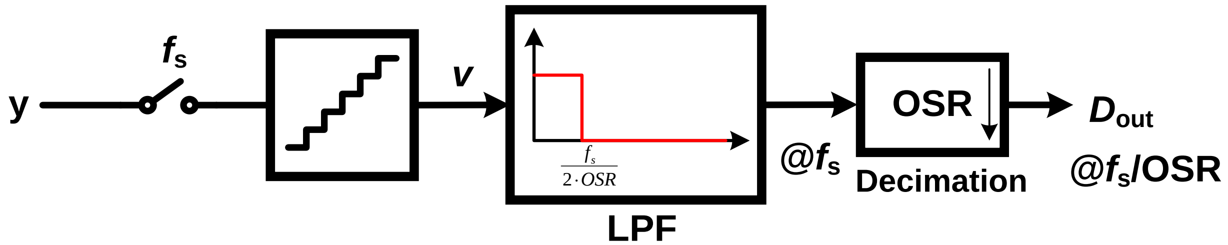
Delta-Sigma Analog-to-Digital Converters

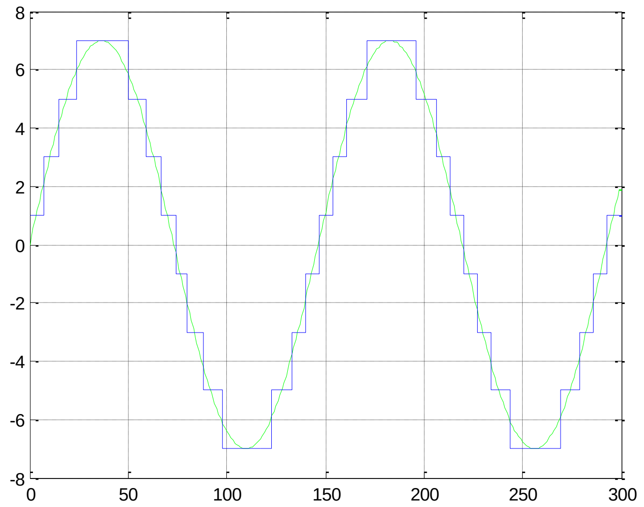
Oversampling Data Converters Basics

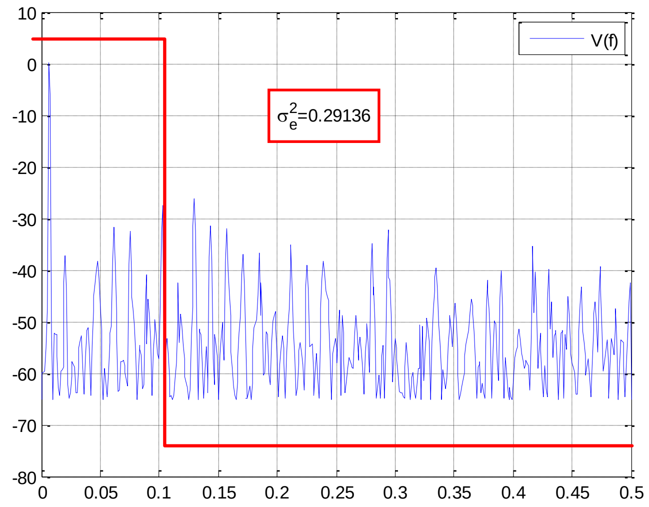
Vishal Saxena

Oversampling and Noise-Shaping

Oversampling

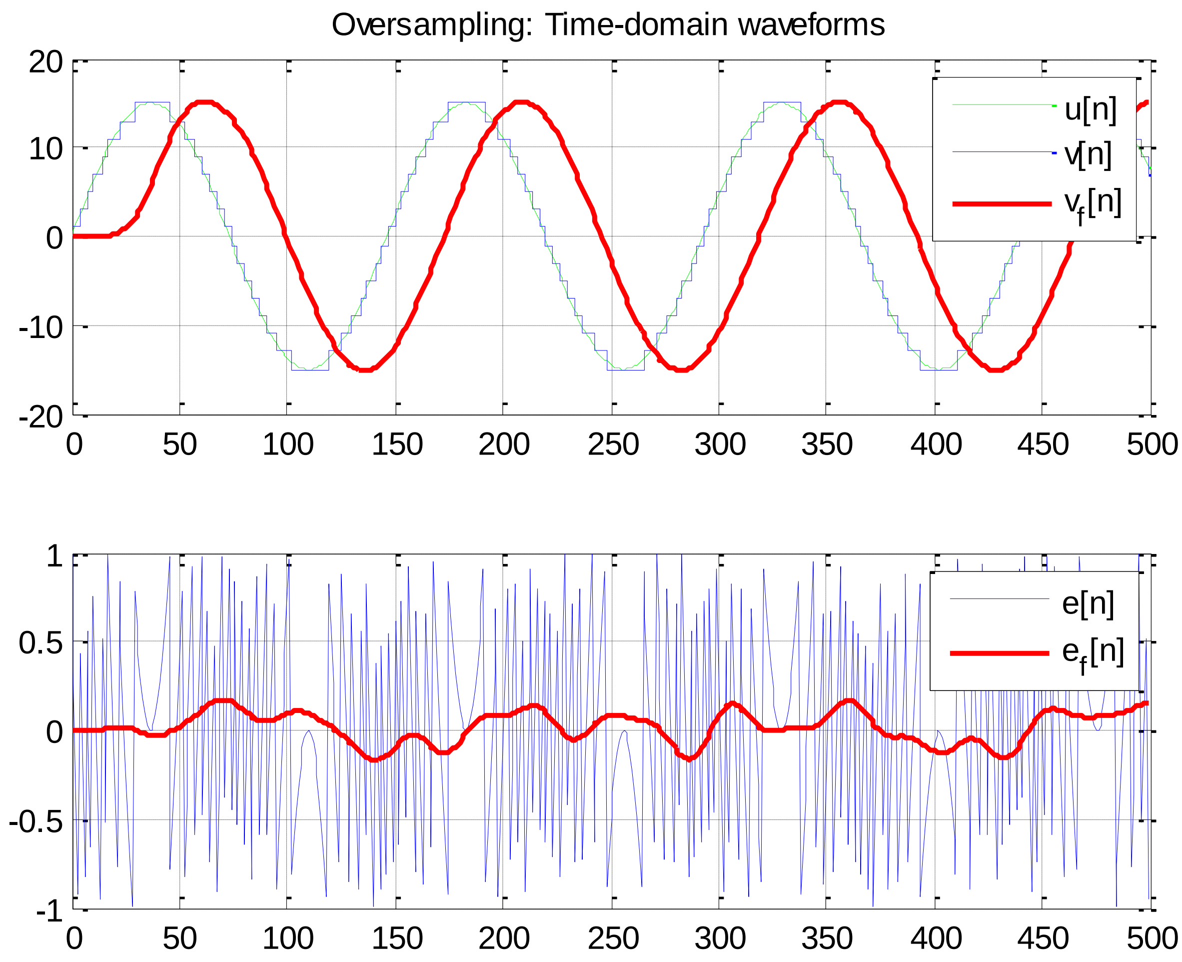






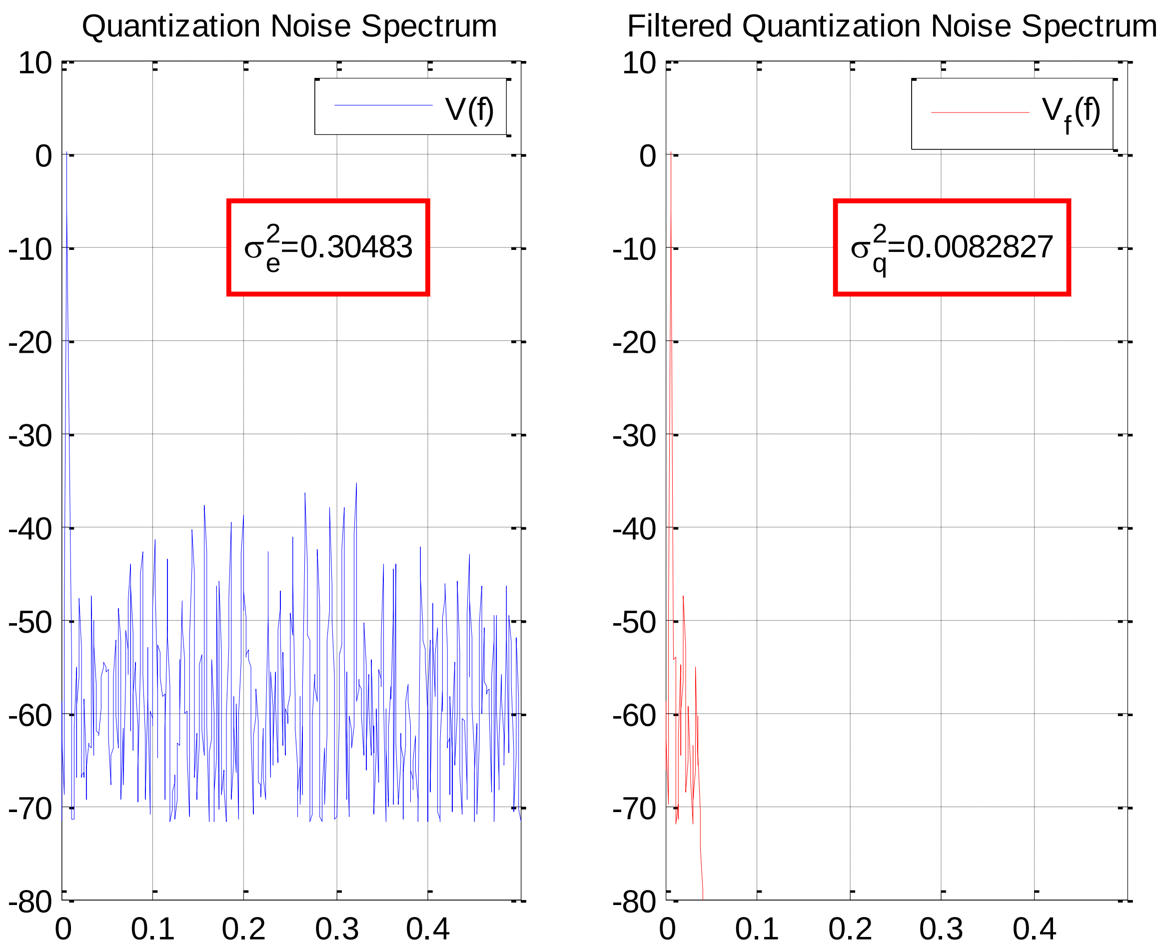
- Oversampling ratio (*OSR*)
- Conversion bandwidth: $f_B = f_s/2 \cdot OSR$
- $SQNR = 6.02 \cdot N + 1.76 + 10 \cdot \log_{10}(OSR)$
- 0.5 bits increase in resolution per doubling in OSR

Oversampling



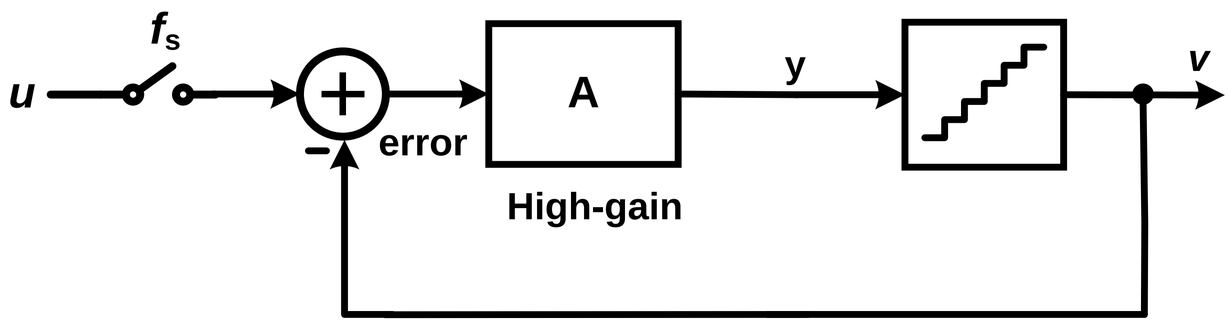
File: oversampling1.m

Oversampling



File: oversampling1.m

Oversampling with Feedback

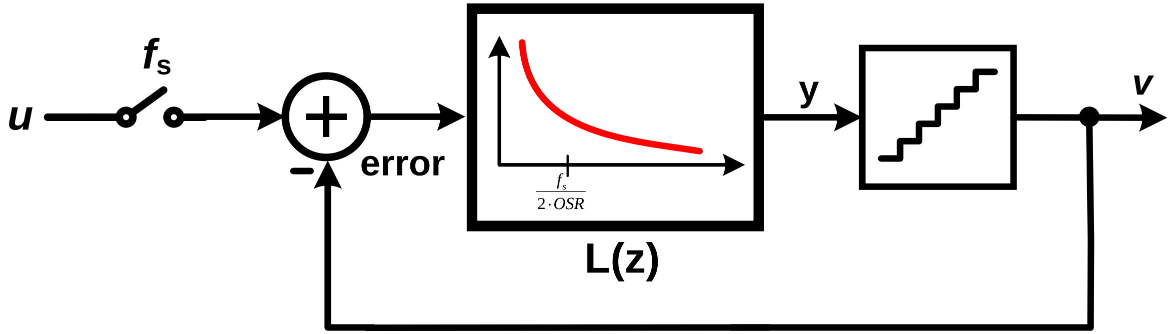


- Can we use feedback with high loop-gain ($A \cdot k_q$) to reduce the error e=|u-v| ?
- Since quantizer output can not be equal to the input

$$A \cdot k_q \to \infty \Rightarrow e = \left| u - v \right| \to \infty$$

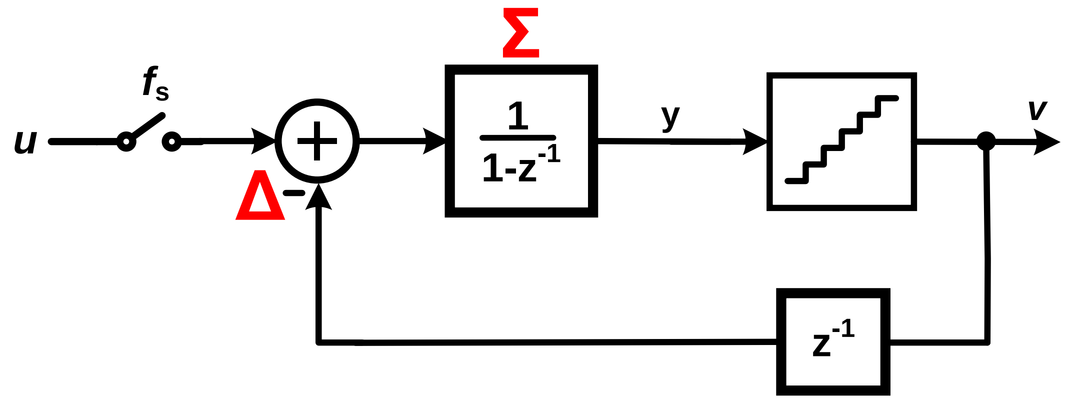
- The loop will be unstable as the error gets unbounded

Oversampling with Feedback



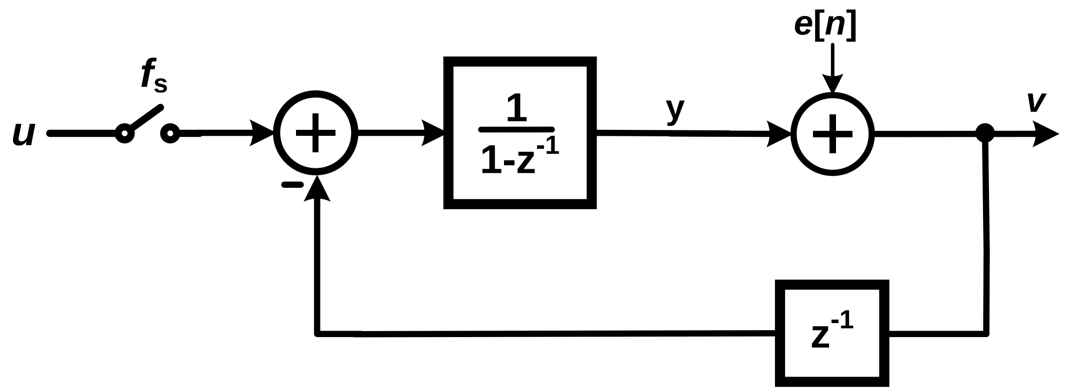
- Use large loop-gain in the signal band and small loop-gain at higher frequencies
- At low frequencies $e = |u - v| \to 0$
- At high frequencies, low loop-gain stabilizes the loop
- $L(z)$ is the loop-filter
- This feedback arrangement is called a (noise) **modulator**

First-order Modulator



- Differencing (Δ) followed by an accumulator (Σ)
  - ΔΣ modulator
- At low frequencies $e = |u - v| \to 0$

