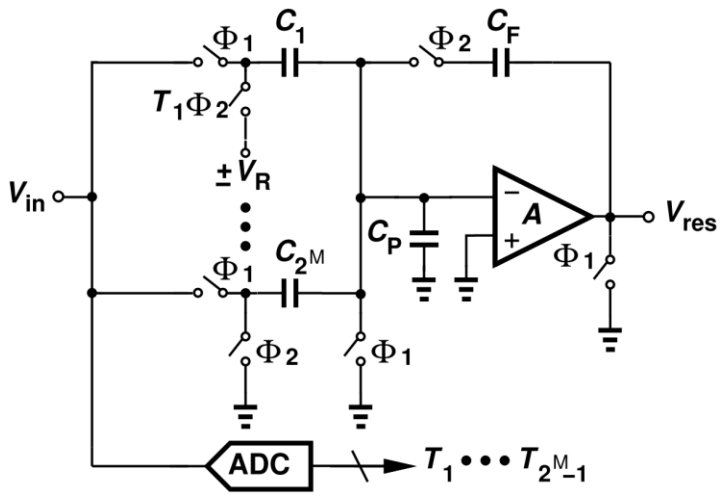
Switched-Capacitor 4-bit Flash

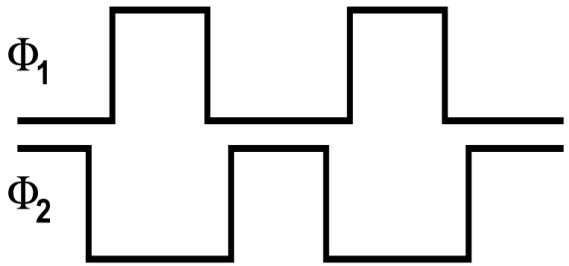


- A 4-bit flash incorporates 15 switched capacitor comparators.
- The threshold voltages V_{TH1} to V_{TH15} are generated by a resistor ladder comprising of 16 equal resistors.
- The outputs of the comparator give a 15-bit wide thermometer code which controls the DAC of the MDAC.
- The thermometer code is converted to 4-bit binary code using an on-chip look-up table, also called read-only-memory (ROM).

Note: A switched capacitor N-bit flash would incorporate 2^{N-1} switched capacitor comparators and a resistor ladder comprising 2^N resistors to generate the 2^{N-1} threshold voltages, V_{TH1} to $V_{TH2^{N-1}}$.

Switched-Capacitor MDAC (1)

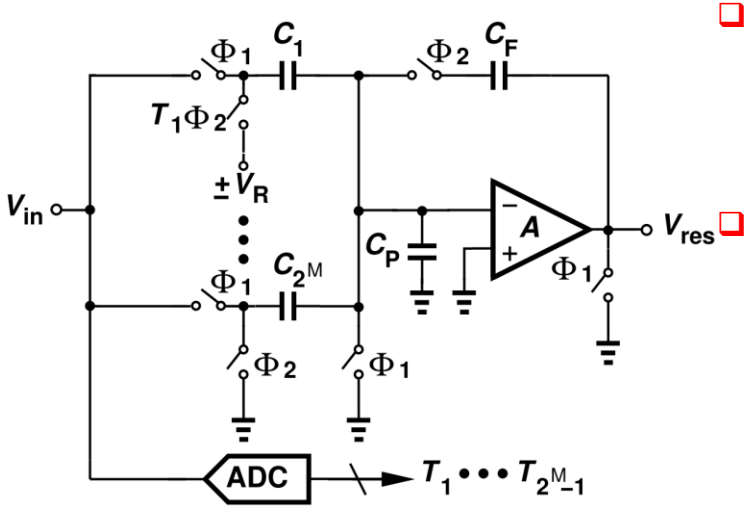


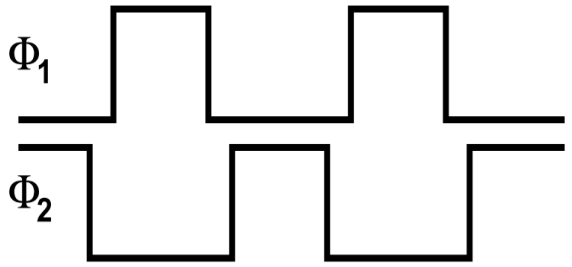


- An M-bit MDAC incorporates 2^M unit capacitors (C_i , i=1 to 2^M), feedback capacitor C_F, switches, and an op amp.
- This block does the following operations:
  - Φ_1 is high (Sample phase/Reset phase):
    - Sampling of input
  - Φ_2 is high (Amplification phase):
    - Digital-to-Analog Conversion **(DAC)**
    - Subtraction ➔ quantization noise generation
    - Amplification (**M**ultiplying) ➔ scaling of the quantization noise to the full-scale for the later stages to digitize to relax the sensitivity requirements of the later stage circuits.

Note: A switched capacitor 4-bit flash would incorporate 2^4 unit capacitors.

Switched-Capacitor MDAC (2)





- During sample phase the sampled charge is,

$$Q_s = \sum_{i=1}^{2^M} C_i V_{in}$$

- During amplification phase the charge is given by,

$$Q_a = \sum_{i=1}^{2^M-1} T_i C_i V_R + C_F V_{res}$$

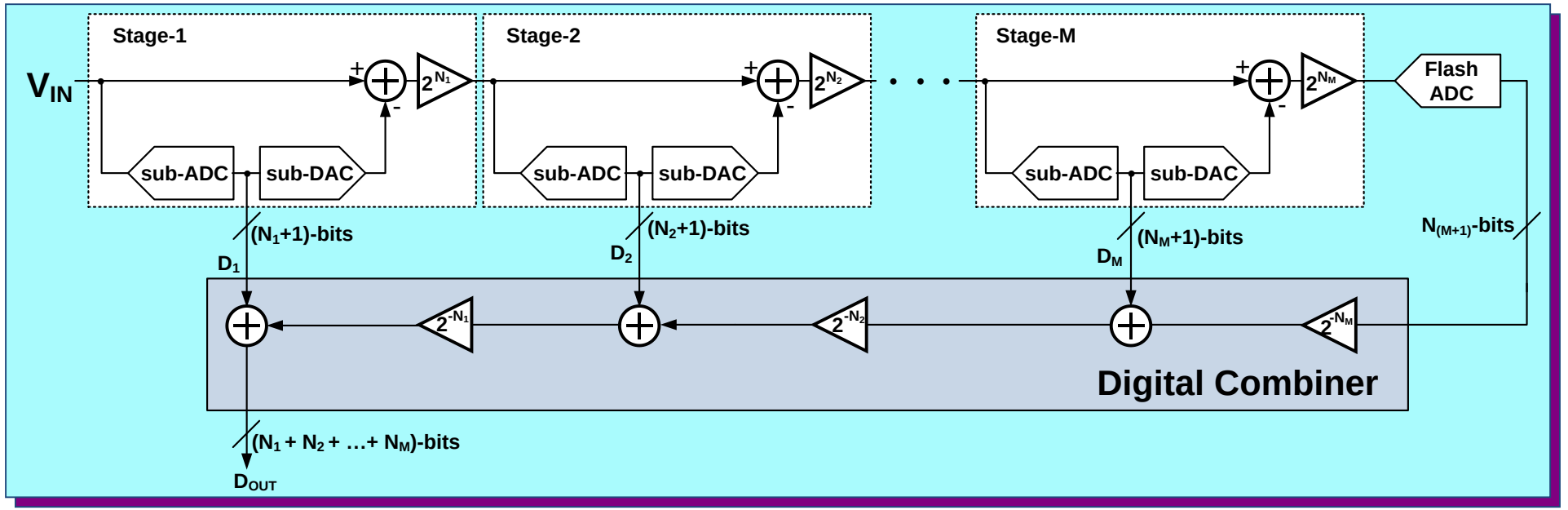
- By conservation of charge we get,

$$V_{res} = \frac{\sum_{i=1}^{2^M} C_i V_{in} - \sum_{i=1}^{2^M-1} B_i C_i V_R}{C_F}$$

where, B_i=1 if T_i=1 and B_i=–1 if T_i=0.

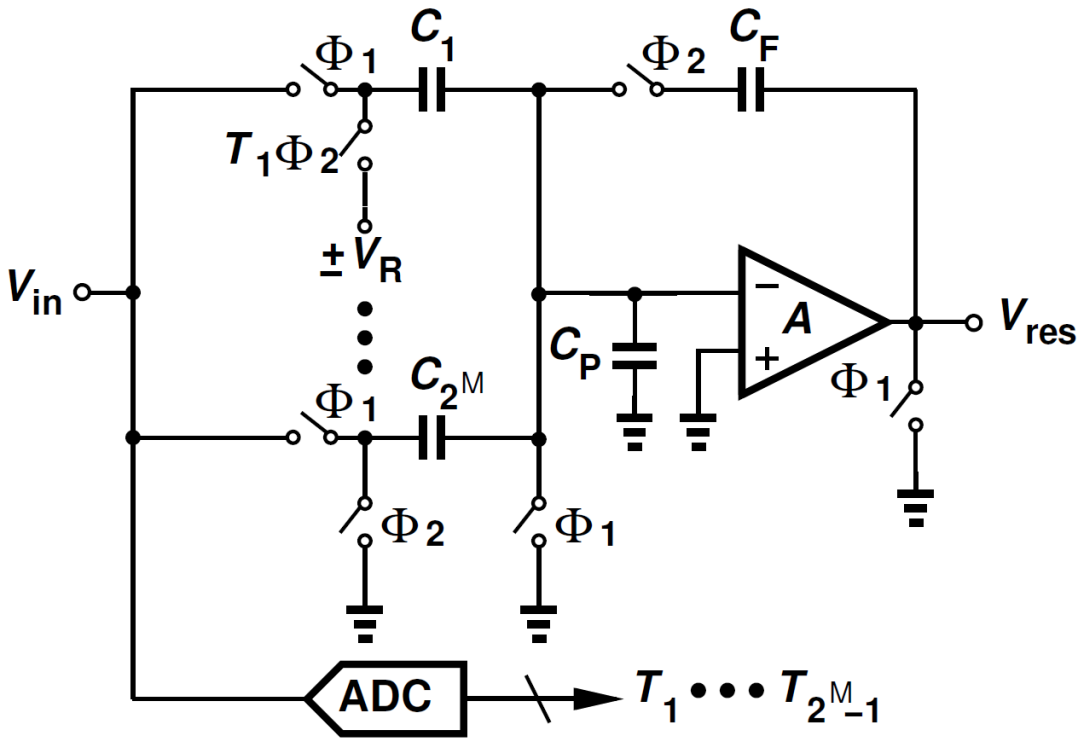
- If $C_F = C_i$ then gain $\approx 2^M$, if $C_F = 2C_i$ then gain $\approx 2^{M-1}$.

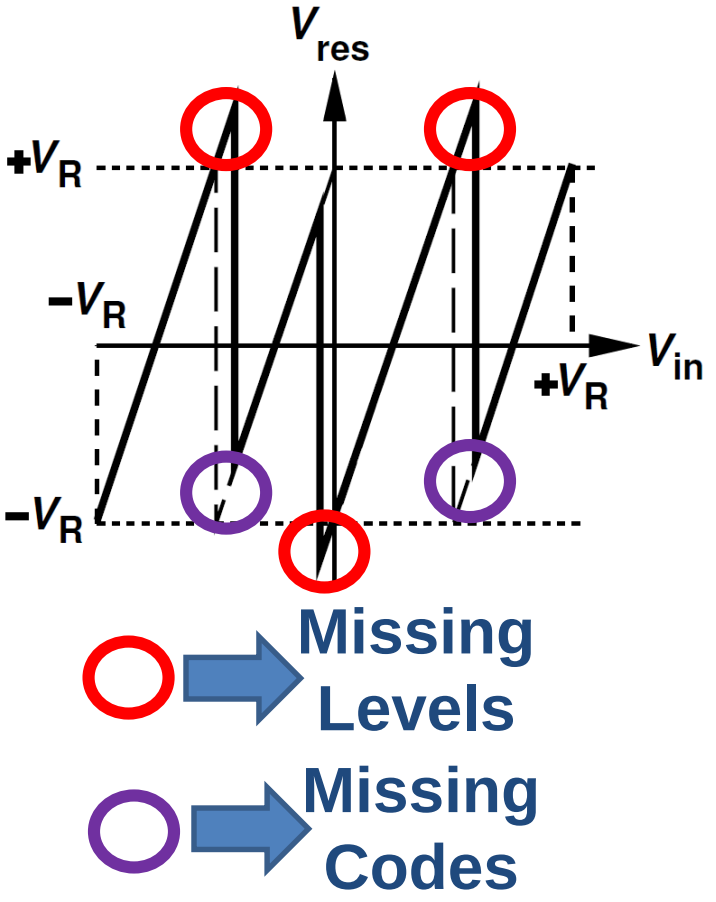
Sources of Errors in Pipelined ADC



▪ Comparator Offset	▪ Op Amp Nonlinearity
▪ Capacitor Mismatch	▪ Op Amp Offset
▪ Op Amp Gain	▪ Charge Injection Mismatch
▪ Op Amp Input Capacitance	▪ Op Amp and kT/C Noise

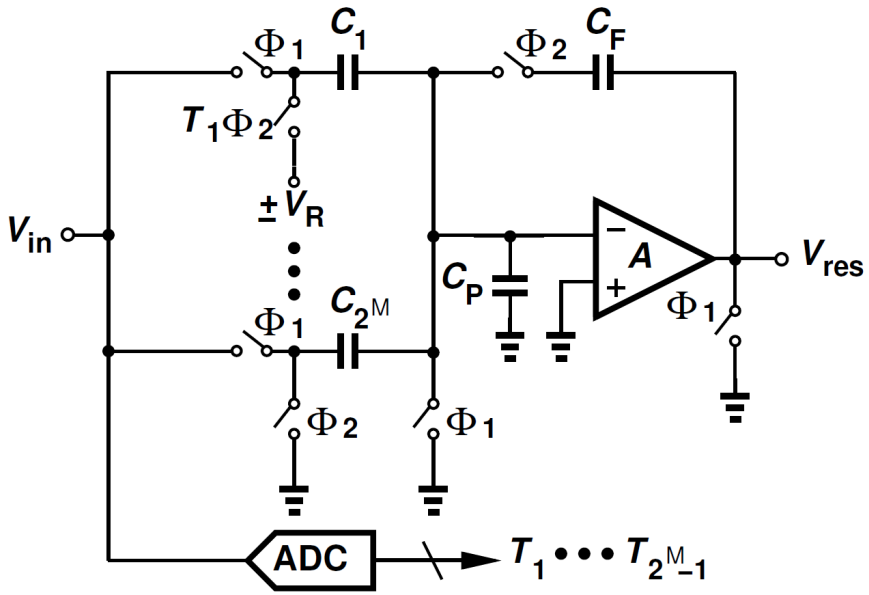
Comparator Offset





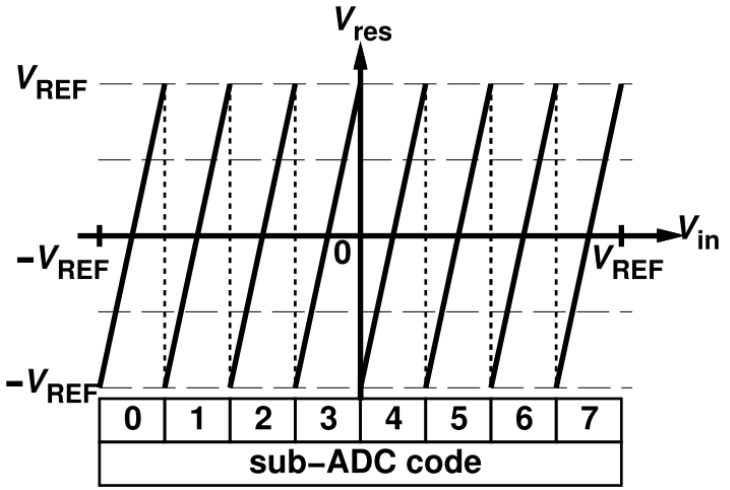
- **Comparator offset saturates the later stage.**
- **Redundancy can overcome this.**

Overcoming Comparator Offset (1)

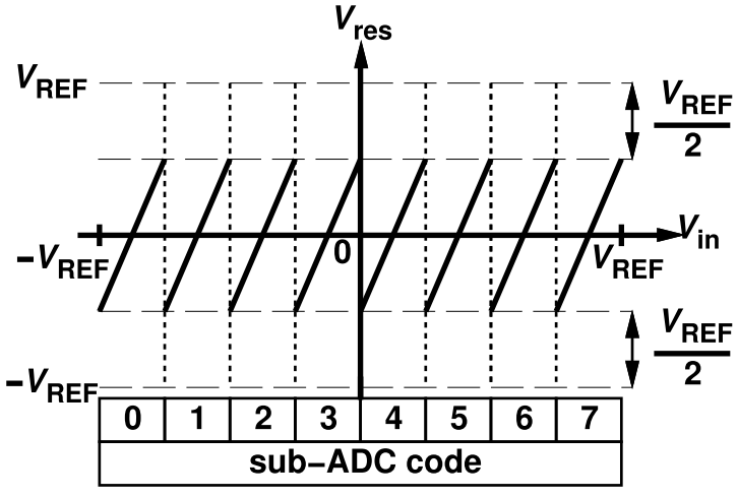


$$V_{res} = \frac{\sum_{i=1}^{2^M} C_i V_{in} - \sum_{i=1}^{2^M-1} B_i C_i V_R}{C_F}$$

If $C_F = 2C_i$ then gain $\approx 2^{M-1}$ resulting in comparator offset tolerance of $\pm V_{REF}/2^M$.

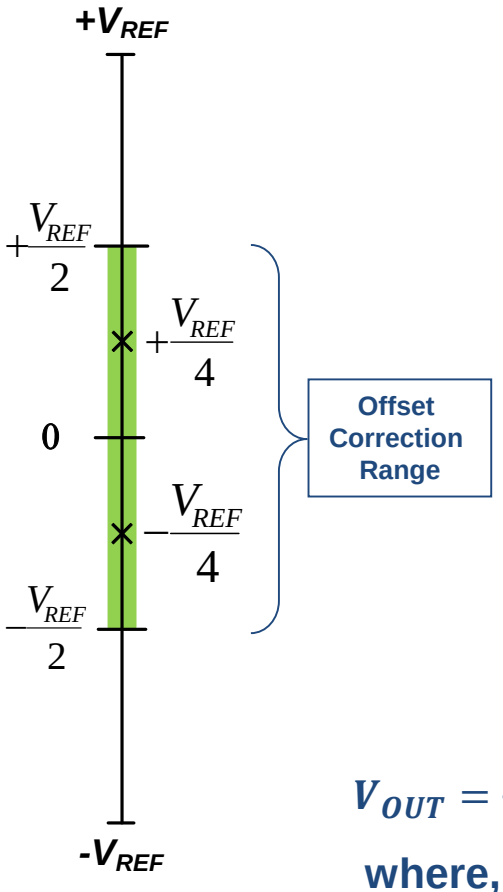


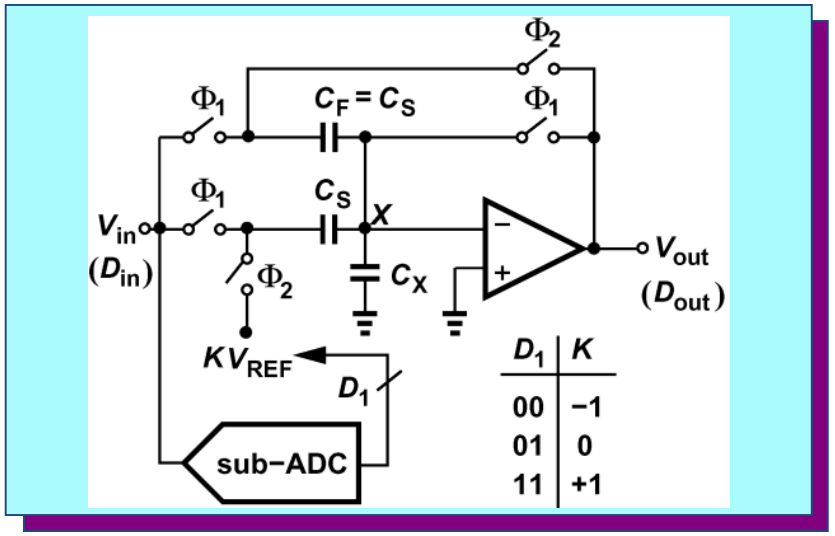
$C_F = C_i$, M=3 ➔ No redundancy



$C_F = 2C_i$, M=3➔ Redundancy

Overcoming Comparator Offset–1.5-bit Architecture–

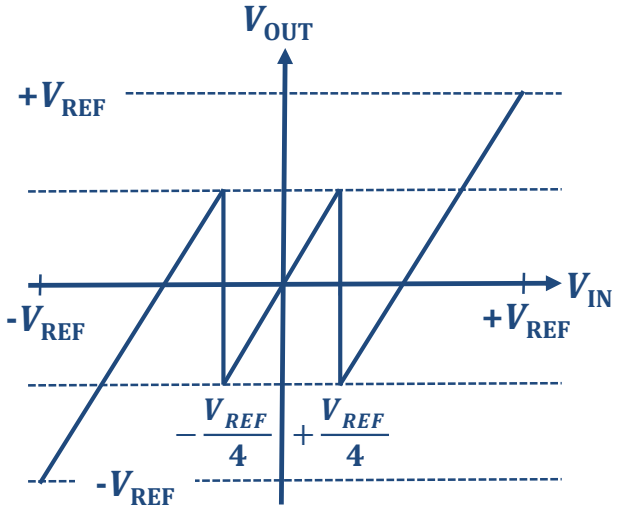




Flip-around Topology

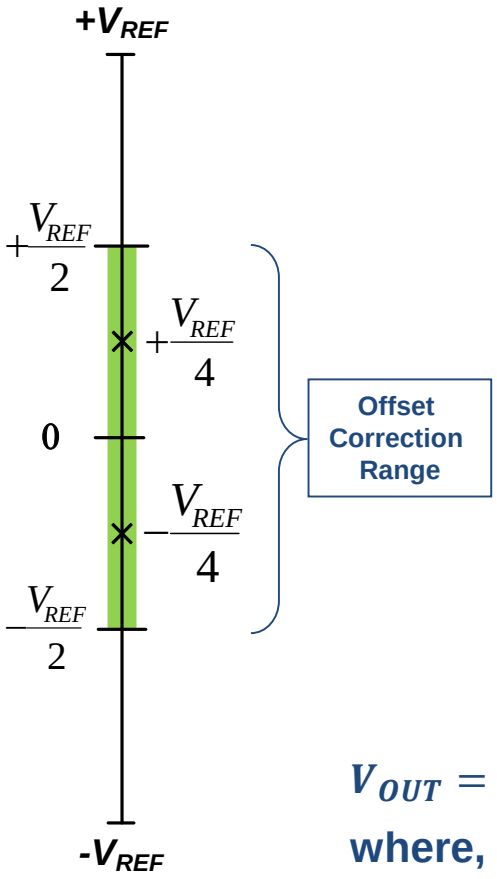
$$V_{OUT} = \frac{(C_S + C_F)V_{IN} - kC_SV_{REF}}{C_F}$$

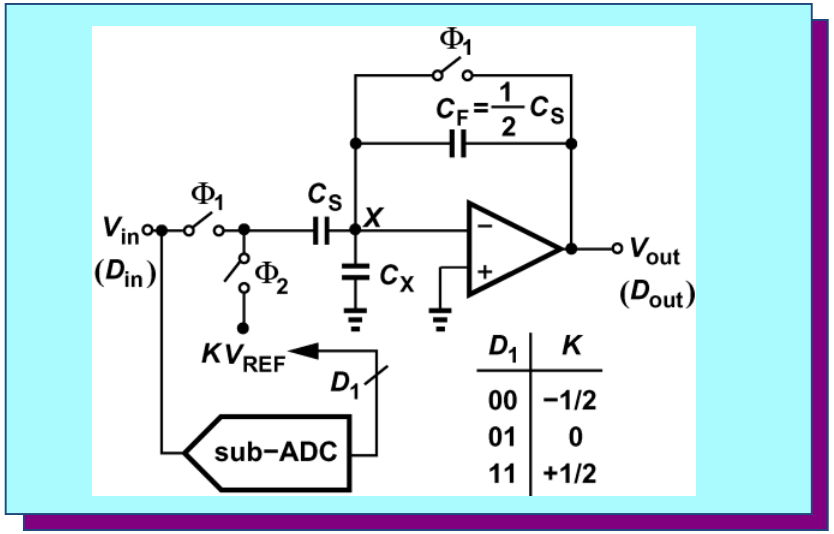
where, $k = \pm 1, 0$



Ref: S. Lewis et al, "A 10-b 20-MS/s Analog-to-Digital Converter," *IEEE JSSC*., pp. 351-358, March 1992

Overcoming Comparator Offset–1.5-bit Architecture–

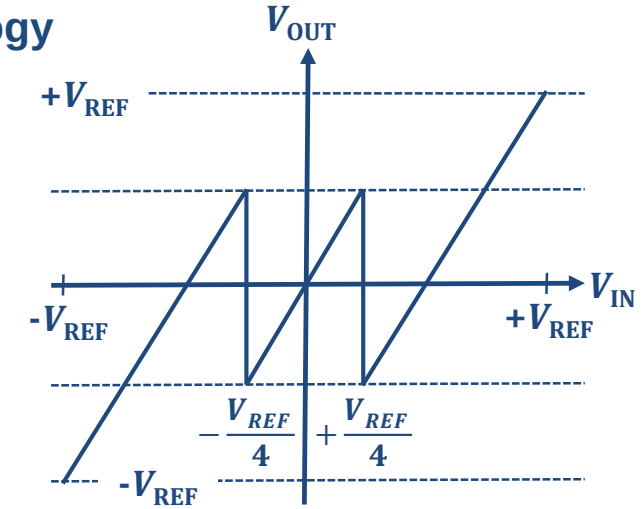




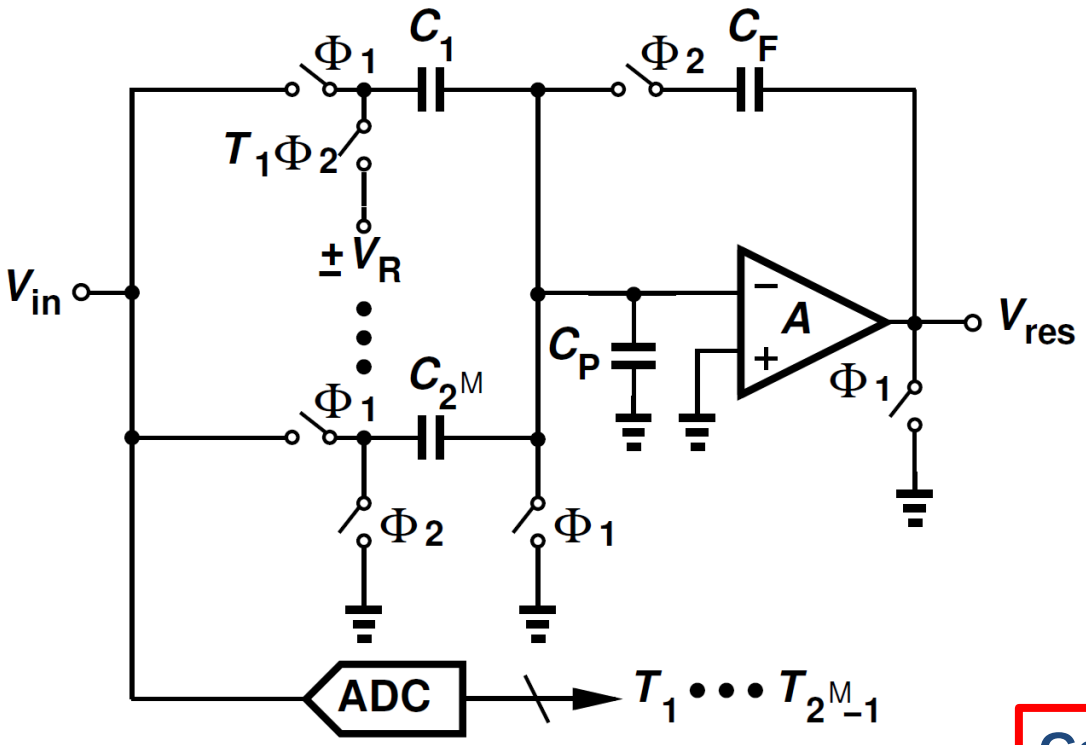
Non-Flip-around Topology

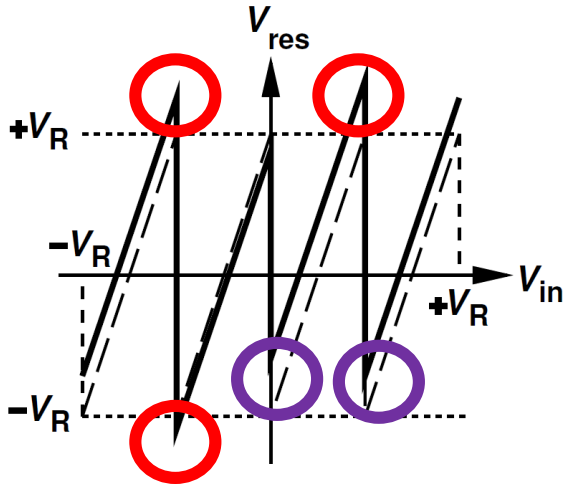
$$V_{OUT} = \frac{C_S}{C_F}(V_{in} - kV_{REF})$$

where, $k = \pm 1/2, 0$



Capacitor Mismatch





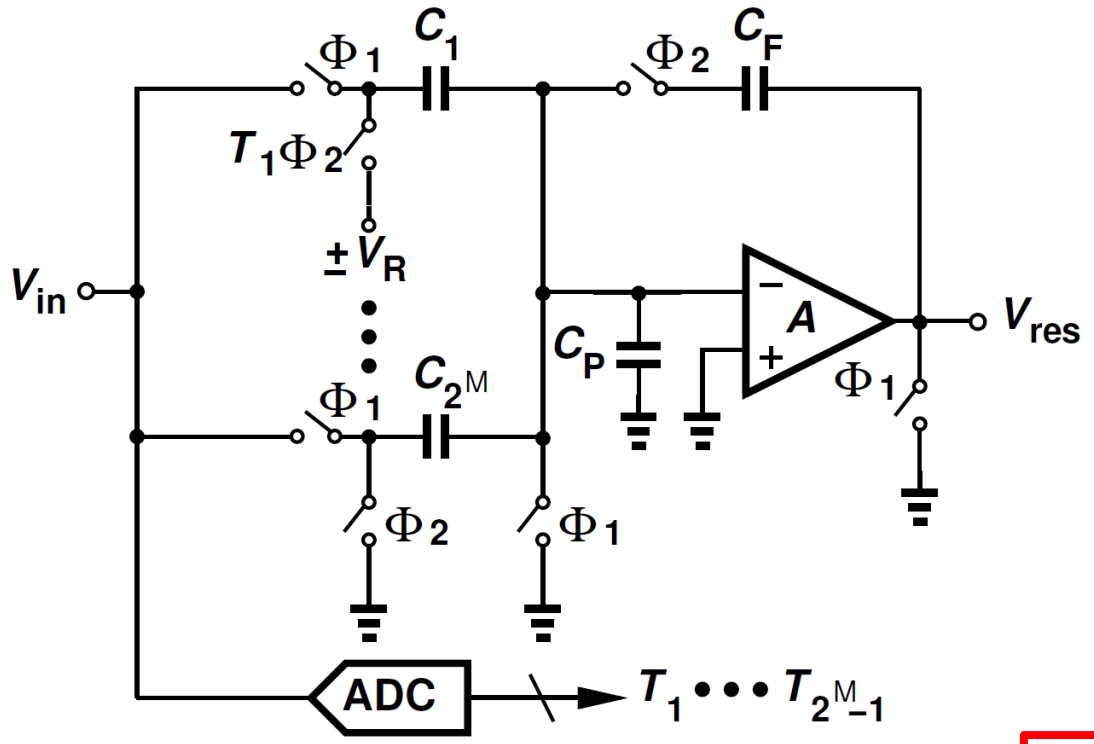
M=2, i.e., 2-bit stage

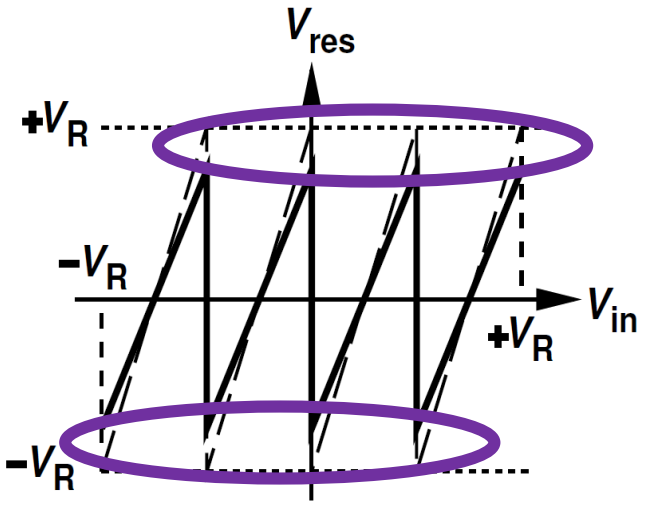
$$V_{res} = \frac{\sum_{i=1}^{2^M} C_i V_{in} - \sum_{i=1}^{(2^M-1)} B_i C_i V_{REF}}{C_F}$$

where, B_i=1 if T_i=1 and B_i=–1 if T_i=0

Capacitor mismatch → Missing Levels, Missing Codes

Finite Op amp Gain





M=2, i.e., 2-bit stage

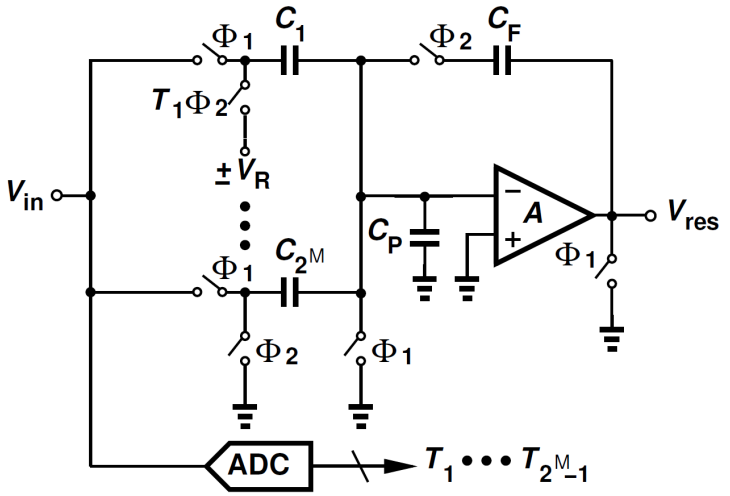
$$V_{res} = \frac{\sum_{i=1}^{2^M} C_i V_{in} + \sum_{i=1}^{(2^M-1)} B_i C_i V_{REF}}{C_F + \frac{\left(C_F + C_P + \sum_{i=1}^{2^M} C_i\right)}{A}}$$