First-order Noise Shaping

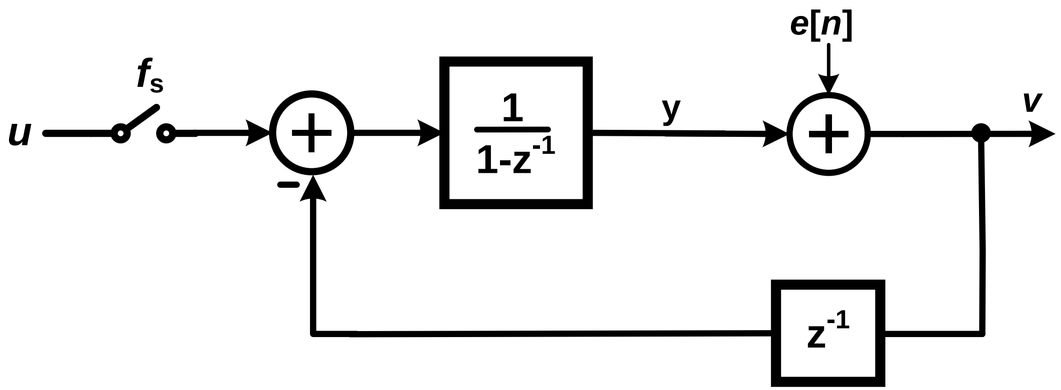


- Linearized model for the modulator

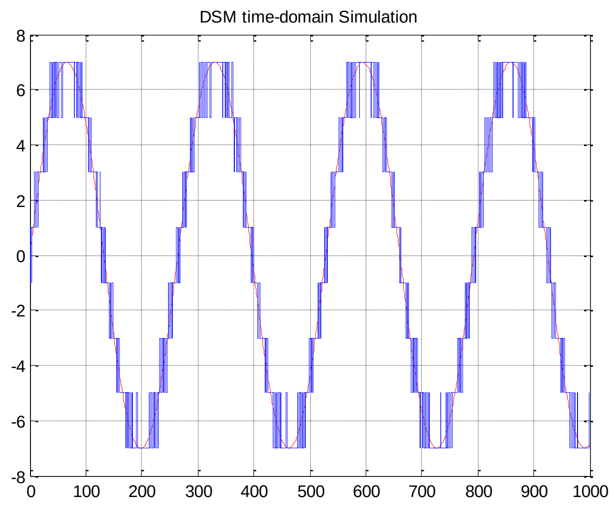
$$V(z) = \underbrace{\boxed{z^{-1}}}_{\text{STF}} U(z) + \underbrace{\boxed{\left(1 - z^{-1}\right)}}_{\text{NTF}} E(z)$$

- Noise transfer function (NTF)
  - $(1\text{-}z^{-1})$: first-order differentiator
  - High-pass shaping of quantization noise
- Signal transfer function (STF)
  - Unit delay

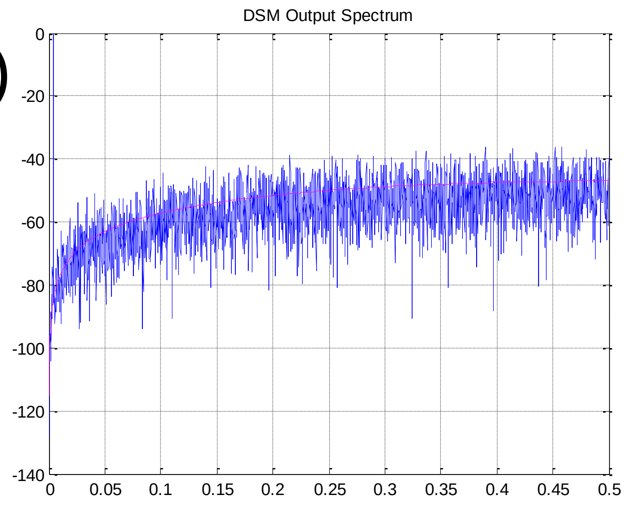
First-order ΔΣ Modulator



v[n]

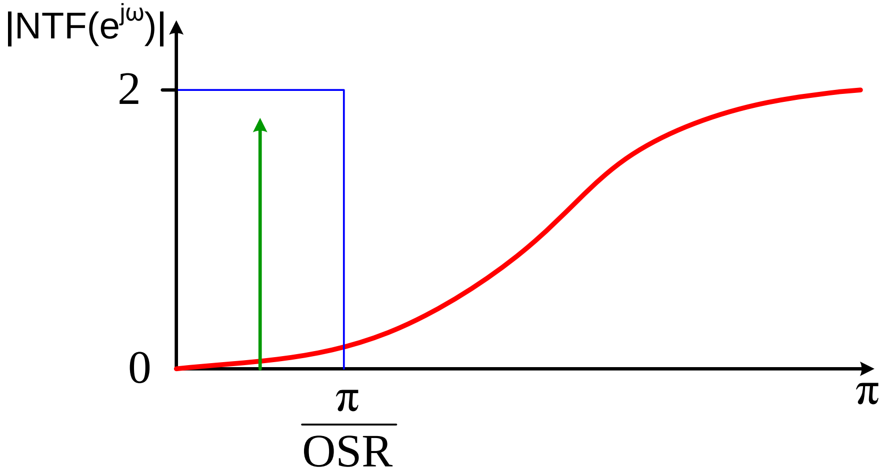


$S_v(f)$



File: First_Order_DSM.m

First-order ΔΣ Modulator SQNR



$$IBN = \int_0^{\frac{\pi}{OSR}} S_v(e^{j\omega}) \cdot d\omega$$

$$= \frac{\Delta^2}{12} \int_0^{\frac{\pi}{OSR}} \left| NTF(e^{j\omega}) \right| \cdot d\omega$$

$$= \frac{\Delta^2}{12} \int_0^{\frac{\pi}{OSR}} 4\sin^2(\omega/2) \cdot d\omega$$

$$\approx \frac{\Delta^2}{12} \int_0^{\frac{\pi}{OSR}} \omega^2 \cdot d\omega$$

$$= \frac{\Delta^2}{12} \cdot \frac{\omega^3}{3} \Bigg|_0^{\frac{\pi}{OSR}}$$

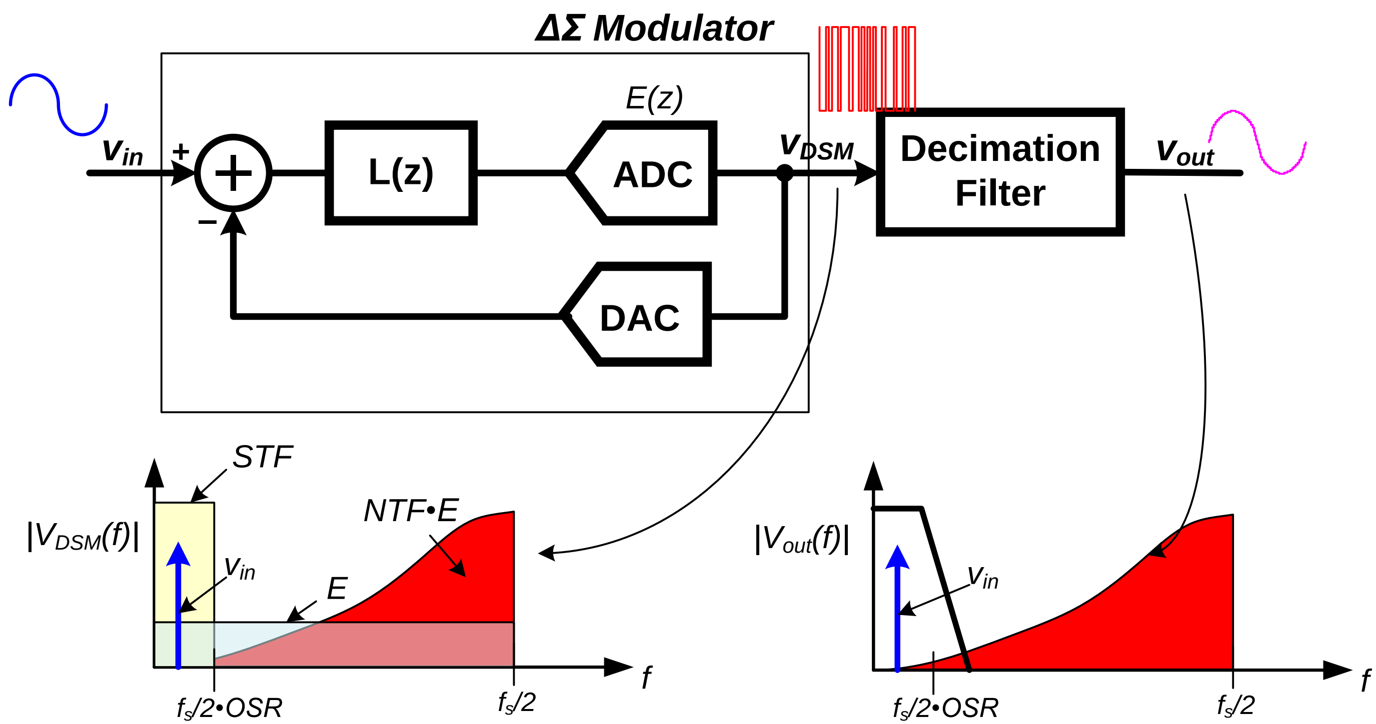
$$= \frac{\Delta^2 \pi^2}{36 \cdot OSR^3}$$

- In-band quantization noise (IBN)

$$\frac{\Delta^2}{12} \cdot \frac{\pi^2}{3} \cdot OSR^{-3}$$

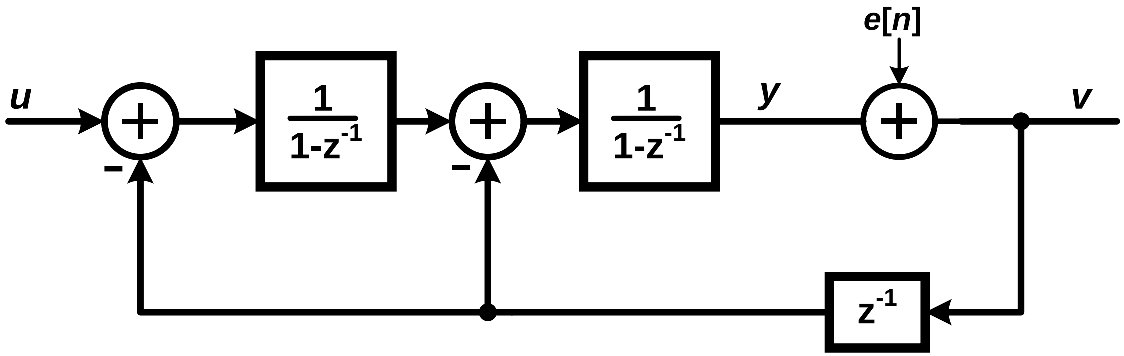
- *SQNR* = 6.02·*N* –3.4 + 30·$\log_{10}$(OSR)
- 1.5 bits increase in resolution per doubling in OSR
- Out-of-band noise is filtered out using a digital filter

Delta-Sigma (ΔΣ) ADC



- Use oversampling ($f_s=2\cdot OSR\cdot BW$) to shape the quantization noise out of the signal band
- Use low-resolution ADC and DAC to higher much higher resolution
- Digitally filter out the out-of band shaped (modulated) noise
- Trades-off SQNR with oversampling ratio (OSR)

Second-Order Noise Shaping

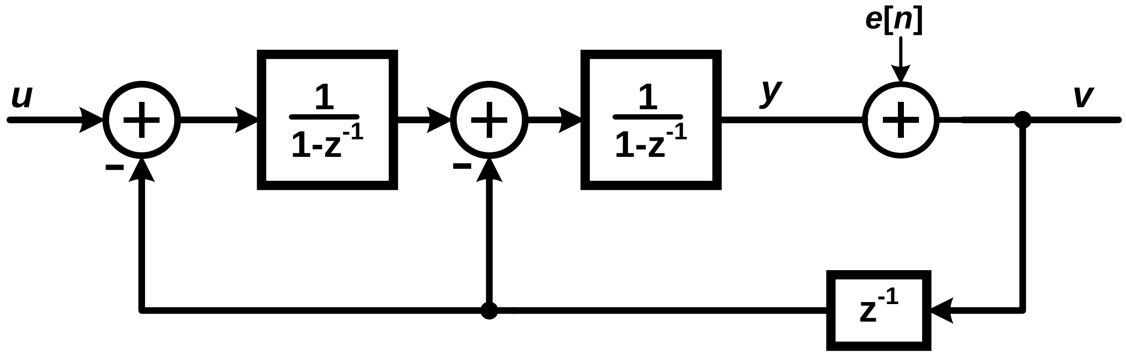


- Linearized model for the modulator

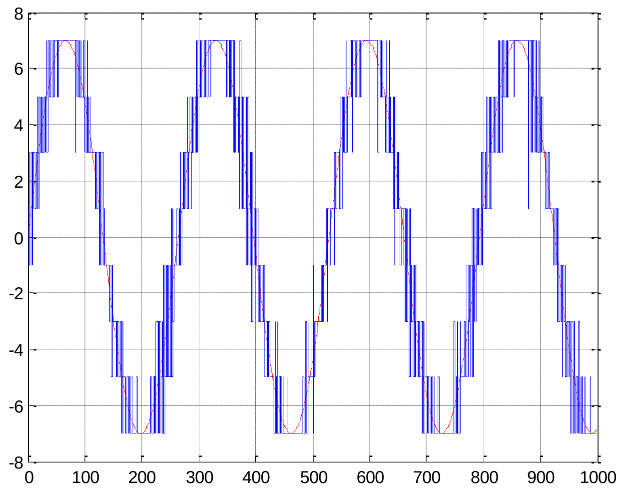
$$V(z) = \underbrace{\boxed{z^{-1}}}_{\text{STF}} U(z) + \underbrace{\boxed{\left(1 - z^{-1}\right)^2}}_{\text{NTF}} E(z)$$

- Second-order noise-shaping
- In-band quantization noise (IBN): $\frac{\Delta^2}{12} \cdot \frac{\pi^4}{5} \cdot OSR^{-5}$
  - 2.5 bits increase in resolution per doubling in OSR
- Can be extended to higher orders

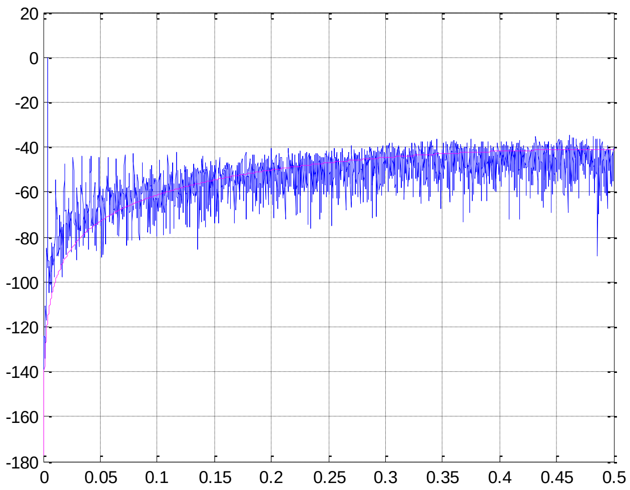
Second-order ΔΣ Modulator



v[n]

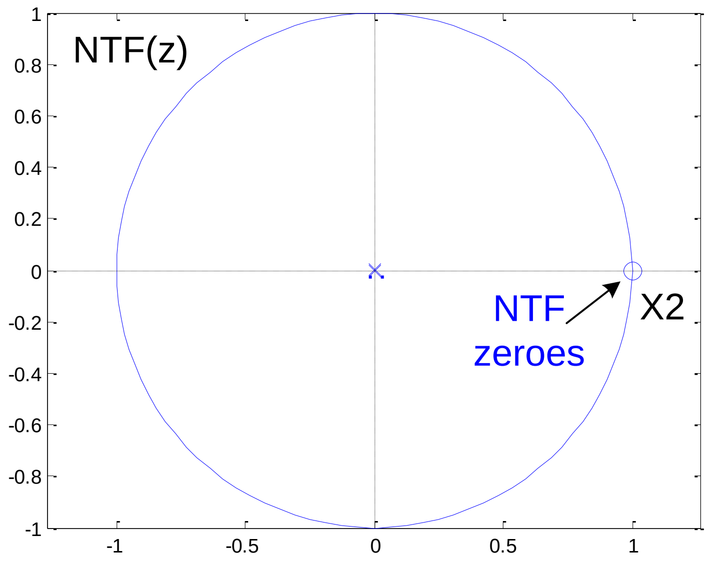


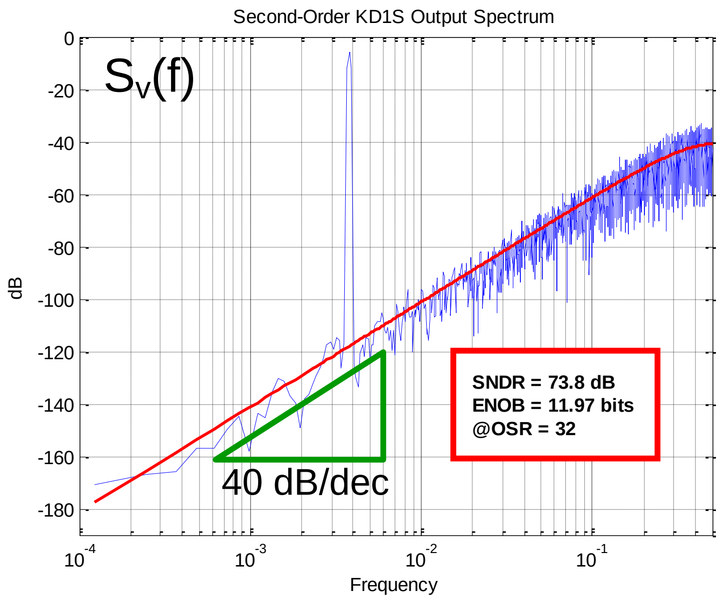
$S_V(f)$



File: Second_Order_DSM.m

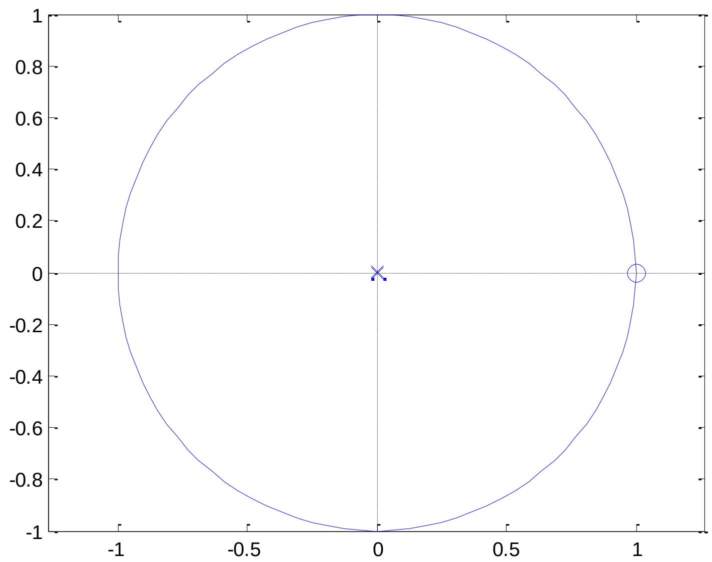
Second-order ΔΣ Modulator

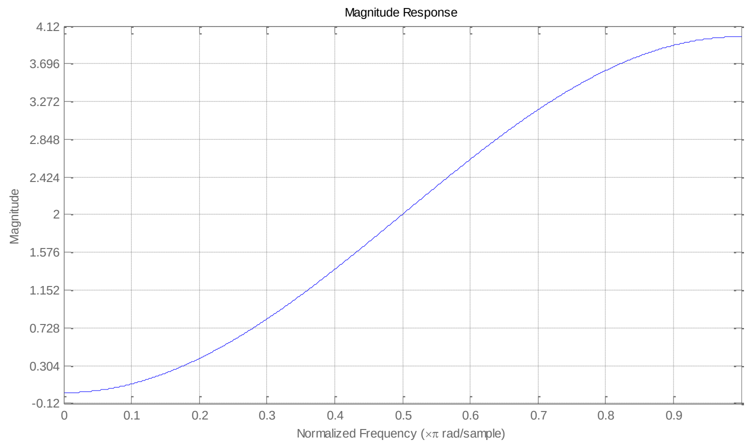


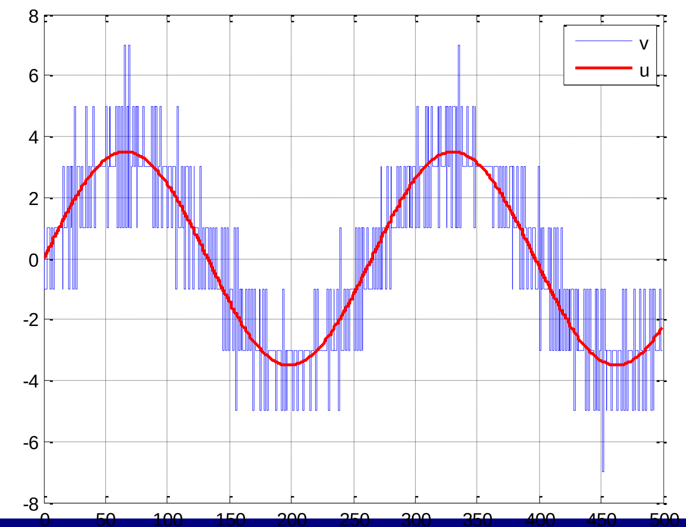


- $NTF(z) = (1-z^{-1})^2$
  - Two zeroes at DC
  - Out-of-Band Gain (OBG) (i.e gain at $\omega \approx \pi$) = 4

2nd order DSM



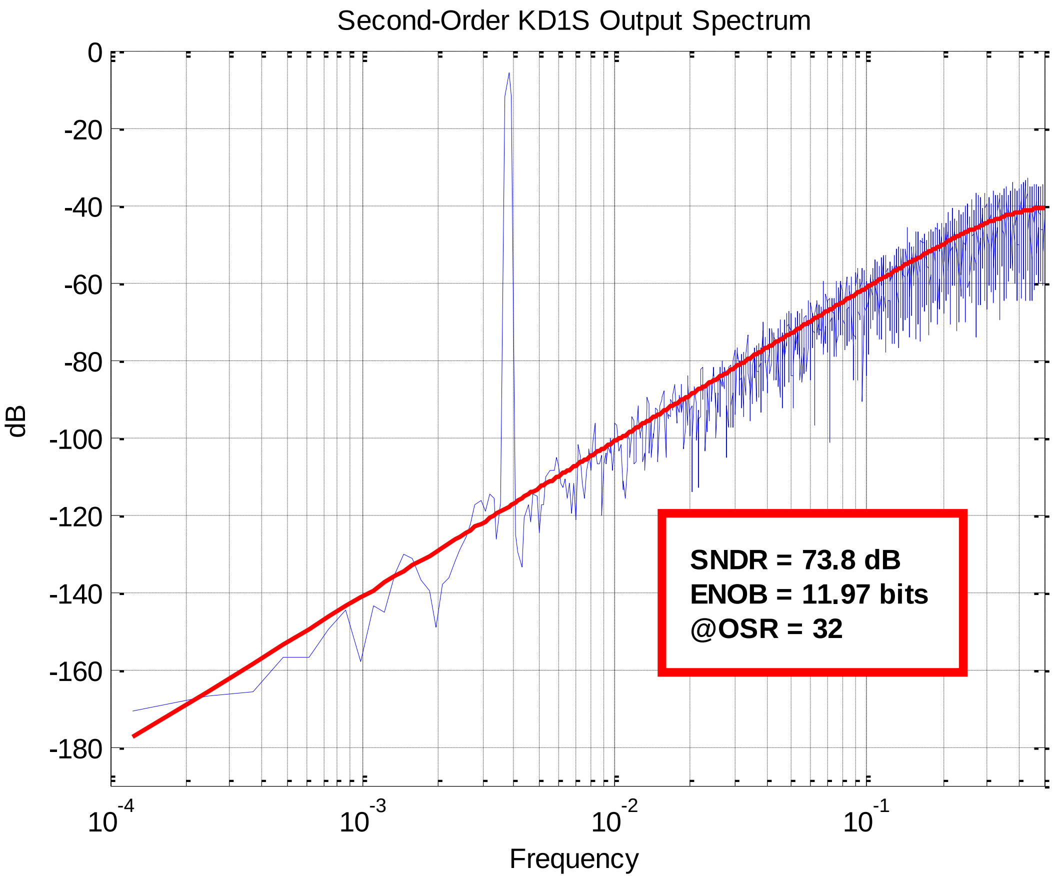




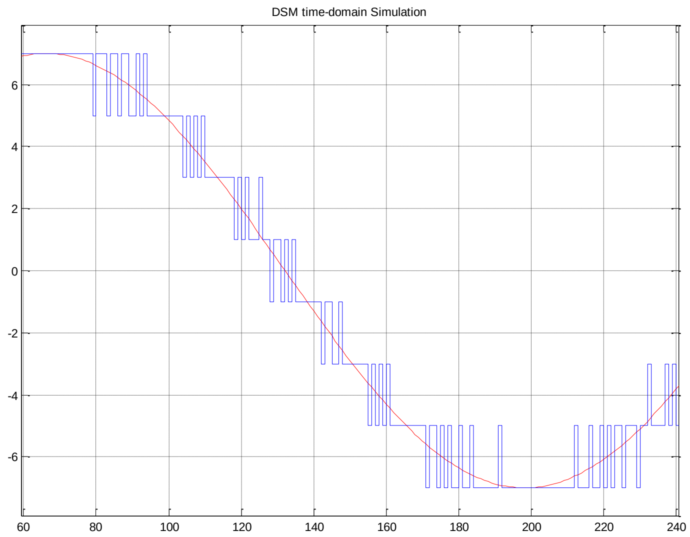
$$NTF(z) = (1 - z^{-1})^2$$

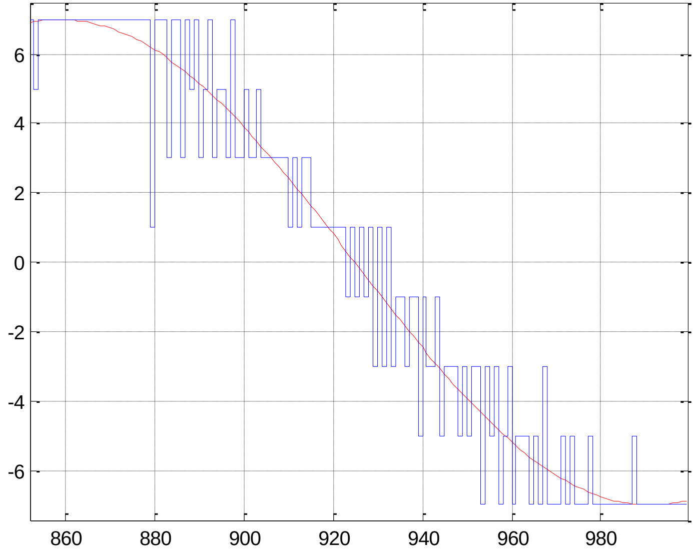
File: Second_Order_DSM_Zero_Opt.m
Set variable opt=0.

2^{nd} order DSM: contd.



Comparison: 1st and 2nd order modulator waveforms



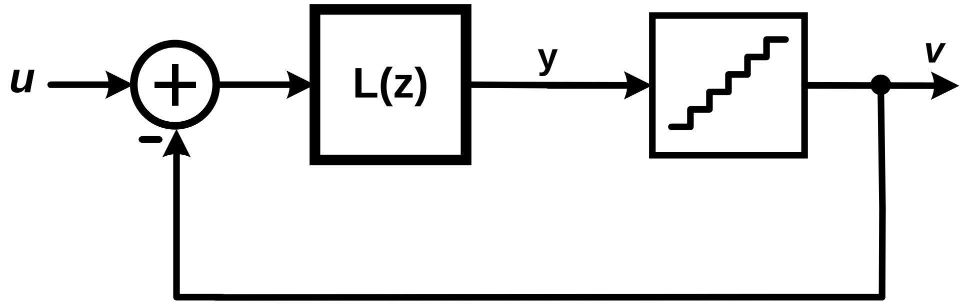


- $NTF(z) = (1-z^{-1})$
- OBG = 2
- Max LSB jump = 1

- $NTF(z) = (1-z^{-1})^2$
- OBG = 4
- Max LSB jump = 3

Higher-Order ΔΣ Modulators

Higher-Order NTFs

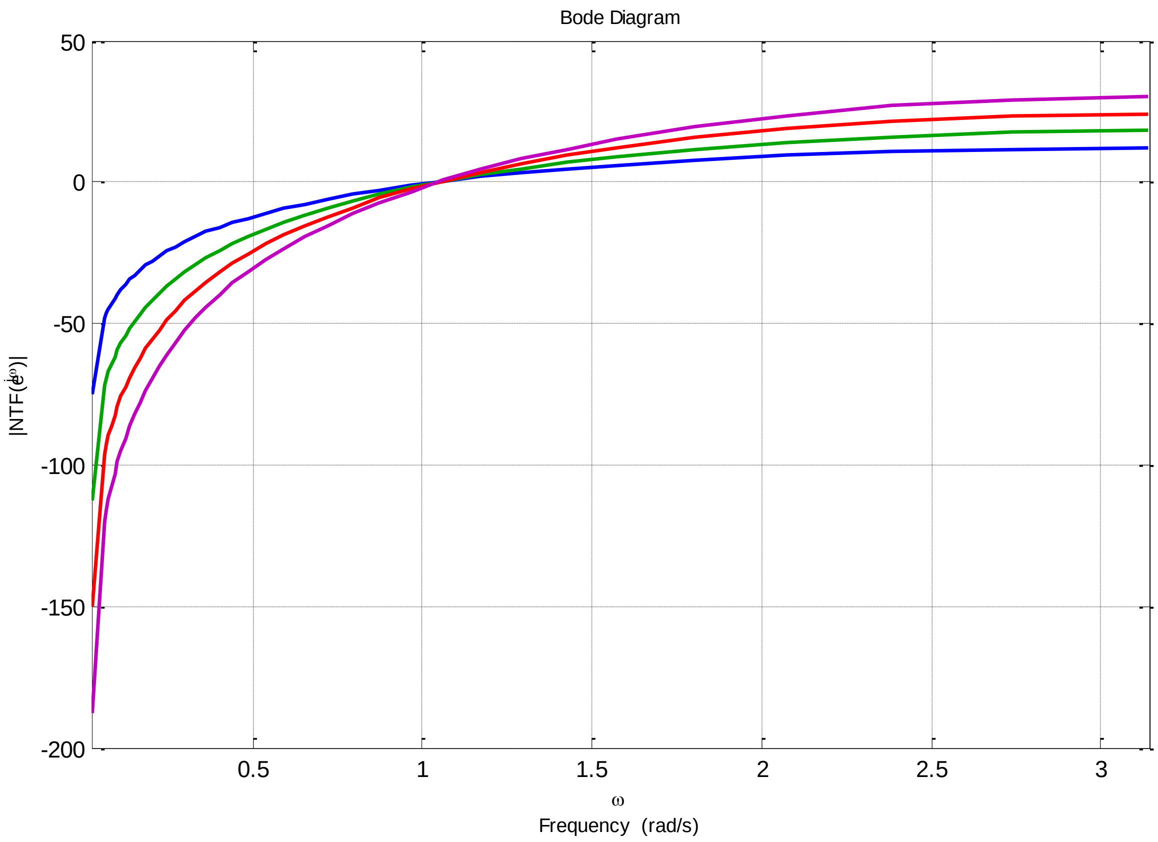


$$V(z) = \boxed{\frac{L(z)}{1+L(z)}} U(z) + \boxed{\frac{1}{1+L(z)}} E(z)$$

STF **NTF**

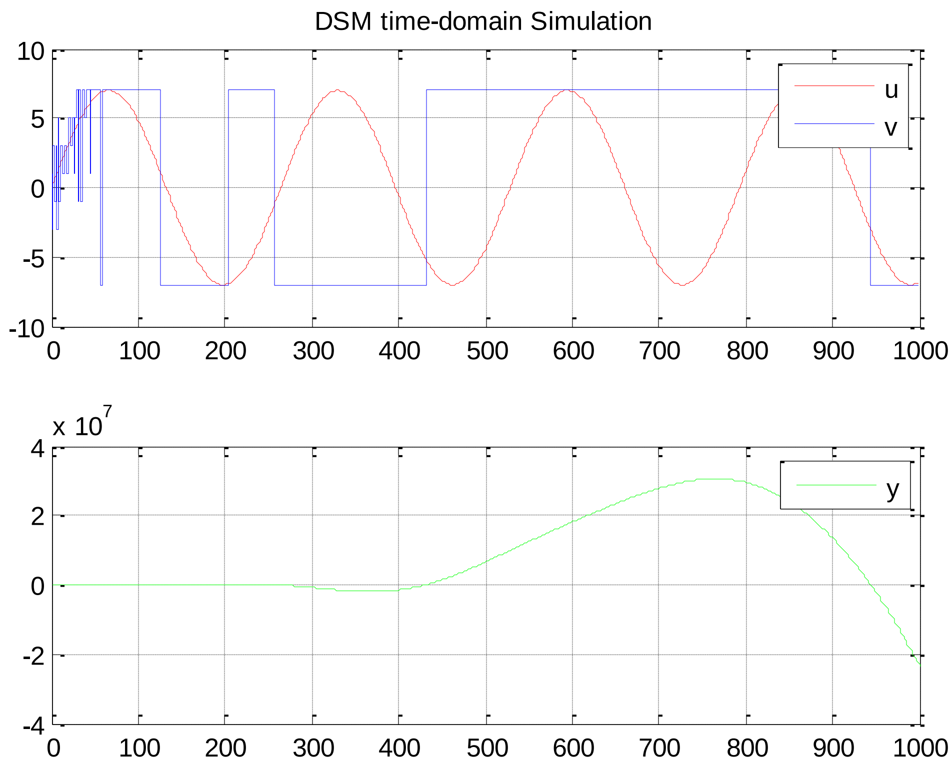
- Higher order noise shaping
  - Reduced in-band noise, higher SQNR
- For *NTF*=(1-z^{-1})$^{-N}$, in-band noise (IBN): $\frac{\Delta^2}{12} \cdot \frac{\pi^{2N}}{(2N+1)} \cdot OSR^{-(2N+1)}$
  - Ideally (N+1/2) bits increase in resolution per doubling in OSR

Higher-Order NTFs



- NTF gain increases at high frequencies (around ω≈π)
- Can we go on increasing the order?

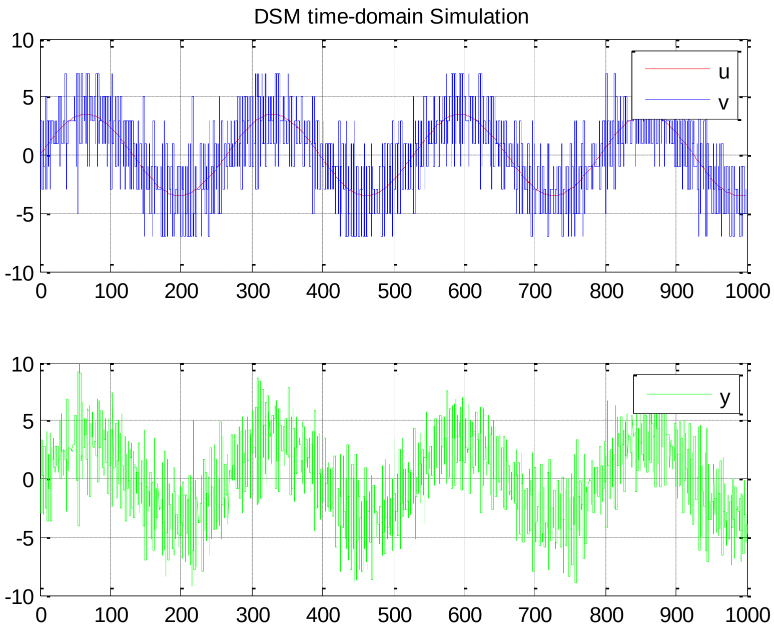
Third-order ΔΣ Modulator Example

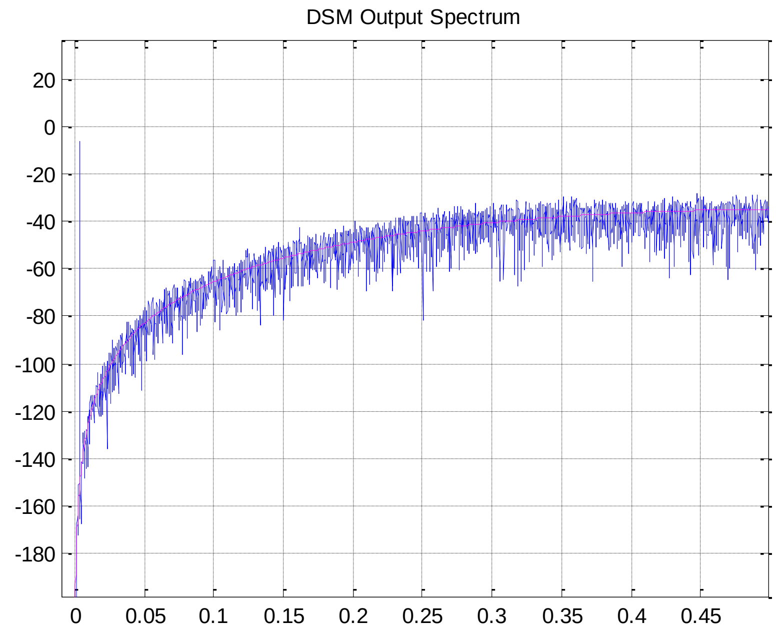


- $NTF(z) = (1\text{-}z^{-1})^3$
- OBG = 8, Full-scale input.
- Unstable after few samples (look at quantize input (*y*) blowing up!).
  - Signature for ΔΣ instability
  - Worst case for a single-bit quantizer.