Finite op amp gain → Missing Codes

where, B_i=1 if T_i=1 and B_i=–1 if T_i=0

and A=

Op amp Nonlinearity (1)



- **Weakly nonlinear op amp open-loop input-output characteristic can be given by a 3rd order polynomial,**

$$V_{OUT} = \alpha_1 V_{in} + \alpha_2 V_{in}^2 + \alpha_3 V_{in}^3$$

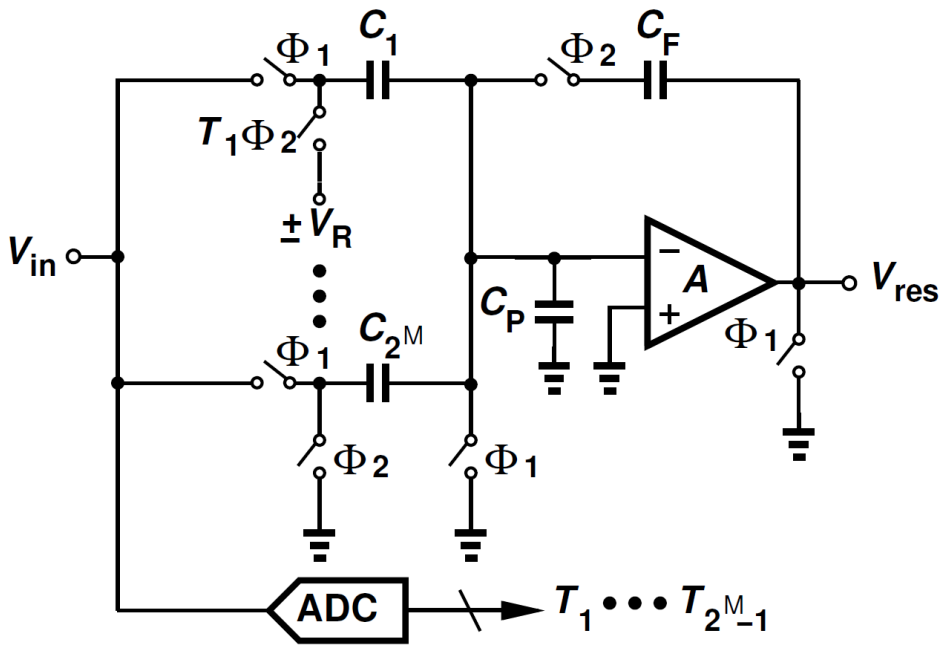
- **The inverse can be given by another polynomial**

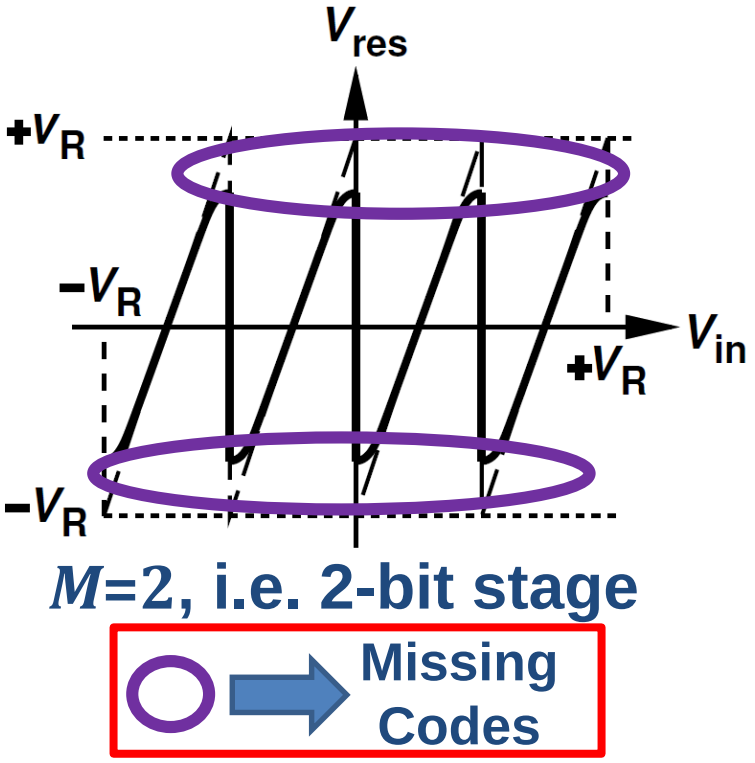
$$V_{in} = \beta_1 V_{out} + \beta_2 V_{out}^2 + \beta_3 V_{out}^3$$

where, $\beta_1 = \frac{1}{\alpha_1}$, $\beta_2 = \frac{-\alpha_2}{\alpha_1^3}$, **and** $\beta_3 = \frac{2\alpha_2^2}{\alpha^5} - \frac{\alpha_3}{\alpha_1^4}$

B. Sahoo and B. Razavi, IEEE JSSC**, vol. 44, pp. 2366-2380, Jan. 2009.**

Op amp Nonlinearity (2)





- The input-output characteristic of the MDAC can be obtained by solving the following non-linear equation:
  - $\sum_{i=1}^{2^M} C_i V_{in} = \sum_{i=1}^{(2^M-1)} C_i V_{REF} + C_F V_{res} - \left(\beta_1 V_{out} + \beta_2 V_{out}^2 + \beta_3 V_{out}^3\right)\left(C_F + C_P + \sum_{i=1}^{2^M} C_i\right)$

Thermal Noise Consideration (1)

- Signal-to-noise ratio of an ADC is given by,

$$SNR = 10log_{10}\left(\frac{P_{sig}}{(Q_N+N_T)}\right)$$

where,

  - P_{sig} = signal power = $\frac{V_{p-p}^2}{8}$ (for input $V_{in} = \frac{V_{p-p}}{2}sin(2\pi ft)$)
  - $Q_N = \frac{\Delta^2}{12}$,
  - N_T = Input referred thermal noise power of the ADC
  - $\Delta = \frac{V_{p-p}}{2^N}$, where N = ADC resolution.

- SNR of Semiconductor ADCs are limited by thermal noise of:
  - Switches
  - Op amps
- Switch thermal noise can be minimized by using large capacitors. The thermal noise of the switches is given by "kT/C", where $k = 1.38 \times 10^{-23}$, T =Temperature in K, and C is the sampling capacitor.
- Op amp thermal noise can be minimized by burning more current.

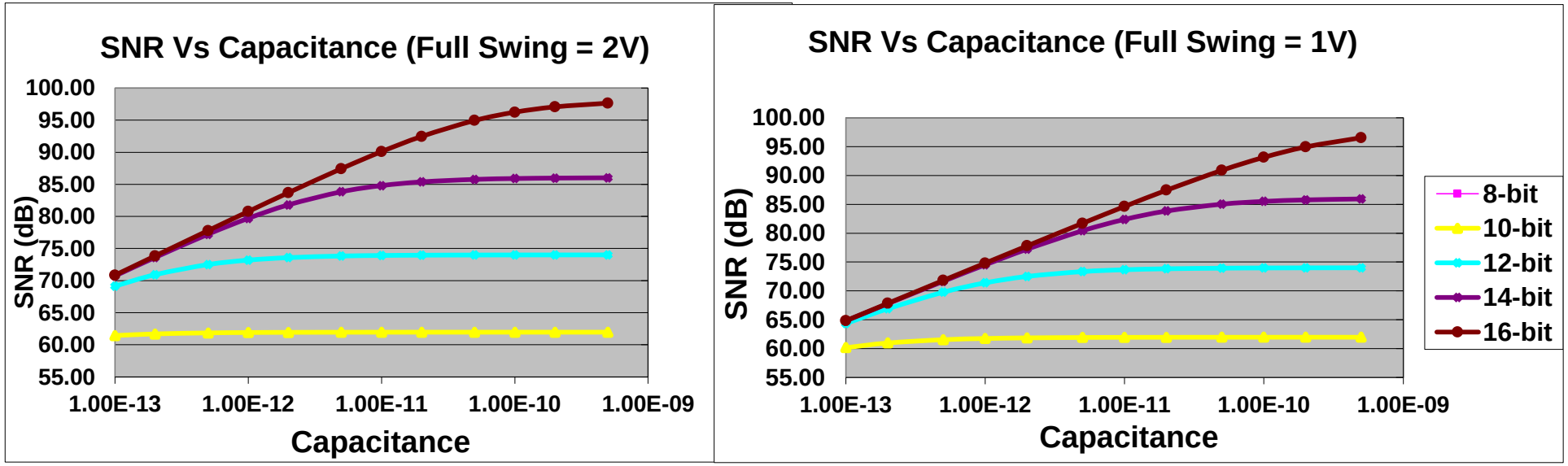
Thermal Noise Consideration (2)

- It is costly in terms of power, area, and speed to make input thermal noise smaller than quantization noise for ADC resolution, $N > 10$–bits.
- For example: If full-scale ADC input is 1 V, then for a 11-bit ADC the quantization noise power is given by:
- $Q_N = \frac{V_{LSB}^2}{12} = \frac{1}{12}\left(\frac{1}{2^{10}}\right)^2 = (141\mu V_{rms})^2$
- If thermal noise voltage power (N_T) is same as quantization noise power then the SNR takes a 3 dB hit.
- If SNR has to take < 1 dB hit then the $N_T \leq \frac{Q_N}{10}$.
- Size of the capacitor required to achieve this for 11 –bit system is $2\ pF$.
- For a 12-bit system the capacitor required would be $8\ pF$ (a large value).
- For a 16-bit system the capacitor size would be $2\ nF$ (almost physically unrealizable on chip).

Thermal Noise Consideration (3)

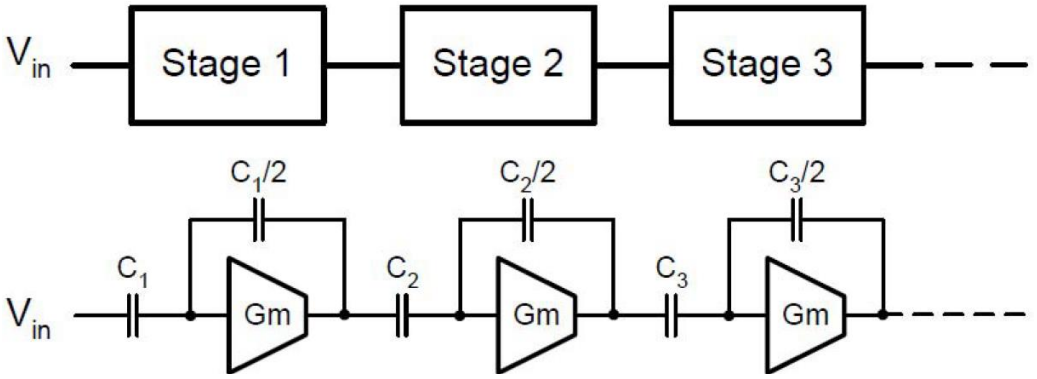
- **Ignoring other noise sources if thermal noise is only modeled by kT/C then the SNR if given by:**

$$SNR = 10log_{10}\left(\frac{P_{sig}}{\left(Q_N+\frac{kT}{C}\right)}\right)$$



Distribution of Thermal Noise

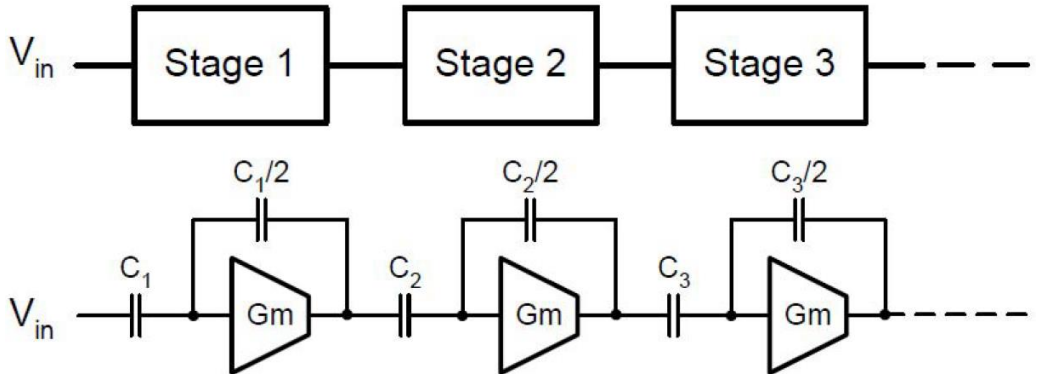
- Each stage contributes to the thermal noise.
- How do we distribute the thermal noise so that the overall input-referred thermal noise is minimized to maximize the SNR?
- Lets consider a pipelined ADC built using 1-bit stages (MDAC gain = 2)



- Considering only kT/C sampled noise the total input referred noise power:

$$N_T \propto kT\left[\frac{1}{C_1}+\frac{1}{G_1^2C_2}+\frac{1}{G_1^2G_2^2C_3}+\cdots+\frac{1}{G_1^2\cdots G_{N-1}^2C_N}\right]$$

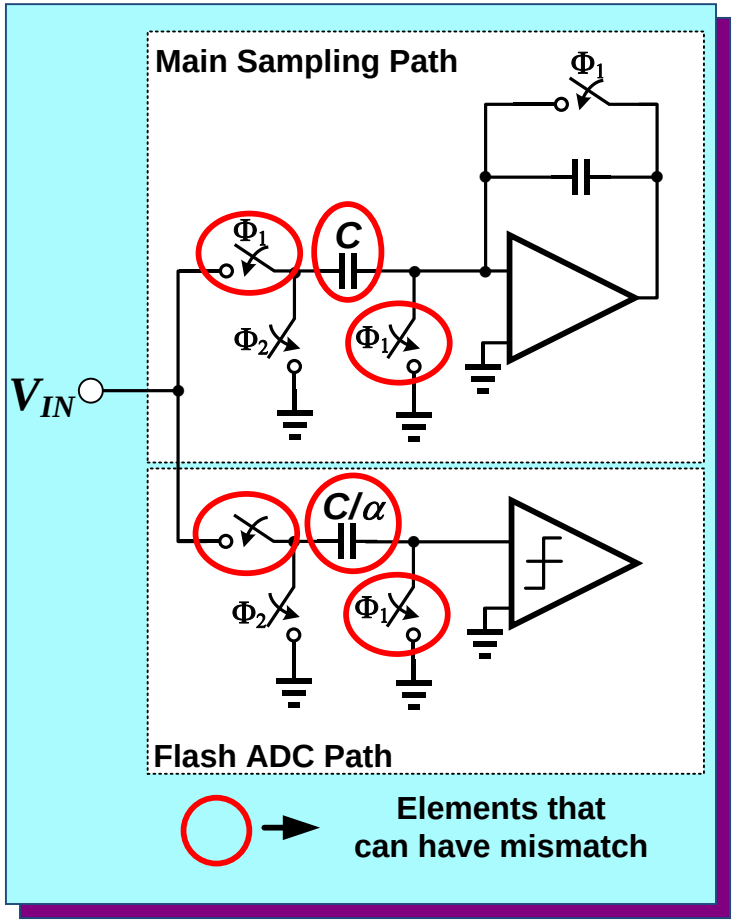
Stage Scaling for Optimal Noise (1)



$$N_T \propto kT\left[\frac{1}{C_1}+\frac{1}{G_1^2C_2}+\frac{1}{G_1^2G_2^2C_3}+\cdots+\frac{1}{G_1^2\cdots G_{N-1}^2C_N}\right]$$

- If $C_1 = C_2 = \cdots CN$ then backend stages contribute very little noise
  - Wasteful as power $\propto G_m \propto C$
- How about scaling by 2^M where M is the resolution of each stage.
  - Same amount of noise from each stage.
  - Power can be reduced.

SHA-less Architecture



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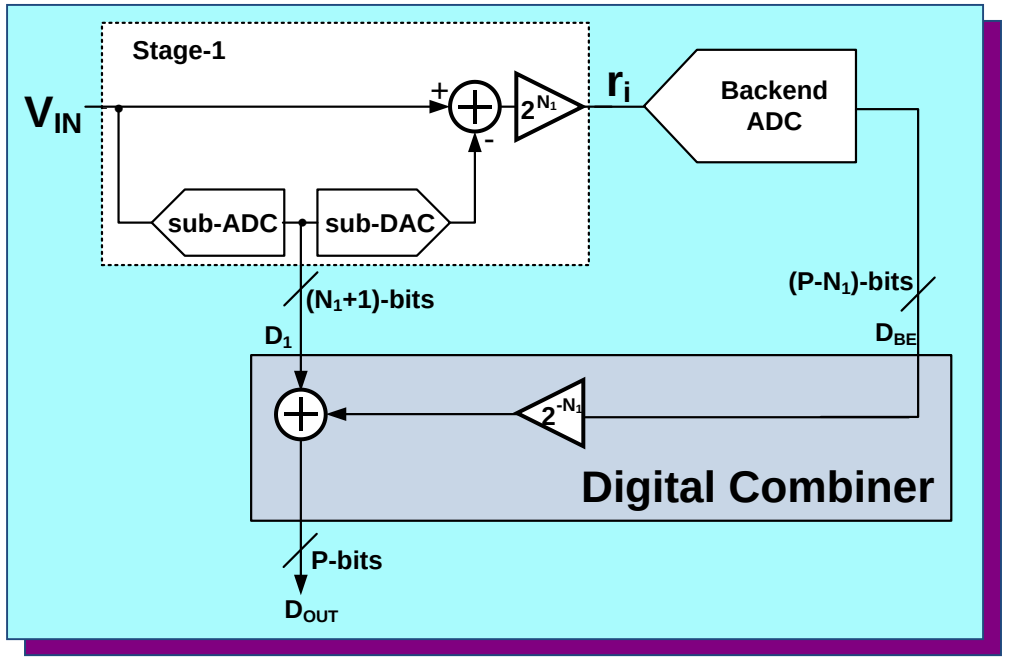
- Any mismatch between the "main sampling path" and "flash ADC path" results in different voltages being sampled on "C" and "C/α".
- The mismatch can be translated to time-constant mismatch (τ).
- For a signal of amplitude "A" and frequency "f_{in}" the difference in voltage sampled on "C" and "C/α" is:
  - $\Delta V = 2\pi f_{in} A\tau$
- Match the flash and MDAC paths.

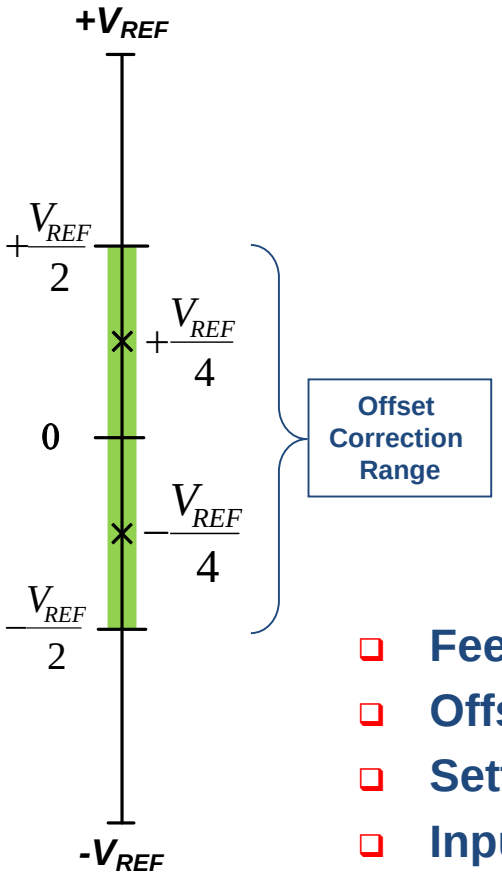
Pipeline ADC
- Area, Power, Speed, Resolution Optimization-

Pipeline ADC – Area, Power, Speed, Resolution Trade-off

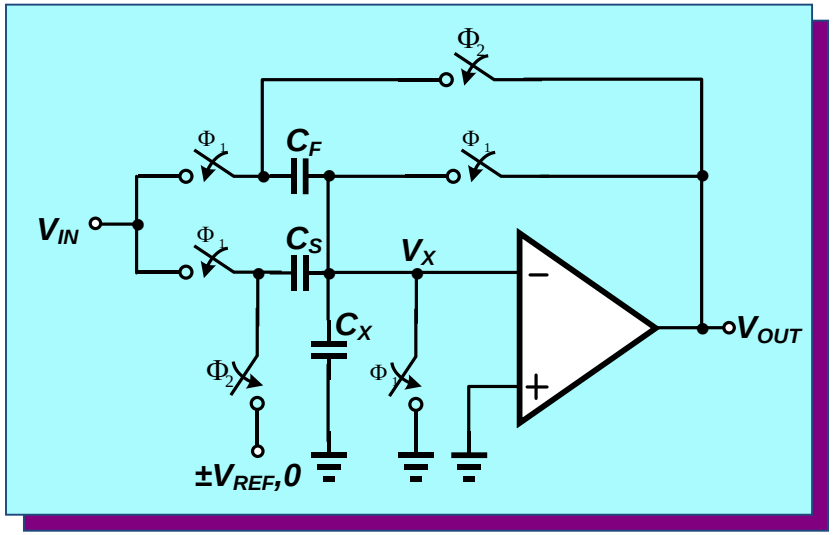


- For a given ADC resolution, the number of stages and number of bits resolved in each stage determines:
  - power consumption
  - area

1.5-bit Stage



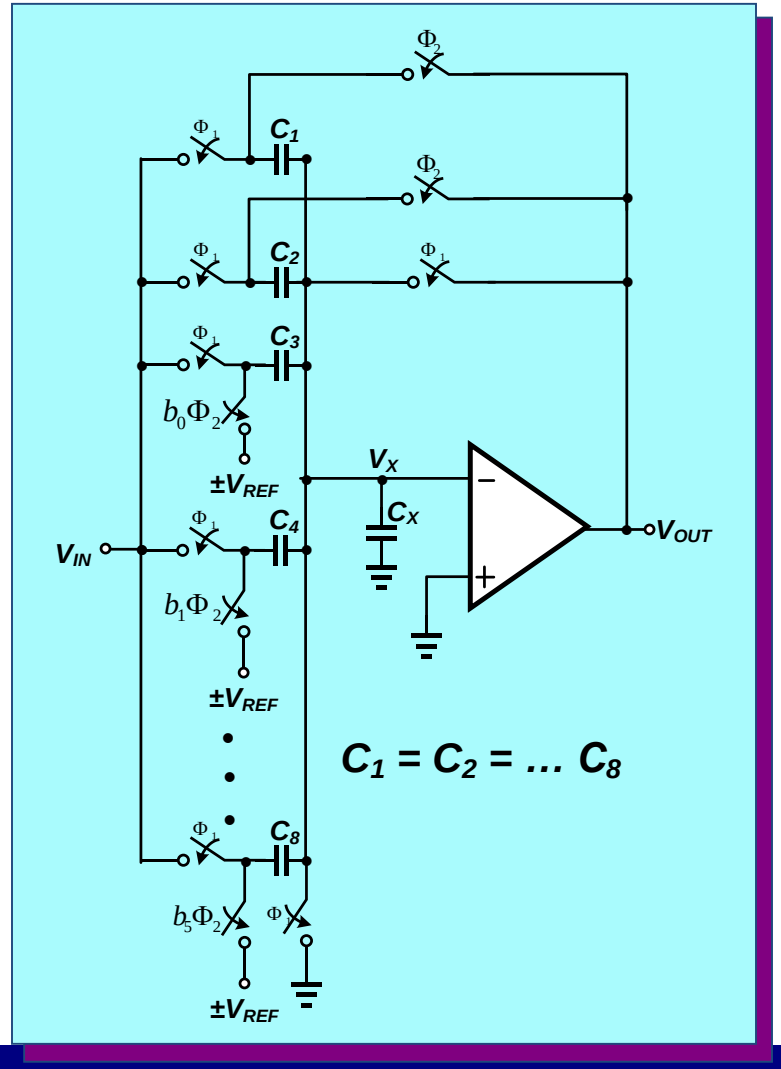
Offset Correction Range

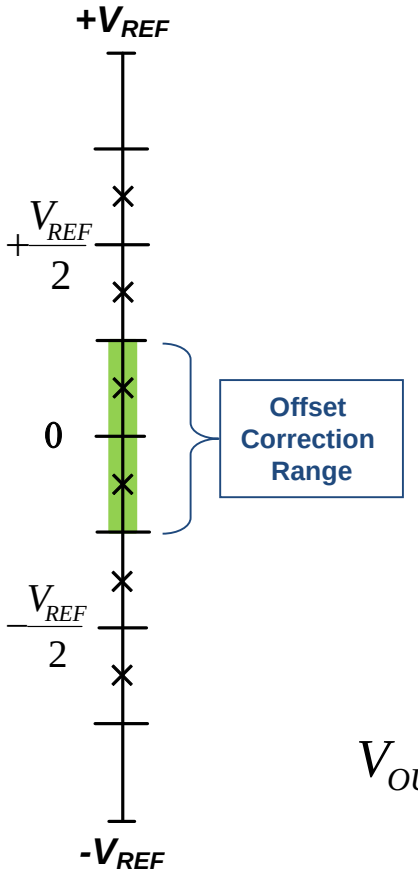


$$V_{OUT} = \frac{(C_S + C_F)V_{IN} - kC_S V_{REF}}{C_F - \left(\frac{kC_S + C_X + C_F}{A}\right)}$$

- Feedback factor = ½.
- Offset correction range = $\pm V_{REF}/4$ (i.e. ±150 mV for V_{REF}=0.6V).
- Settling Requirement on the op amp reduced by 1-bit.
- Input referred noise = ½ of output noise.

2.5-bit Stage

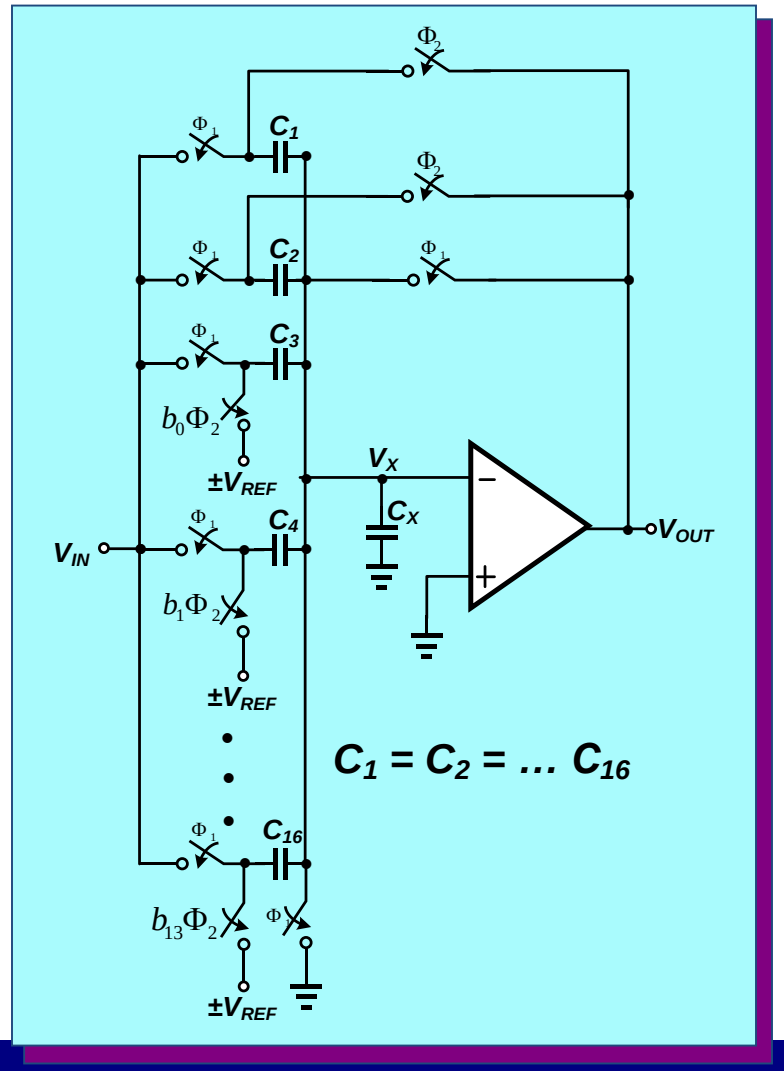


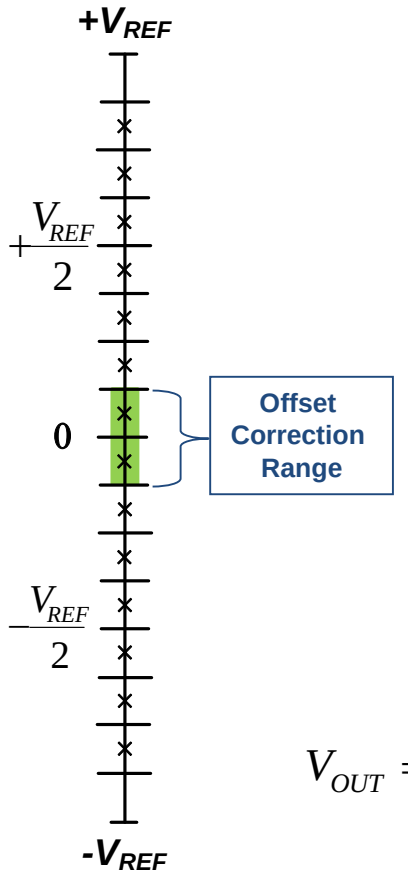


- Feedback factor =1/4.
- Offset correction range = $\pm V_{REF}/8$ (i.e. ±75 mV for V_{REF}=0.6V).
- Settling Requirement on the op amp reduced by 2-bits.
- Input referred noise is ¼ of output noise.
- Input-Output transfer function is:

$$V_{OUT} = \frac{\sum_{i=1}^{8} C_i V_{IN} - \sum_{i=0}^{5} C_{i+3} b_i V_{REF}}{C_1 + C_2 + \frac{C_1 + C_2 + C_X}{A}}$$

3.5-bit Stage





- Feedback factor = 1/8.
- Offset correction range = $\pm V_{REF}/16$ (i.e. ±37.5 mV for V_{REF}=0.6V).
- Settling Requirement on the op amp reduced by 3-bits.
- Input referred noise is 1/8 of output noise.
- Input-Output transfer function is:

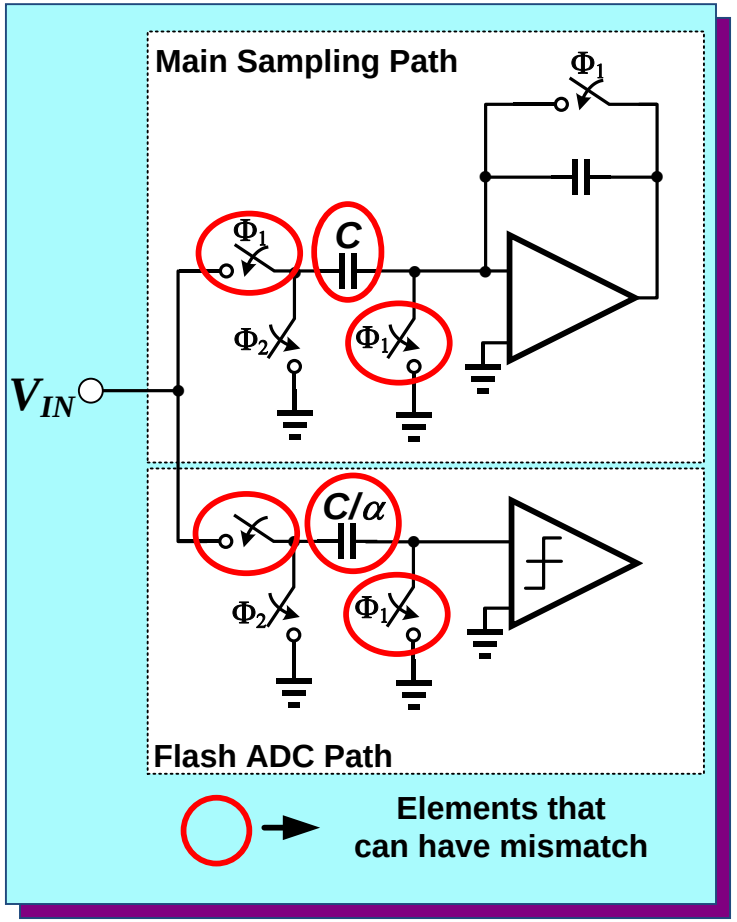
$$V_{OUT} = \frac{\sum_{i=1}^{16} C_i V_{IN} - \sum_{i=0}^{13} C_{i+3} b_i V_{REF}}{C_1 + C_2 + \frac{C_1 + C_2 + C_X}{A}}$$

Architecture Summary

Summary of ADC Stage Architectures				
	1.5-bit Stage	*2.5-bit Stage*	*3.5-bit Stage*	*Parameter effected*
Feedback Factor	$1/2$	$1/4$	$1/8$	*Speed and Power*
Offset Correction Range	$\pm V_{REF}/4$	$\pm V_{REF}/8$	$\pm V_{REF}/16$	*Linearity of ADC*
Reduction in Settling Requirement	1-bit	2-bits	*3-bits*	*Speed and Power*
Noise Scaling	$1/2$	$1/4$	$1/8$	*SNR, Power, & Area*
Reduction in Capacitor Matching Requirement	*1-bit*	*2-bits*	*3-bits*	*Power and Area*

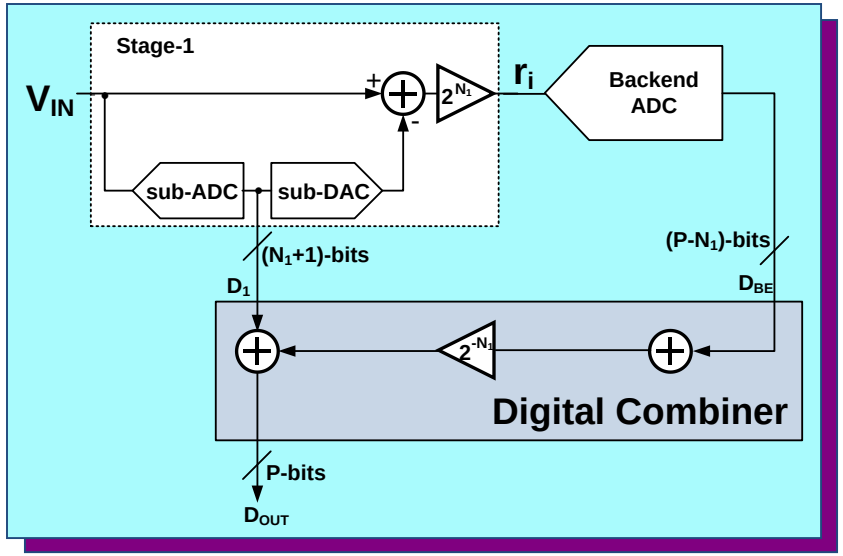
- For resolutions more than 10-bits it is better to resolve more bits in the first stage:
  - relaxing op amp settling.
  - capacitor matching.
  - reducing capacitance ➔ input referred noise is reduced.
  - DOES NOT relax the op amp open loop DC gain requirement (more later).