File: Third_Order_DSM.m

Third-order ΔΣ Modulator Example





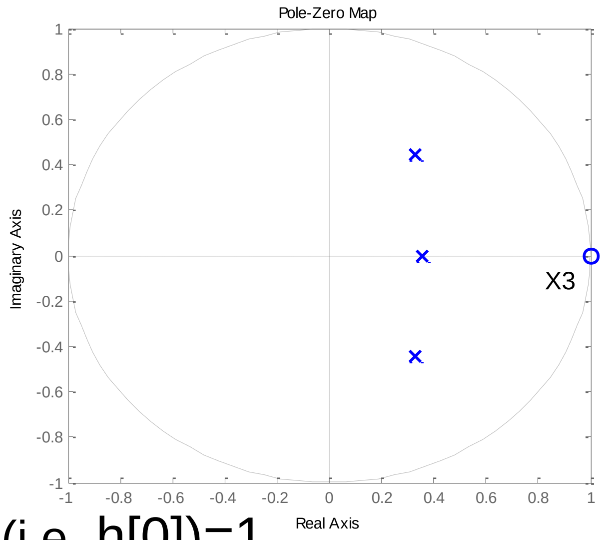
- Stable for 50% of full-Scale amplitude
- Signal dependent stability
  - Need to develop intuition for modulator stability
  - Reference: Stability theory from the Yellow Bible of delta-sigma

File: Third_Order_DSM.m

Systematic NTF Design

- NTFs of the form $(1-z^{-1})^N$ have stability issues
  - The OBG (2^N) are too high
- A larger OBG causes more wiggling at the quantizer input
  - This saturates the quantizer for even smaller inputs
  - Irrecoverable quantizer saturation causes loop instability
- For high-OBG the maximum stable (input) amplitude (MSA) is small
- The stability is worse for low quantizer resolutions
- Thus we need to reduce OBG while maintaining high in-band noise shaping

Systematic NTF Design Procedure



- Introduce poles into the NTF

- $$NTF(z) = \frac{(1-z^{-1})^N}{D(z)}$$

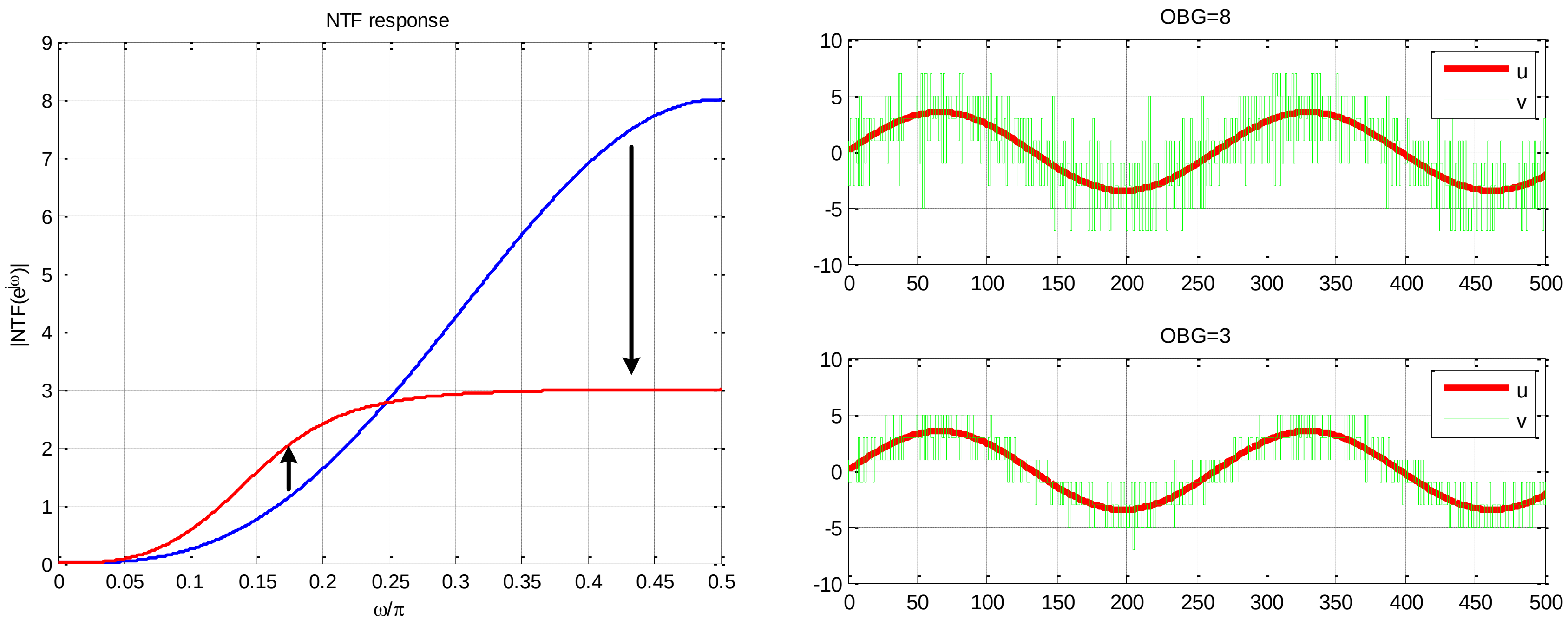
- NTF realizability criterion
  - No delay-free loops in the modulator
    - First sample of the NTF impulse response (i.e. h[0])=1

$\Rightarrow$ NTF(∞)=1

$\Rightarrow$ D(z=∞)=1

- Commonly used pole positions: Butterworth, Inverse Chebyshev and maximally flat poles (maxflat)

NTF Response with Poles



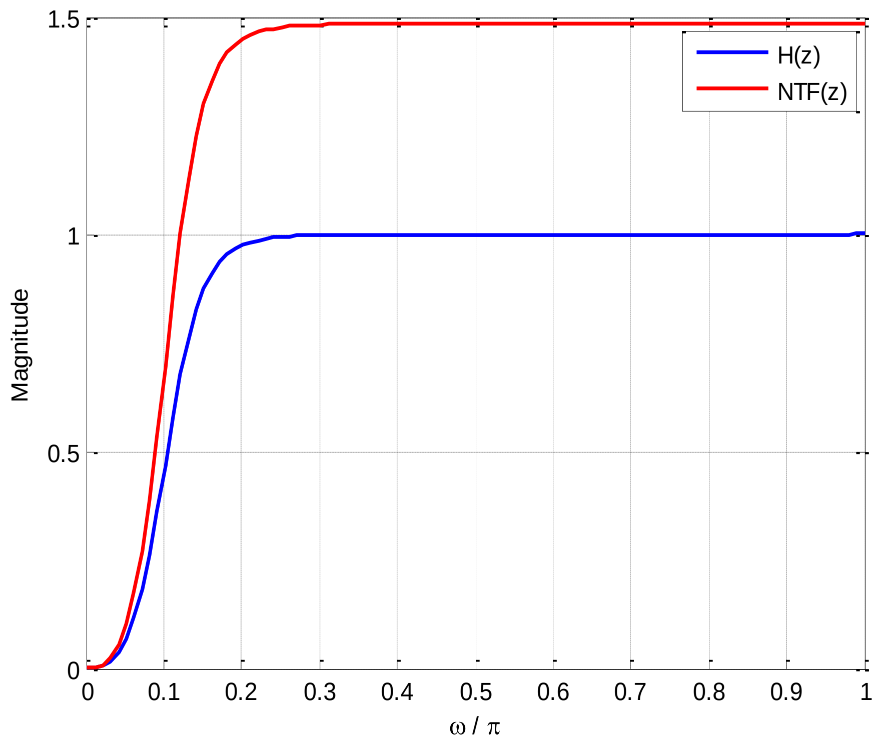
- Select appropriate OBG for the NTF to assure stability
- Trade-off between stability and increased in-band noise
  - MSA vs SQNR for a given order and quantizer resolution

Systematic NTF Design Example

- Specifications
  - SQNR > 120 dB
  - A signal bandwidth which results in an OSR = 64
    - Study optimal clock rate for the given process and quantizer design.
- Designer's Choice
  - Order = 3
  - Quantizer levels (nLev) = 16
  - Butterworth high-pass response for the NTF

- Use MATLAB for finding coefficients of the HPF response.
  - [b,a] = butter(order, ω_{3dB}, ‘high’)
  - The cutoff frequency ω_{3dB} specifies the transfer function.

Systematic NTF Design contd.

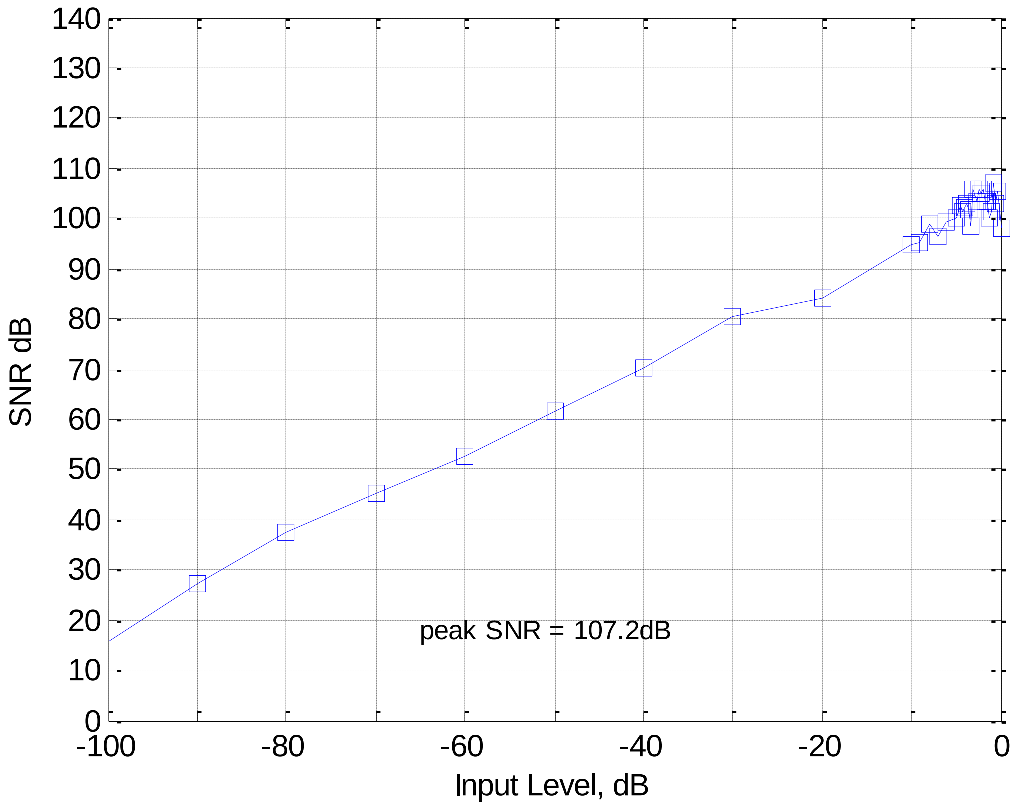


- Start with cutoff frequency $\omega_{3dB}=\pi/8$, for the butterworth HPF H(z).
- Derive a realizable NTF using $NTF(z)=H(z)/b_0$

Systematic NTF Design contd.

- Map the NTF response to a loop-filter architecture (details later)
- Simulate the modulator for all possible amplitudes and input tone frequencies.
- Compute the peak SQNR and MSA.
  - simulateDSM function in the toolbox.
  - Can use Risbo's method shown later

Systematic NTF Design contd.

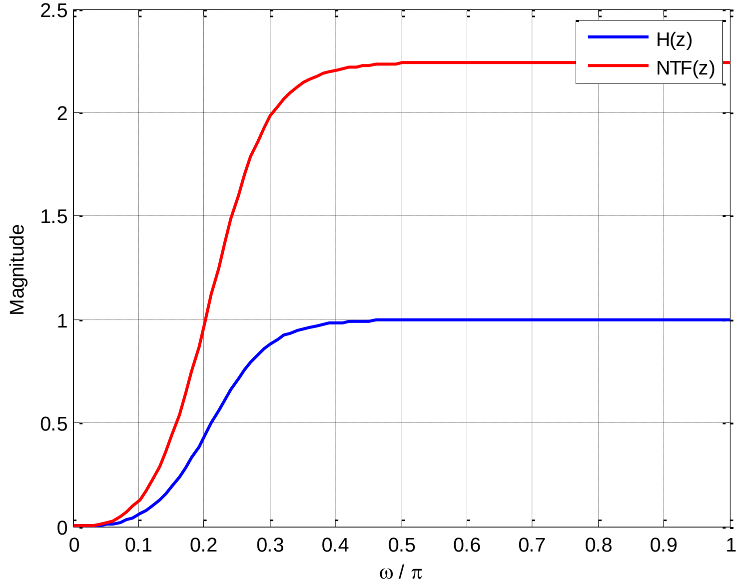


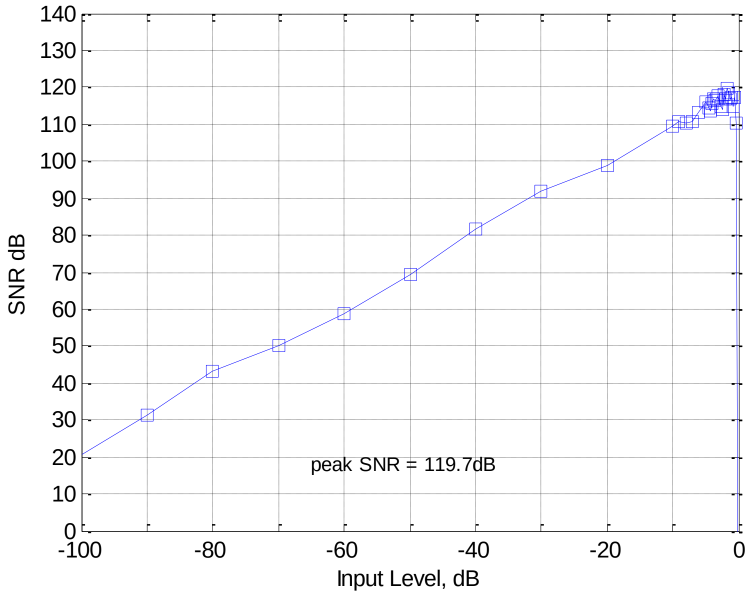
- Peak SNR = 107 dB
- MSA = 0.9

Systematic NTF Design contd.

- If SNR is not enough, repeat the entire procedure with a higher cutoff frequency for the Butterworth HPF
  - IBN ↓, SQNR ↑
  - OBG ↑ and MSA ↓
- If SNR is too high, repeat the entire procedure with a lower cutoff frequency for the Butterworth HPF
  - IBN ↑, SQNR ↓
  - OBG ↓ and MSA ↑

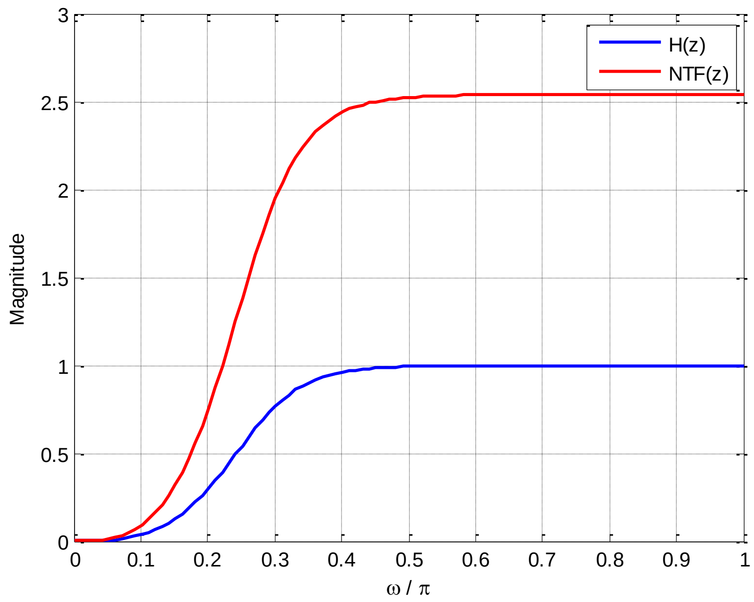
Systematic NTF Design contd.

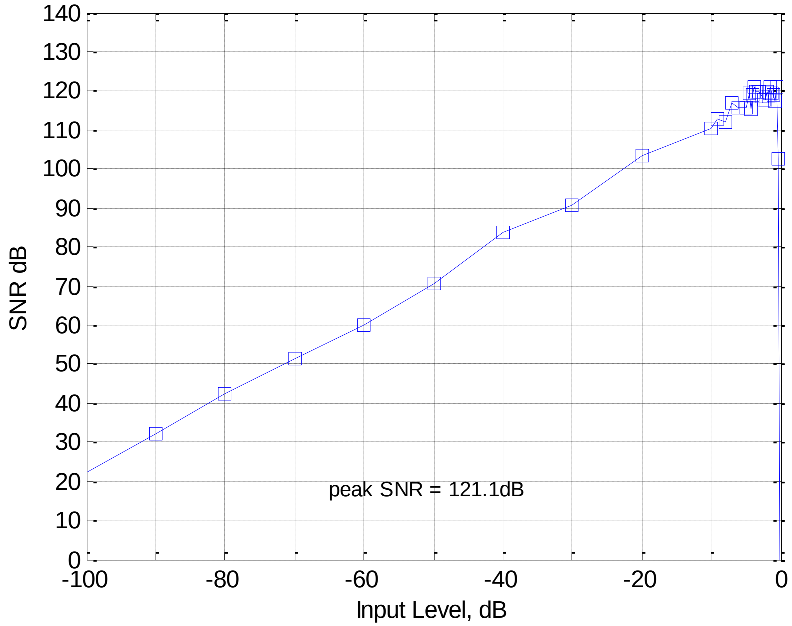




- $\omega_{3dB}=\pi/4$.
- Peak SNR = 119 dB, OBG = 2.25, MSA = 0.8

Systematic NTF Design contd.





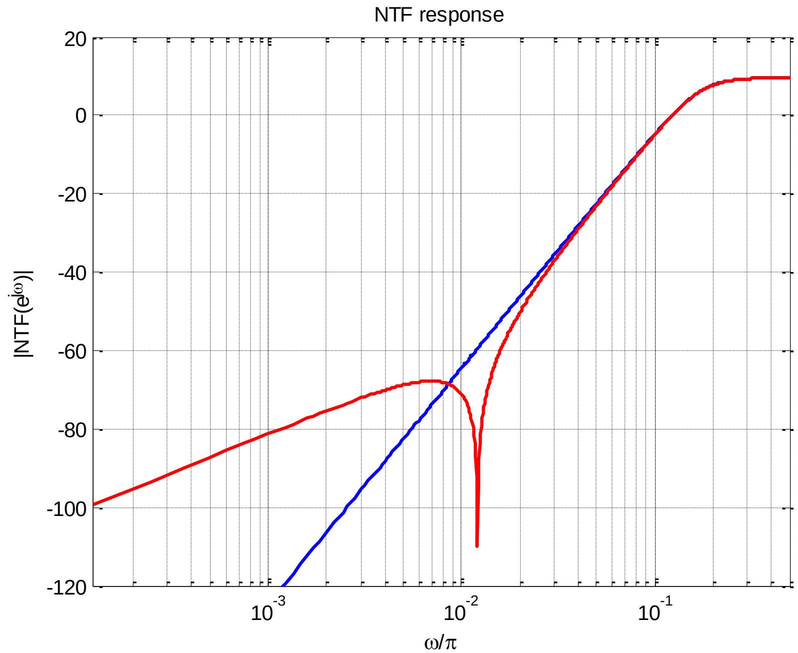
- $\omega_{3dB}=2\pi/7$.
- Peak SNR = 121 dB, OBG = 2.54, MSA = 0.8.
  - Design closed !

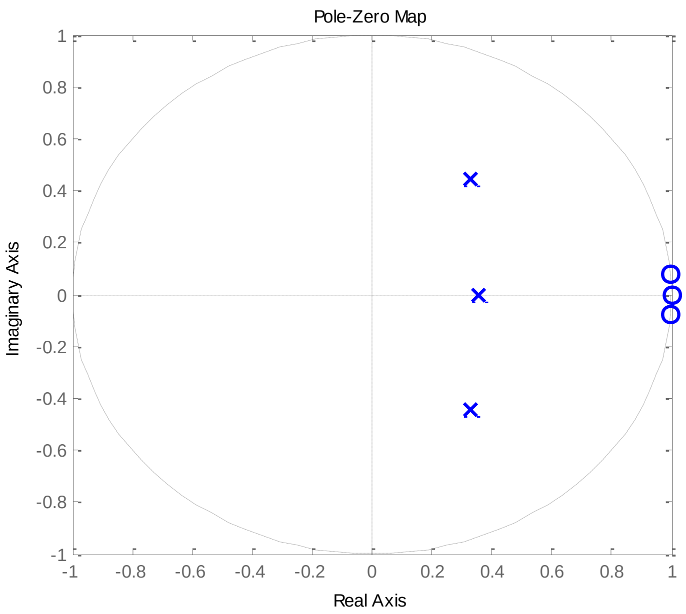
Systematic NTF Design contd.

- An advanced version of this iterative process is implemented as the function synthesizeNTF in the delta-sigma Toolbox.
  - Several ‘opt’ params for NTF zero (and pole) optimization
  - Use synthesizeChebyshevNTF for low OSR and low OBG designs.

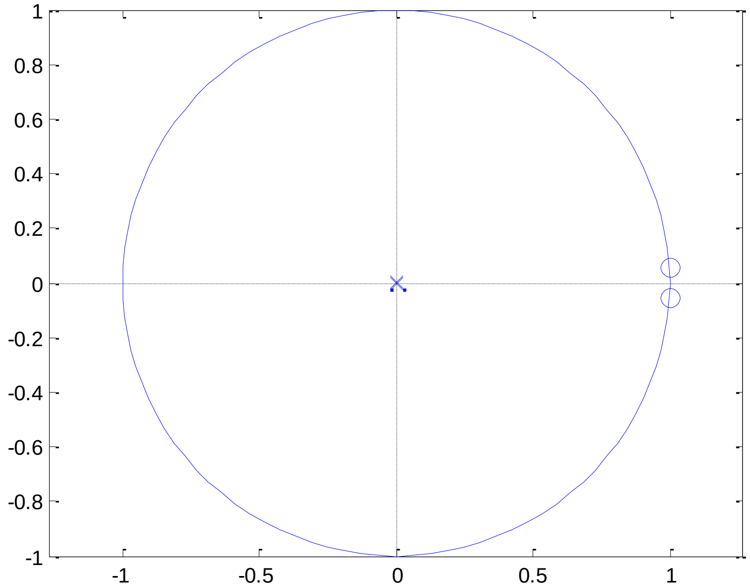
NTF-Zero Optimization

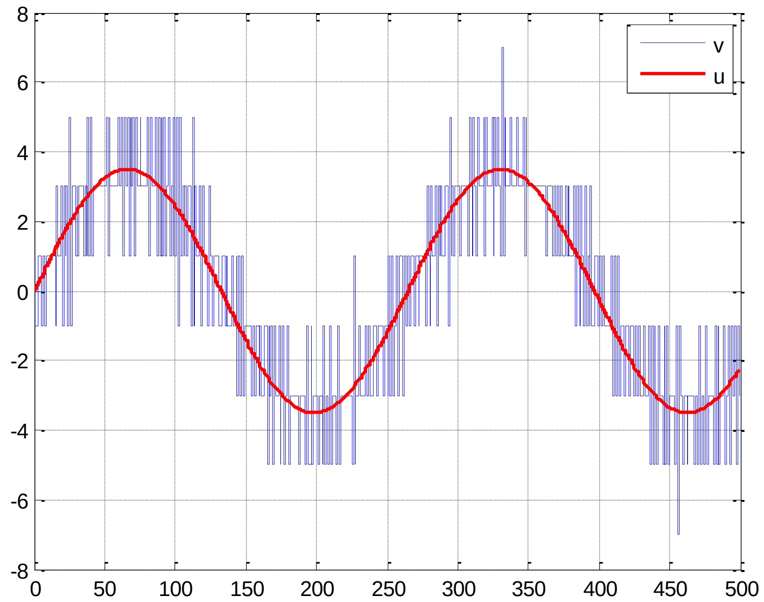
- Spread zeros in the signal band to minimize in-band noise
  - Complex zeros on the unit circle
  - 8dB increase in SQNR for 3rd order modulator
- Bandwidth normalized NTF-zero locations obtained by toolbox function ds_optzeros(order, 1)
- Already implemented in synthesizeNTF function for opt=1





2^{nd} order DSM: NTF Zero Optimization

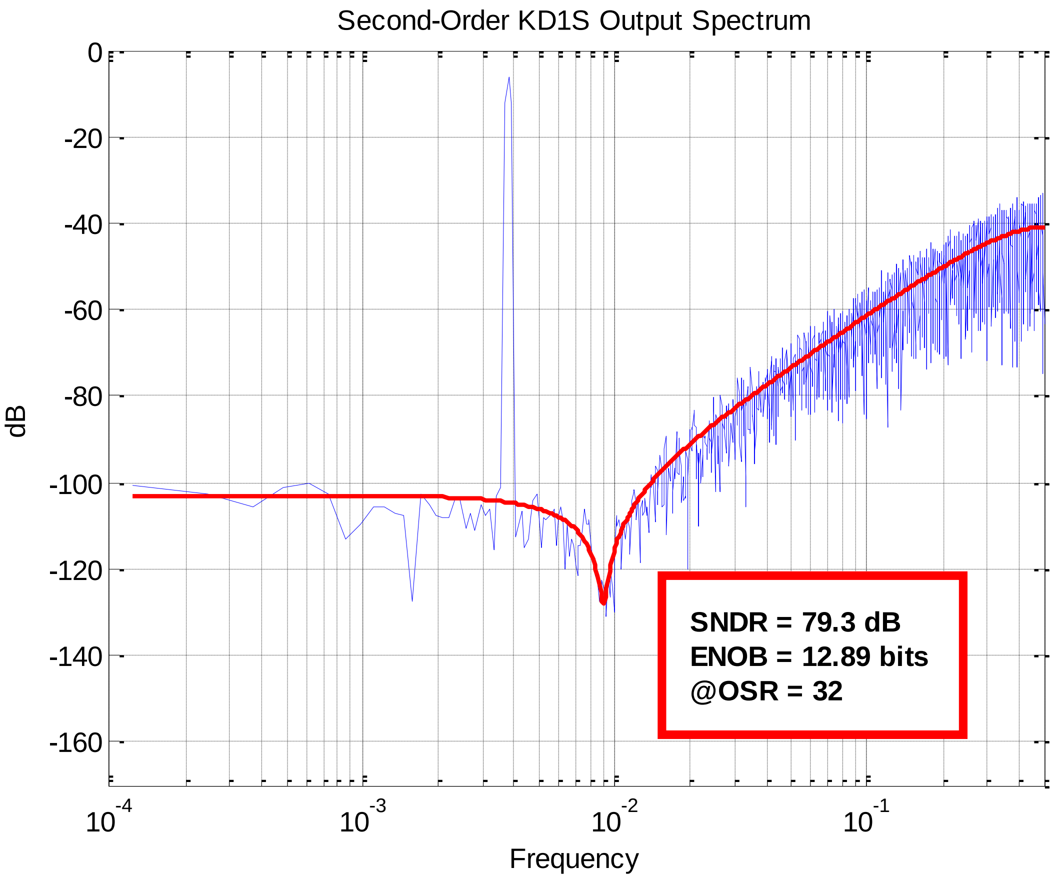




$$NTF(z) = (1 - e^{j0.06} z^{-1})(1 - e^{-j0.06} z^{-1})$$

File: Second_Order_DSM_Zero_Opt.m
Set variable opt=1.

2nd order DSM: NTF Zero Optimization contd.



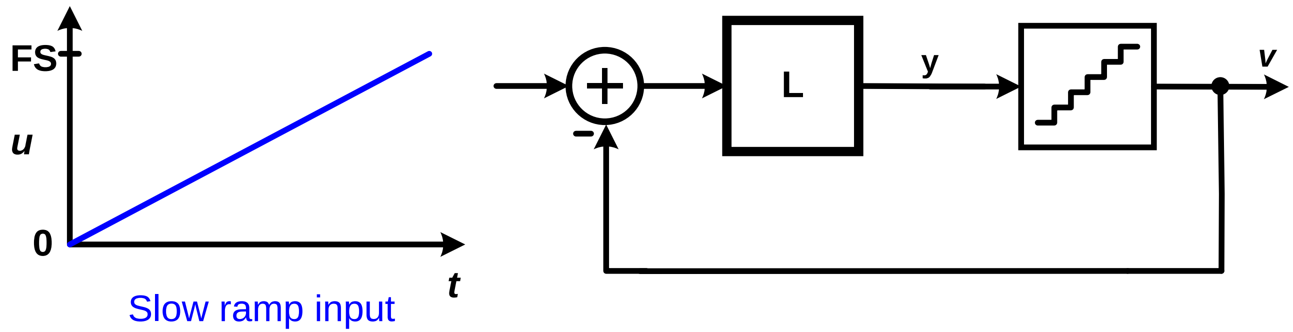
•5.5 dB increase in SQNR.

NTF pole (if any) optimization to be discussed later.

Estimating MSA (Maximum Stable Amplitude)

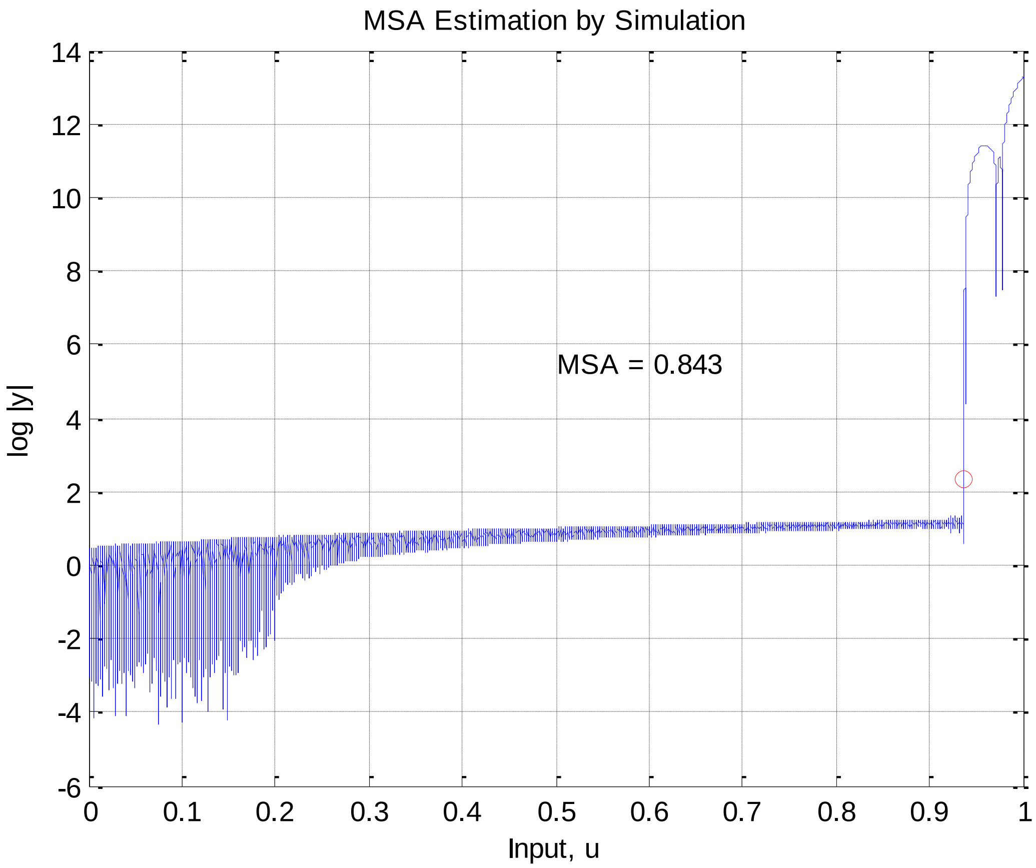
- MSA is found through extensive simulation
- Simulate for input sinusoids of varying amplitudes for all possible signal frequencies in the signal band.
  - For every input amplitude compute in-band SNR.
  - Beyond the MSA, the NTF poles move out of the unit circle.
  - Noise shaping is disrupted and the in-band SNR drops.
  - At this point the quantizer input (y[n]) blows up.
- simulateSNR function in the toolbox does exactly the same
- Time consuming and often impractical for iterative design

Estimating MSA using Risbo's Method



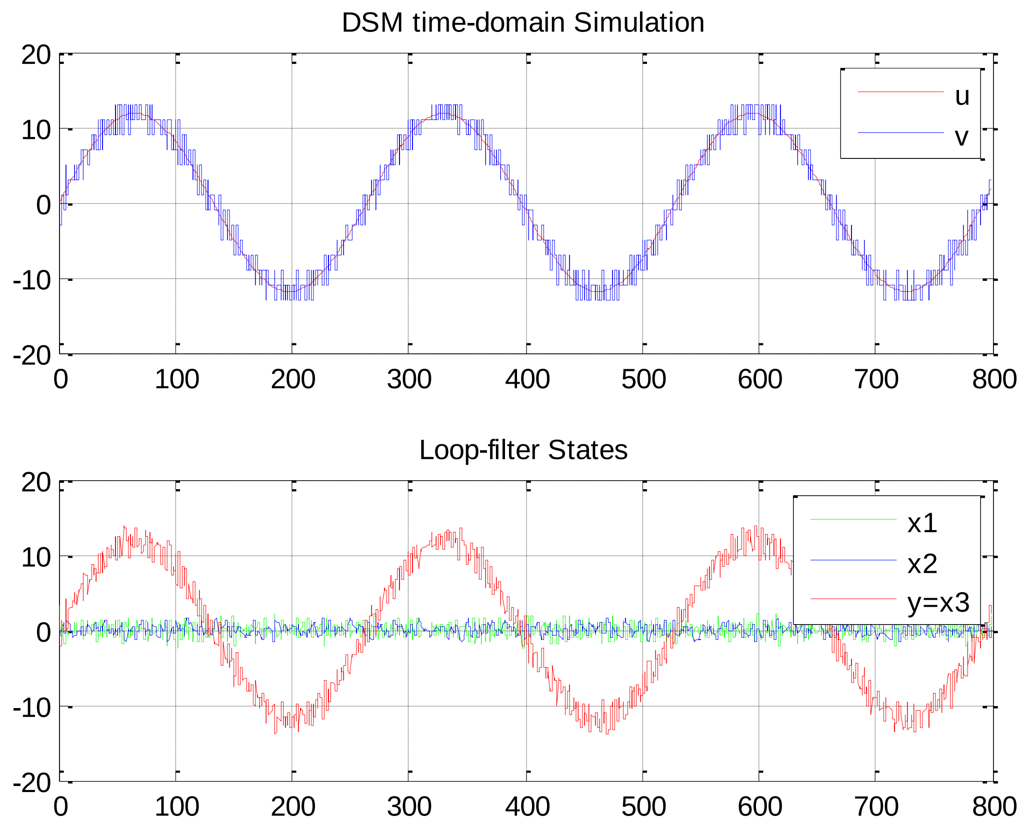
- Use a slow ramp input from 0 to FS value.
  - Plot $\log_{10}|y[n]|$. Observe where this plot blows up.
  - Take 90% of the input amplitude where $\log_{10}|y[n]|$ blows up as a conservative estimate for MSA.
  - Estimated MSA is close to that predicted by the sinewave input method.
- Much quicker than the sinewave technique (simulateSNR function)