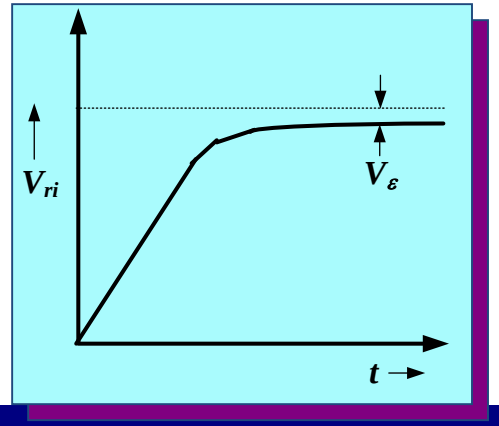
Why not resolve more bits in 1st Stage?



- Any mismatch between the "main sampling path" and "flash ADC path" results in different voltages being sampled on "C" and "C/α".
- The mismatch can be translated to time-constant mismatch (τ).
- The difference in voltage should be within the offset correction range of the Flash ADC.
- Resolving more bits in the 1st stage reduces the offset-correction range and hence could result in missing codes.
- Offset correction range should include:
  - Comparator offsets in the flash.
  - Time constant mismatch (τ).
- For a signal of amplitude "A" and frequency "f_{in}" the difference in voltage sampled on "C" and "C/α" is:
  - $\Delta V = 2\pi f_{in} A\tau$

DC Gain Requirement of op amp in each stage





- Residue voltage V_{ri} has to settle to $LSB/2$ of the backend-ADC.
- Gain error:

$$\frac{V_\varepsilon}{V_{ideal}} = \frac{1}{1+\beta A_{DC}} < \frac{1}{2^{(P-N_1+1)}}$$

- Resolution reduces but the feedback factor also reduces by the same amount ⇒ DC gain is defined by the resolution of the ADC and not the resolution of the backend ADC that follows.
- The above holds true for the op amps in the later stages of the pipeline.

Architecture Optimization (1)

- **Some expressions used for architecture optimization i.e. number of pipeline stages and number of bits/stage:**
  - **Settling time for *N*-bit accuracy:**

$$t_{settle} = (N+1) \bullet \tau \bullet \ln(2)$$

  - **Two stage op amp poles and unity gain bandwidths:**

$$\omega_{p1} = \frac{1}{R_1 g_{m2} R_2 C_C},\ \omega_{p2} = \frac{g_{m2}}{C_L},\ \omega_u = \frac{g_{m1}}{C_C},\ \text{and}\ \omega_{p2} \approx 5 \bullet \omega_u$$

  - **Variance of input referred sampled noise:**

$$\sigma_{IN}^2 = 2 \bullet \left[\frac{kT}{C_1} + \sum_{i=2}^{N} \frac{kT}{C_i} \frac{1}{G_{i-1}^2} \right] + \sigma_{op}^2 + \sigma_{ref}^2 + \sigma_{jitter}^2$$

  where, C_i = sampling caps in each stage, and G_i = gain of each stage.

  - **2nd stage of the op amp is a common source stage. For maximum output swing at the highest speed typical gain in the 2nd stage is ≈ 10.**
  - **Overdrive voltage to maximize swing is chosen to be around V_{OV}=150 mV and hence current in each branch in the two stages are $I_{D1} = g_{m1} \bullet V_{OV}/2$ and $I_{D2} = g_{m2} \bullet V_{OV}/2$.**

Architecture Optimization (2)

- **Signal swing = ±*750 mV* for *1.5 V* supply**
- **Resolution = *12-bits* (determines quantization noise)**
- f_{MAX_IN} **= *100 MHz***
- f_S **= *200 MHz***
- $t_{slewing}$ **= *0.5 ns***
- $t_{non\_overlap}$ **= *0.2 ns***
- $t_{settling}$ **= *1.8 ns***
- **Noise Budget:**
  - **Quantization Noise**
  - **Sampled Thermal Noise**
  - **Op amp Noise**
  - **Reference Noise**
  - **Jitter Noise**
  - **Input signal buffer Noise**

Architecture Optimization (3) Jitter Specification

- The variance of jitter voltage is given by:

$$\sigma_{jitter} = \sqrt{2}\,\pi\, t_j\, f_{in} A$$

where, t_j = variance of jitter.

f_{in} = frequency of the input signal.

A = amplitude of the input signal.

- For maximum input frequency of 100 MHz and jitter limited SNR of 80 dB the required rms jitter is $700\ fs$.

Architecture Optimization (4) Noise Budget

Noise Budget	
LSB (Δ)	$\frac{V_{p-p}}{2^{12}} = \frac{1.5}{2^{12}} = 366\ \mu V$
Reference Noise	90 µV
Op Amp Noise	120 µV
Sampled Noise (kT/C)	64 µV (2 pF)
Jitter Noise $(\sqrt{2}\pi t_j f_{in} V_{p-p}/2)$	66 µV (200 *fs* RMS jitter)
Overall SNR	67.8 dB (in 100 MHz band)

Architecture Optimization (5)

Sl. No.	*Architecture*	*Sampling Capacitor (pF)*	*Capacitance switching to Reference (pF)*	*Power (mW)*
1	9, 1.5-bit stages, 3-bit flash	3.0	4.0	138
2	4, 2.5-bit stages, 4-bit flash	1.5	2.5	120
3	3, 3.5-bit stages, 3-bit flash	1.0	2.0	140
4	2.5-bit 1st stage, 6, 1.5-bit stages, and 4-bit flash	2.0	2.5	77
5	3.5-bit 1st stage, 5, 1.5-bit stages, and 4-bit flash	1.0	1.5	50

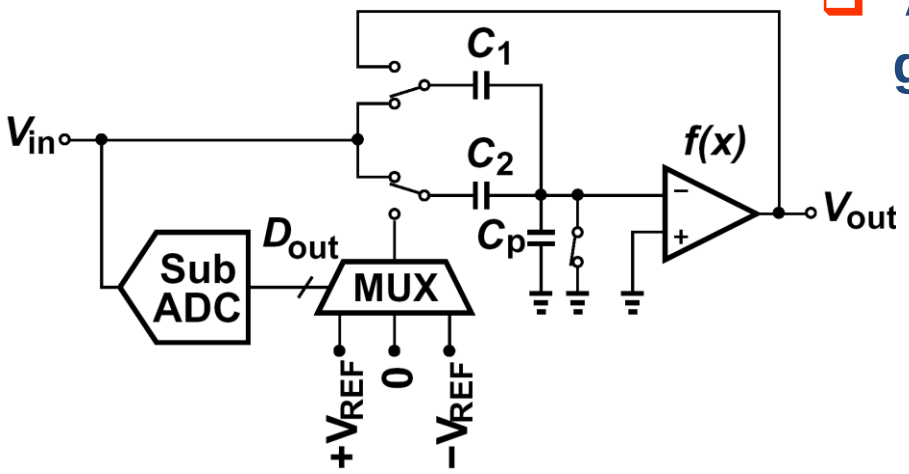
- Optimization based on the following:
  - $V_{in(p-p)(diff)}$=1.5 V
  - Quantization noise is at 12-bit level.
  - Thermal noise limited to 66 dB in 100MHz band.

- **Architecture 5 is optimal.**

Calibration : A Necessity

Why Calibrate?

Basic Pipeline Stage



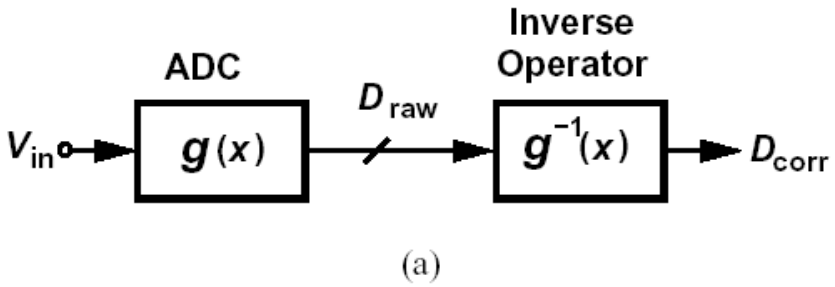
$$A_1 = \frac{3\left(1-2^{-(N+1)}\right)}{2^{-(N+1)}}$$

- As technology scales it is difficult to get:
  - get high op amp gain to
    1. remove gain error
    2. suppress nonlinearity
  - low op amp offset.
  - capacitor matching to remove DAC nonlinearity.
- For example, op amp gain in a 12-bit system should exceed 12000 ≈ 81 dB.

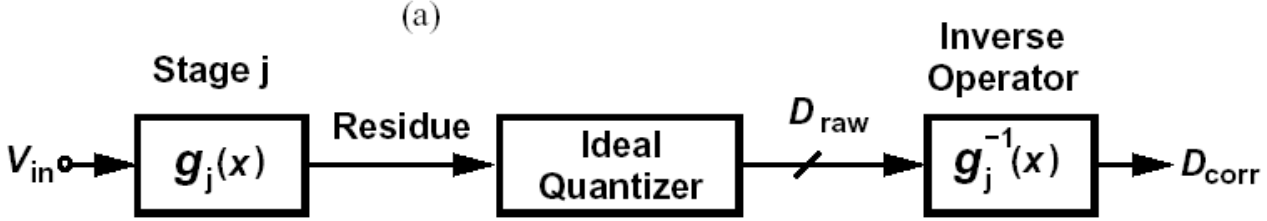
Current ADC Design Trends

- **Choose capacitors to satisfy kT/C noise, not matching.**
- **Choose op amp with high swing**
  - **→ kT/C noise relaxed**
  - **→ power consumption reduced.**
  - **→ Relaxes op amp linearity requirement**
- **Choose best trade-off between speed, power, and noise of op amp regardless of its gain.**
- **Digitally correct for everything!**

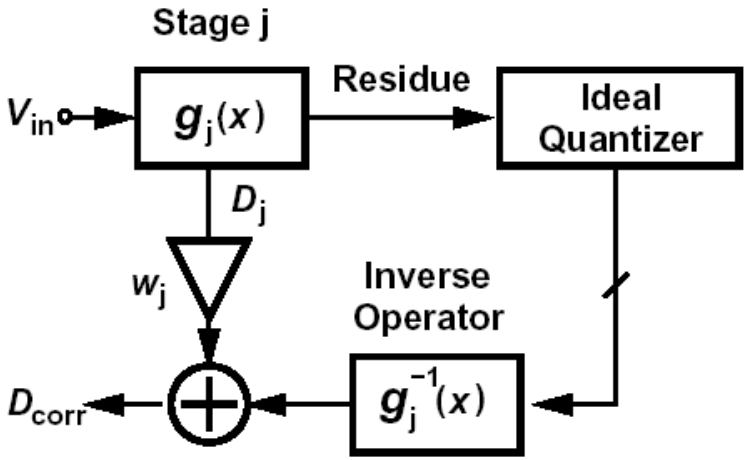
How to Calibrate?



(a)



(b)

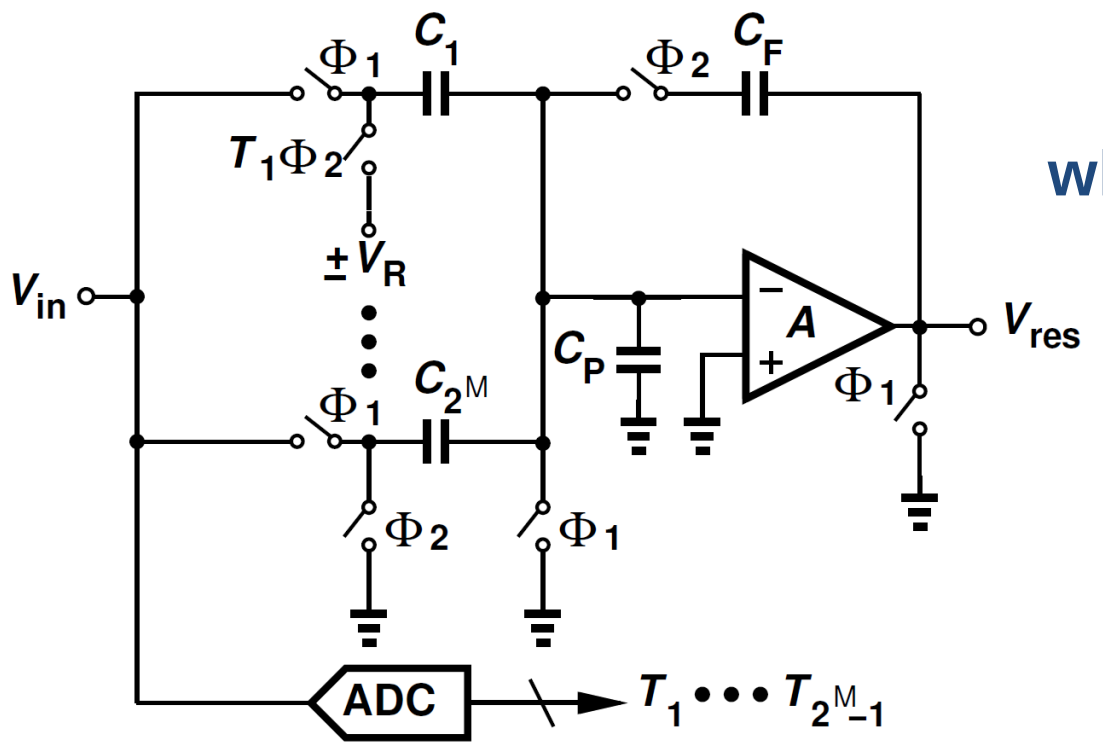


(c)

- **Inverse Operator estimation can be done in:**
  - **Background**
  - **Foreground**

Capacitor Mismatch Calibration

Comparator Forcing Based Calibration



$$V_{res} = \alpha V_{in} - \beta_j V_R$$

where,

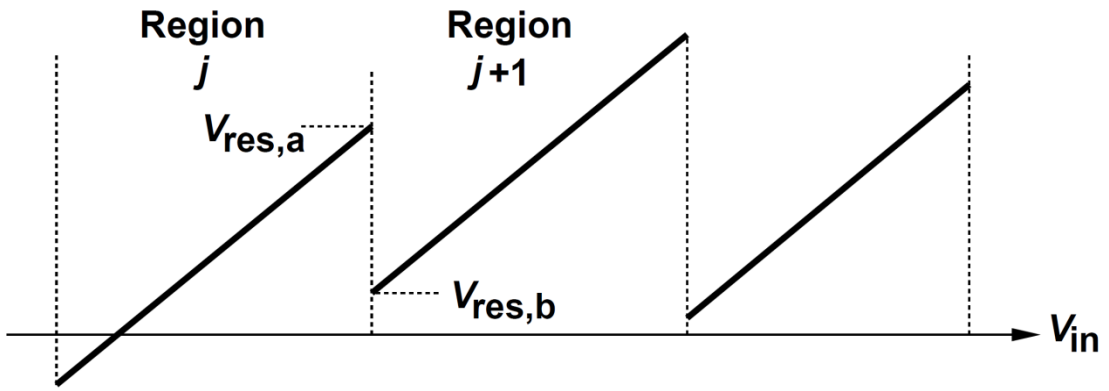
$$\alpha = \frac{\sum_{i=1}^{2^M} C_i}{C_{eq}},$$

$$\beta_j = \frac{\sum_{i=1}^{2^M-1} T_i C_i V_R}{C_{eq}}$$

and $$C_{eq} = C_F + \frac{1}{A}\left(C_F + \sum_{i=1}^{2^M} C_i + C_P\right)$$

B. Sahoo and B. Razavi, IEEE JSSC**, vol. 48, pp. 1442-1452, Jan. 2013.**

Computation of β_j (I)



$$V_{res,b} - V_{res,a} = (\beta_j - \beta_{j+1})V_{REF}$$

$$D_{j,b} - D_{j,a} = \beta_j - \beta_{j+1}$$

In other words,

$$D_{1,b} - D_{1,a} = \beta_1 - \beta_2$$

$$D_{2,b} - D_{2,a} = \beta_2 - \beta_3$$

$$\vdots$$

$$D_{15,b} - D_{15,a} = \beta_{15} - \beta_{16}$$

Computation of β_j (II)

$$
\begin{aligned}
D_{1,b} - D_{1,a} &= \beta_1 - \beta_2 \\
D_{2,b} - D_{2,a} &= \beta_2 - \beta_3 \\
&\vdots \\
D_{15,b} - D_{15,a} &= \beta_{15} + \beta_1
\end{aligned}
$$

$$
\begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{15} \end{bmatrix} = \begin{bmatrix} D_{1,b} - D_{1,a} \\ D_{2,b} - D_{2,a} \\ \vdots \\ D_{15,b} - D_{15,a} \end{bmatrix}
$$

In other words,

$$
\Longrightarrow \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{15} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ -1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} D_{1,b} - D_{1,a} \\ D_{2,b} - D_{2,a} \\ \vdots \\ D_{15,b} - D_{15,a} \end{bmatrix}
$$

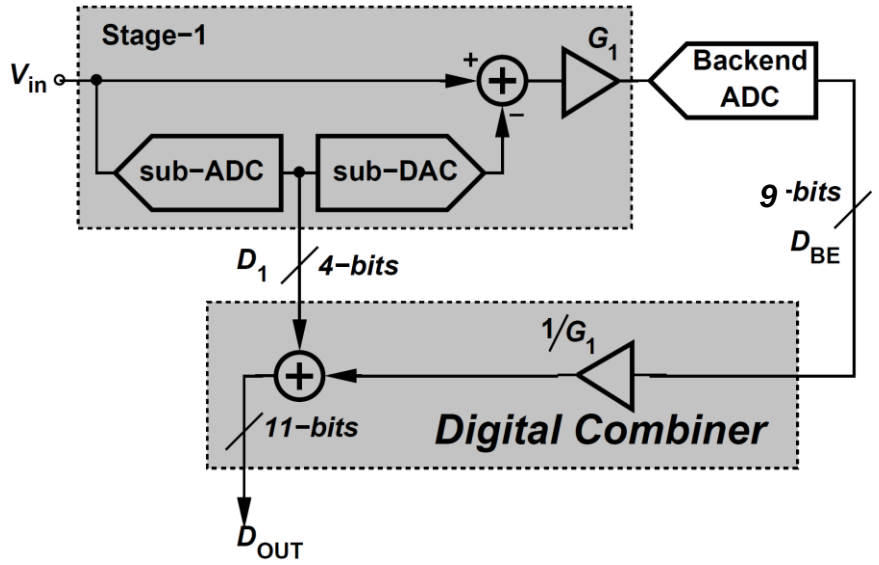
β_j's can be calculated using adders and right shifts.

B. Sahoo and B. Razavi, "A 10-b 1-GHz 33-mW CMOS ADC", IEEE Journal of Solid-State Circuits, vol. 48, pp. 1442-1452, Jun. 2013.

A. Karanicolas, et. al., "A 15-b 1-Msample/s digitally self-calibrated pipeline ADC, ", IEEE Journal of Solid-State Circuits, vol. 28, pp. 1207-1215, Dec. 1993.

Capacitor Mismatch Calibration (1)

- The input output characteristic of a 4-bit stage is:



$$V_{OUT} = \frac{\sum_{m=1}^{16} C_m V_{IN} - \sum_{m=1}^{15} C_m A_{m,j} V_R}{C_F - \frac{C_F + C_P + \sum_{m=1}^{16} C_m}{A}}$$

$$\Rightarrow V_{OUT} = \alpha V_{IN} - \frac{\sum_{m=1}^{15} C_m A_{m,j} V_R}{C_F - \frac{C_F + C_P + \sum_{m=1}^{16} C_m}{A}}$$

$$\Rightarrow V_{OUT} = \alpha V_{IN} - \beta_j V_R ,$$

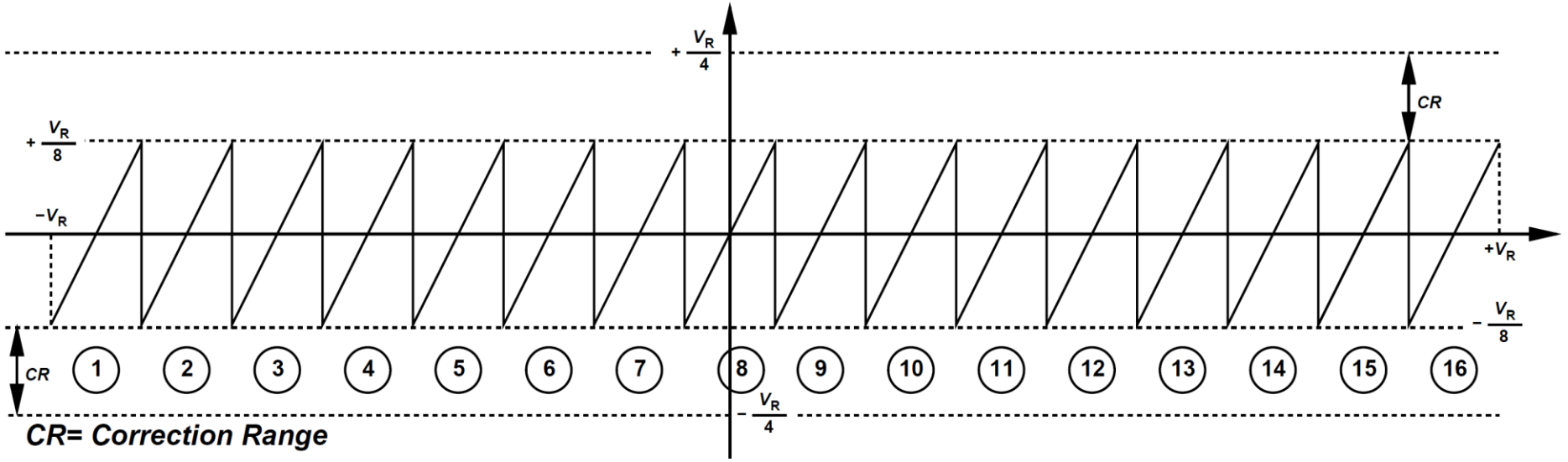
where $\beta_j = \frac{\sum_{m=1}^{15} C_m A_{m,j} V_R}{C_F - \frac{C_F + C_P + \sum_{m=1}^{16} C_m}{A}}$ and $\alpha = \frac{\sum_{i=1}^{16} C_m}{C_F - \frac{C_F + C_P + \sum_{m=1}^{16} C_m}{A}}$.

- Dividing both sides by V_R we get,
  - D_{BE}=backend digital output

$$D_{BE} = \alpha D_{IN} - \beta_j$$

$$\Rightarrow D_{IN} = \frac{D_{BE}}{\alpha} + \frac{\beta_j}{\alpha}$$

Capacitor Mismatch Calibration (2)



$$\text{Region } 1: D_{BE} = \alpha D_{IN} + \frac{(C_1 + C_2 + \bullet\bullet\bullet + C_{15})}{C_F - \dfrac{C_F + C_P + \sum_{i=1}^{16} C_i}{A}} = \alpha D_{IN} - \beta_1$$

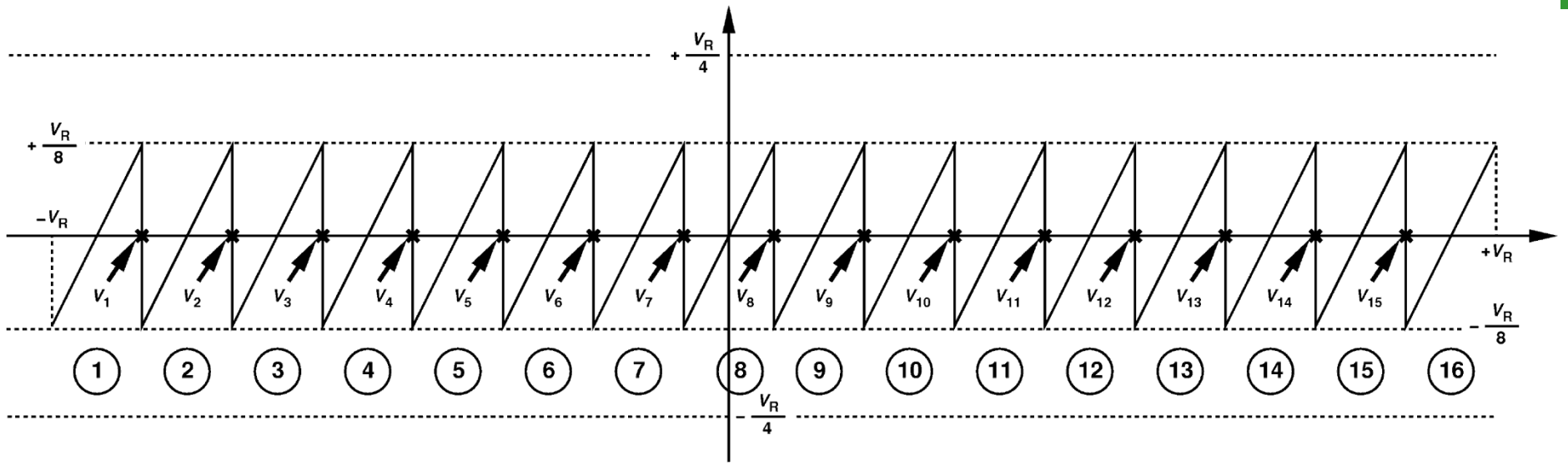
$$\text{Region } 2: D_{BE} = \alpha D_{IN} - \frac{[C_1 - (C_2 + \bullet\bullet\bullet + C_{15})]}{C_F - \dfrac{C_F + C_P + \sum_{i=1}^{16} C_i}{A}} = \alpha D_{IN} - \beta_2$$

$$\text{Region } 3: D_{BE} = \alpha D_{IN} - \frac{[C_1 + C_2 - (C_3 + \bullet\bullet\bullet + C_{15})]}{C_F - \dfrac{C_F + C_P + \sum_{i=1}^{16} C_i}{A}} = \alpha D_{IN} - \beta_3$$

$$\text{Region } 15: D_{BE} = \alpha D_{IN} - \frac{[C_1 + C_2 + \bullet\bullet\bullet + C_{14} - C_{15}]}{C_F - \dfrac{C_F + C_P + \sum_{i=1}^{16} C_i}{A}} = \alpha D_{IN} - \beta_{15}$$

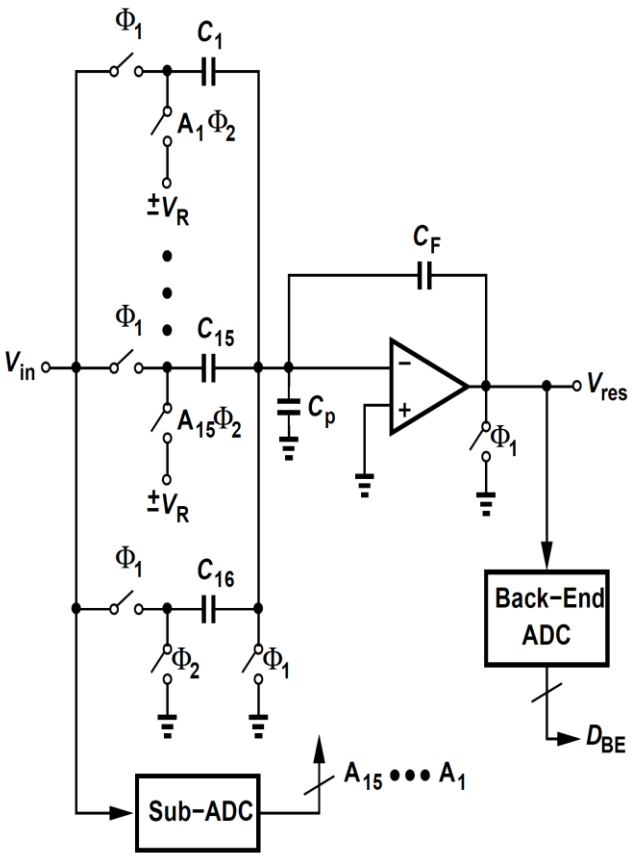
$$\text{Region } 16: D_{BE} = \alpha D_{IN} - \frac{(C_1 + C_2 + \bullet\bullet\bullet + C_{15})}{C_F - \dfrac{C_F + C_P + \sum_{i=1}^{16} C_i}{A}} = \alpha D_{IN} - \beta_{16}$$

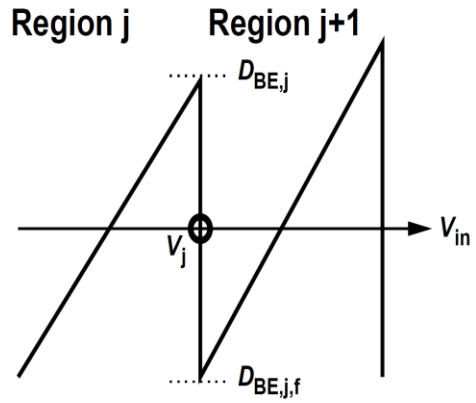
Capacitor Mismatch Calibration (3)



- The digital output goes from 0 to 15 when the input changes from $-V_R$ to $+V_R$.
- Apply V_j close to the comparator threshold and force the flash ADC output so that the residue is once in region j and then in region (j+1).
- The redundancy/offset correction range in the architecture prevents the ADC from clipping.
- The backend ADC gives two different codes for the same input voltage.