Estimating MSA using Risbo's Method



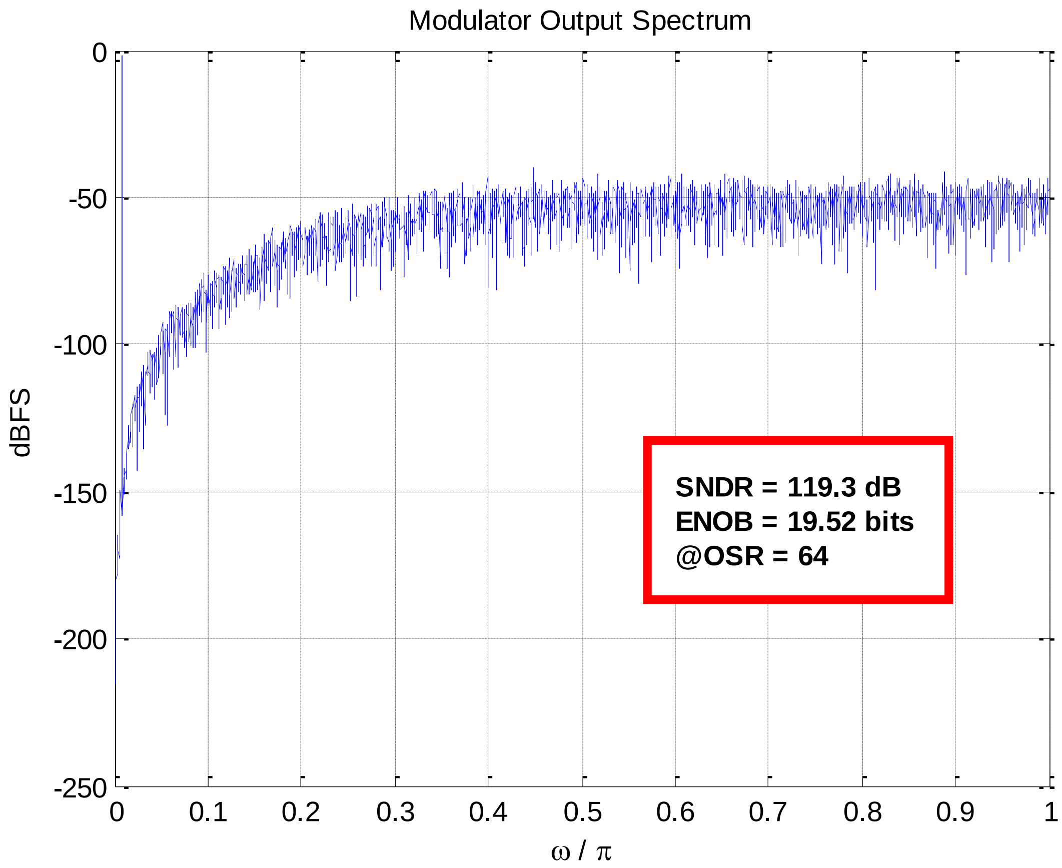
File: MSA_Risbo_Method.m

Simulation with input with MSA



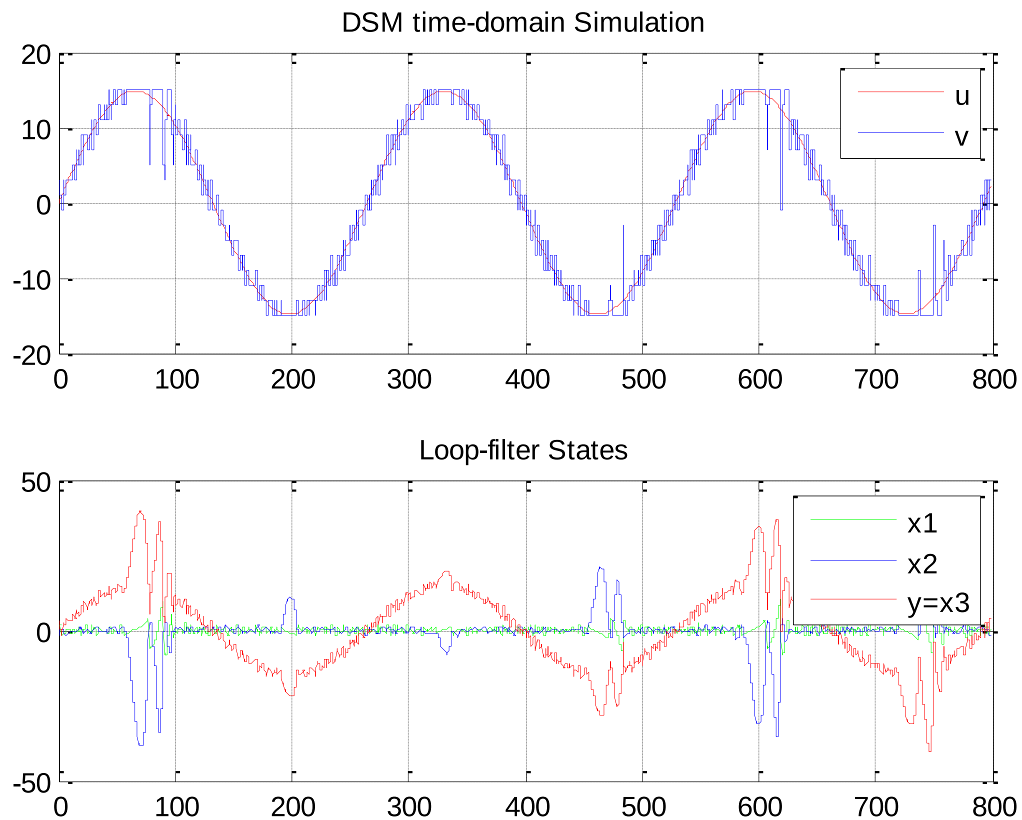
File: MSA_Risbo_Method.m

Simulated SNR with input with MSA



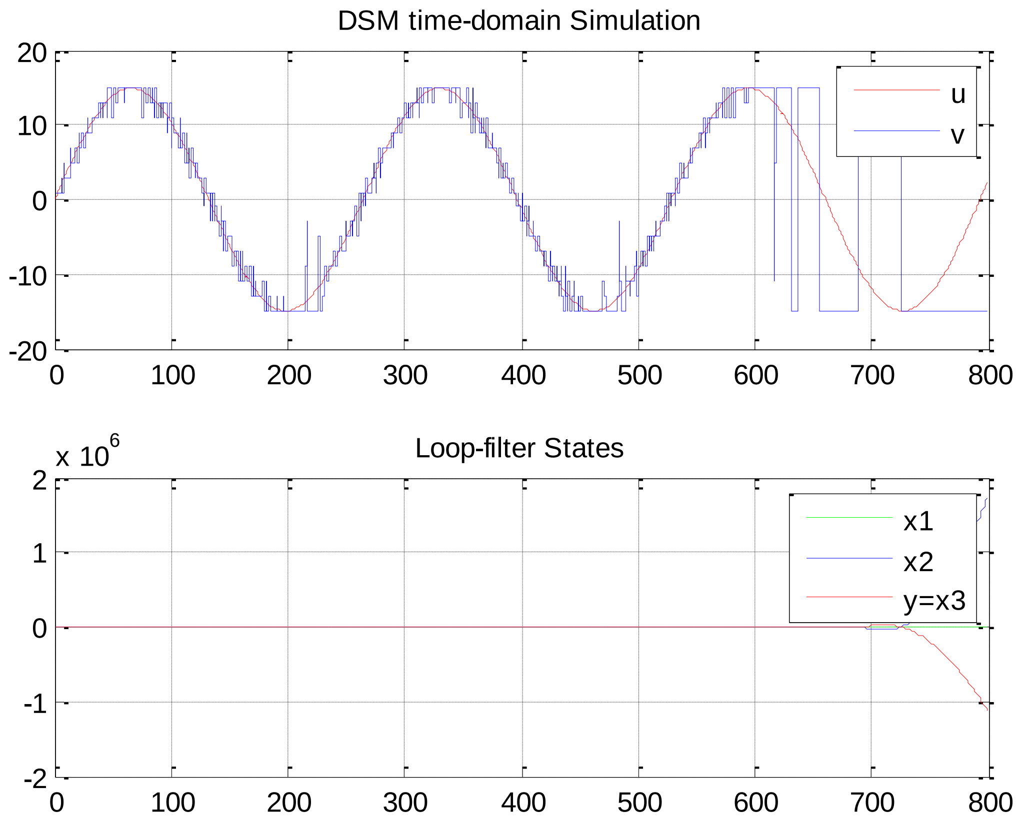
File: MSA_Risbo_Method.m

Simulation with input with 1.2*MSA



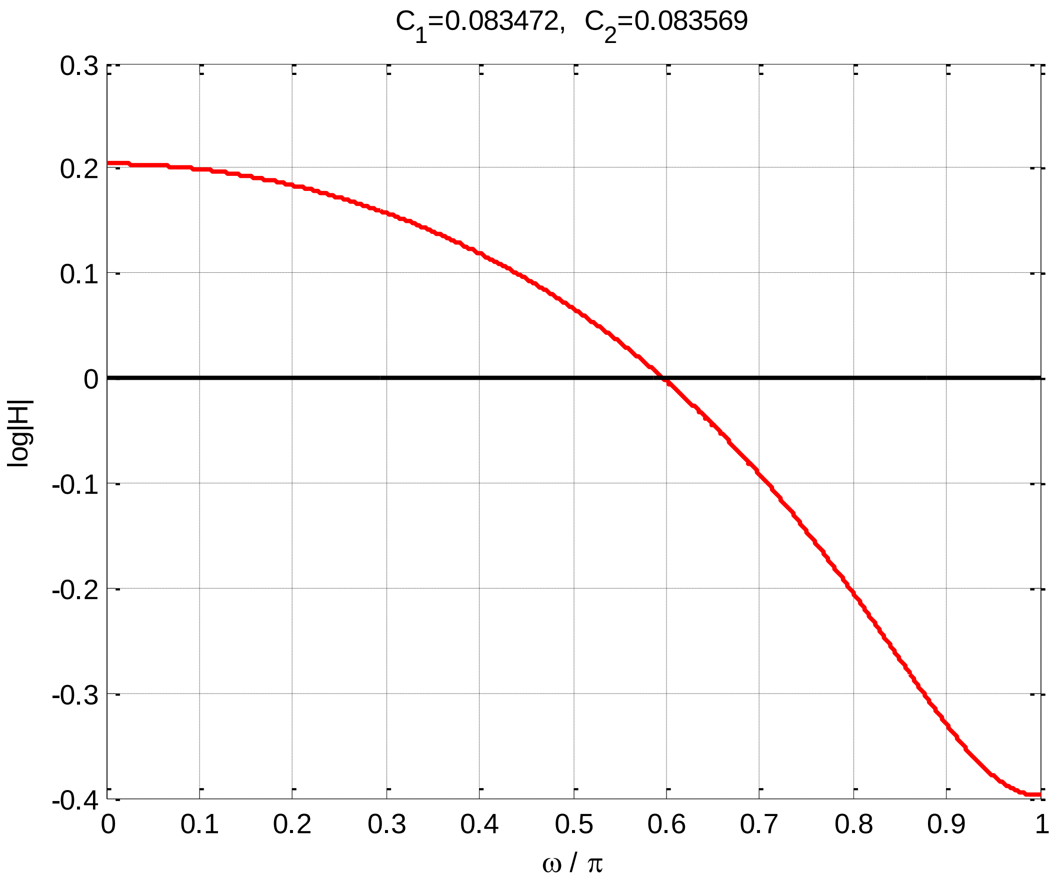
File: MSA_Risbo_Method.m

Simulation with input with 1.2*MSA



File: MSA_Risbo_Method.m

Bode Sensitivity Integral

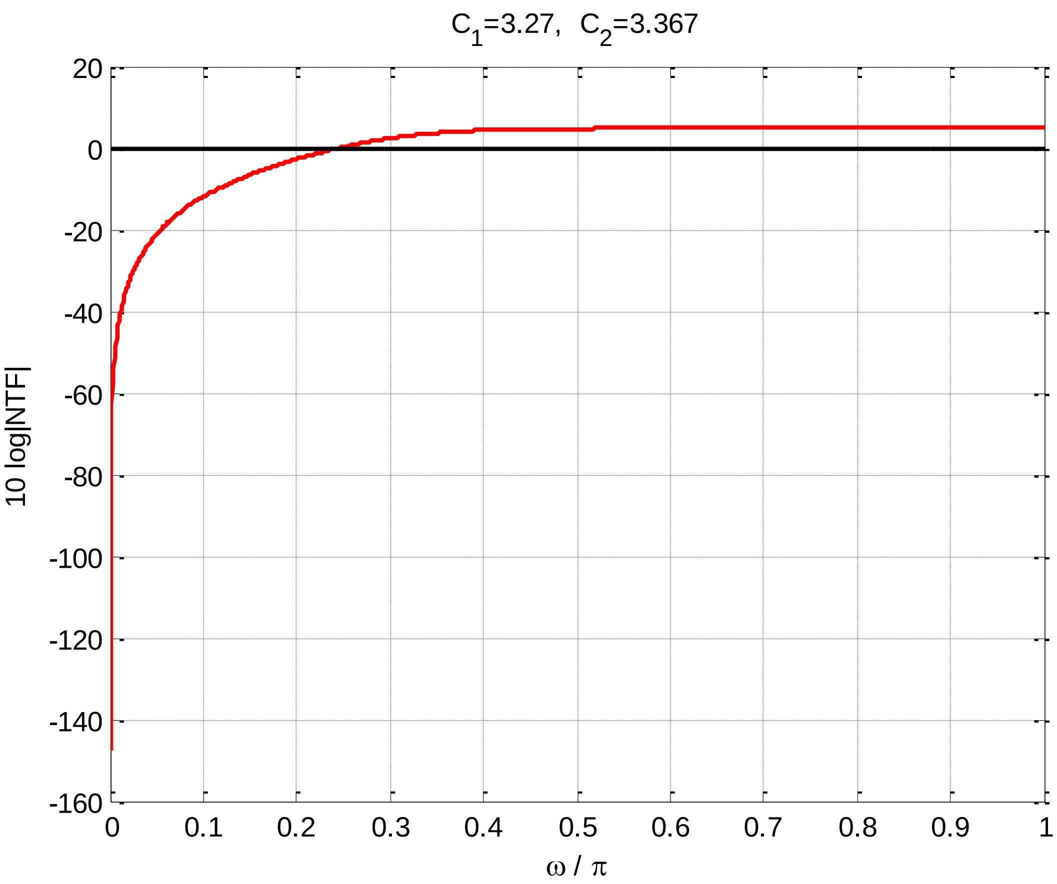


Single pole/zero transfer function with pole/zero inside the unit circle.

Area above and below the 0-dB axis are equal.

File: BodeSensitivity1.m

Bode Sensitivity Integral

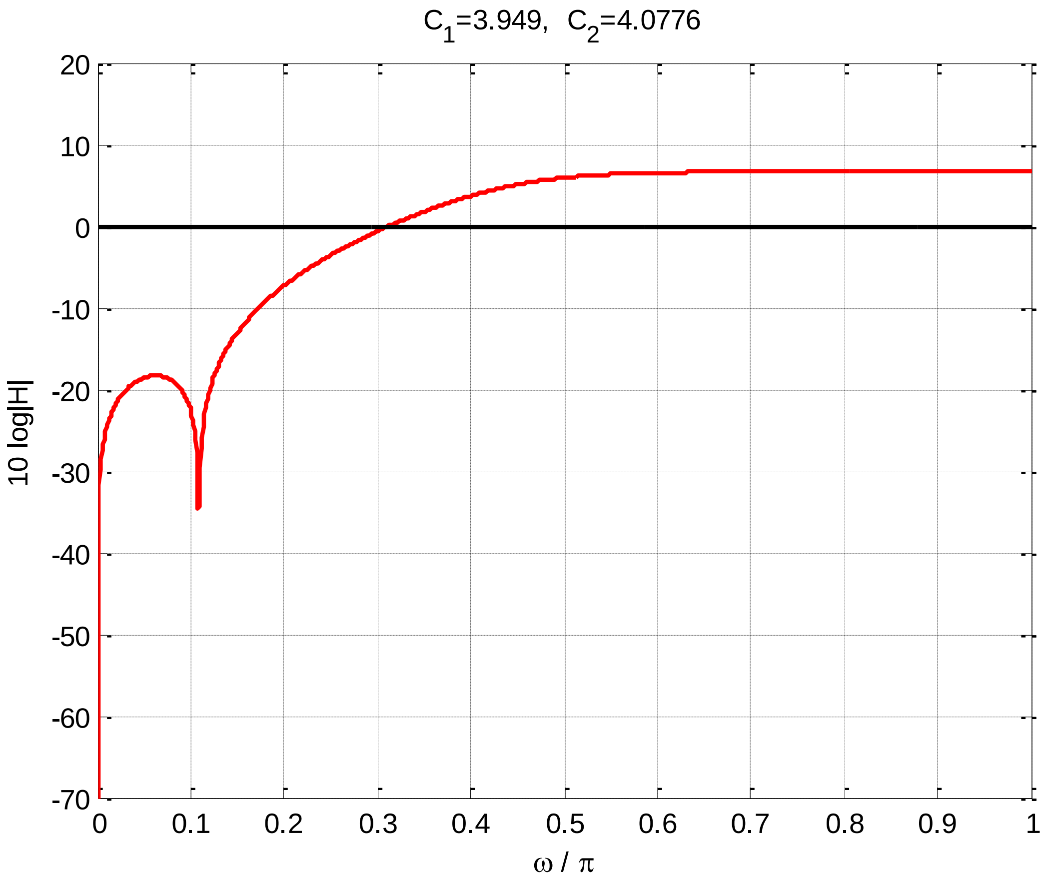


Butterworth NTF.

Area above and below the 0-dB axis are equal.