Capacitor Mismatch Calibration (4)





$$\alpha = \sum_{m=1}^{16} C_m / \gamma \qquad \beta_j = \sum_{m=1}^{15} A_{m,j} C_m / \gamma$$

$$\gamma = C_F - (C_p + C_F + \sum_{m=1}^{16} C_m) / A_{opamp}$$

- Applying V_j to the ADC in region j we get,

$$D_j = \frac{D_{BE,j}}{\alpha} + \frac{\beta_j}{\alpha}$$

- Similarly applying V_j and forcing the flash ADC output to be in region (j+1) we get,

$$D_{j,f} = \frac{D_{BE,j,f}}{\alpha} + \frac{\beta_{j+1}}{\alpha}$$

- Since, same voltage is applied we can equate both of them:

$$D_{BE,j,f} - D_{BE,j} = \beta_j - \beta_{j+1}$$

which is not dependent on gain error.

- Repeat the above steps for *j=1 to 15*.

Capacitor Mismatch Calibration (5)

- **Thus we end up with:**

$$
\begin{aligned}
&D_{BE,1,f} - D_{BE,1} = \beta_1 - \beta_2 \\
&D_{BE,2,f} - D_{BE,2} = \beta_2 - \beta_3 \\
&D_{BE,3,f} - D_{BE,3} = \beta_3 - \beta_4 \\
&D_{BE,4,f} - D_{BE,4} = \beta_4 - \beta_5 \\
&D_{BE,5,f} - D_{BE,5} = \beta_5 - \beta_6 \\
&D_{BE,6,f} - D_{BE,6} = \beta_6 - \beta_7 \\
&D_{BE,7,f} - D_{BE,7} = \beta_7 - \beta_8 \\
&D_{BE,8,f} - D_{BE,8} = \beta_8 - \beta_9 \\
&D_{BE,9,f} - D_{BE,9} = \beta_9 - \beta_{10} \\
&D_{BE,10,f} - D_{BE,10} = \beta_{10} - \beta_{11} \\
&D_{BE,11,f} - D_{BE,11} = \beta_{11} - \beta_{12} \\
&D_{BE,12,f} - D_{BE,12} = \beta_{12} + \beta_{13} \\
&D_{BE,13,f} - D_{BE,13} = \beta_{13} - \beta_{14} \\
&D_{BE,14,f} - D_{BE,14} = \beta_{14} - \beta_{15} \\
&D_{BE,15,f} - D_{BE,15} = \beta_{15} + \beta_1
\end{aligned}
\quad \Rightarrow \quad
\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \\ \beta_8 \\ \beta_9 \\ \beta_{10} \\ \beta_{11} \\ \beta_{12} \\ \beta_{13} \\ \beta_{14} \\ \beta_{15}
\end{bmatrix}
=
\begin{bmatrix}
D_{BE,1,f} - D_{BE,1} \\
D_{BE,2,f} - D_{BE,2} \\
D_{BE,3,f} - D_{BE,3} \\
D_{BE,4,f} - D_{BE,4} \\
D_{BE,5,f} - D_{BE,5} \\
D_{BE,6,f} - D_{BE,6} \\
D_{BE,7,f} - D_{BE,7} \\
D_{BE,8,f} - D_{BE,8} \\
D_{BE,9,f} - D_{BE,9} \\
D_{BE,10,f} - D_{BE,10} \\
D_{BE,11,f} - D_{BE,11} \\
D_{BE,12,f} - D_{BE,12} \\
D_{BE,13,f} - D_{BE,13} \\
D_{BE,14,f} - D_{BE,14} \\
D_{BE,15,f} - D_{BE,15}
\end{bmatrix}
$$

- **Solving for β_j is straight forward and does not require multiplication.**

Capacitor Mismatch Calibration (6)

- Thus β_j can be obtained as follows without the need of multipliers:

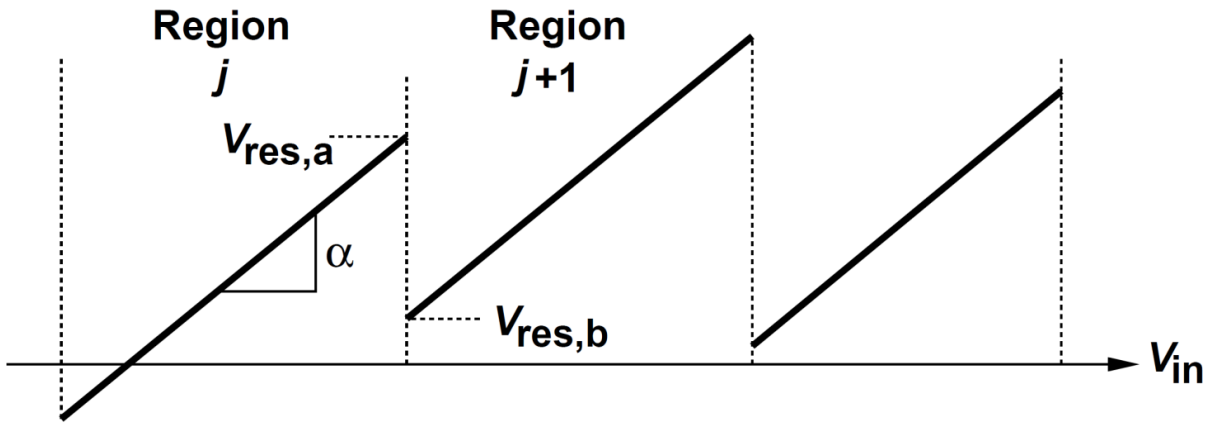
$$\begin{bmatrix}\beta_1\\ \beta_2\\ \beta_3\\ \beta_4\\ \beta_5\\ \beta_6\\ \beta_7\\ \beta_8\\ \beta_9\\ \beta_{10}\\ \beta_{11}\\ \beta_{12}\\ \beta_{13}\\ \beta_{14}\\ \beta_{15}\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1\\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1\\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1\\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1\\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1\\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1\end{bmatrix}\begin{bmatrix}D_{BE,1,f}-D_{BE,1}\\ D_{BE,2,f}-D_{BE,2}\\ D_{BE,3,f}-D_{BE,3}\\ D_{BE,4,f}-D_{BE,4}\\ D_{BE,5,f}-D_{BE,5}\\ D_{BE,6,f}-D_{BE,6}\\ D_{BE,7,f}-D_{BE,7}\\ D_{BE,8,f}-D_{BE,8}\\ D_{BE,9,f}-D_{BE,9}\\ D_{BE,10,f}-D_{BE,10}\\ D_{BE,11,f}-D_{BE,11}\\ D_{BE,12,f}-D_{BE,12}\\ D_{BE,13,f}-D_{BE,13}\\ D_{BE,14,f}-D_{BE,14}\\ D_{BE,15,f}-D_{BE,15}\end{bmatrix}$$

- Combining the bits with appropriate β_j:
  - Flash ADC output tells us which region the analog voltage is in.
  - The above information can be used appropriately combine the bits.

Gain Calibration for Multi-bit MDAC

Computation of α

$$\alpha = \frac{C_1 + \cdots + C_{16}}{C_{eq}}$$



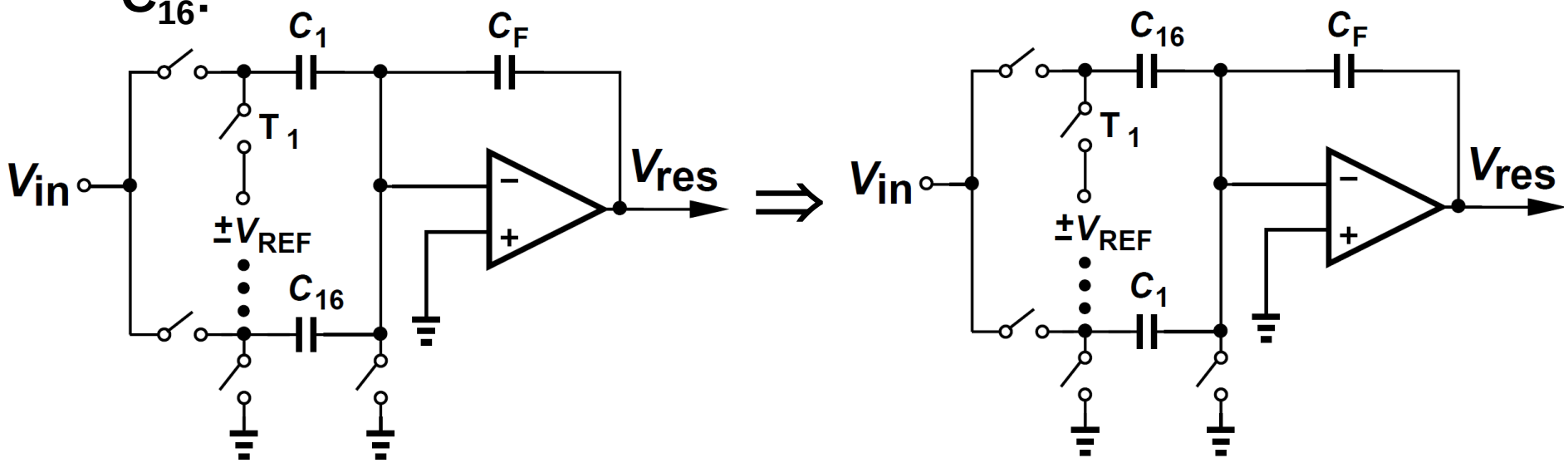
- Need to obtain C_1/C_{eq}, C_2/C_{eq},…, C_{16}/C_{eq}.
- Fortunately, $$\beta_{j+1} - \beta_j = \frac{C_1 + \cdots + C_{j+1}}{C_{eq}} - \frac{C_1 + \cdots + C_j}{C_{eq}} = \frac{C_{j+1}}{C_{eq}}$$

- We already have these values from previous measurements

$$\begin{aligned} D_{1,b} - D_{1,a} &= \beta_1 - \beta_2 \\ D_{2,b} - D_{2,a} &= \beta_2 - \beta_3 \\ &\vdots \\ D_{15,b} - D_{15,a} &= \beta_{15} + \beta_1 \end{aligned}$$

Computation of C_{16}/C_{eq}

- **Swap C_1 and C_{16}:**

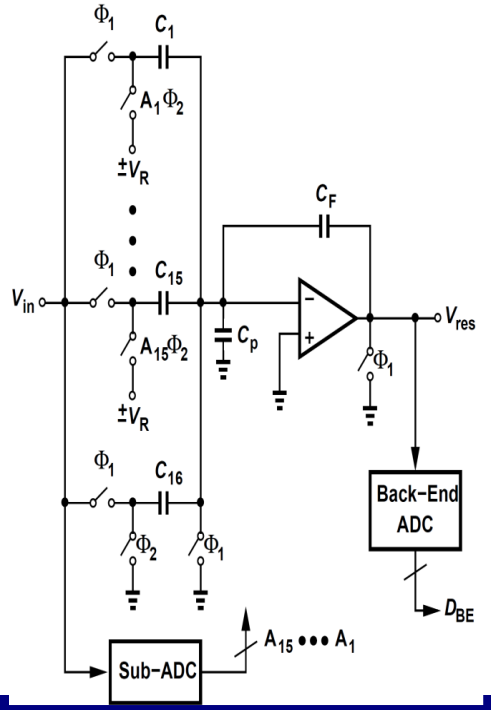


$$\Rightarrow \beta_{16} = \frac{C_2 + \cdots + C_{16}}{C_{eq}} \Rightarrow \frac{C_{16}}{C_{eq}} = \beta_{16} - \beta_{15}$$

- **Thus** $\alpha = \frac{C_1 + \cdots + C_{16}}{C_{eq}}$ **is obtained.**

Gain Error Calibration (1)

- We can obtain a similar set of measurements by connecting C_{16} to $\pm V_R$ (controlled by A_1) and C_1 to V_{CM}.
- Instead of β_1 to β_{15} we can define η_1 to η_{15} as shown on the side.
- Similarly we can solve for η_1 to η_{15} by matrix inversion.



$$\text{Region } 1: D_{BE} = \alpha D_{IN} + \frac{(C_{16} + C_2 + \bullet\bullet\bullet + C_{15})}{C_F - \dfrac{C_F + C_P + \sum_{i=1}^{16} C_i}{A}} = \alpha D_{IN} - \eta_1$$

$$\text{Region } 2: D_{BE} = \alpha D_{IN} - \frac{[C_{16} - (C_2 + \bullet\bullet\bullet + C_{15})]}{C_F + \dfrac{C_F + C_P + \sum_{i=1}^{16} C_i}{A}} = \alpha D_{IN} - \eta_2$$

$$\text{Region } 3: D_{BE} = \alpha D_{IN} - \frac{[C_{16} + C_2 - (C_3 + \bullet\bullet\bullet + C_{15})]}{C_F + \dfrac{C_F + C_P + \sum_{i=1}^{16} C_i}{A}} = \alpha D_{IN} - \eta_3$$

$$\vdots$$

$$\text{Region } 15: D_{BE} = \alpha D_{IN} - \frac{[C_{16} + C_2 + C_3 + \bullet\bullet\bullet - C_{15}]}{C_F + \dfrac{C_F + C_P + \sum_{i=1}^{16} C_i}{A}} = \alpha D_{IN} - \eta_{15}$$

$$\text{Region } 16: D_{BE} = \alpha D_{IN} - \frac{(C_{16} + C_2 + C_3 + \bullet\bullet\bullet + C_{15})}{C_F + \dfrac{C_F + C_P + \sum_{i=1}^{16} C_i}{A}} = \alpha D_{IN} + \eta_1$$

Gain Error Calibration (2)

- We can rewrite β_1 to β_{15} in terms C_1 to C_{16} as shown below:

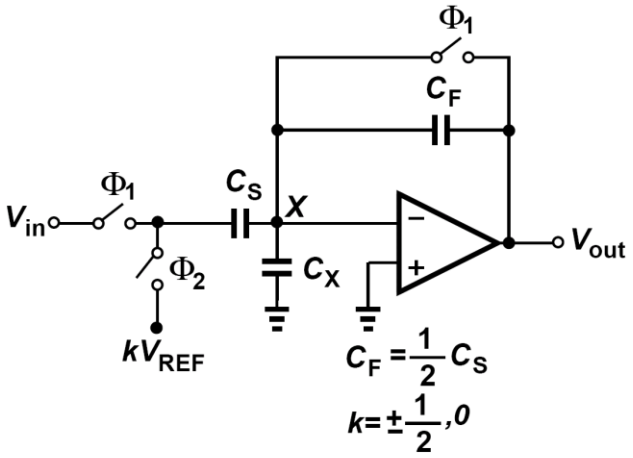
$$\frac{1}{\left(C_F - \frac{C_F + C_P + \sum_{i=1}^{16} C_i}{A}\right)} \begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \\ C_{11} \\ C_{12} \\ C_{13} \\ C_{14} \\ C_{15} \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \\ \beta_8 \\ \beta_9 \\ \beta_{10} \\ \beta_{11} \\ \beta_{12} \\ \beta_{13} \\ \beta_{14} \\ \beta_{15} \end{bmatrix}$$

- Similarly, we can rewrite η_1 to η_{15} in terms C_2 to C_{16}.
- Solving the two matrices we can obtain $C_i/(C_F-C_X)$ where, for i=1 to 16.
- Gain error, $\alpha = \sum_{i=1}^{16} \frac{C_i}{C_F - C_X}$

$$C_X = \frac{C_F + C_P + \sum_{i=1}^{16} C_i}{A}$$

Gain Error Calibration – 1.5 bit stages

- Backend stages ➔ need gain error calibration.
- Perturbation based calibration [1]:
  - Applying V_{IN} we get D_0 and D_{BE0}.
  - Applying $(V_{IN} + \Delta)$ we get D_1 and D_{BE1}.
  - Applying Δ we get D_0 and $D_{BE\Delta}$.
- V_{IN} and $(V_{IN} + \Delta)$ should produce different codes.
- Thus, gain error γ is obtained as follows:



$$(V_{IN} + \Delta) - V_{IN} - \Delta = 0$$

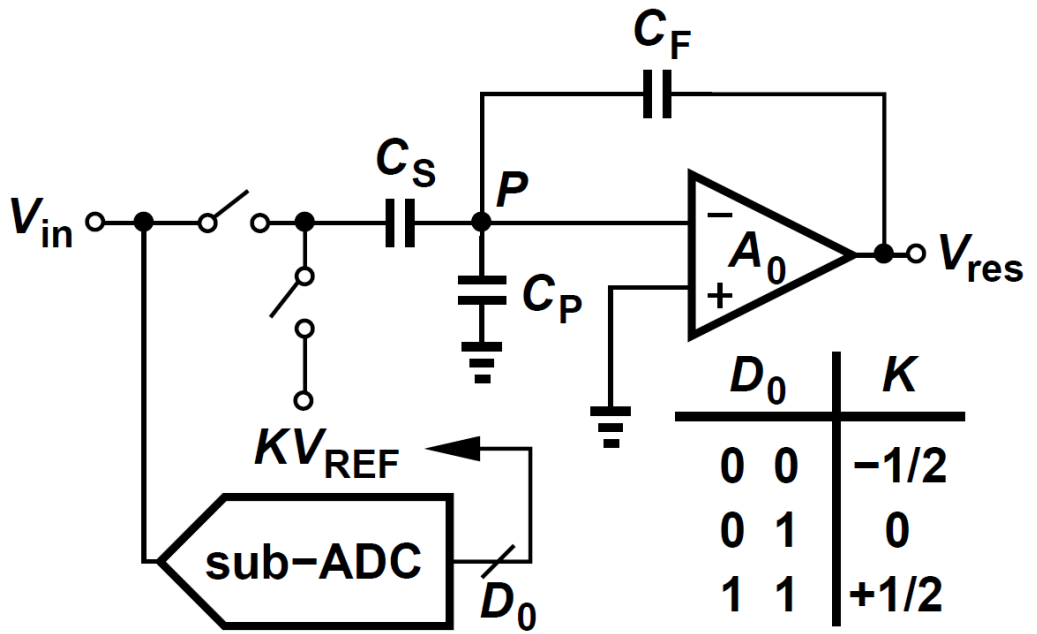
$$\Rightarrow D_1 + \gamma D_{BE1} - D_0 - \gamma D_{BE0} - D_0 - \gamma D_{BE\Delta} = 0$$

$$\Rightarrow \gamma = \frac{D_1}{D_{BE0} + D_{BE\Delta} - D_{BE1}}$$

[1] B. Sahoo and B. Razavi, "A 12-Bit 200-MHz CMOS ADC," IEEE Journal of Solid-State Circuits, vol. 44, pp. 2366-2380, Sept. 2009

Gain Calibration for 1.5-bit Non-flip-around MDAC

1.5-Bit Stages



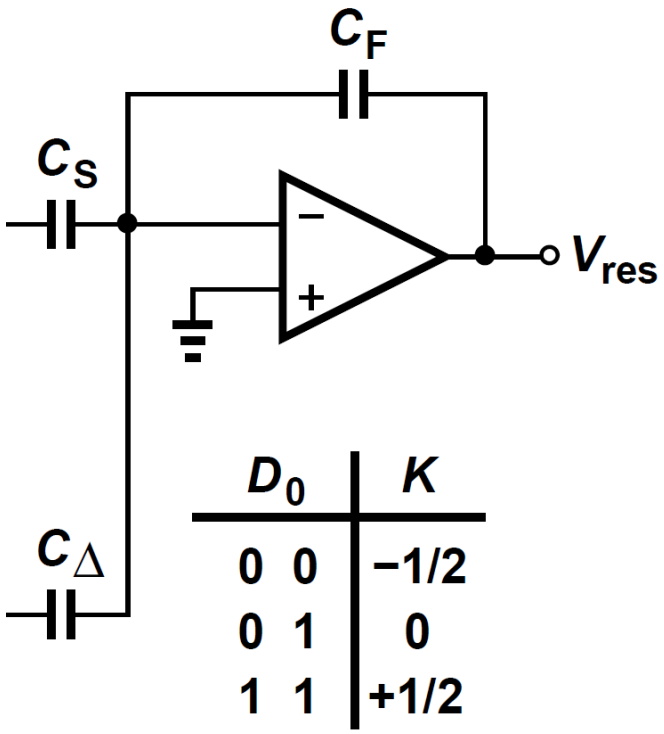
D_0	K
0 0	−1/2
0 1	0
1 1	+1/2

$$V_{res} = \frac{C_S}{C_F + \frac{C_S+C_F+C_P}{A_0}}(V_{in} - KV_{REF}) \Rightarrow D_{in} = K + \frac{1}{\alpha}D_{BE}$$

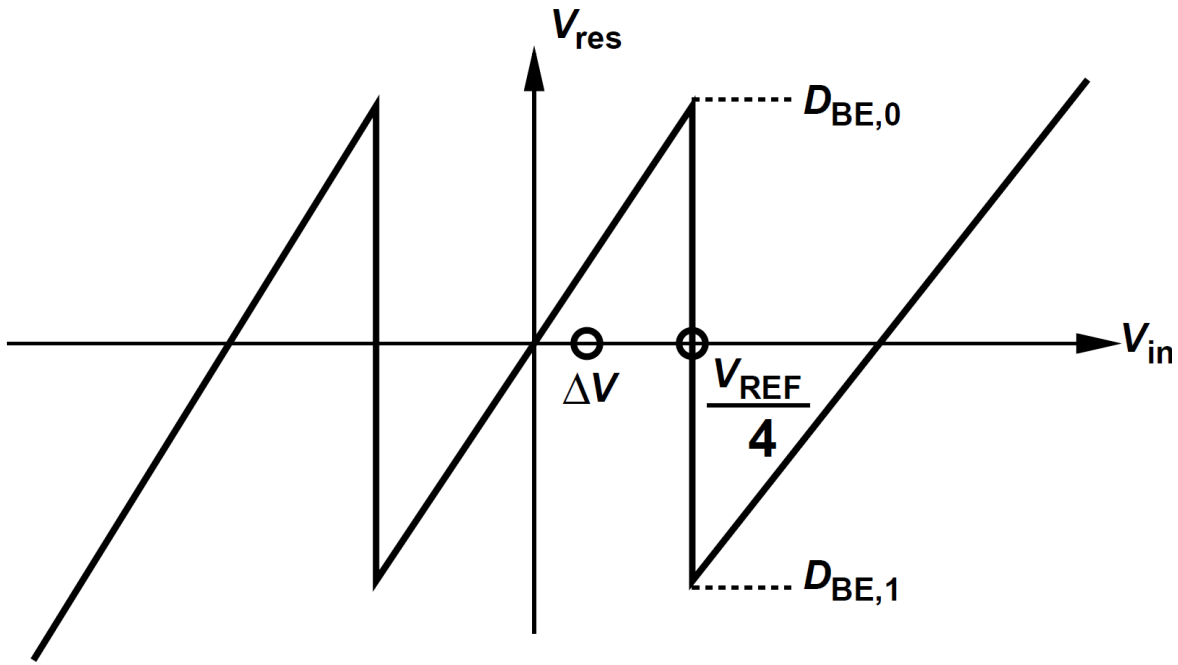
where, $\alpha = \dfrac{C_S}{C_F + \frac{C_S+C_F+C_P}{A_0}}$ is the gain.

- C_S and C_F cannot be swapped to obtain gain as it would lead to over-range.

Calibration Algorithm* (1.5-Bit Stages)



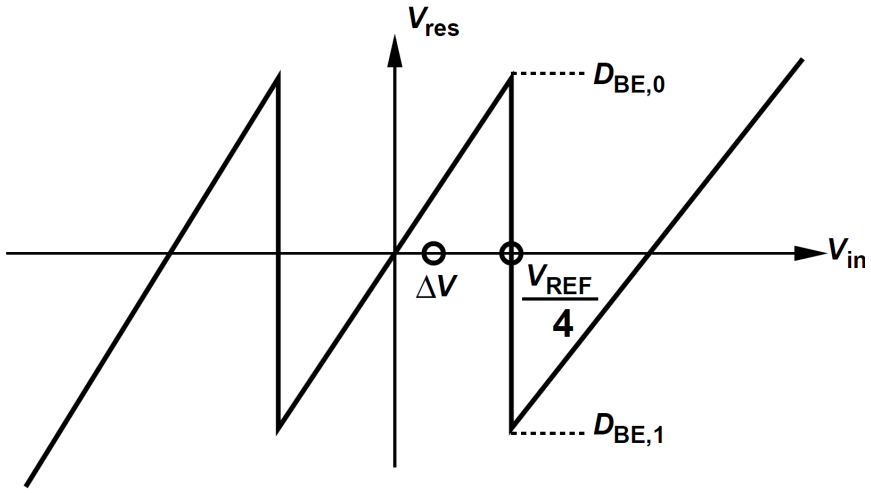
D_0		K
0	0	−1/2
0	1	0
1	1	+1/2



- **Apply $\Delta V \approx 10$ mV**
- **Apply $V_{REF}/4$**
- **Apply $V_{REF}/4+\Delta V$**

*** B. Sahoo and B. Razavi, *IEEE Journal of Solid-State Circuits*, vol. 44, pp. 2366-2380, Sept. 2009**

1.5-Bit Stages - Computing Inverse Gain (1/α)



- **Apply ΔV**

$$D_a = D_{sub,a} + \frac{1}{\alpha} D_{BE,a}$$

- **Apply V_{REF}/4, force comparator output to be "0"**

$$D_b = D_{sub,b} + \frac{1}{\alpha} D_{BE,b}$$

- **Apply (V_{REF}/4+ΔV), force comparator output to be "1"**

$$D_c = D_{sub,c} + \frac{1}{\alpha} D_{BE,c}$$

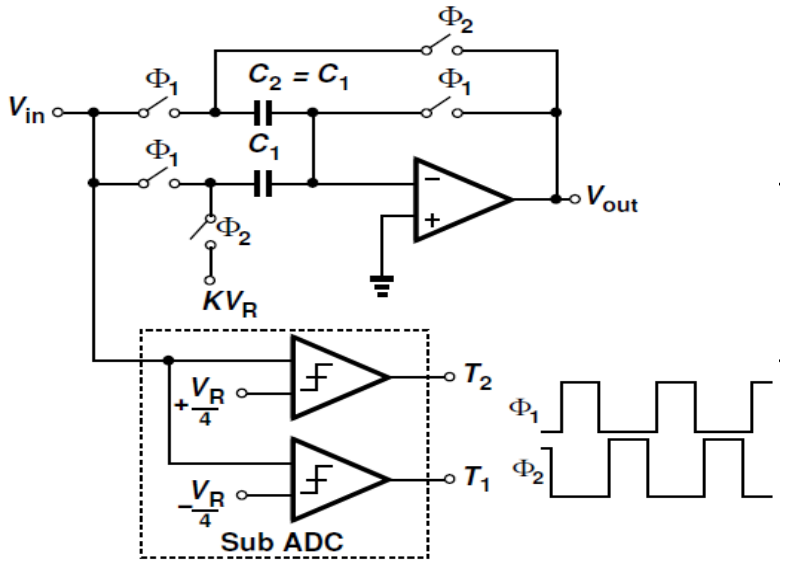
Inverse Gain :

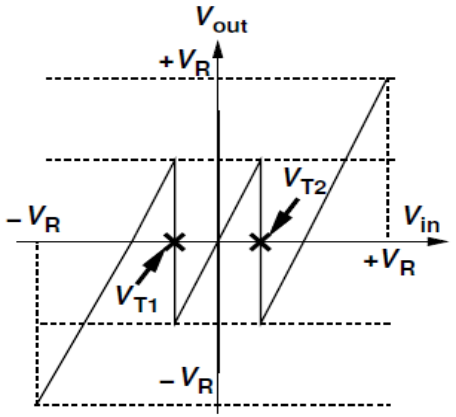
$$\frac{1}{\alpha} = \frac{D_{sub,c}}{D_{BE,a} + D_{BE,b} - D_{BE,c}}$$

- **Obtained using Newton-Raphson iterative method instead of division.**

Gain Calibration for 1.5-bit Flip-around, 2.5-bit, etc. MDAC

Gain Calibration for 1.5-bit Flip-around MDAC (1)





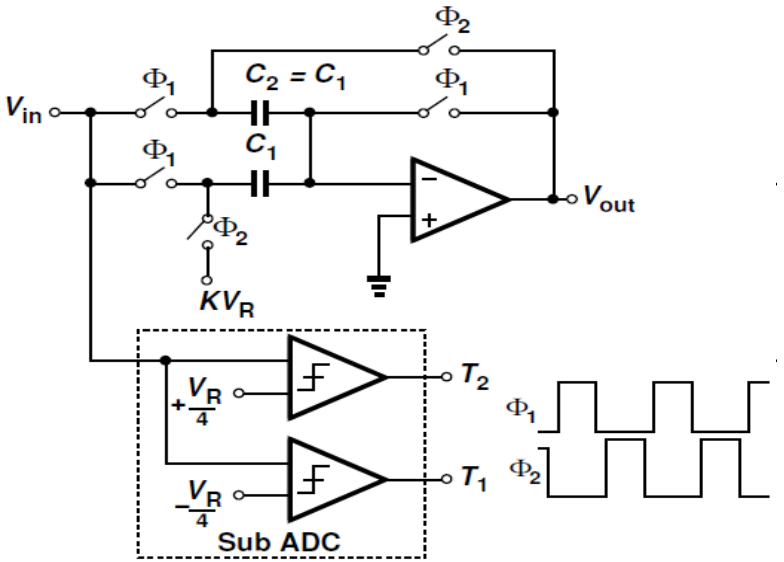
$$V_{out} = \frac{C_1+C_2}{C_2+\frac{C_1+C_2+C_P}{A}} V_{in} - \frac{C_1}{C_2+\frac{C_1+C_2+C_P}{A}} KV_R, \text{ where } K = \pm 1, 0$$

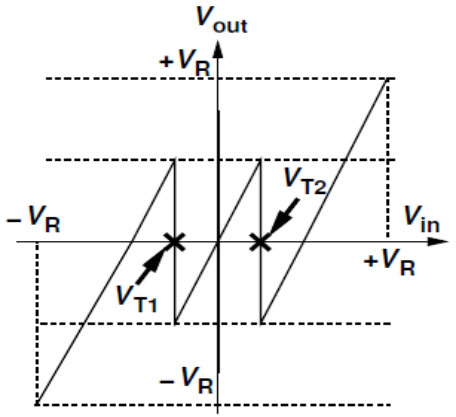
$$\Longrightarrow V_{out} = \alpha V_{in} - K\beta V_R, \text{ where } \alpha = \frac{C_1+C_2}{C_2+\frac{C_1+C_2+C_P}{A}}, \text{ and } \beta = \frac{C_1}{C_2+\frac{C_1+C_2+C_P}{A}}$$

- β can be solved by applying V_{T1} or V_{T2} and forcing the corresponding comparator to "1" or "0".
- Unlike, an N-bit architecture as mentioned earlier we cannot swap the capacitors here to solve for α.
- Swapping capacitors changes the denominator $C_2 + \frac{C_1+C_2+C_P}{A}$ to $C_1 + \frac{C_1+C_2+C_P}{A}$

* C. Ravi, V. Sarma, and B. Sahoo," IEEE NEWCAS, June 2015

Gain Calibration for 1.5-bit Flip-around MDAC (2)





$$V_{out} = \frac{C_1+C_2}{C_2+\frac{C_1+C_2+C_P}{A}} V_{in} - \frac{C_1}{C_2+\frac{C_1+C_2+C_P}{A}} KV_R, \text{ where } K = \pm 1, 0$$

$$\Rightarrow V_{out} = \alpha V_{in} - K\beta V_R, \text{ where } \alpha = \frac{C_1+C_2}{C_2+\frac{C_1+C_2+C_P}{A}}, \text{ and } \beta = \frac{C_1}{C_2+\frac{C_1+C_2+C_P}{A}}$$

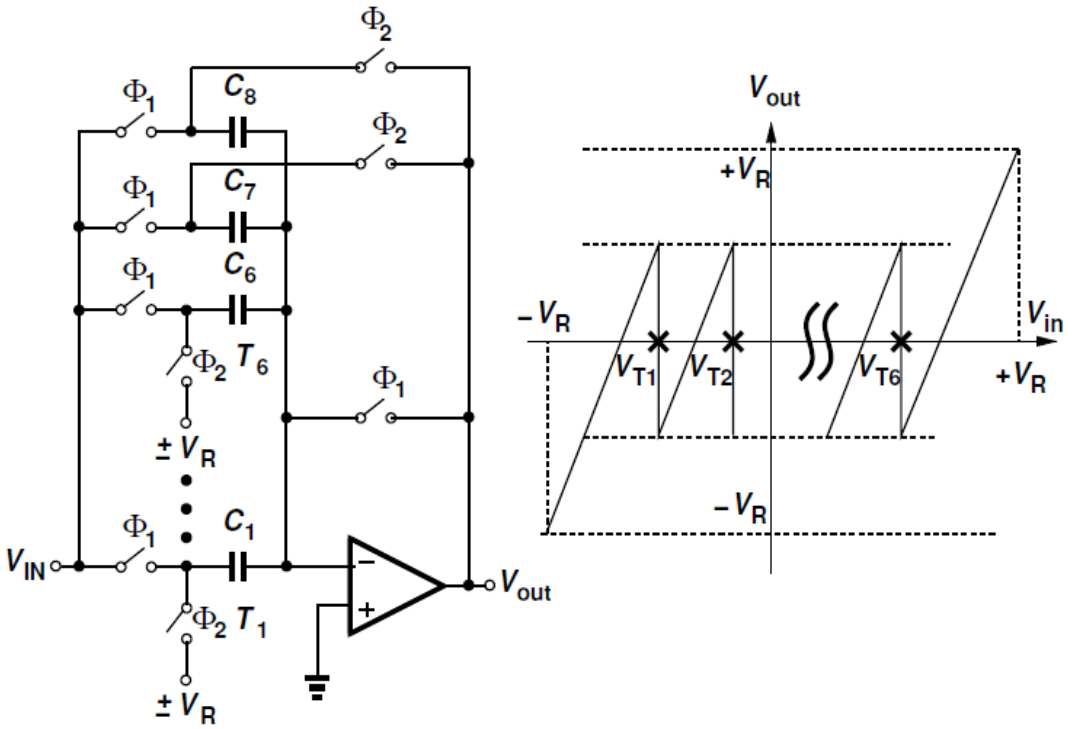
- Applying V_R the back-end ADC output can be given as:

$$V_{out} = \alpha V_R - \beta V_R \Rightarrow V_{out} = \frac{C_2}{C_2 + \frac{C_1 + C_2 + C_P}{A}} V_R \Rightarrow D_{BE} = \frac{C_2}{C_2 + \frac{C_1 + C_2 + C_P}{A}}$$

- The β obtained using the comparator forcing algorithm can be added to the above D_{BE} measurement to obtain α

* C. Ravi, V. Sarma, and B. Sahoo,“ IEEE NEWCAS, June 2015

Gain Calibration for 2.5-bit Flip-around MDAC



$$V_{out} = \frac{\sum_{i=1}^{8} C_i V_{in}}{C_7 + C_8 + \frac{\sum_{i=1}^{8} C_i + C_P}{A}} - \frac{\sum_{i=1}^{6} T_i C_i}{C_7 + C_8 + \frac{\sum_{i=1}^{8} C_i + C_P}{A}} V_R$$

$$\Longrightarrow V_{out} = \alpha V_{in} - \beta_i V_R$$

- Comparator forcing based calibration technique is used to obtain β_1 to β_6.
- Just as in 1.5-bit flip-around topology swapping capacitor changes the denominator and hence cannot be used to solve for the gain α.
- Applying the full-scale input to the MDAC and digitizing the output using the backend we obtain,

$$D_{BE} = \frac{C_7 + C_8}{C_7 + C_8 + \frac{\sum_{i=1}^{8} C_i + C_P}{A}}$$

- Now, $\beta_6 = \frac{\sum_{i=1}^{6} C_i}{C_7 + C_8 + \frac{\sum_{i=1}^{8} C_i + C_P}{A}}$.
- $\alpha = D_{BE} + \beta_6$
- Can be extended to 3.5-bit.

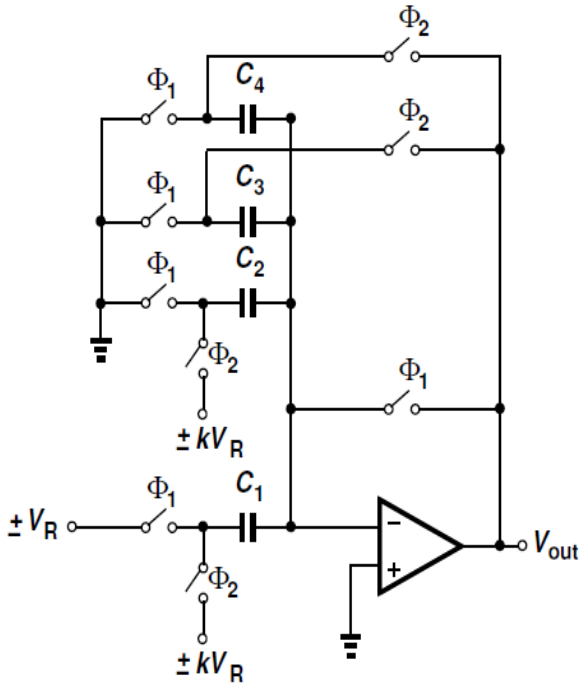
* C. Ravi, V. Sarma, and B. Sahoo," IEEE NEWCAS, June 2015

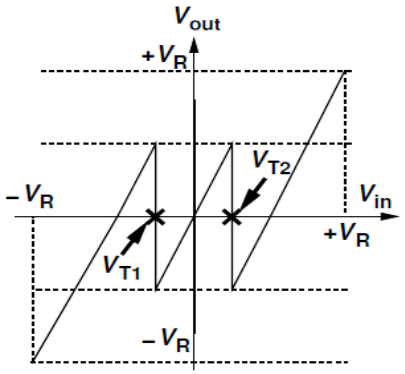
Calibration at Full-Speed

- **Speed of existing calibration methods are limited by**
  - **Circuitry which applies the calibration inputs**
- **Calibration at low speed doesn't capture the error in residue**
  - **Due to insufficient settling of the op amp at high frequency**
  - **Incorrect gain estimation**
- **In order to facilitate calibration at full-speed the calibration voltages have to be generated using capacitors switching to $\pm V_R$.**
- **This eliminates the resistor ladder to generate the calibration voltages.**

*** C. Ravi, V. Sarma, and B. Sahoo,“ IEEE NEWCAS, June 2015**

Calibration Signal Generation for 1.5-bit Stage (1)

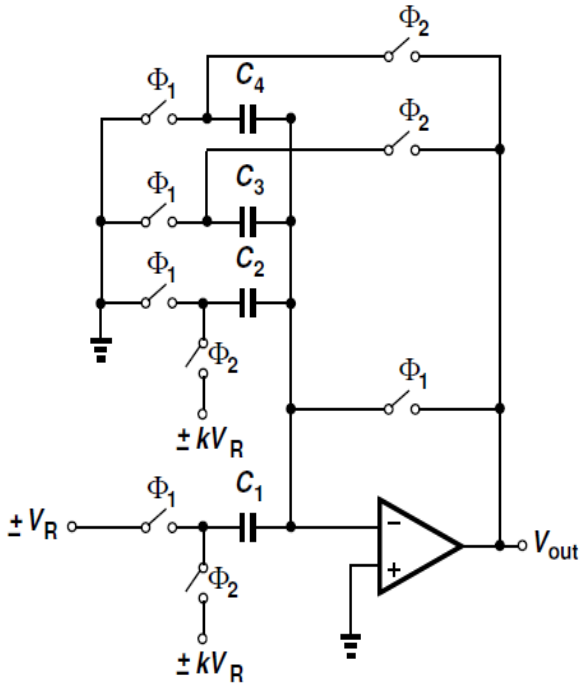


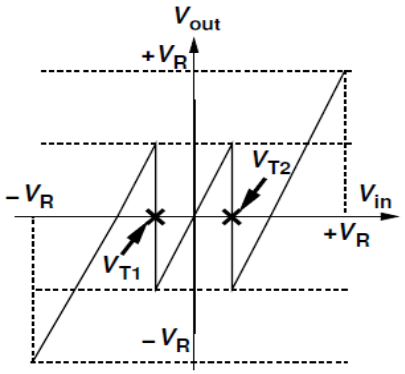


- **Split the sampling capacitor and the feedback capacitor into two equal unit capacitors**
- **During normal operation**
  - **Sampling phase: Input is sampled onto all the capacitors**
  - **Amplification phase: 2 capacitors are flipped around**
  - **Remaining two capacitors switch to KV_R**

*** C. Ravi, V. Sarma, and B. Sahoo,“ IEEE NEWCAS, June 2015**

Calibration Signal Generation for 1.5-bit Stage (2)





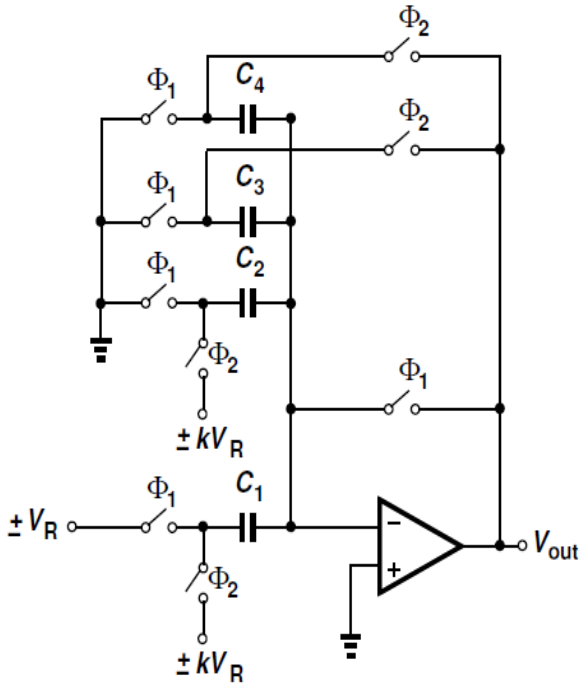
- During Calibration,
  - Sampling phase: $-VR$ sampled onto one sampling capacitors
  - Remaining capacitors connected to ground for applying $V_{T1} = -VR/4$.
  - Amplification phase: Two capacitors connected to KV_R
  - Two capacitors flipped around
- Resulting residue voltage is

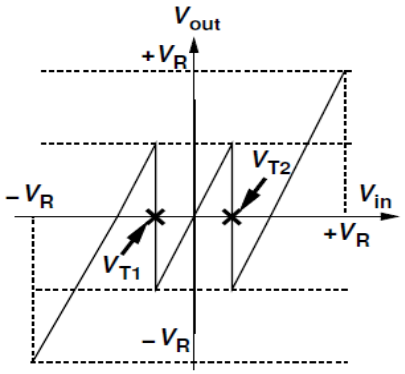
$$V_{out} = \frac{-V_R C_1 + KV_R(C_1 + C_2)}{C_3 + C_4 + \frac{C_1 + C_2 + C_3 + C_4}{A}}$$

- This residue is same as if $V_{IN} = -VR/4$ is applied

* C. Ravi, V. Sarma, and B. Sahoo," IEEE NEWCAS, June 2015

Calibration Signal Generation for 1.5-bit Stage (3)





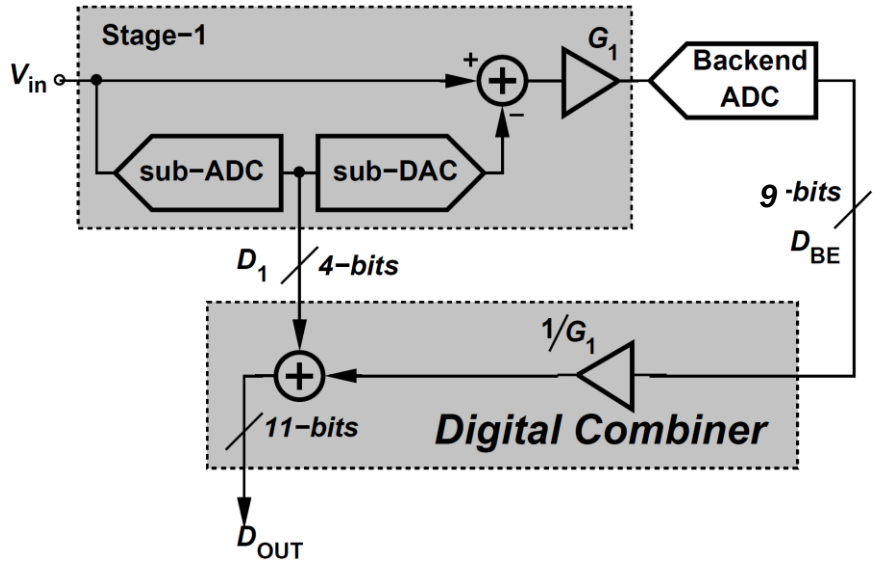
- **Similarly, we can mimic the generation of $V_{T2} = VR/4$ by**
  - **Applying V_R to one sampling capacitor**
  - **Remaining connected to ground**

*** C. Ravi, V. Sarma, and B. Sahoo,“ IEEE NEWCAS, June 2015**

Background Gain Calibration for Multi-bit, 1.5-bit, 2.5-bit, etc. MDACs

Pipeline Stage I/O Characteristic

- The input output characteristic of a 4-bit stage is:



$$V_{OUT} = \frac{\sum_{m=1}^{16} C_m V_{IN} - \sum_{m=1}^{15} C_m A_{m,j} V_R}{C_F - \frac{C_F + C_P + \sum_{m=1}^{16} C_m}{A}}$$

$$\Rightarrow V_{OUT} = \alpha V_{IN} - \frac{\sum_{m=1}^{15} C_m A_{m,j} V_R}{C_F - \frac{C_F + C_P + \sum_{m=1}^{16} C_m}{A}}$$

$$\Rightarrow V_{OUT} = \alpha V_{IN} - \beta_j V_R ,$$

$$\text{where } \beta_j = \frac{\sum_{m=1}^{15} C_m A_{m,j} V_R}{C_F - \frac{C_F + C_P + \sum_{m=1}^{16} C_m}{A}} \text{ and } \alpha = \frac{\sum_{i=1}^{16} C_m}{C_F - \frac{C_F + C_P + \sum_{m=1}^{16} C_m}{A}}.$$

- Dividing both sides by V_R we get,

$$D_{BE} = \alpha D_{IN} - \beta_j$$

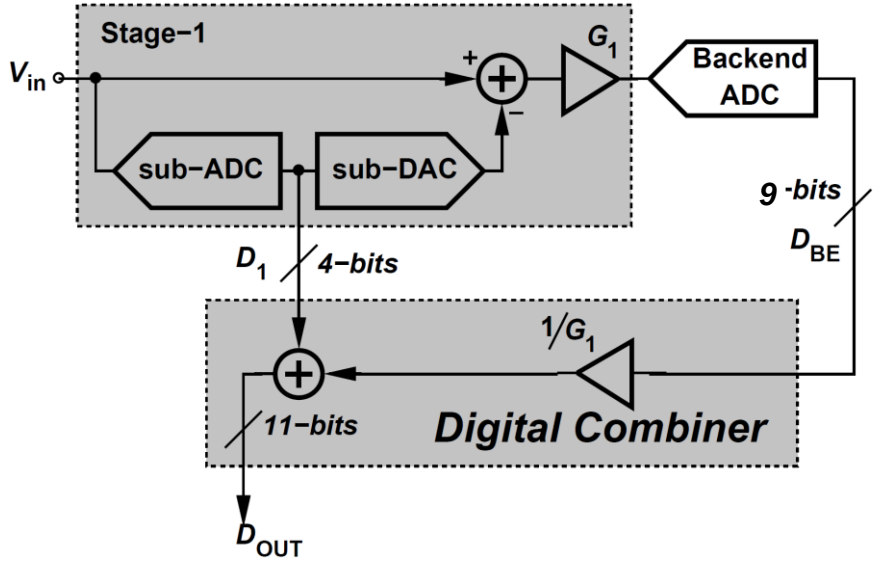
$$\Rightarrow D_{IN} = \frac{D_{BE}}{\alpha} + \gamma_j$$

where,

- γ_j is the capacitor mismatch ➔ independent of op amp gain
- α is the gain (G_1)➔ function of op amp gain.

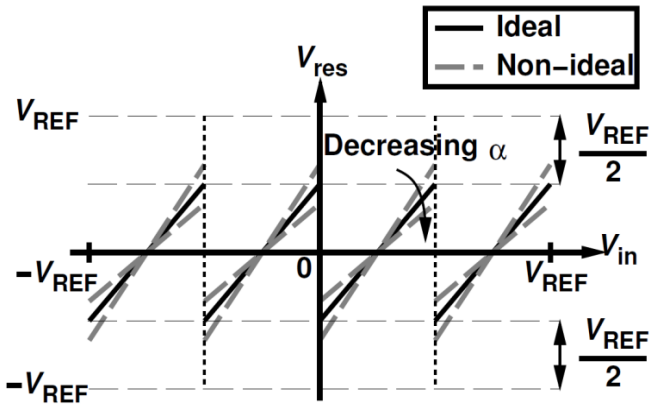
Proposed Calibration Algorithm

- **Initially estimate the gain (α=G_1) and the capacitor mismatch (γ_j) in the foreground using the calibration technique in [ϒ].**
- **Then estimate the inter-stage gain α, in the background.**

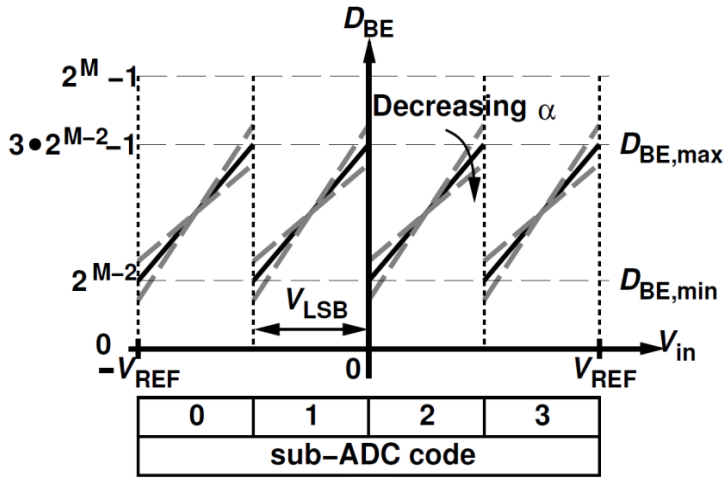


ϒB. Sahoo, and B. Razavi, ”A 10-bit 1-GHz 33-mW CMOS ADC,” *IEEE JSSC*, June 2013.

Pipelined Stage Residue Characteristic with MDAC



2-bit MDAC residue characteristics V_{res} Vs V_{in} with Gain variation



2-bit MDAC residue characteristics D_{BE} Vs V_{in} with Gain variation

- **MDAC gain (α) changes ➔ slope of the residue characteristic changes.**
- **Residue quantized by an ideal M-bit back-end to give a digital estimate, D_{BE},**

$$D_{BE,\min} = 2^{M-2} \text{ and } D_{BE,\max} = 3 \times 2^{M-2} - 1$$

- **Ideally $\alpha = D_{BE,\max}/VLSB/2$.**
- **Parameter drift changes $D_{BE,\min}$ and $D_{BE,\max}$.**

Calibration Algorithm

- Estimate the MDAC gain, α in the foreground mode using technique in [ϒ].
- Estimate $D_{BE,max1}$ in the background mode, immediately after the foreground calibration is done. Thus, $\alpha = D_{BE,max1}/V_{LSB}/2$.
- Calibration engine keeps on estimating $D_{BE,max}$. If the gain drifts a new back-end maximum, $D_{BE,max2}$ is obtained, resulting in $\alpha_{new} = D_{BE,max2}/V_{LSB}/2$.
- Thus,

$$\frac{\alpha_{new}}{\alpha} = \frac{D_{\max 2}}{D_{\max 1}} \Rightarrow \alpha_{new} = \alpha \frac{D_{\max 2}}{D_{\max 1}}$$

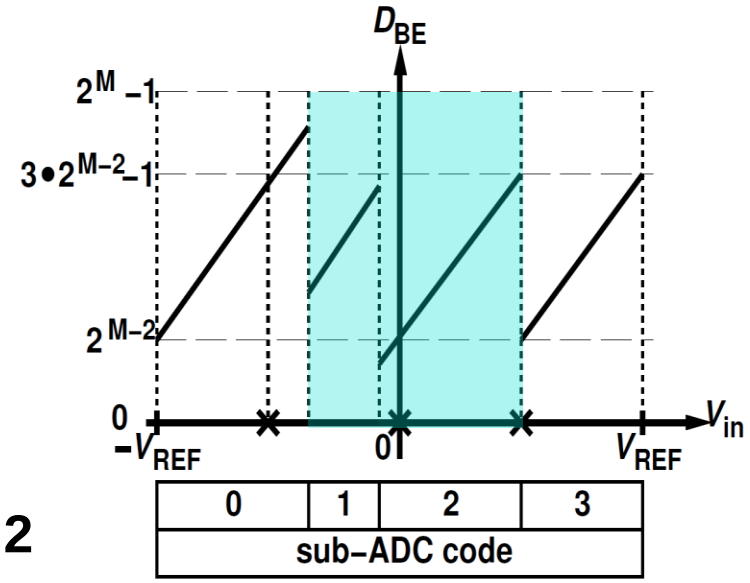
[ϒ] B. Sahoo, and B. Razavi, "A 10-bit 1-GHz 33-mW CMOS ADC," *IEEE JSSC*, June 2013.

Effect of Non-Idealities

- **The estimation of $D_{BE,\,MAX}$ can be corrupted due to the following non-idealities:**
  - **Comparator Offset**
  - **Capacitor mismatch**
  - **Thermal Noise**

Effect of Comparator Offset

- With comparator offset maximum back-end code changes from region to region, but slope in each region is the same.
- Maximum in any one region gives the accurate estimate of inter-stage gain
- The region should be such that the calibration can work even with lower signal swing
- For 2-bit MDAC, characteristic corresponding to output code of 1 or 2 is chosen

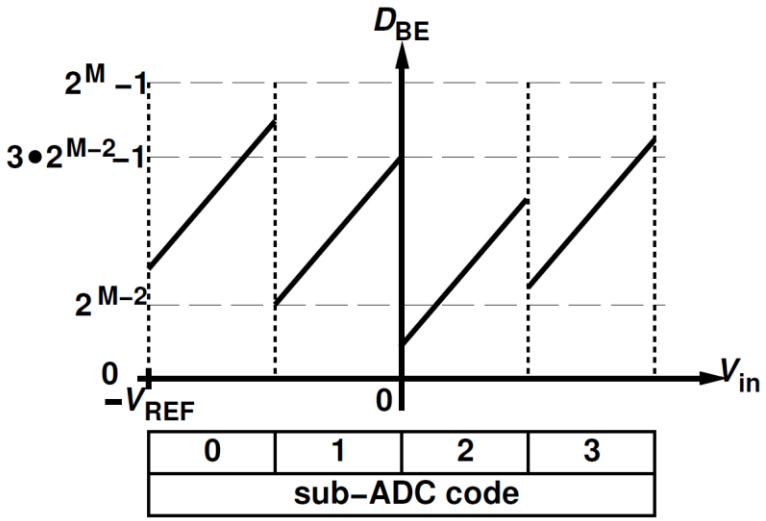


- For 3-bit and 4-bit MDACs calibration would work for 1/4th and 1/8th of the signal swing.

Proposed calibration would thus require a minimum swing that is either 12 dB or 18 dB below full scale.

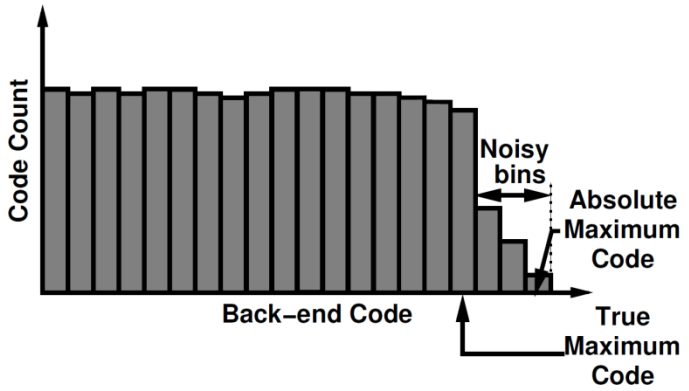
Effect of Capacitor Mismatch

- Capacitor mismatch changes the residue/back-end characteristic.
- Although the slope is the same in each region the maximum in each region is different.
- Calibration obtains the maximum back-end code for a particular region



Effect of Thermal Noise

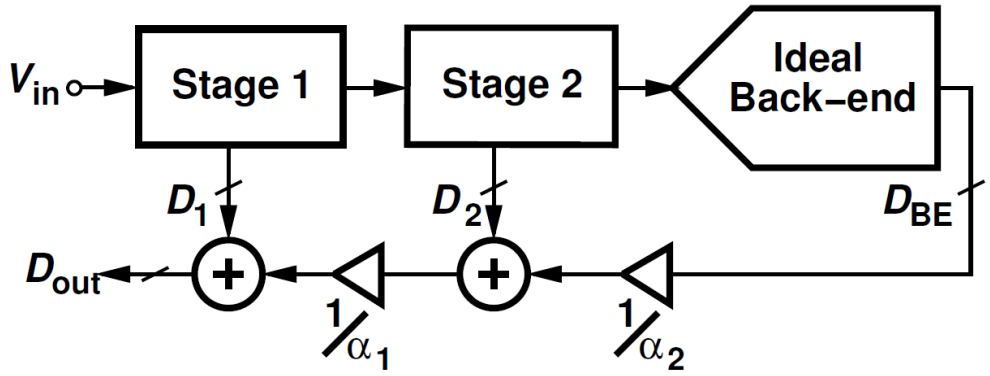
- Thermal noise corrupts the measurement of $D_{BE,max}$.
- Histogram of the back-end code estimates the true maximum code and eliminates the absolute maximum code.



- For a noisy bin to have the same height as that of a noiseless bin, the thermal noise should have a variance, σ_{NTH} > 10 LSB ➔ SNR degradation of approx. 30 dB.

➔ **Noisy bins cannot be of the same height as noiseless bins**

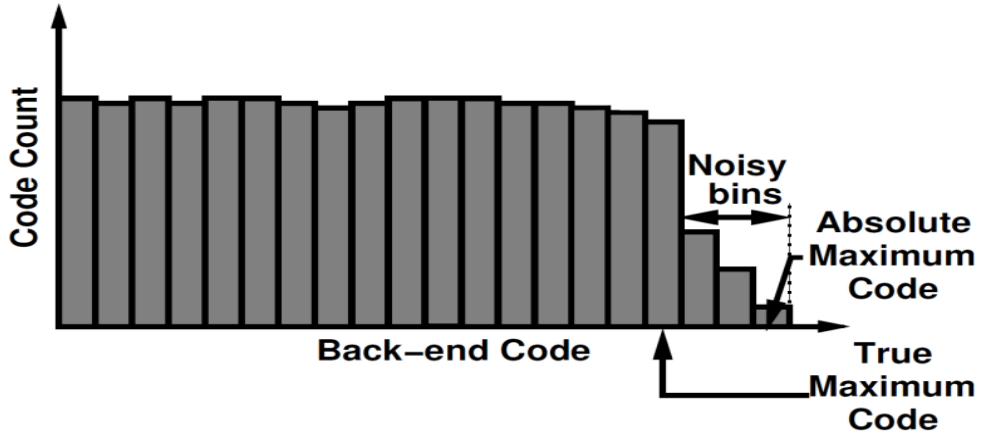
Multi-stage Gain Calibration



- **Algorithm first calibrates the 2nd stage that has an ideal back-end**
- **Consider the 2nd stage onwards as an ideal back-end and calibrate the 1st stage**
- **Calibration starts from the later stages and moves to the 1st stage**

Digital Hardware Complexity

- Histogram requires counters and finding the maximum requires comparators.
- For M-bit back-end, do we need 2^M comparators and counters! **No ☺**



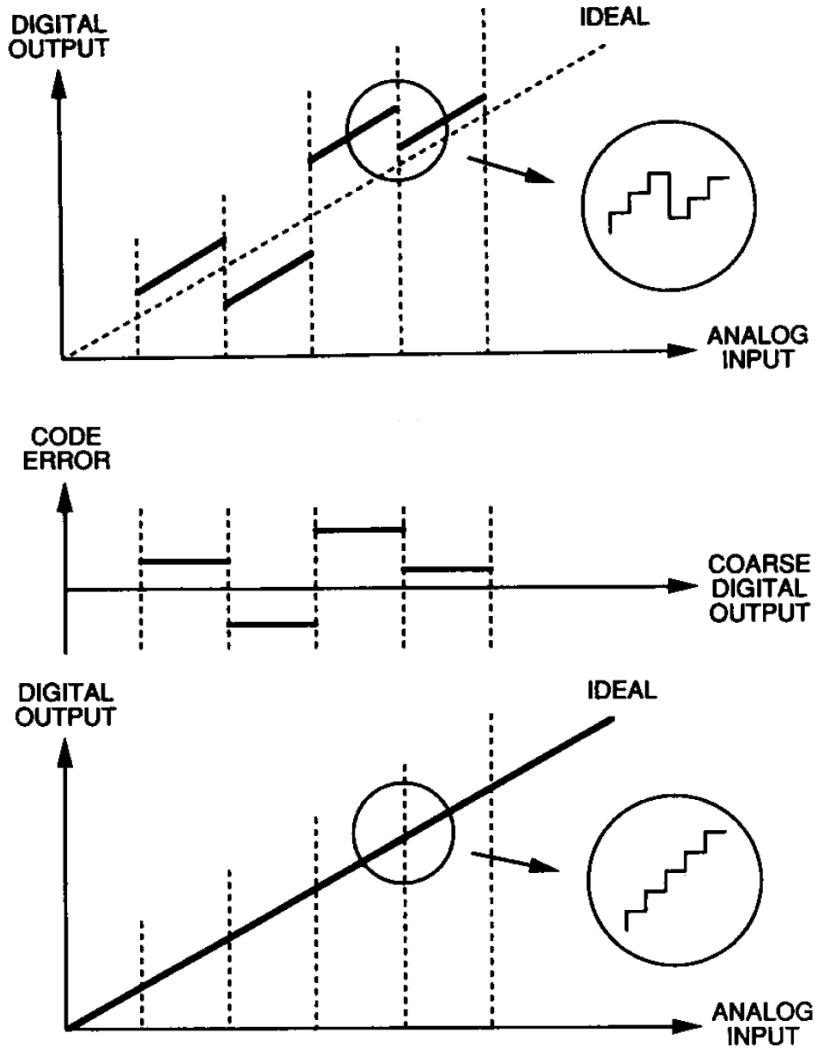
- Foreground calibration gives an initial estimate of $D_{BE,max}$ and noise corrupts this by maximum of ±10 to ±20 back-end codes
- Hence maximum of 40 digital comparators and counters used
- Division operation is realized using Newton-Raphson technique, which requires a multiplier and adder

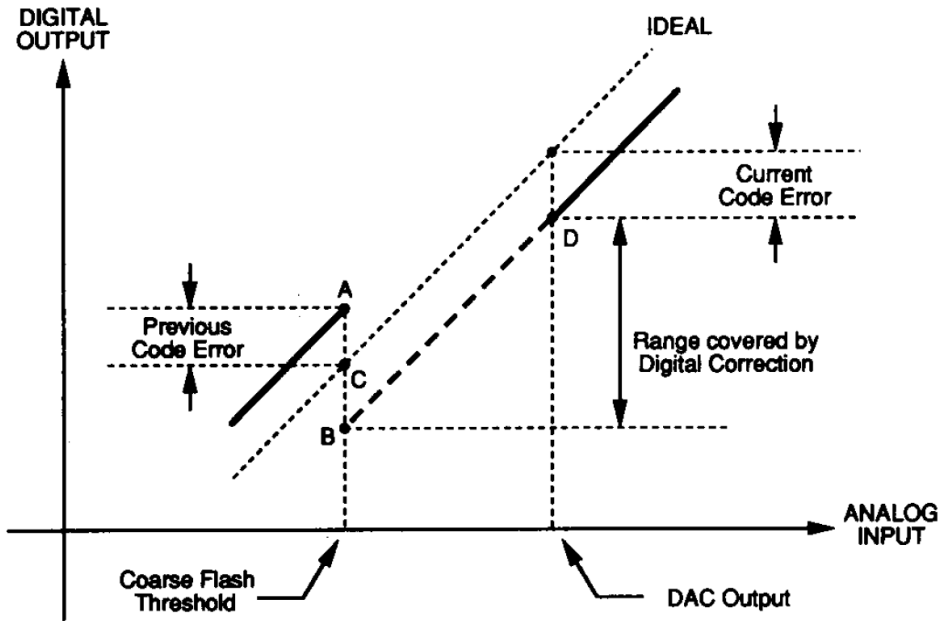
Survey of Digital Calibration Techniques