File: BodeSensitivity2.m

Bode Sensitivity Integral

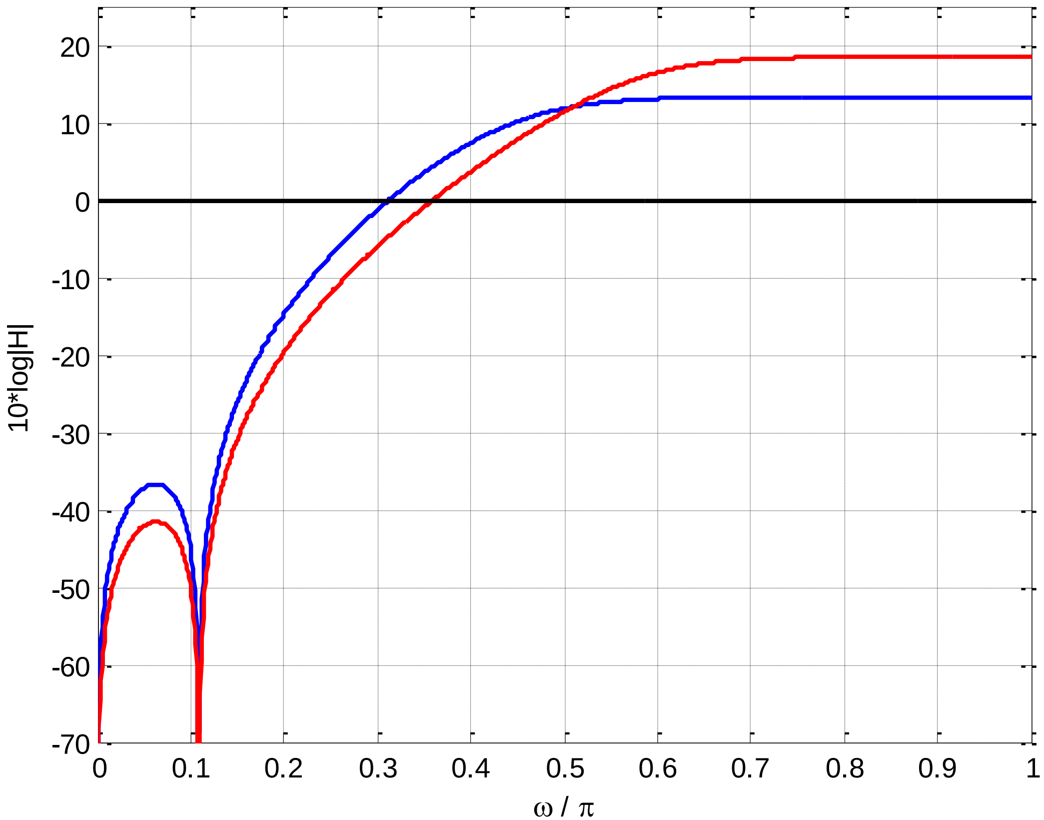


Inverse Chebyshev NTF.

Area above and below the 0-dB axis are equal.

File: BodeSensitivity3.m

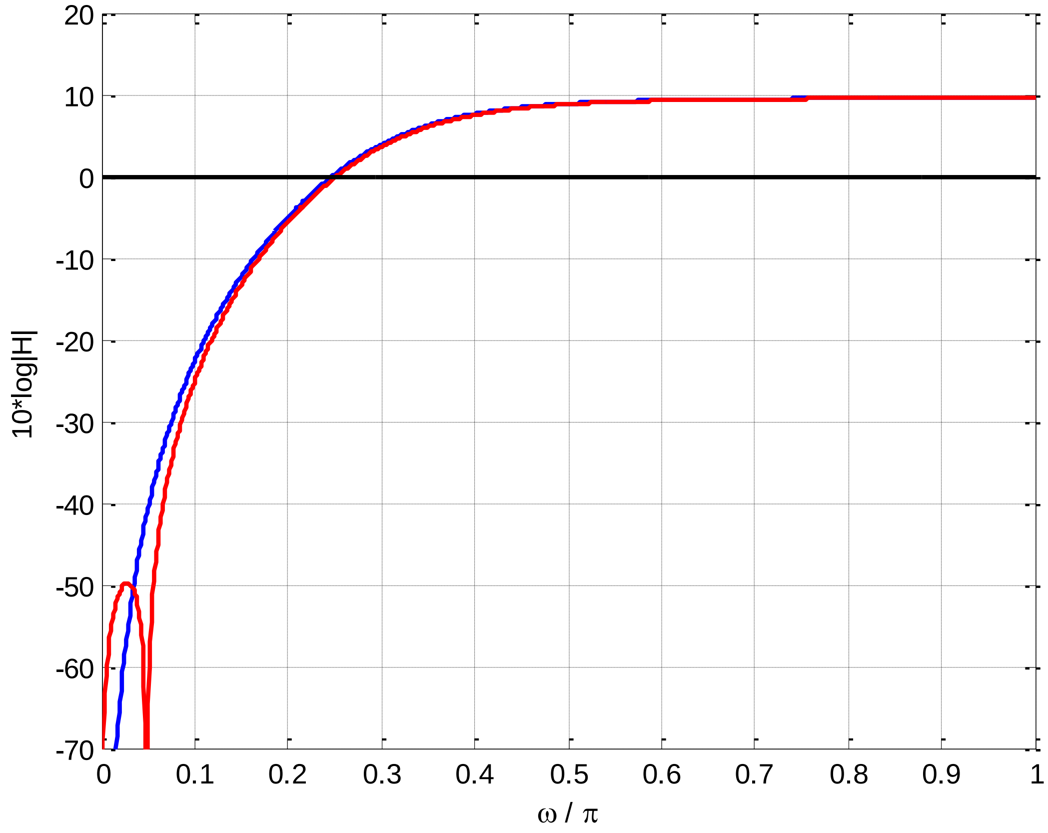
Bode Sensitivity Integral



Better in-band performance results in worse out-of-band performance.

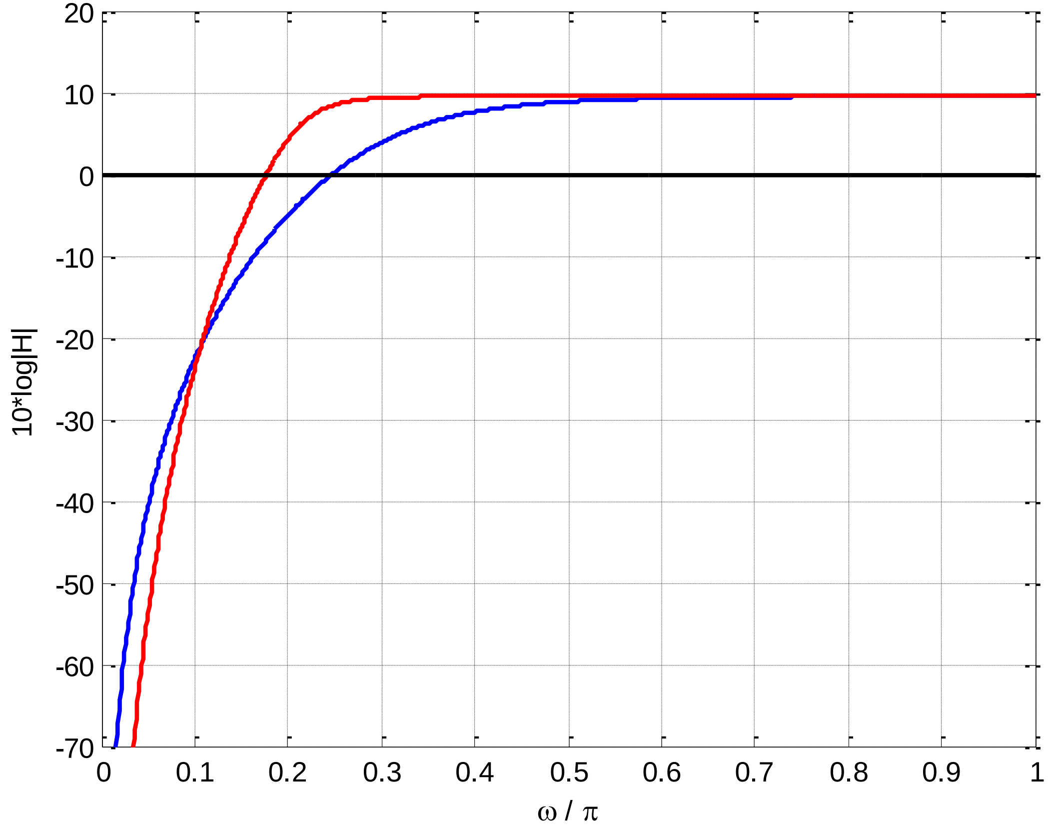
File: BodeSensitivity4.m

Bode Sensitivity Integral



Complex NTF zeros result in better in-band performance for the same OBG.

Bode Sensitivity Integral



Higher-order NTF results in better in-band performance for the same OBG.

Loop Filter Architectures

Loop-Filter Architectures

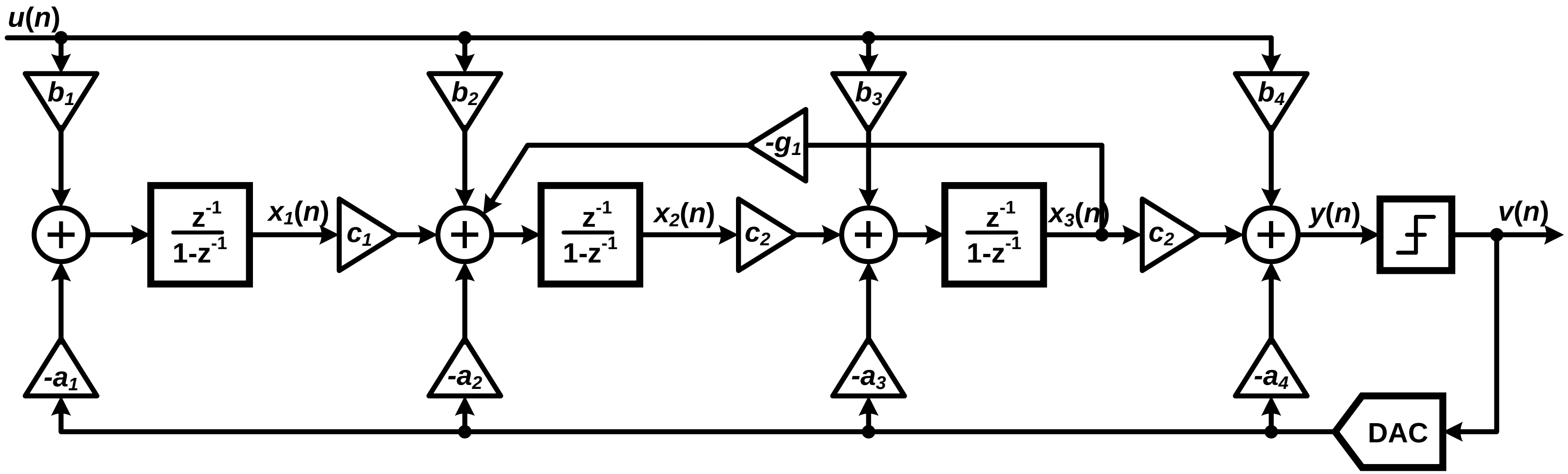
- Several loop-filter discrete-time architectures possible
- Toolbox function realizeNTF maps the synthesized NTF to loop-filter co-efficients

```
[a,g,b,c] = realizeNTF(H, form);
```

- Dynamic Range Scaling (DRS) performed to scale loop-filter states to a bounded value
  - Scaling performed using ABCD matrix representation of the loop-filter
  - See any introductory text on Linear Systems

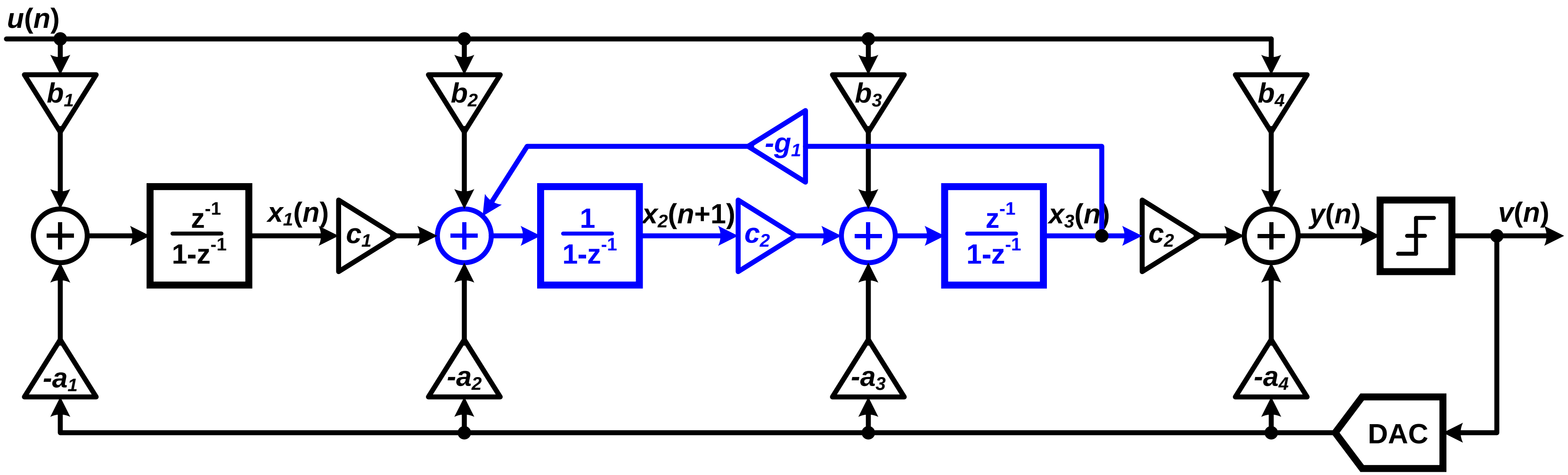
```
ABCD = stuffABCD(a,g,b,c,form);
[ABCDs umax] = scaleABCD(ABCD, nLev, f0, xLim);
[a,g,b,c] = mapABCD(ABCDs, form);
```

CIFB (Cascade of Integrators with Distributed Feedback)



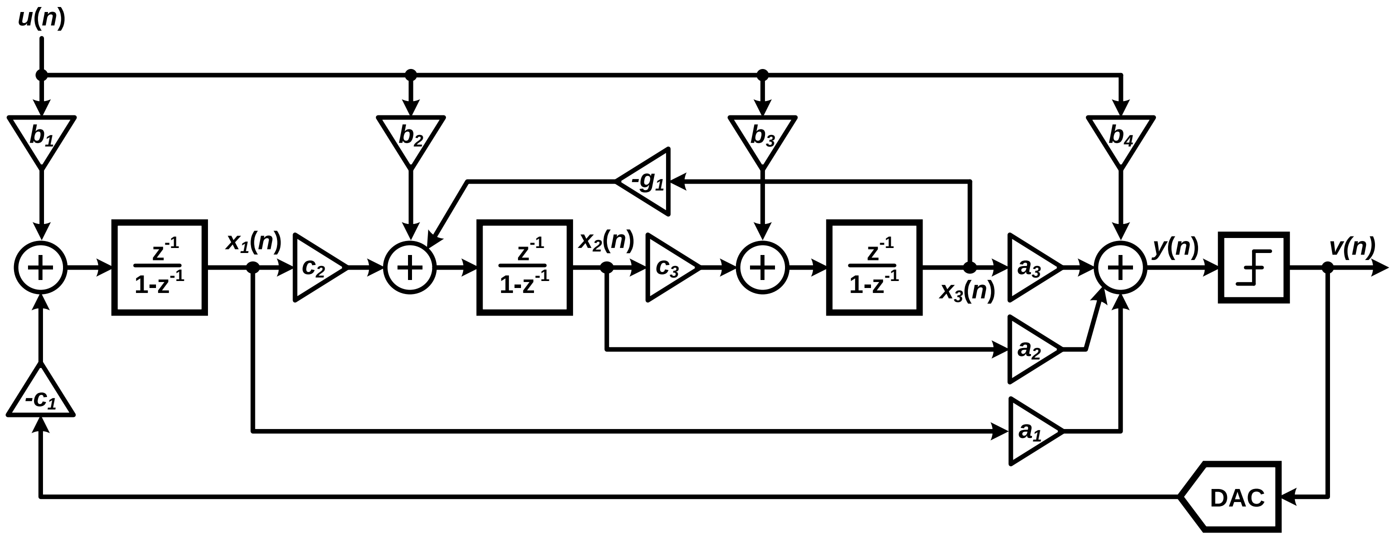
- Cascade of delaying integrators:
  - Feedback coefficients **a**'s realize the zeros of L_1 and thus the NTF and STF poles.
  - Feed-in coefficients **b**'s determine zeros of L_0 and thus the STF zeros.
  - State scaling coefficients **c**'s are used for dynamic range scaling.