Survey of Calibration Techniques

Sl. No.	Author (Year)	Type of MDAC	Foreground/ Background	Notes
1.	Lee (1992)	Multi-bit	Foreground	Capacitor mismatch and gain error
2.	Karanicolas (1992)	1-bit	Foreground	Gain error
3.	Erdogan (1999)	1-bit	Background	Gain error
4.	Ming (2001)	1.5-bit non-flip around	Background	Gain error
5.	Li (2003)	1.5-bit flip-around	Background	Gain error
6.	Murmann (2003)	3-bit	Background	Capacitor mismatch, op amp nonlinearity, and gain error
7.	Wang (2004)	1.5-bit flip-around	Background	Gain error and capacitor mismatch.
8.	Verma (2009)	1.5-bit flip-around	Foreground	Gain error, op amp nonlinearity, and capacitor mismatch

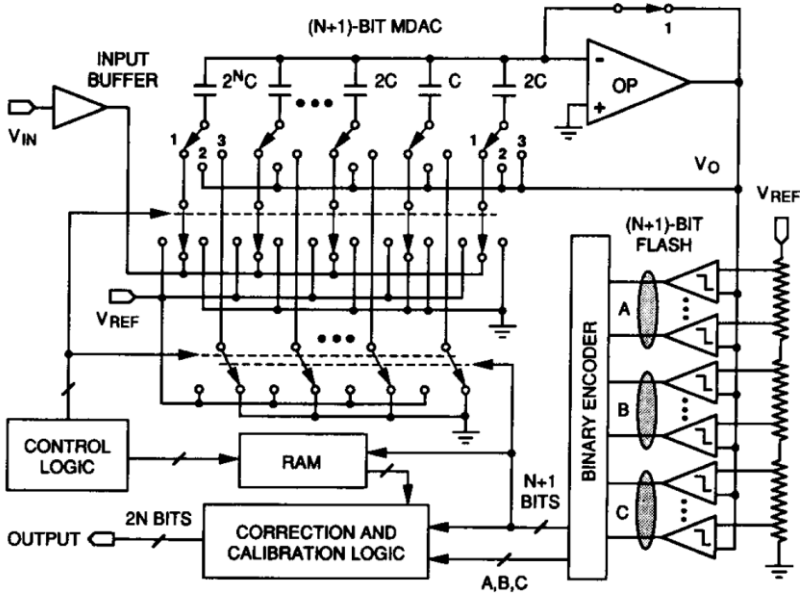
Calibration of Multistep ADC (1)

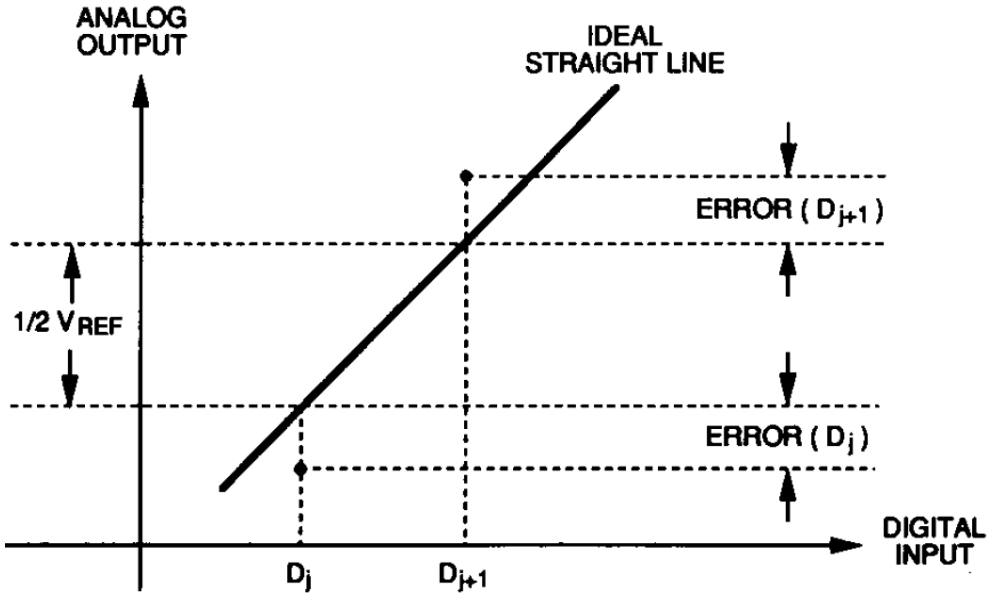




S-H. Lee and B-S. Song, IEEE JSSC**, vol. 27, pp. 1679-1688, Dec. 1992.**

Calibration of Multistep ADC (2)

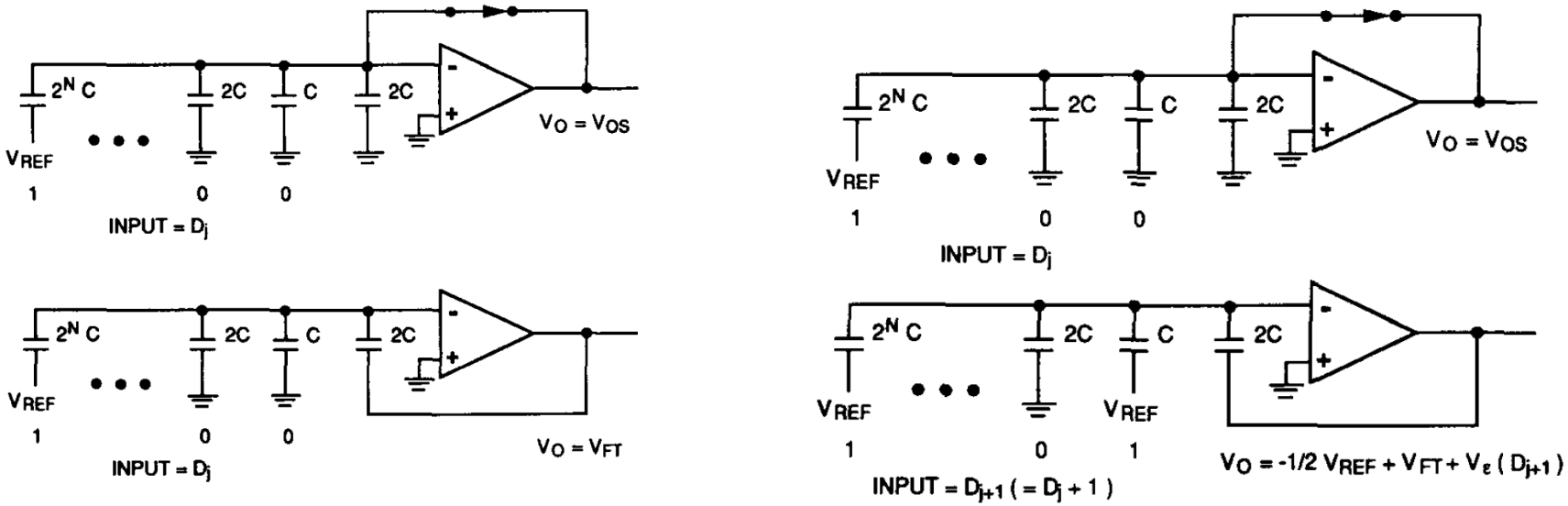




- Error (D_j) and Error (D_{j+1}) are the errors with digital codes D_j and D_{j+1}.

S-H. Lee and B-S. Song, IEEE JSSC, vol. 27, pp. 1679-1688, Dec. 1992.

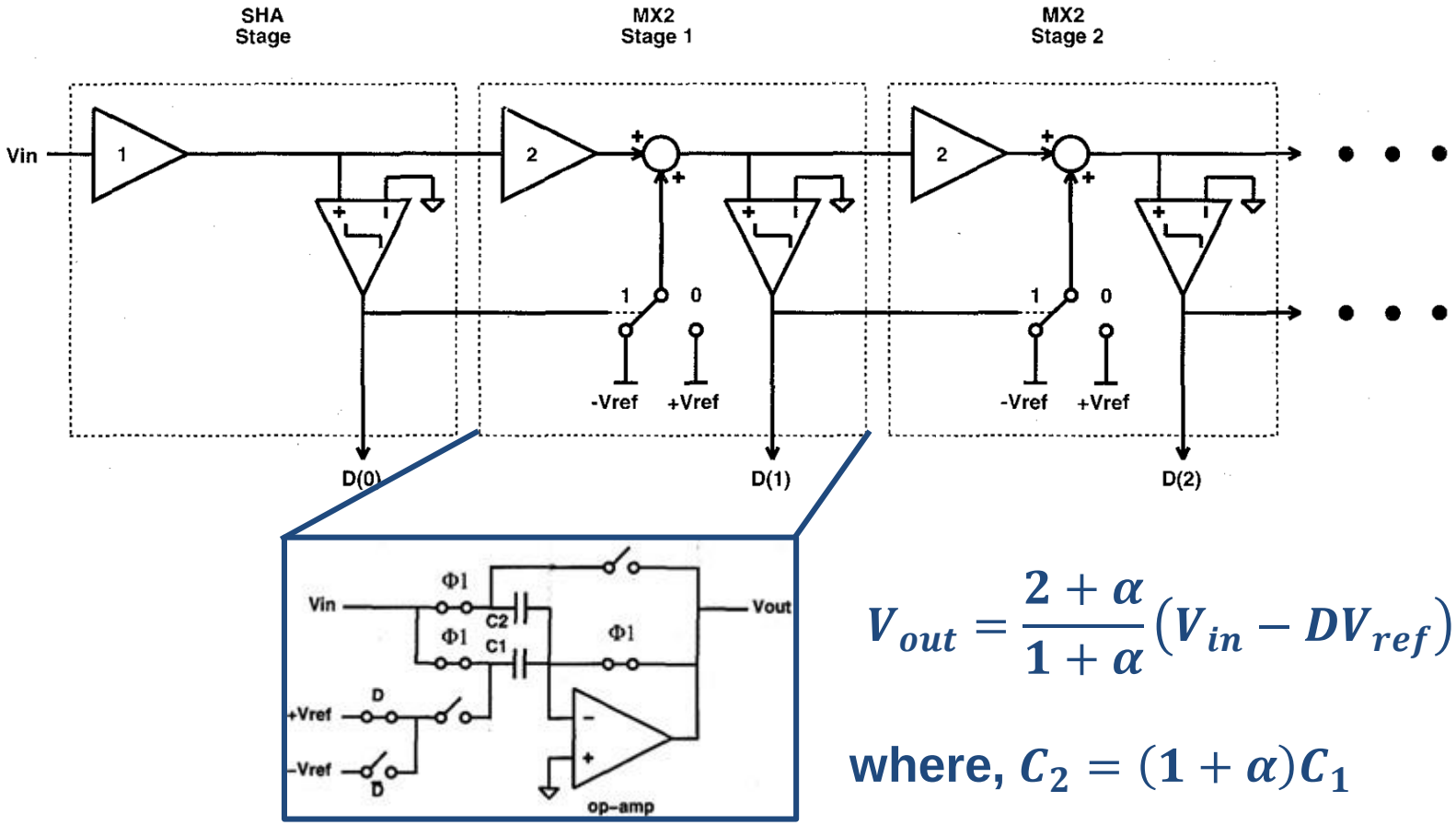
Calibration of Multistep ADC (3)



- Measure the feedthrough voltage, i.e. offset, charge-injection, etc.
- Change the digital code by "1" and measure the output voltage.
- When digital code is changed by "1" then the change in the output should be exactly ½ V_{REF}.
- Thus the Error (D_{j+1}) can be obtained from the above measurement and stored in memory.

S-H. Lee and B-S. Song, IEEE JSSC, vol. 27, pp. 1679-1688, Dec. 1992.

15-bit Self Calibrated Pipelined ADC (1)



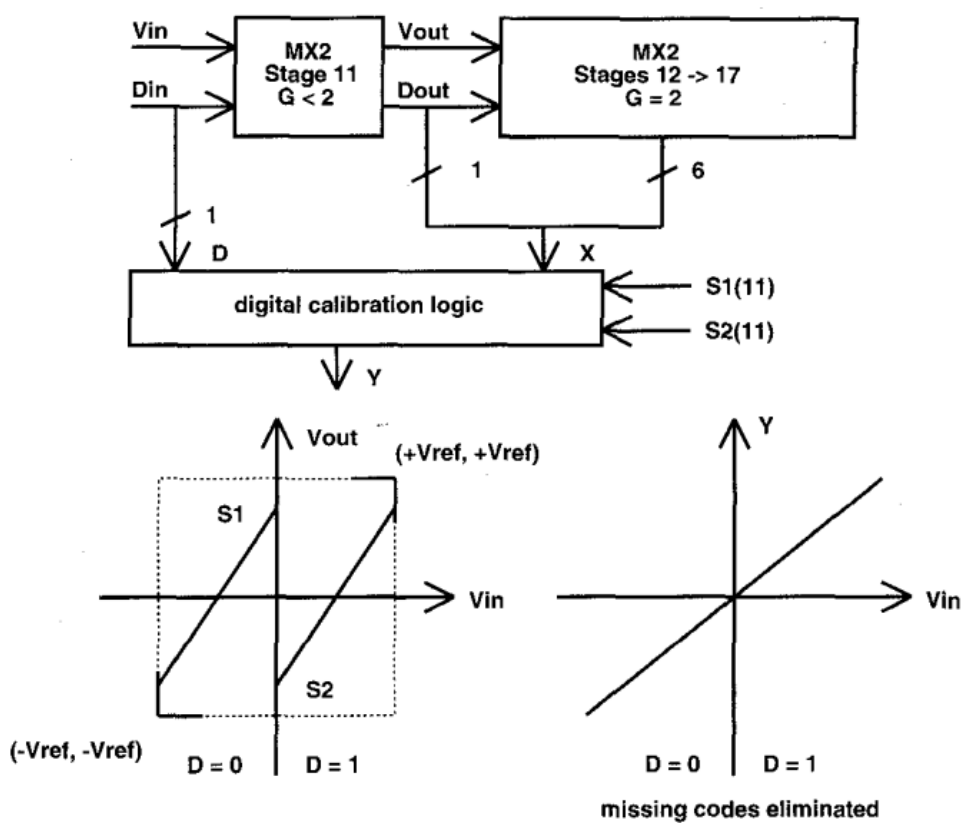
$$V_{out} = \frac{2+\alpha}{1+\alpha}\left(V_{in} - DV_{ref}\right)$$

where, $C_2 = (1+\alpha)C_1$

- Capacitor mismatch is merged with the gain term.

A. Karanicolas and H. S. Lee, IEEE JSSC, vol. 28, pp. 1207-1215, Dec. 1993.

15-bit Self-Calibrated Pipelined ADC (2)



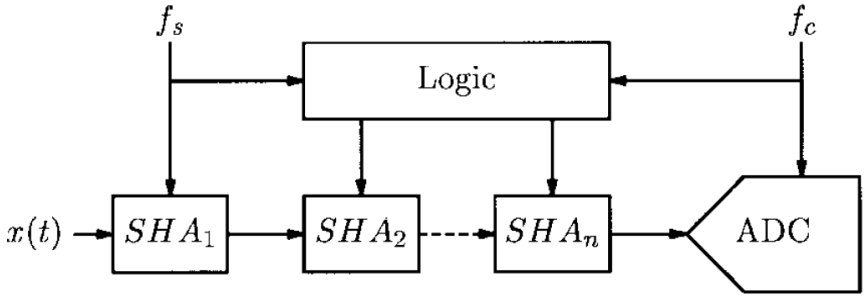
- Measure two quantities S_1 and S_2 by applying V_{in}=0 and forcing D=0 and D=1.
- Thus, the output can be given by,

$$Y = X \text{ if } D = 0$$
$$= X + S_1 - S_2 \text{ if } D = 1.$$

- Calibration estimates only S_1 and S_2 for each stage, stores them and then uses them in the digital calibration logic.
- Calibration does not require multiplication.
- Calibration starts from the later stages and moves to the earlier stages.
- Difficult for a multi-bit stage.

A. Karanicolas and H. S. Lee, IEEE JSSC, vol. 28, pp. 1207-1215, Dec. 1993.

Queue Based Algorithmic ADC Calibration (1)



- Queue based background gain error calibration.
- Having "n" sample-and-hold (SHA) and choosing $f_c > f_s$, time slots for calibrating the ADC can be obtained without compromising the normal operation of the ADC.
- The number of SHAs is given by:

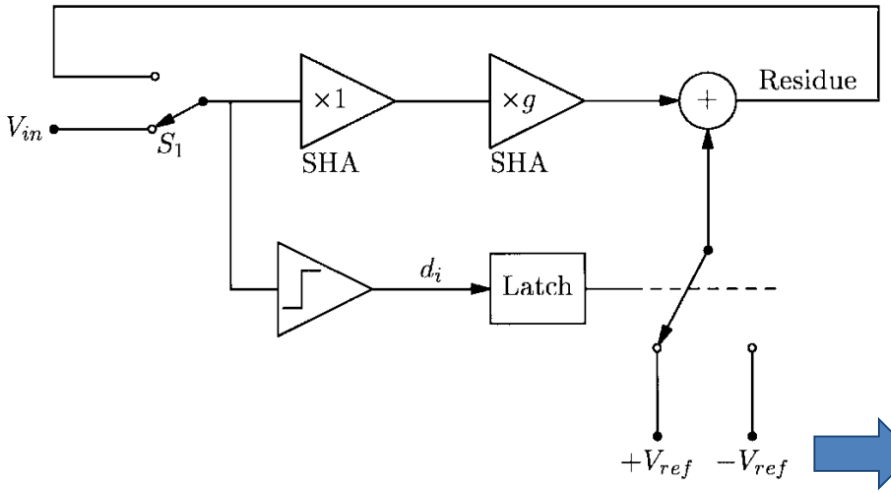
$$n \geq \left\lceil \frac{T_{cal}}{T_S} \right\rceil$$

where, T_{cal} is the calibration time and $T_s = 1/f_s$.

- Each of the SHA adds noise and degrades the SNR of the ADC.
- Also the additional SHA's consume significant power.
- The paper demonstrates this for a Algorithmic ADC.
- It can also be extended to a pipelined ADC.

O. E. Erdogan, P. J. Hurst, and S. H. Lewis, IEEE JSSC, vol. 34, pp. 1812-1820, Dec. 1999.

Queue Based Algorithmic ADC Calibration (2)



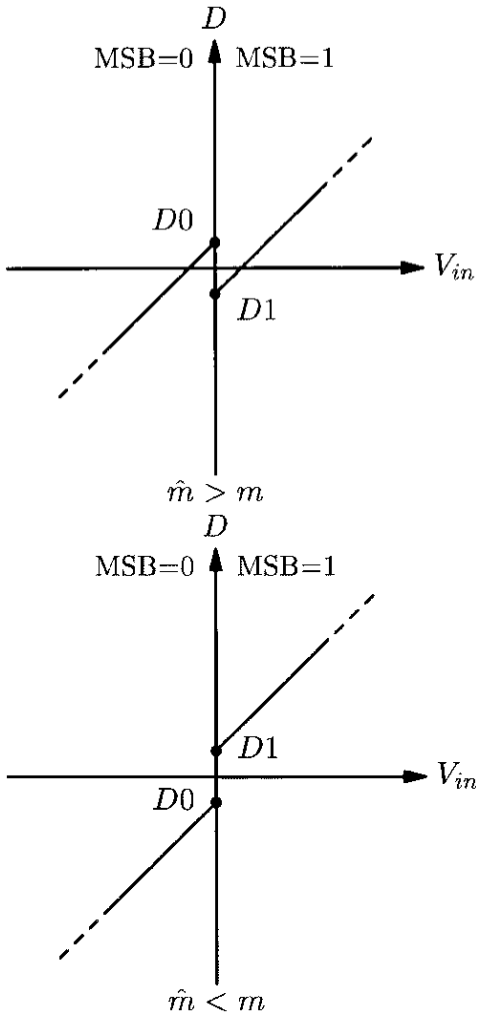
$$\begin{aligned}
\text{res}_1 &= g\,V_{\text{in}} - d_1 V_{\text{ref}} \\
\text{res}_2 &= g\,\text{res}_1 - d_2 V_{\text{ref}} \\
&= g^2 V_{\text{in}} - g\,d_1 V_{\text{ref}} - d_2 V_{\text{ref}} \\
&\vdots \\
\text{res}_N &= V_{\text{in}} g^N - d_1 g^{N-1} V_{\text{ref}} - \cdots - d_N V_{\text{ref}} \\
&= g^N \left(V_{\text{in}} - \sum_{i=1}^{N} g^{-i} d_i V_{\text{ref}} \right)
\end{aligned}$$

$$\frac{V_{\text{in}}}{V_{\text{ref}}} = \sum_{i=1}^{N} g^{-i} d_i + \epsilon \quad |\epsilon| \le \frac{1}{g^N}$$

$$D = \sum_{i=1}^{N} m^i d_i \quad m = \frac{1}{g}$$

O. E. Erdogan, P. J. Hurst, and S. H. Lewis, IEEE JSSC, vol. 34, pp. 1812-1820, Dec. 1999.

Queue Based Algorithmic ADC Calibration (3)



- After the queue is empty the ADC goes into calibration mode.
- The ADC uses a 1-bit architecture just like in Karanicolas 1993.
- The nominal gain "$m < 2$" to make sure that there are no missing levels.
- Since the actual value of "m" is not known an initial estimate of "$\hat{m}$" is used to obtain the digital output:

$$D = \sum_{i=1}^{N} \hat{m}^i \, d_i$$

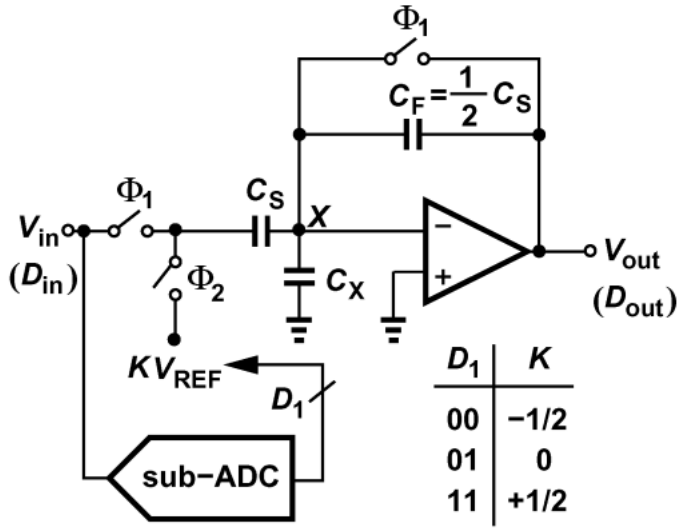
- During calibration an input of 0 V is applied and the comparator output is forced to "1" and "0" to obtain respectively D_1 and D_0.
- LMS is used to estimate "$\hat{m}$" as per the following:

$$\hat{m}(j+1) = \hat{m}(j) + \mu(D_1 - D_0 - 1LSB)$$

O. E. Erdogan, P. J. Hurst, and S. H. Lewis, IEEE JSSC, vol. 34, pp. 1812-1820, Dec. 1999.

8-bit Pipelined ADC With Background Calibration (1)

1.5-bit MDAC architecture

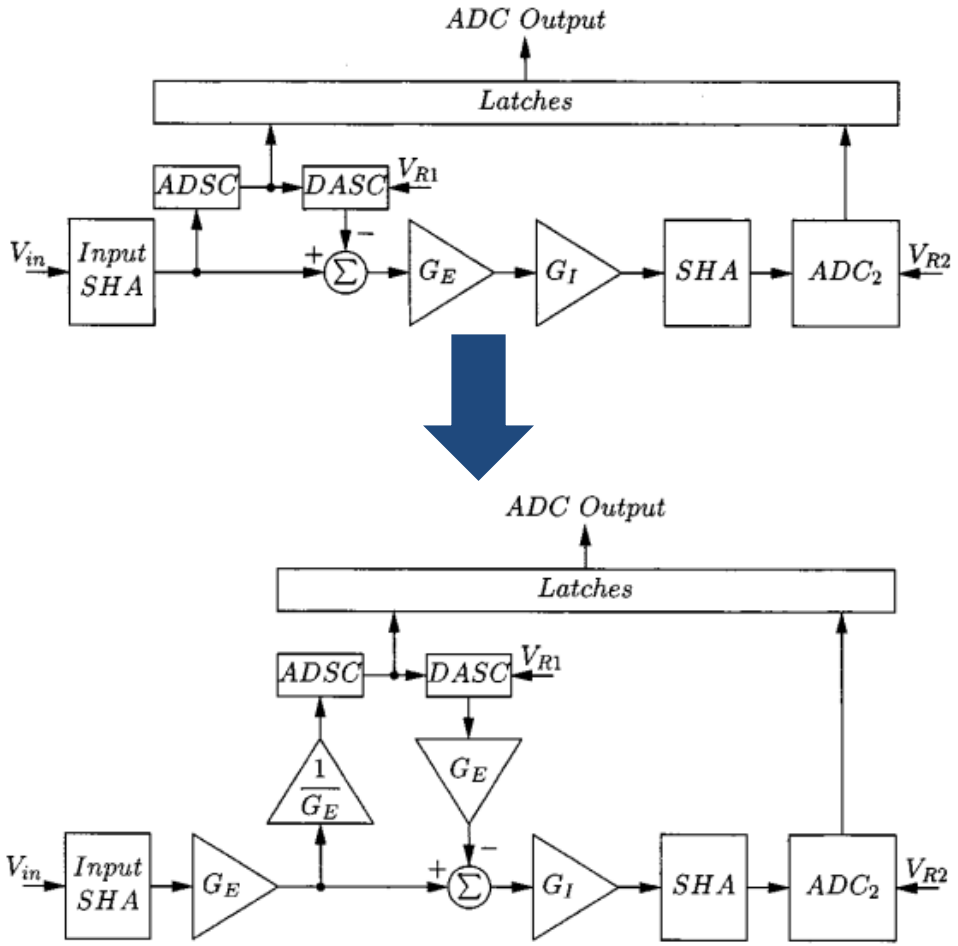


D_1	K
00	−1/2
01	0
11	+1/2

$$V_{out} = \frac{C_S}{C_F}\left[\frac{1}{1+\frac{1}{A}+\frac{C_S+C_X}{AC_F}}\right](V_{in} - kV_{REF})$$

➔ $V_{out} = G_I G_E (V_{in} - kV_{REF})$

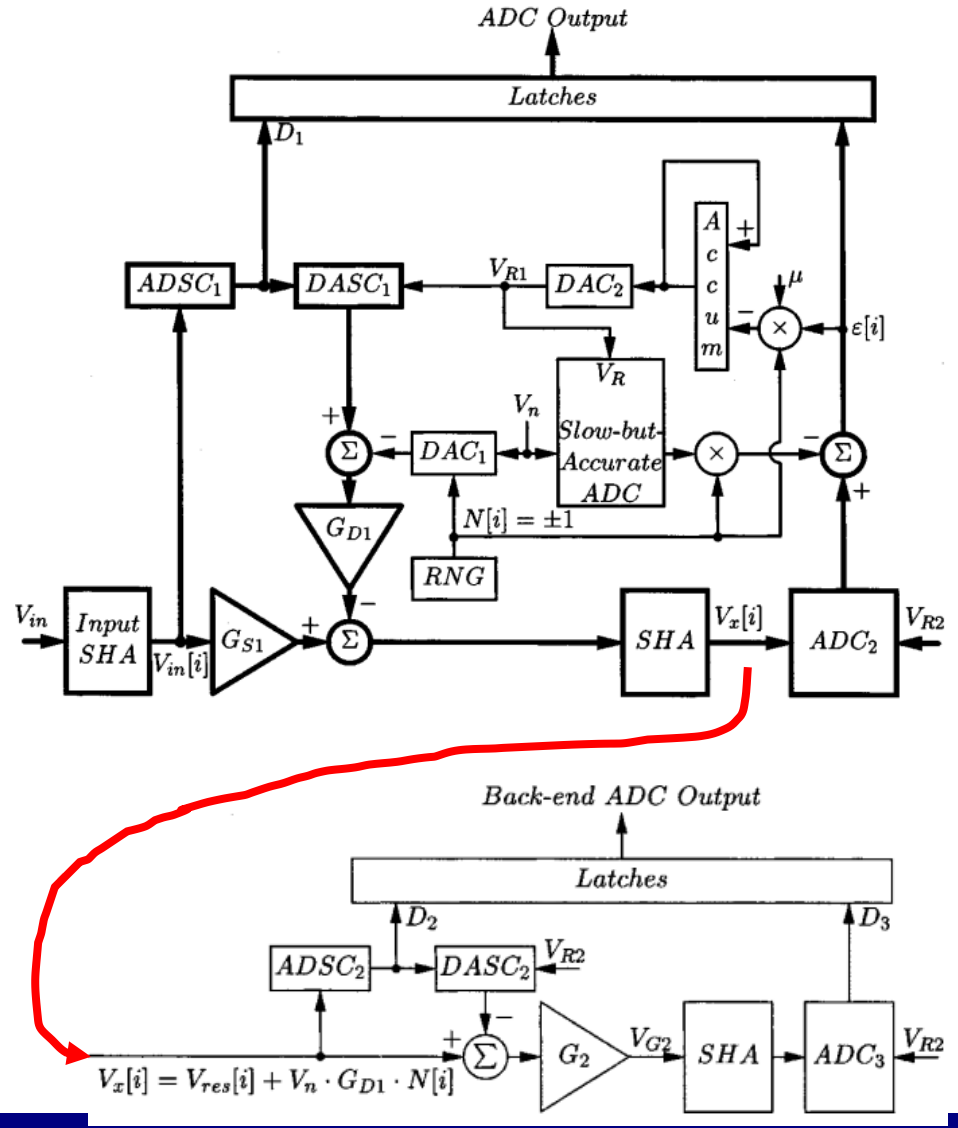
where, $G_I = \frac{C_S}{C_F}$ and $G_E = \frac{1}{1+\frac{1}{A}+\frac{C_S+C_X}{AC_F}}$



Adjust $V_{R1} = \frac{V_{R2}}{G_E}$ to overcome the gain-error.

J. Ming and S. H. Lewis, IEEE JSSC, vol. 36, pp. 1489-1497, Oct. 2001.

8-bit Pipelined ADC With Background Calibration (2)

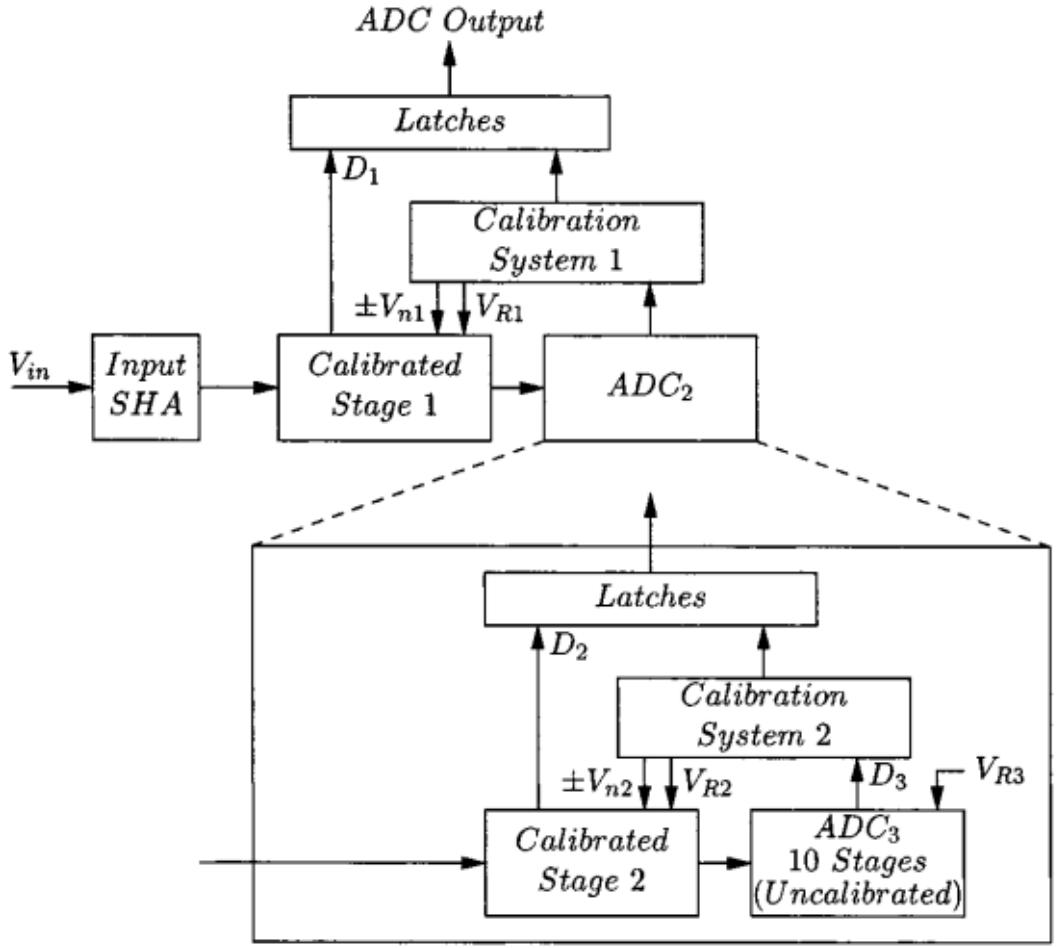


- A pseudo-random generator generates a ± 1 digital number.
- The random number is converted to analog by DAC_1.
- The output of DAC_1 is digitized by the back-end ADC and by a slow-but-accurate ADC.
- The slow-but-accurate ADC output should be subtracted from the back-end ADC output to recover the digital representation of Vin.
- The gain error of the stage can be obtained if ε_i does not contain the random input $N(i)$. This is possible if:

$$\frac{V_n G_{D1}}{V_{R2}} - \frac{V_n}{V_{R1}} = 0 \rightarrow V_{R1} = \frac{V_{R2}}{G_{D1}}$$

J. Ming and S. H. Lewis, IEEE JSSC, vol. 36, pp. 1489-1497, Oct. 2001.

8-bit Pipelined ADC With Background Calibration (3)

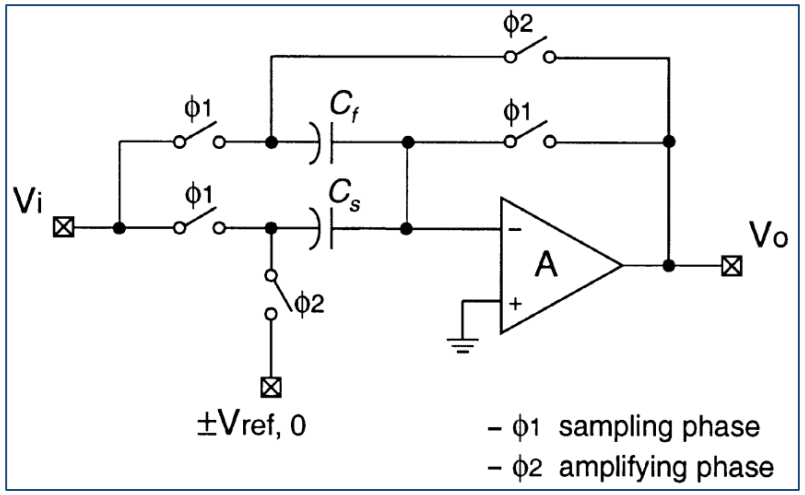


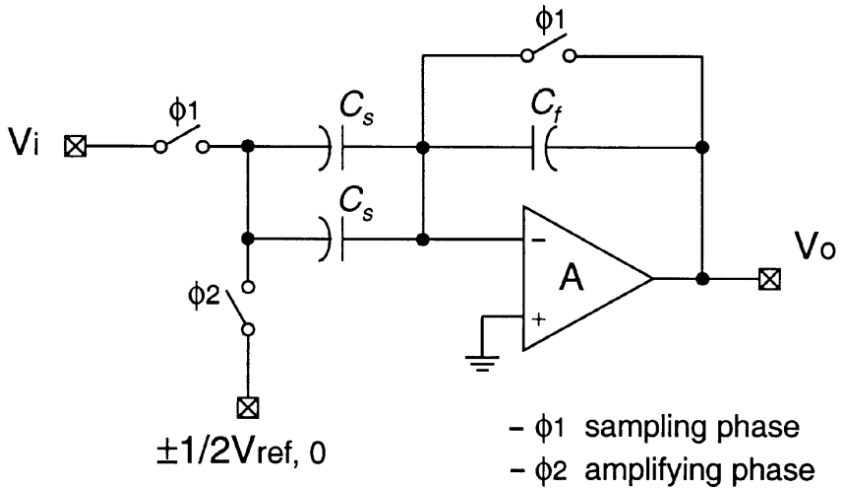
Multi-stage calibration

- Needs extensive analog hardware for calibration.
- The DAC in the calibration system should be accurate ➔ difficult to calibrate high resolution ADCs (> 10-bits).
- Calibration technique can effectively calibrate 1.5-bit/stage architecture and not multi-bit architecture.
- Extension to multi-bit architecture is very hardware intensive.

J. Ming and S. H. Lewis, IEEE JSSC, vol. 36, pp. 1489-1497, Oct. 2001.

Radix Based Calibration (1)





$$V_O = \frac{1}{1+\frac{C_S+C_F}{AC_F}}\left(\frac{C_S+C_F}{C_F}V_i - \frac{C_S}{C_F}DV_{ref}\right)$$

$$\rightarrow V_O = (1+\delta)(2+\alpha)\left\{V_i - \frac{(1+\beta)}{(2+\alpha)}DV_{ref}\right\}$$

where, $(1+\delta) = \frac{1}{1+\frac{C_S+C_F}{AC_F}}$, $(2+\alpha) = \frac{C_S+C_F}{C_F}$,

and $(1+\beta) = \frac{C_S}{C_F}$

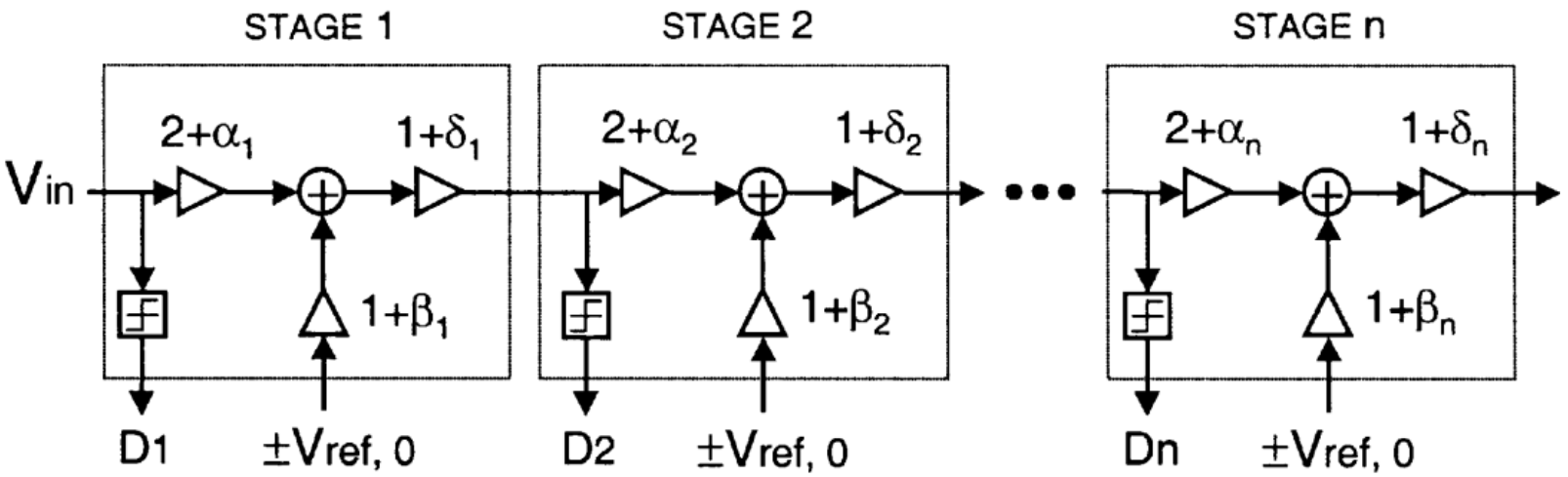
$$V_O = \frac{2C_S}{C_F}\frac{1}{1+\frac{2C_S+C_F}{AC_F}}\left(V_i - \frac{1}{2}DV_{ref}\right)$$

$$\rightarrow V_O = (1+\delta)(2+\alpha)\{V_i - DV_{ref}\}$$

where, $(1+\delta) = \frac{1}{1+\frac{C_S+C_F}{A}}$ and $(2+\alpha) = \frac{C_S+C_F}{C_F}$,

J. Li and Un-Ku Moon, IEEE TCAS-I, vol. 50, pp. 531-538, Sept. 2003.

Radix Based Calibration (2)



Representation of the pipelined ADC with each stage using 1.5-bit non-flip around topology.

- The digital output can be represented as:

$$D_O = D_n + D_n(ra) + D_n(ra)^2 + \cdots + D_1(ra)^{n-1}$$

where, $ra = (1+\delta)(2+\alpha)$ and reference voltage of each stage is scaled.

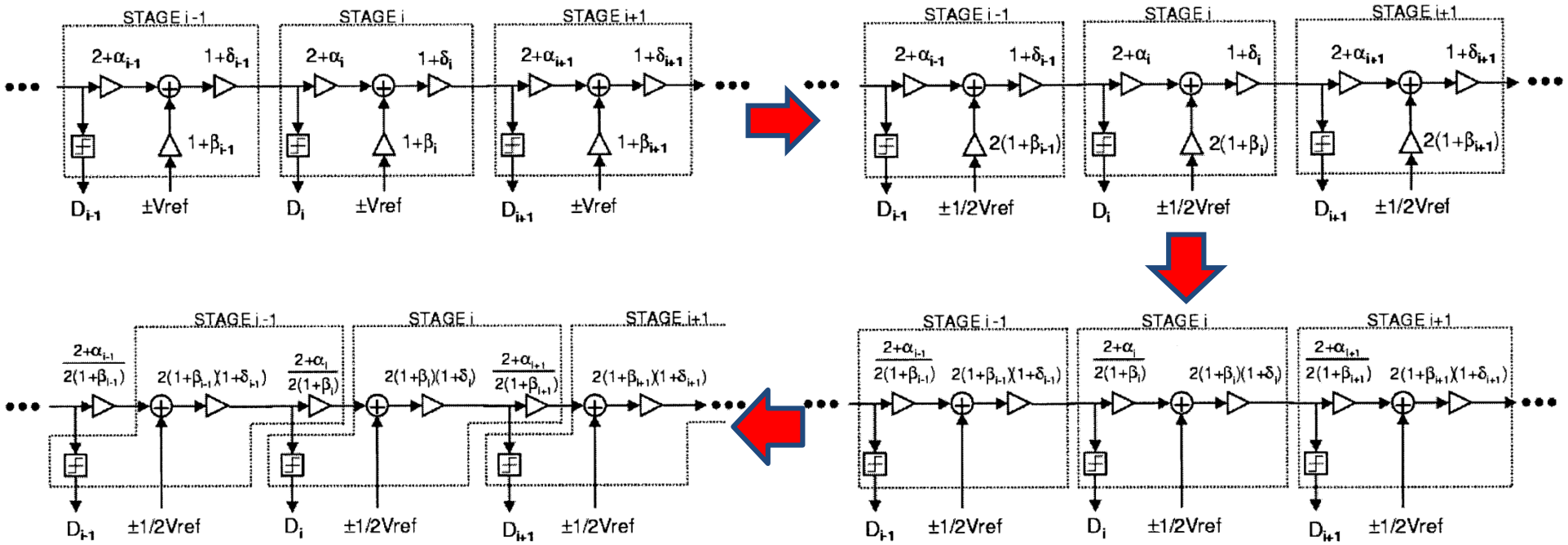
- Since the reference is scaled for each stage this is not attractive.
- However if each stage uses a non-flip-around topology then,

$$D_O = D_n + D_n(ra) + D_n(ra)^2 + \cdots + D_1(ra)^{n-1}$$

where, $ra = (1+\delta)(2+\alpha)$ and reference voltage of each stage is not scaled.

J. Li and Un-Ku Moon, IEEE TCAS-I, vol. 50, pp. 531-538, Sept. 2003.

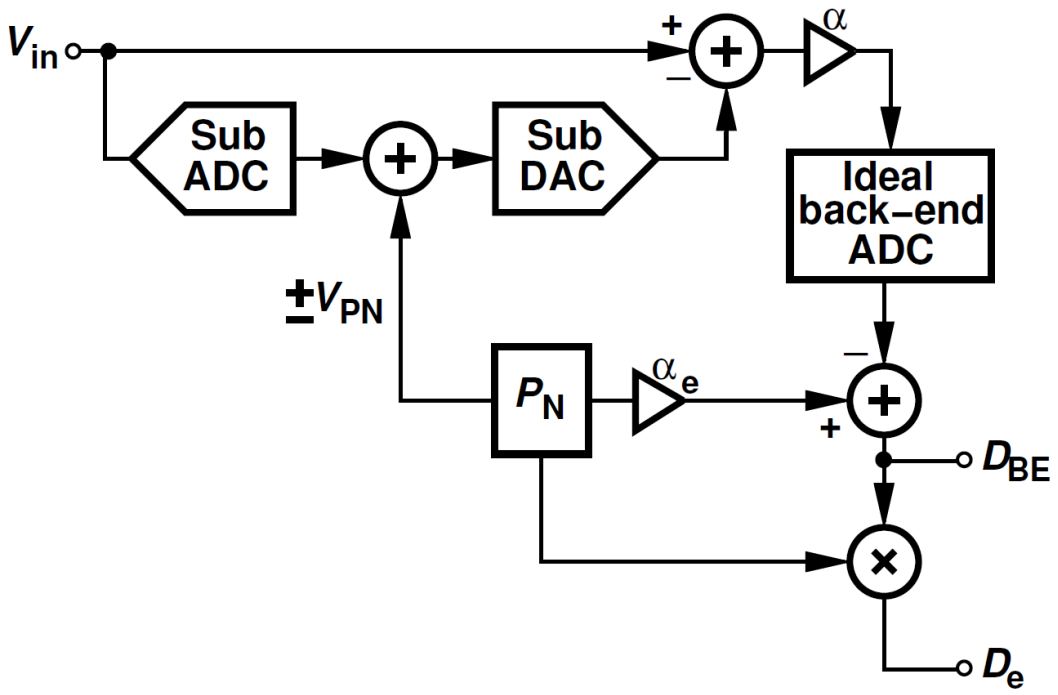
Radix Based Calibration (3)



- The new radix is . $ra = (1+\beta_i)(1+\delta_i)\frac{(2+\alpha_{i+1})}{(1+\beta_{i+1})}$.
- The reference voltage is not scaled from stage-to-stage.

J. Li and Un-Ku Moon, IEEE TCAS-I, vol. 50, pp. 531-538, Sept. 2003.

Radix Based Calibration (4)



$$D_{BE} = P_N \cdot \alpha_e - D_{res} - P_N \cdot \alpha$$
$$\Rightarrow D_e = \alpha_e - \alpha - P_N \otimes D_{res}$$
$$\Rightarrow \alpha = \alpha_e - D_e$$

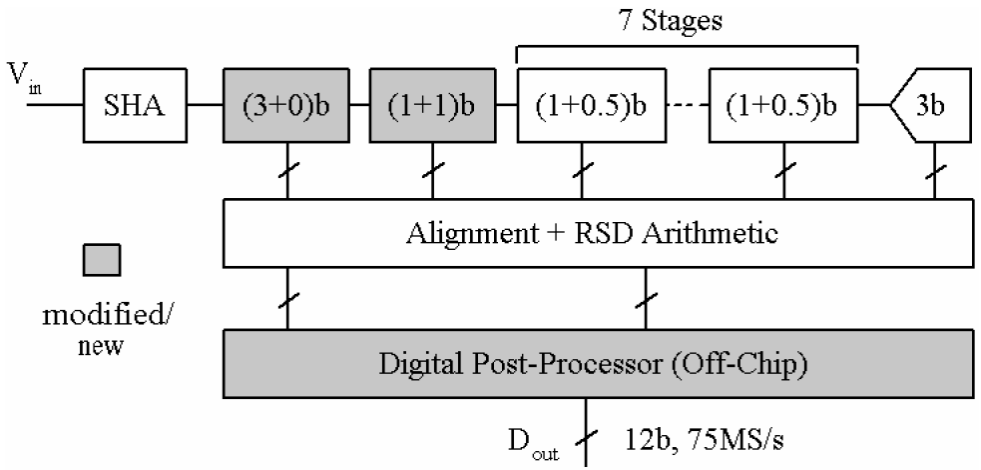
J. Li and Un-Ku Moon, IEEE TCAS-I, vol. 50, pp. 531-538, Sept. 2003.

Radix Based Calibration (5)

- Large convergence time as D_{BE} has to be correlated for a long time to guarantee that $P_N \otimes D_{res}$ vanishes.
- Generation of precise analog voltage $\pm V_{PN}$ whose digital value is PN.
- Reduction in dynamic range of the ADC due to injection of pseudorandom voltage $\pm V_{PN}$.

J. Li and Un-Ku Moon, IEEE TCAS-I, vol. 50, pp. 531-538, Sept. 2003.

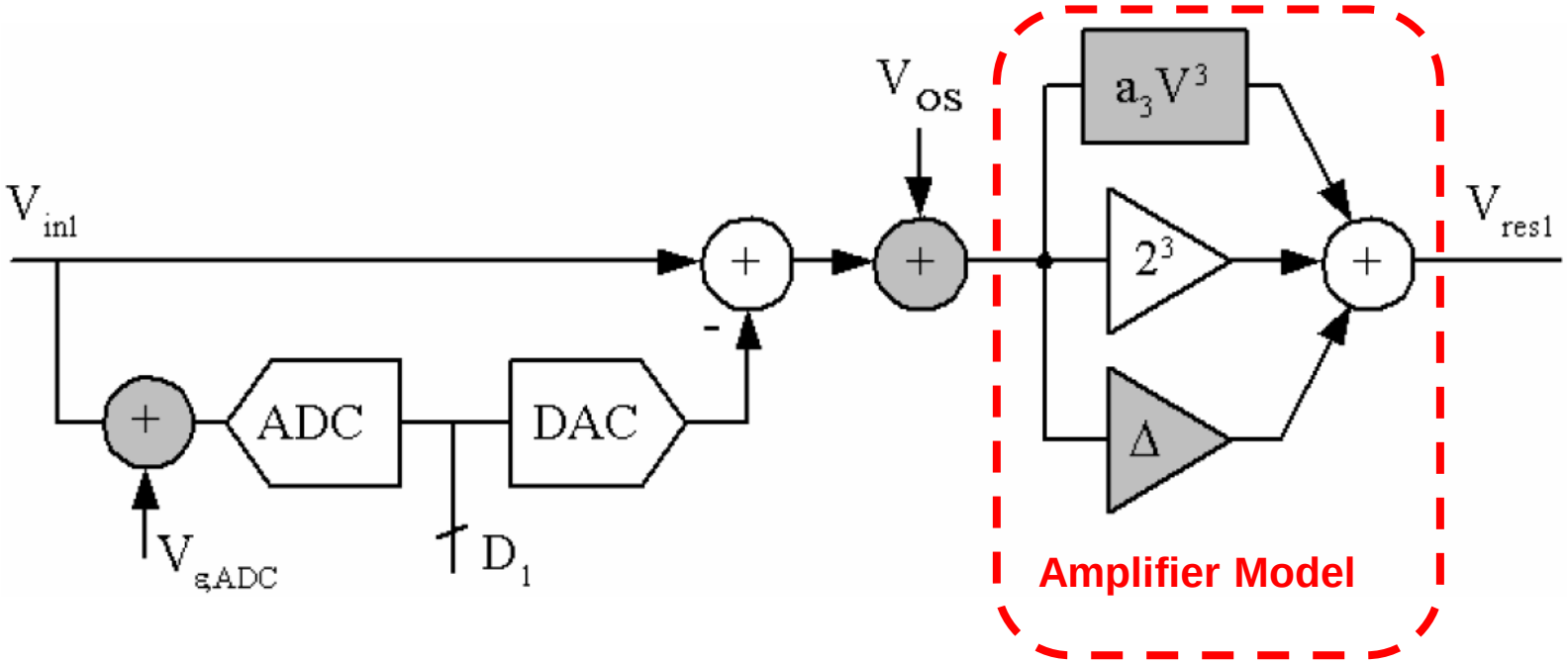
Open Loop Op amp Nonlinearity Calibration (1)



B. Murmann and B. E. Boser, IEEE JSSC**, vol. 38, pp. 2040-2050, Dec. 2003.**

- **The 12-bit pipelined ADC incorporates:**
  - **3-bit stage-1 realized using open-loop amplifier.**
  - **1-bit stage-2 (with 1 redundant bit to incorporate the signal injection for calibration).**
  - **Seven 1.5-bit stages**
  - **3-bit flash ADC.**
- **Although 14-bits of raw data the last two bits are used for calibration purpose.**
- **Only stage-1 is calibrated for linear gain error and non-linearity.**
- **All other stages form an ideal Back-end ADC.**

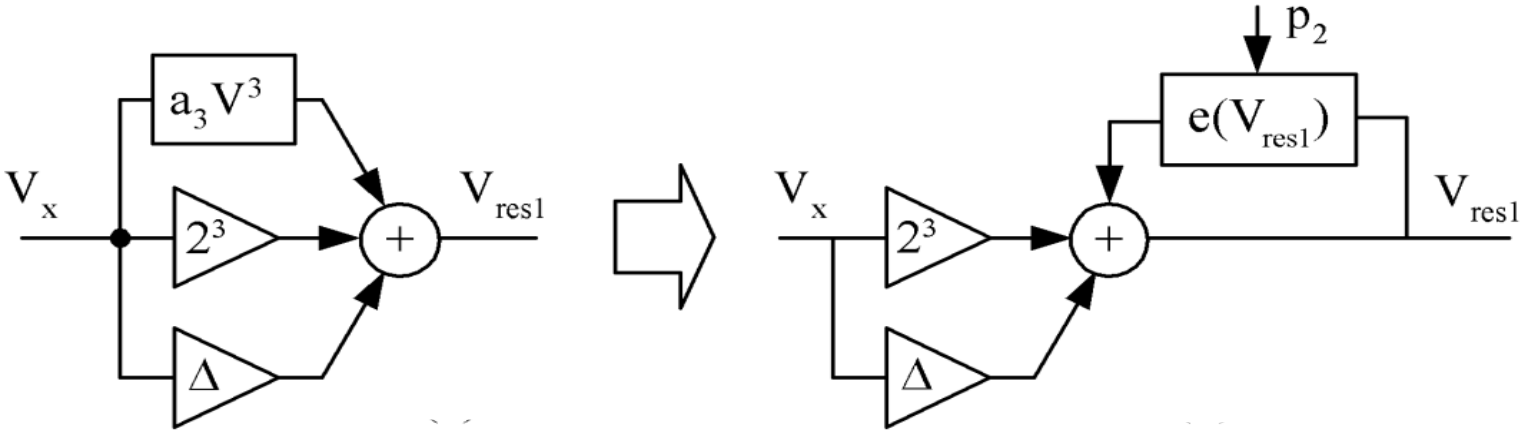
Open Loop Op amp Nonlinearity Calibration (2)



- **Stage-1 that incorporates an open-loop amplifier is modeled as per the above block diagram with various error sources:**
  - V_{OS} ➔ **op amp offset**
  - Δ ➔ **gain error ➔ modeled by calibration parameter p_1.**
  - a_3 ➔ **3rd order nonlinear term of the open-loop op amp.**

B. Murmann and B. E. Boser, IEEE JSSC**, vol. 38, pp. 2040-2050, Dec. 2003.**

Open Loop Op amp Nonlinearity Calibration (3)



Amplifier model with input referred nonlinearity

Amplifier model with output referred nonlinearity

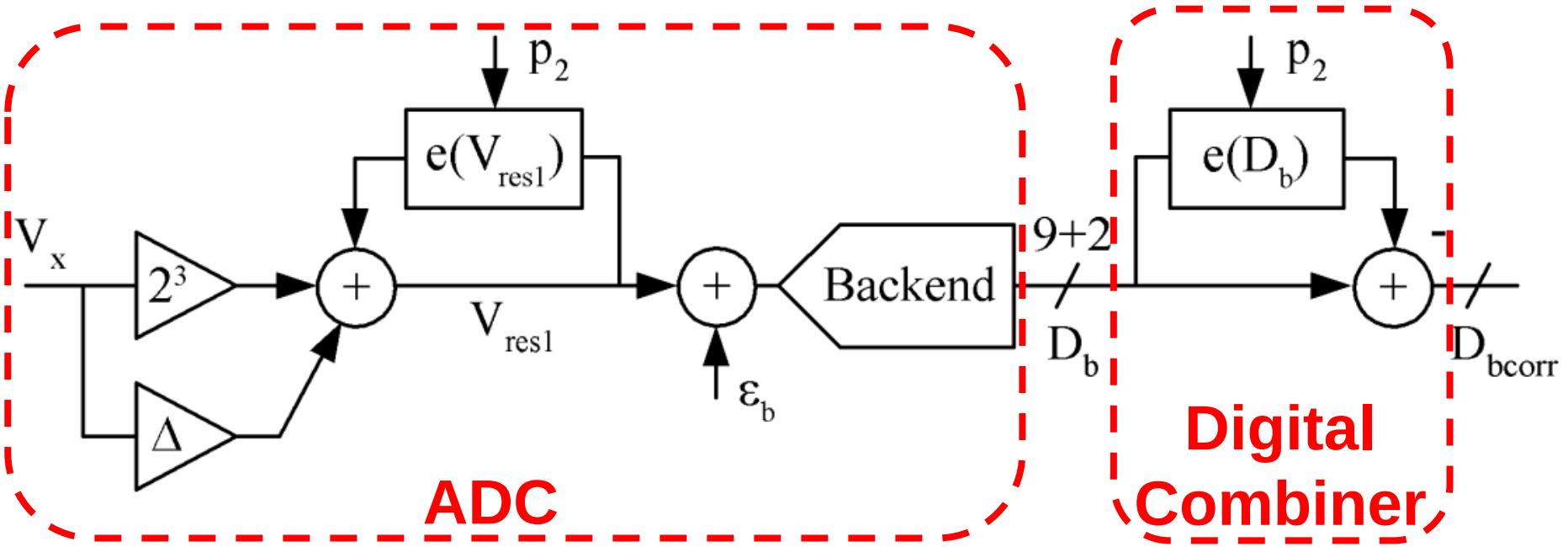
$$e(V_{res1}) = V_{res1} - 2\sqrt{-\frac{1}{3p_2}}\cos\left[\frac{\pi}{3} + \frac{1}{3}\cos^{-1}\left(\frac{V_{res1}}{2\sqrt{-\frac{1}{27p_2}}}\right)\right]$$

where, V_{res1} is digitized by the back-end ADC and

$$p_2 = \frac{a_3}{(2^3 - \Delta)^3}$$

B. Murmann and B. E. Boser, IEEE JSSC**, vol. 38, pp. 2040-2050, Dec. 2003.**

Open Loop Op amp Nonlinearity Calibration (4)

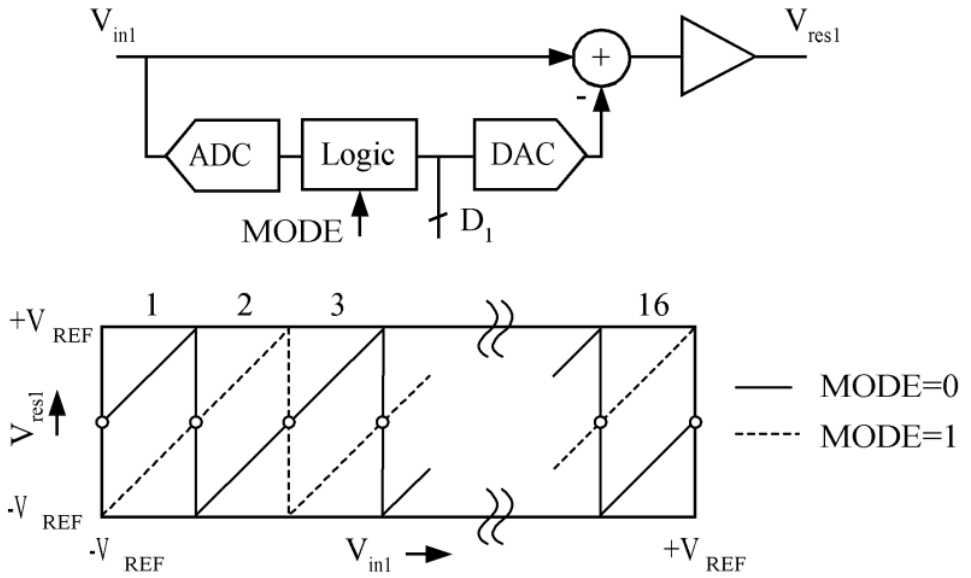


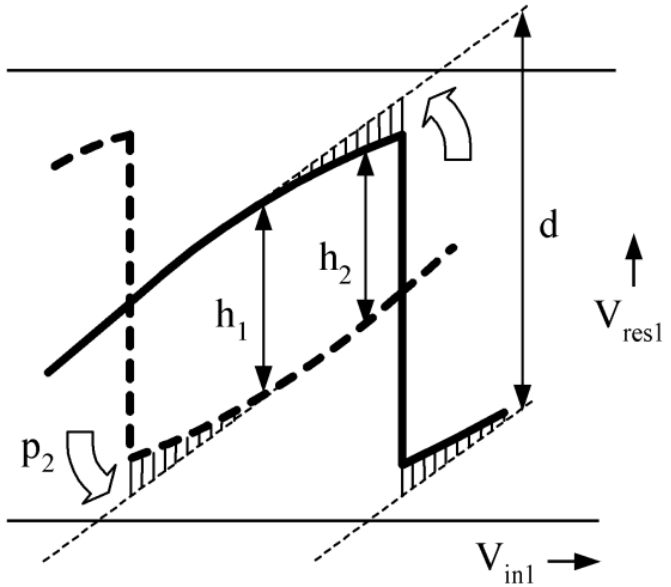
$$e(D_b) = D_b - 2\sqrt{-\frac{1}{3p_2}}\cos\left[\frac{\pi}{3} + \frac{1}{3}\cos^{-1}\left(\frac{D_b}{2\sqrt{-\frac{1}{27p_2}}}\right)\right]$$

where, D_b is the back-end ADC output and the calibration engine estimates p_2.

B. Murmann and B. E. Boser, IEEE JSSC, vol. 38, pp. 2040-2050, Dec. 2003.

Open Loop Op amp Nonlinearity Calibration (5)

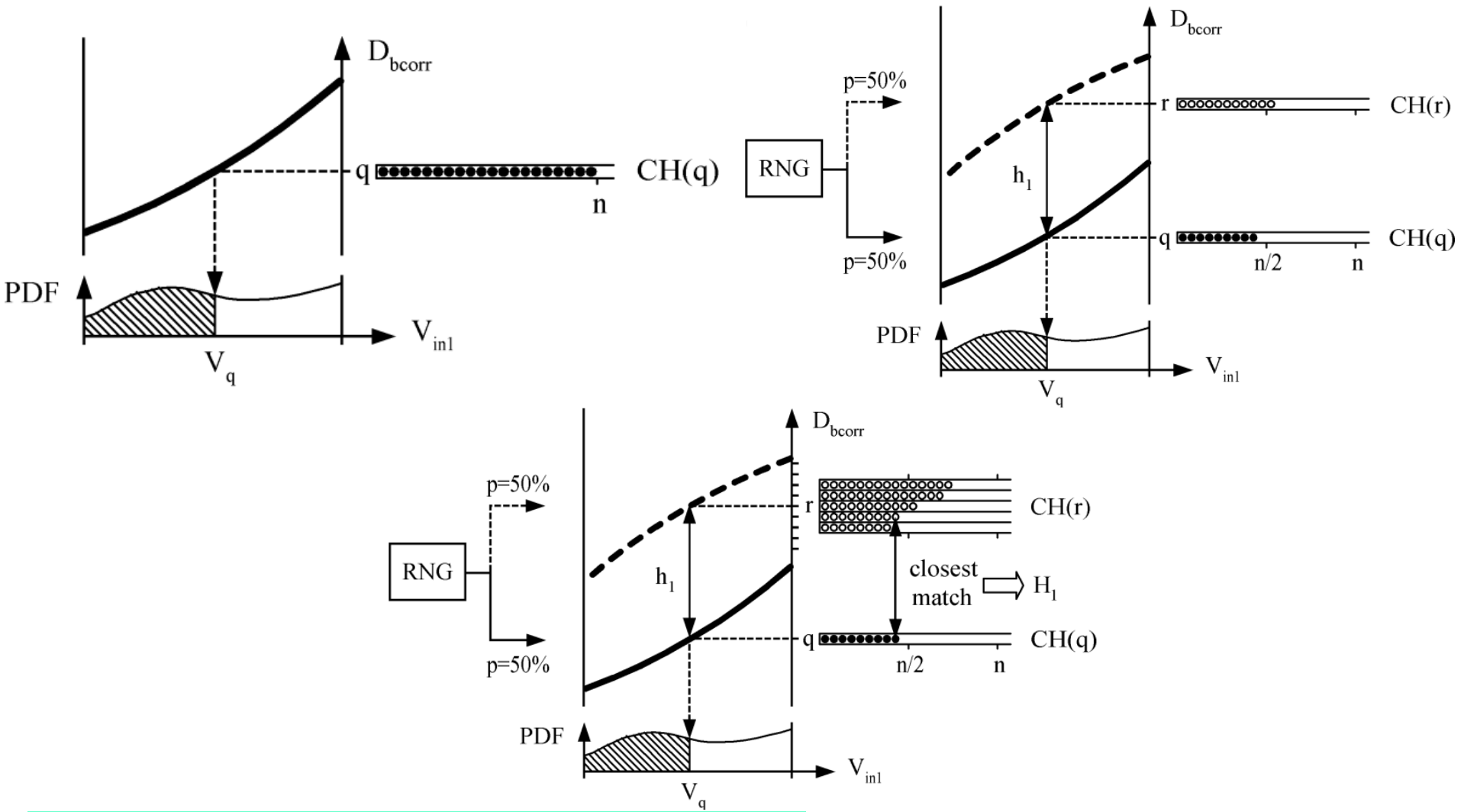




- V_{res1} can generate two curves based on the digital random-bit MODE.
- In order to accommodate the two transfer curves and not saturate the back-end ADC stage-2 has 1-bit of redundancy.
- The residue characteristic with nonlinearity shows compression.
- Nonlinearity is overcome if $h_1 = h_2$, i.e. the distance between the two residue characteristic is constant at all points.
- Its sufficient to estimate the distance at the center and at the extremes.

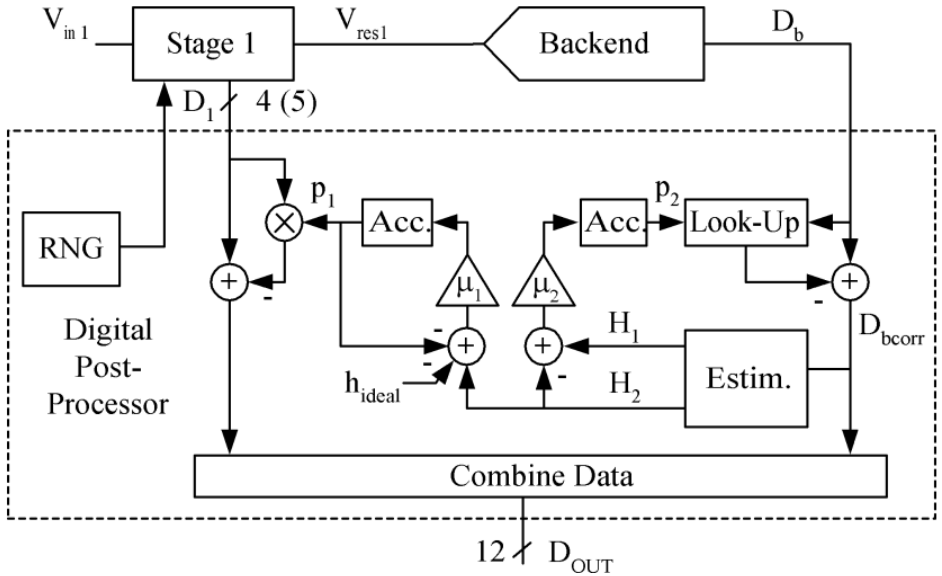
B. Murmann and B. E. Boser, IEEE JSSC, vol. 38, pp. 2040-2050, Dec. 2003.

Open Loop Op amp Nonlinearity Calibration (6)



B. Murmann and B. E. Boser, IEEE JSSC, vol. 38, pp. 2040-2050, Dec. 2003.

Open Loop Op amp Nonlinearity Calibration (7)



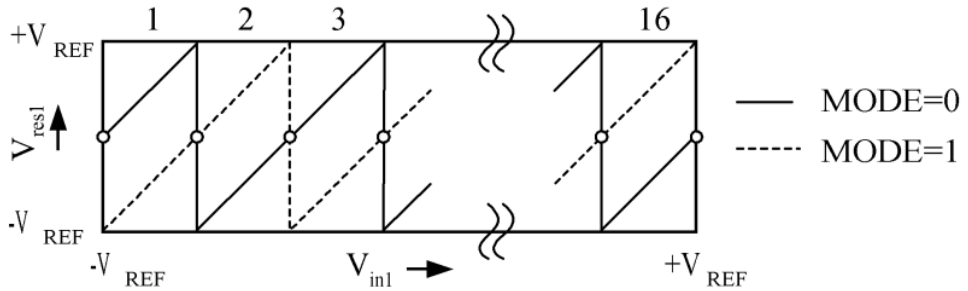
- Estimation of the gain error (p_1) is similar to the method in Li2003.
- Estimation of nonlinearity (p_2) is based on an LMS method which minimizes the MSE of $(H_1 - H_2)$.
- $(H_1 - H_2)$ is a function of p_2 as per

$$e(D_b) = D_b - 2\sqrt{-\frac{1}{3p_2}}\cos\left[\frac{\pi}{3} + \frac{1}{3}\cos^{-1}\left(\frac{D_b}{2\sqrt{-\frac{1}{27p_2}}}\right)\right]$$

B. Murmann and B. E. Boser, IEEE JSSC, vol. 38, pp. 2040-2050, Dec. 2003.

Open Loop Op amp Nonlinearity Calibration (2)

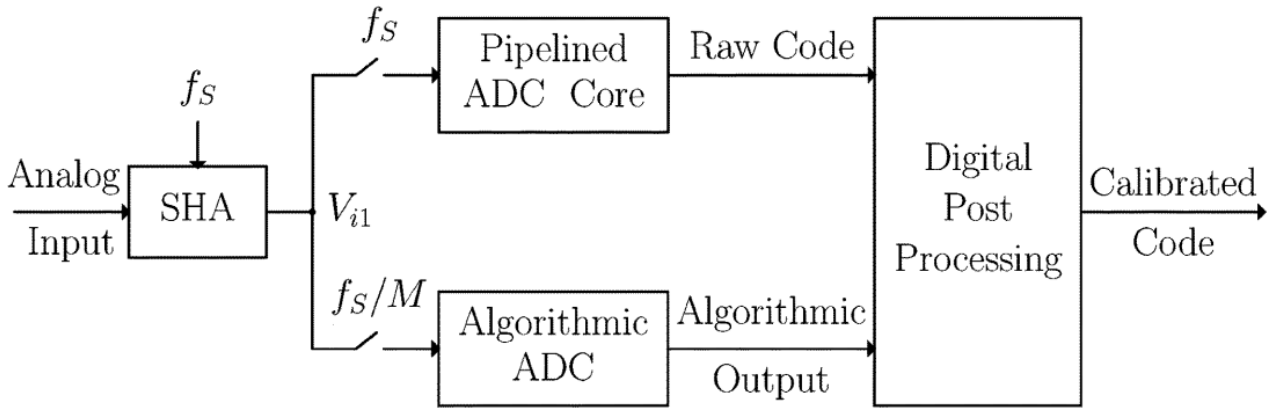
- Requires that the inputs be sufficiently busy, i.e., the analog input to the ADC be such that it exercises all the ADC levels. If the signal is not full scale then the calibration cannot estimate the nonlinearity.
- As shown below in the residue characteristic of stage-1, if the signal is within 1/16th of the full scale then also it exercises the full-scale of the back-end ADC and hence estimates the nonlinearity.



- The open-loop amplifier is very susceptible to gain variation due to temperature. If the LMS loop has a smaller time-constant as opposed to the gain variation then the calibration works.

B. Murmann and B. E. Boser, IEEE JSSC, vol. 38, pp. 2040-2050, Dec. 2003.

Nested Digital Background Calibration (1)



- The algorithmic ADC and Pipelined ADC are realized using 1.5-bit flip-around topology whose input-output characteristic is given by,

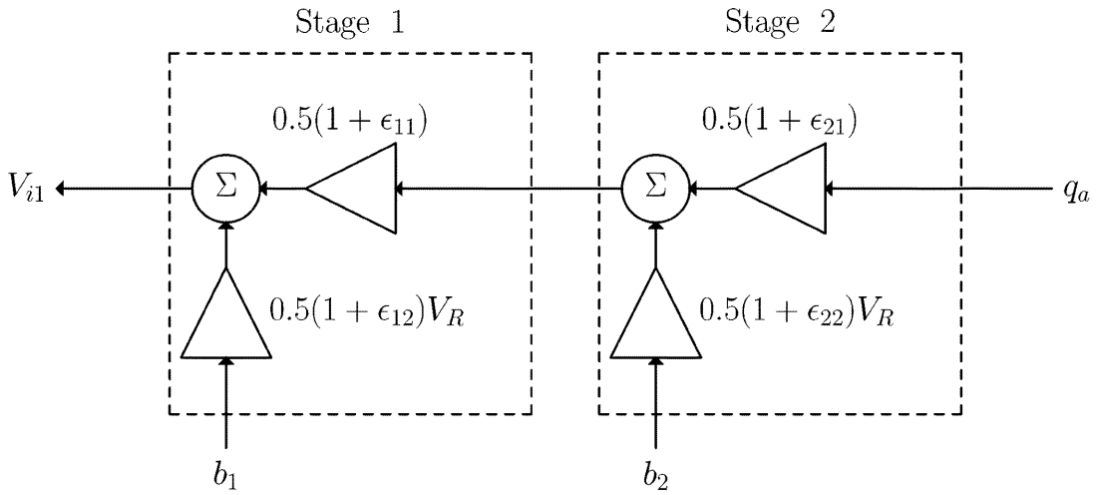
$$V_O = \frac{1}{1+\frac{C_S+C_F}{AC_F}}\frac{C_S+C_F}{C_F}V_i - \frac{1}{1+\frac{C_S+C_F}{AC_F}}\frac{C_S}{C_F}DV_{ref}$$

$$\rightarrow V_O = (2+\epsilon_g)V_i - (1+\epsilon_{DAC})DV_{ref}$$

where, $(2+\epsilon_g) = \frac{1}{1+\frac{C_S+C_F}{AC_F}}\frac{C_S+C_F}{C_F}$, and $(1+\epsilon_{DAC}) = \frac{1}{1+\frac{C_S+C_F}{AC_F}}\frac{C_S}{C_F}$

X. Wang, P. J. Hurst, and S. H. Lewis, IEEE JSSC, vol. 39, pp. 1799-1808, Nov. 2004.

Nested Digital Background Calibration (2)



- Digital output computation from the raw bits,

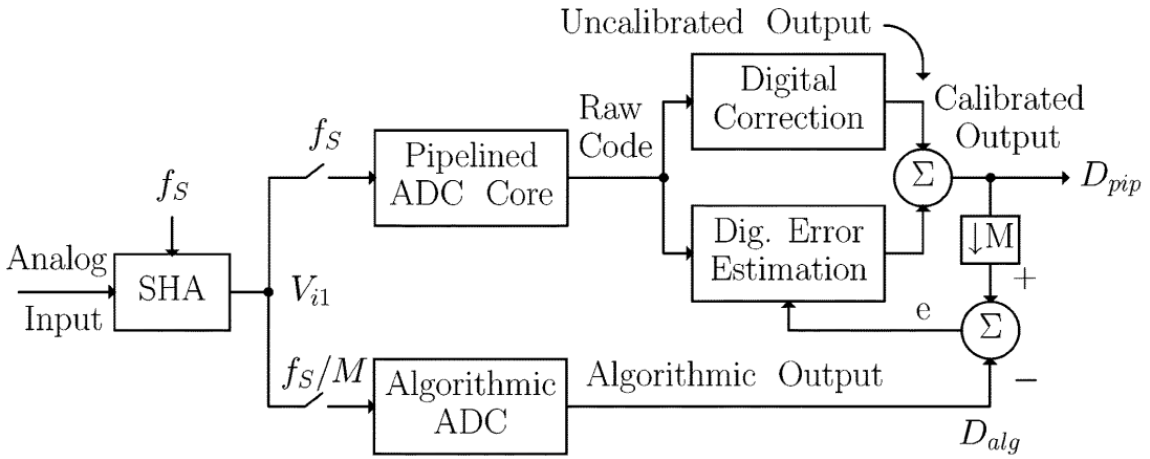
$$V_{i1} = V_R\left[0.5(1+\epsilon_1)b_1 + 0.5^2(1+\epsilon_2)b_2\right] + q_a 0.5^2(1+\epsilon_3)$$

where, $\epsilon_1 = \epsilon_{12}$, $\epsilon_2 = \epsilon_{11} + \epsilon_{22} + \epsilon_{11}\epsilon_{12}$, $\epsilon_3 = \epsilon_{11} + \epsilon_{21} + \epsilon_{11}\epsilon_{21}$, and q_a is the quantization noise.

- So 4 parameters are needed for a 2-stage pipeline
  - Three (ϵ_1, ϵ_2, and ϵ_3) for the gain errors
  - One more for the overall offset of the pipelined stages.
- For a K-stage ADC, (K+2) parameters need to be estimated, including the offset term.

X. Wang, P. J. Hurst, and S. H. Lewis, IEEE JSSC, vol. 39, pp. 1799-1808, Nov. 2004.

Nested Digital Background Calibration (3)



- Pipelined ADC core has 13-stages.
- First 5-stages are calibrated.

- Digital output computation from the raw bits,

$$D_{pip}(V_{i1}) = \sum_{i=1}^{13} 0.5^i b_i + \sum_{i=1}^{5} 0.5^i \epsilon_i b_i + 0.5^5 q + offset$$

- LMS optimization can be used to minimize the error,

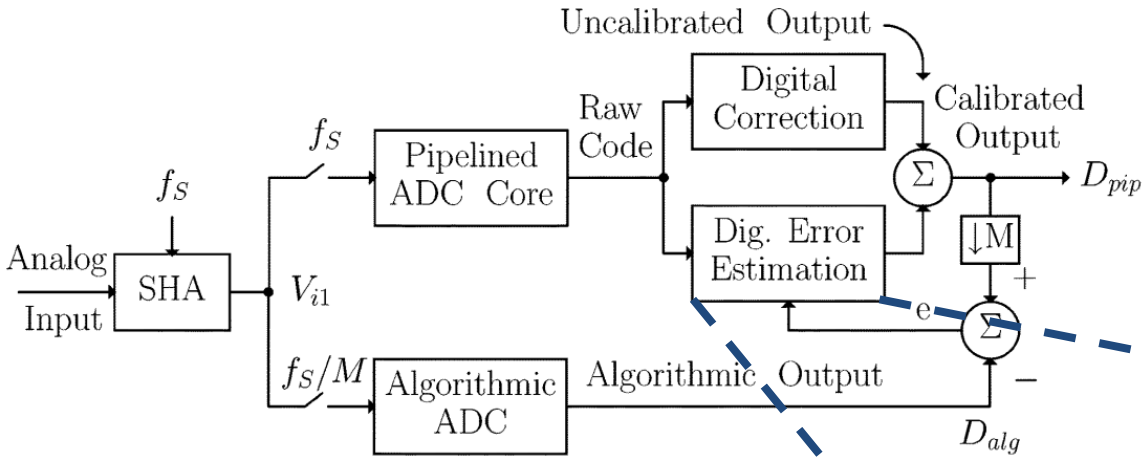
$$e = D_{pip} - D_{alg}$$

- The update equation is,

$$\epsilon_i(m+1) = \epsilon_i(m) + \mu_{pip} e \frac{\partial e}{\partial \epsilon_i}$$

X. Wang, P. J. Hurst, and S. H. Lewis, IEEE JSSC, vol. 39, pp. 1799-1808, Nov. 2004.

Nested Digital Background Calibration (4)



- For ϵ_1 to ϵ_5, the derivative of MSE is zero,

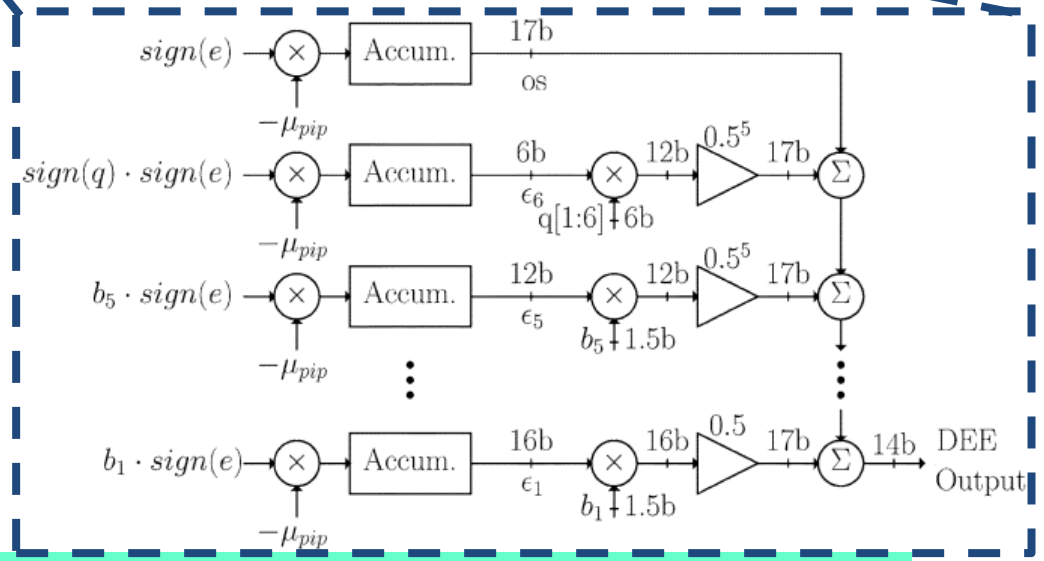
$$\frac{\partial E(e^2)}{\partial \epsilon_i} = E\left(2e\frac{\partial e}{\partial \epsilon_i}\right) = E\left(2e\frac{\partial D_{pip}}{\partial \epsilon_i}\right) = 2(0.5)^i E(e \cdot b_i) = 0 \quad 1 \le i \le 5.$$

- For ϵ_6 ,

$$\frac{\partial E(e^2)}{\partial \epsilon_6} = E\left(2e\frac{\partial e}{\partial \epsilon_6}\right) = E\left(2e\frac{\partial D_{pip}}{\partial \epsilon_6}\right) = 2(0.5)^5 E(e \cdot q) = 0$$

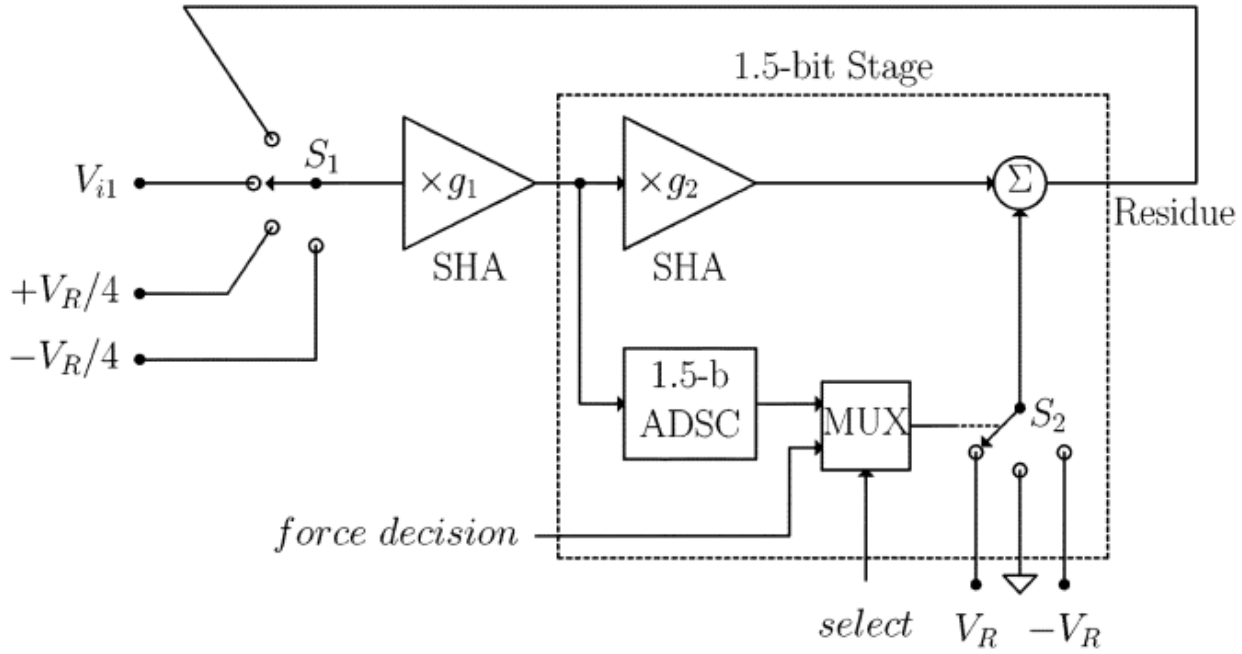
- For offset,

$$\frac{\partial E(e^2)}{\partial (os)} = E\left(2e\frac{\partial e}{\partial (os)}\right) = E\left(2e\frac{\partial D_{pip}}{\partial (os)}\right) = 2E(1 \cdot e) = 0$$



X. Wang, P. J. Hurst, and S. H. Lewis, IEEE JSSC, vol. 39, pp. 1799-1808, Nov. 2004.

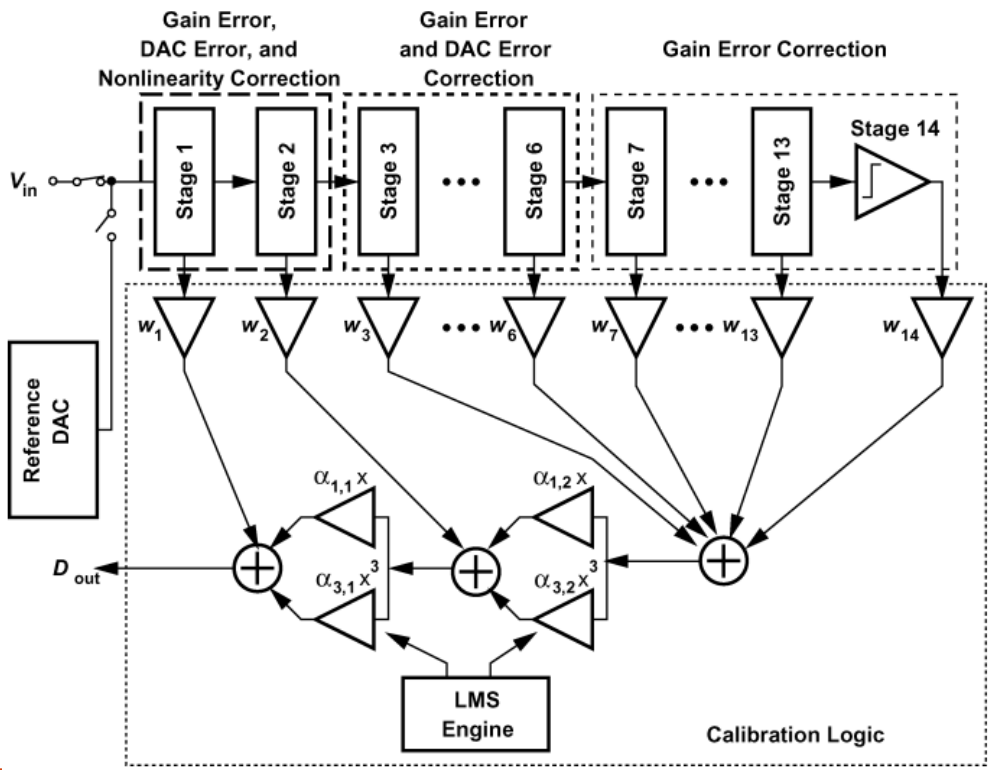
Nested Digital Background Calibration (5)



- The algorithmic ADC is calibrated based on the calibration technique proposed by Erdogan 1999.
- Erdogan used a 1-bit architecture for the stage. Here 1.5-bit flip-around topology is used instead.

X. Wang, P. J. Hurst, and S. H. Lewis, IEEE JSSC, vol. 39, pp. 1799-1808, Nov. 2004.

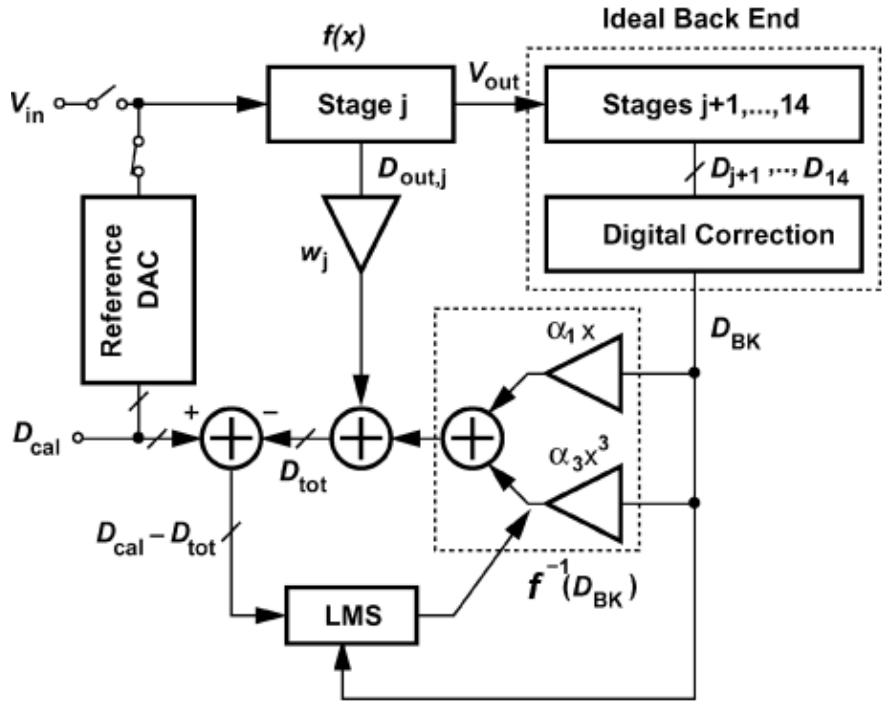
10-bit 500 MHz 55 mW CMOS ADC (1)



A. Verma and B. Razavi, IEEE JSSC, vol. 44, pp. 3039-3050, Nov. 2009.

- All stages use 1.5-bit flip-around topology.
- Gain error, capacitor mismatch, and op amp nonlinearity correction done in the 1st two stages.
- Gain error and capacitor mismatch calibration done in stage 3 to 6.
- No calibration for the remaining stages.
- LMS is used to do gain error and op amp nonlinearity calibration.

10-bit 500 MHz 55 mW CMOS ADC (3)



- Calibration requires a precision DAC.
  - For the 10-bit system here the reference DAC has to be 11-bit linear
- Calibration applies $\pm VR/2$, $\pm VR/4$, and 0 from the reference DAC to stage-j for calibration.
- Stage-j is configured in multiply-by-2 configuration.
- The digitized output of stage-j is given by

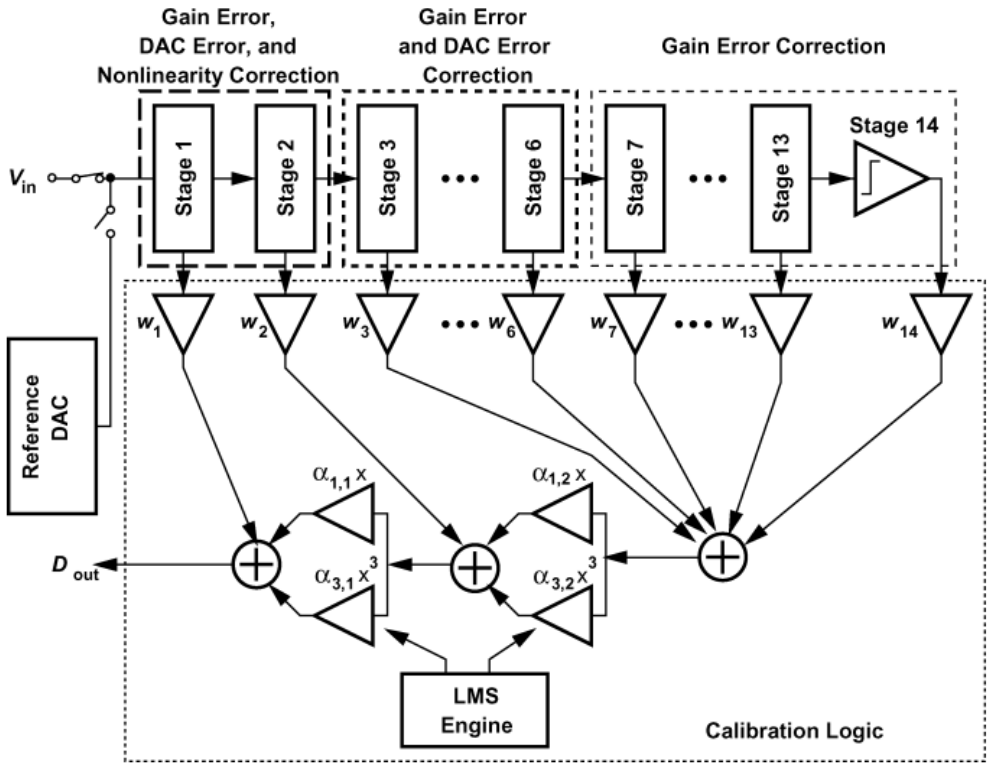
$$D_{tot} = \alpha_1 D_{BK} + \alpha_3 D_{BK}^3$$

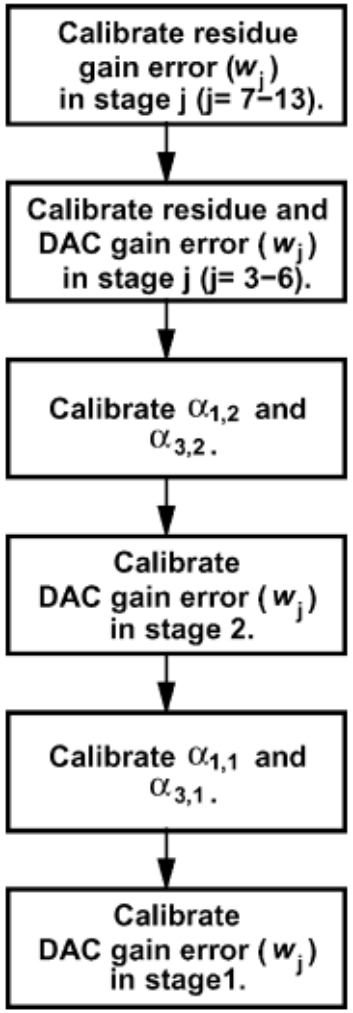
- α_1 and α_3 are updated using the following LMS equation:

$$\alpha_1(k+1) = \alpha_1(k) + \mu(D_{cal} - D_{tot})D_{BK}$$
$$\alpha_3(k+1) = \alpha_3(k) + \mu(D_{cal} - D_{tot})D_{BK}^3$$

A. Verma and B. Razavi, IEEE JSSC, vol. 44, pp. 3039-3050, Nov. 2009.

10-bit 500 MHz 55 mW CMOS ADC (3)





A. Verma and B. Razavi, IEEE JSSC**, vol. 44, pp. 3039-3050, Nov. 2009.**

10-bit 500 MHz 55 mW CMOS ADC (5)

- Calibration requires a precision DAC.
  - For a10-bit system the reference DAC has to be 11-bit linear
  - For a 12-bit system the reference DAC has to be 13-bit linear
  - Difficult to realize highly linear and precise DACs
  - The calibration technique cannot be used to calibrate more than 10-bit systems.
- Calibration applies signals from the resistor ladder ➔ calibration cannot be run at the full-speed of the ADC because of the RC-settling issue.
- High frequency settling behavior of the op amps is not captured.

A. Verma and B. Razavi, IEEE JSSC, vol. 44, pp. 3039-3050, Nov. 2009.

Conclusion

- In the last 20 years various digital calibration techniques have been developed.
- Goal is to overcome various circuit non-idealities like finite op amp gain, op amp nonlinearity, and capacitor mismatch.
- Digital calibration techniques can be categorized as:
  - Background
  - Foreground
- Digital calibration technique calibrates for:
  - Capacitor mismatch
  - Linear Gain Error
  - Op amp nonlinearity

Thank You

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