CRFB (Cascade of Resonators with Distributed Feedback)



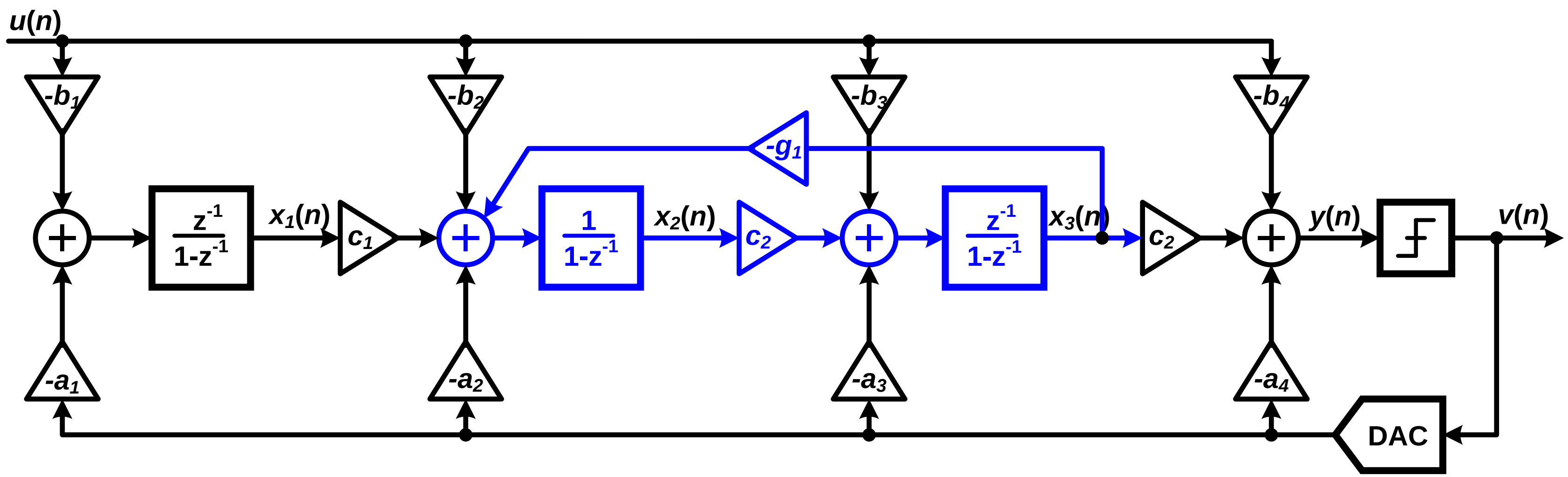
- Combine a non-delaying and a delaying integrator with local feedback around them, to form a stable resonator
  - Local feedback coefficients **g**'s realize the complex zeros in the NTF.
  - Implements NTF with complex zeros $z_i = e^{\pm j\sqrt{g_1}}$
- For odd-order, use an integrator in the front to avoid noise coupling due to g

CIFF (Cascade of Integrators with Feed-Forward Summation)



- Feedforward summation of states
- For $b_1=b_{N+1}=1$ and b_2 to $b_N=0$, STF=1
  - Loop-filter only processes quantization noise, low power and distortion
- Feedforward loop-filters typically result in lower-power implementation

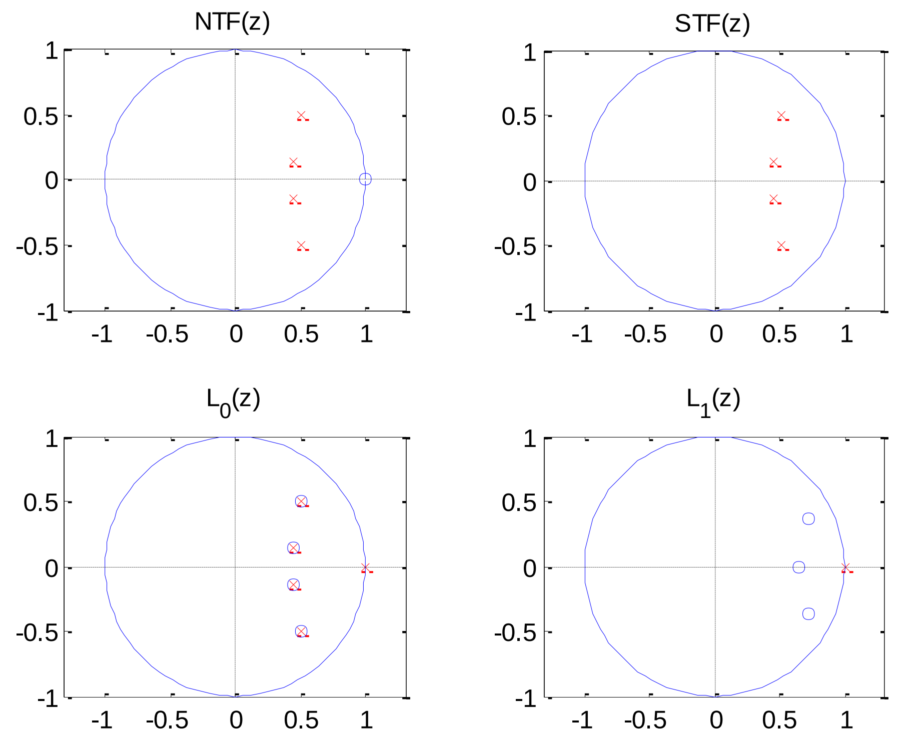
CRFF (Cascade of Resonators with Feed-Forward Summation)



- Use resonators with feedforward summation
  - Implements NTF with complex zeros
- For odd-order, use an integrator in the front to avoid noise coupling due to g

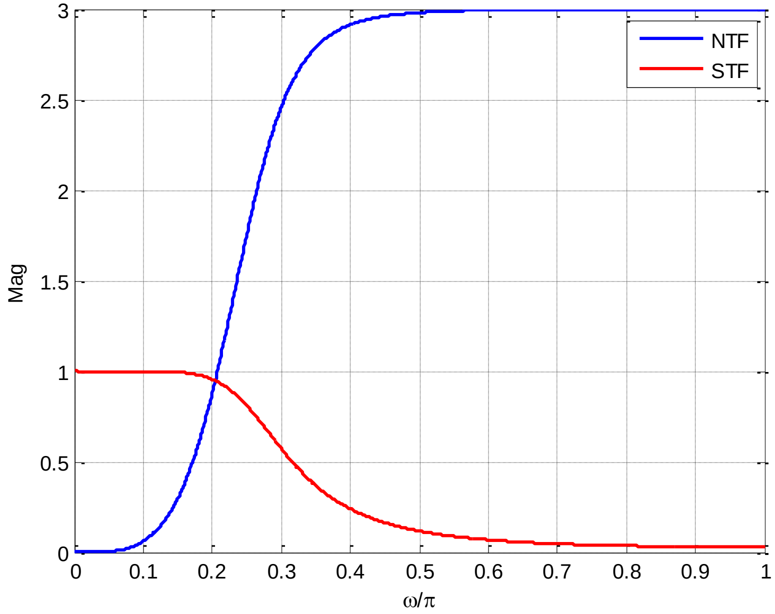
CIFB Example 1

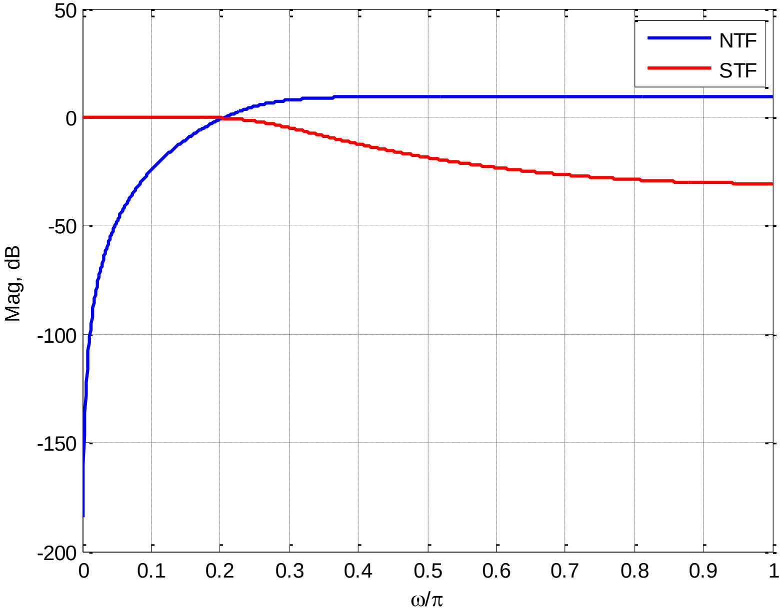
- CIFB, order = 4
- All NTF zeros at z=1, i.e. opt =0.
- OBG = 3, OSR = 16, nLev = 15.
- Only single input coupling is used
  - b(2:end) = 0
  - Maxflat poles in STF
- **a** = [0.16 0.86 1.9 2.1]
- **b** = [0.16 0 0 0]
- **c** = [1 1 1 1]
- **g** = [0 0]



File: CIFB_4th_Order_1.m

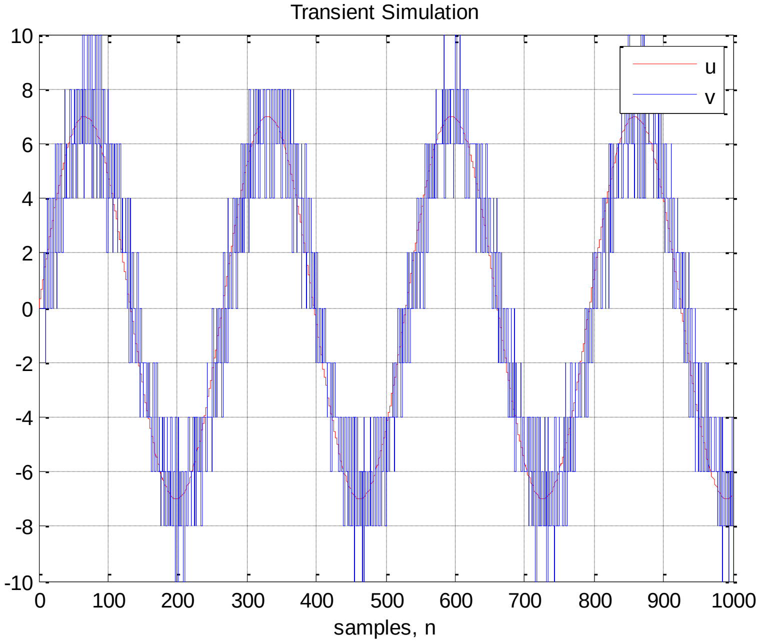
CIFB Example 1 contd. : NTF and STF

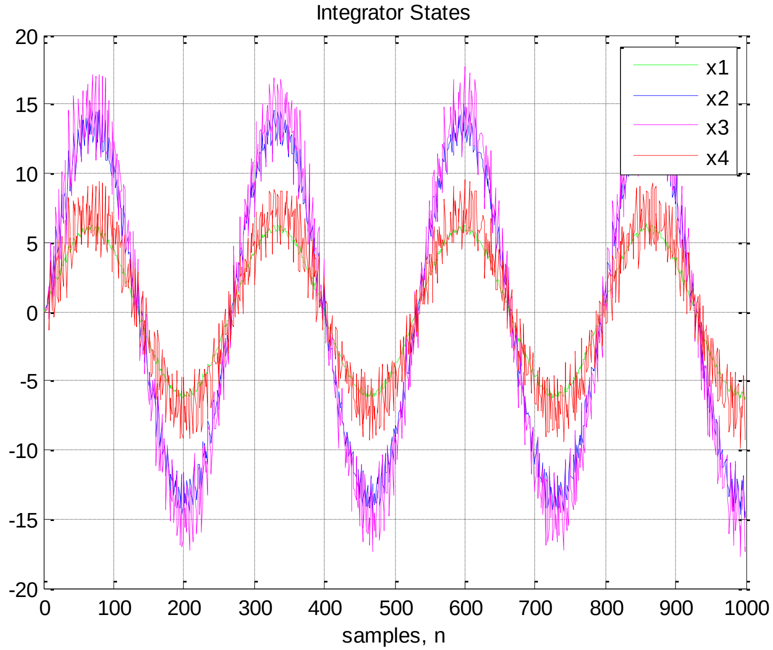




File: CIFB_4th_Order_1.m

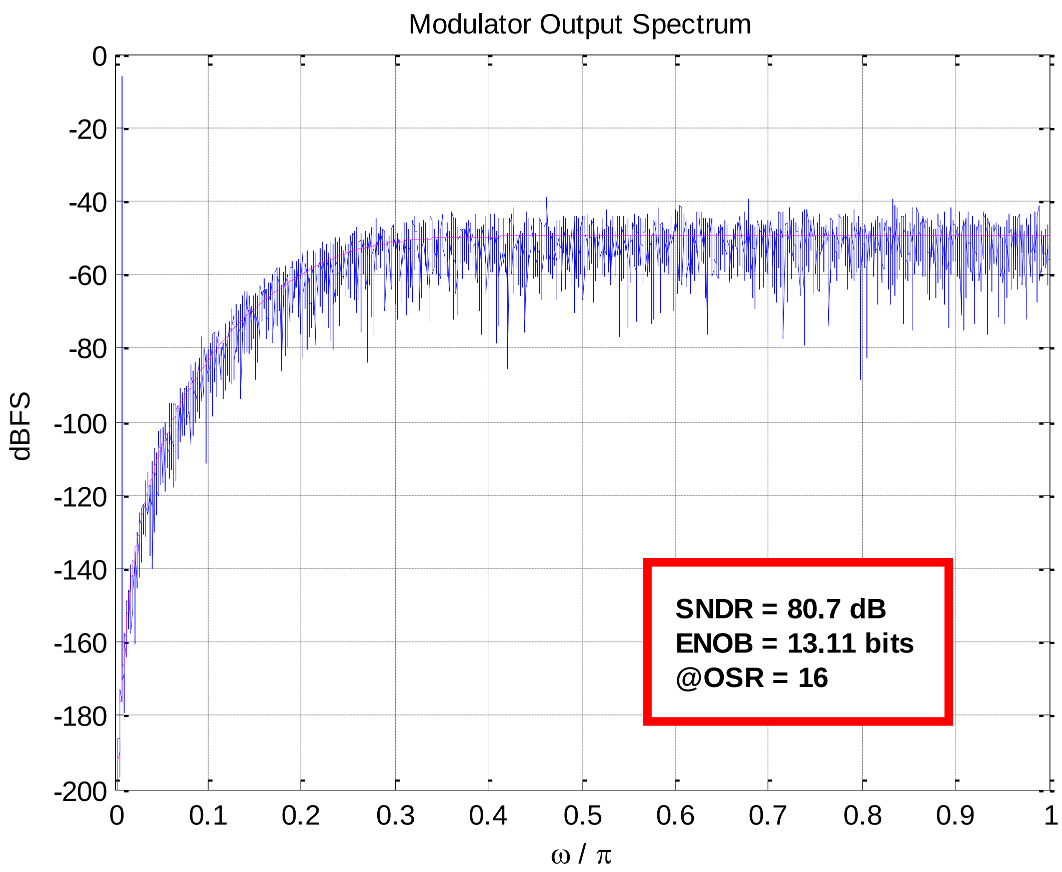
States





File: CIFB_4th_Order_1.m

Spectrum



File: CIFB_4th_Order_1.m

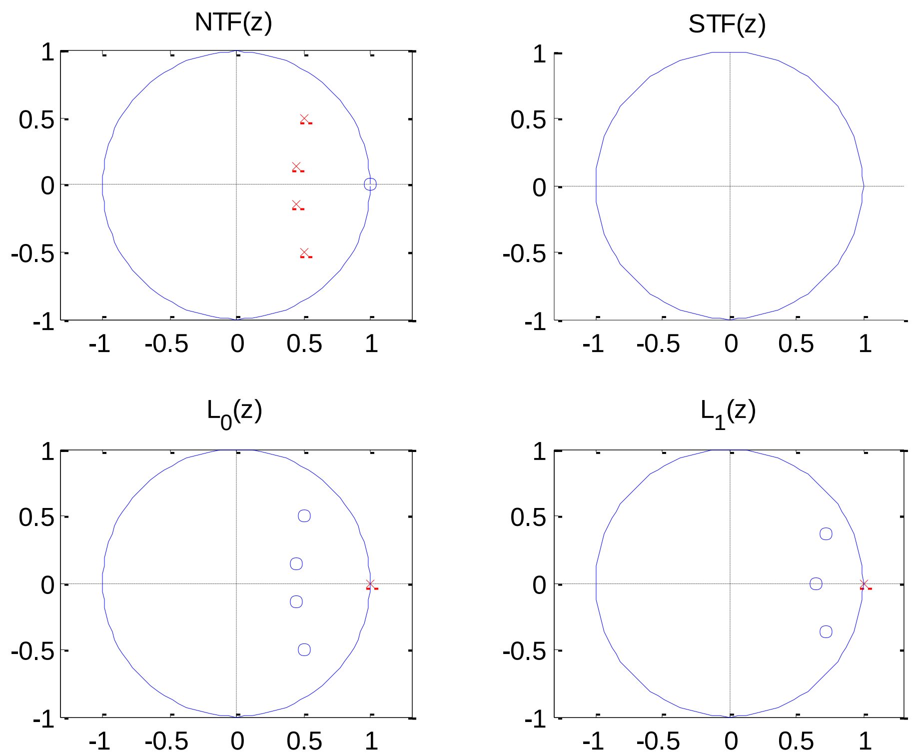
Topologies

- CRFB with single feed-in
  - CRFB_4th_Order_1.m
- Low-distortion CRFB topology
  - CRFB_4th_Order_2.m
- CIFB with single feed-in and optimized NTF zeros
  - CIFB_Opt_4th_Order_1.m
- Low-distortion CIFB topology with optimized NTF zeros
  - CIFB_Opt_4th_Order_2.m

File: CIFB_4th_Order_2.m

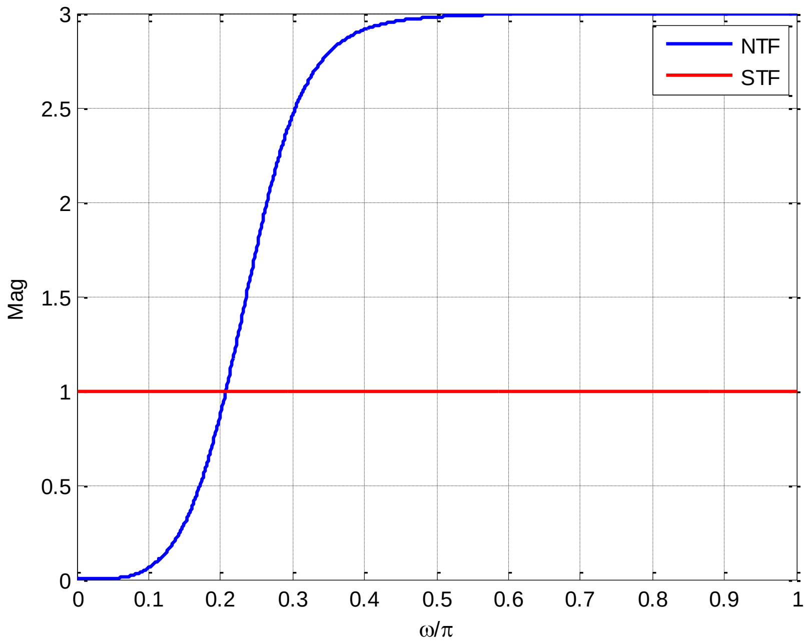
CIFF Example 1

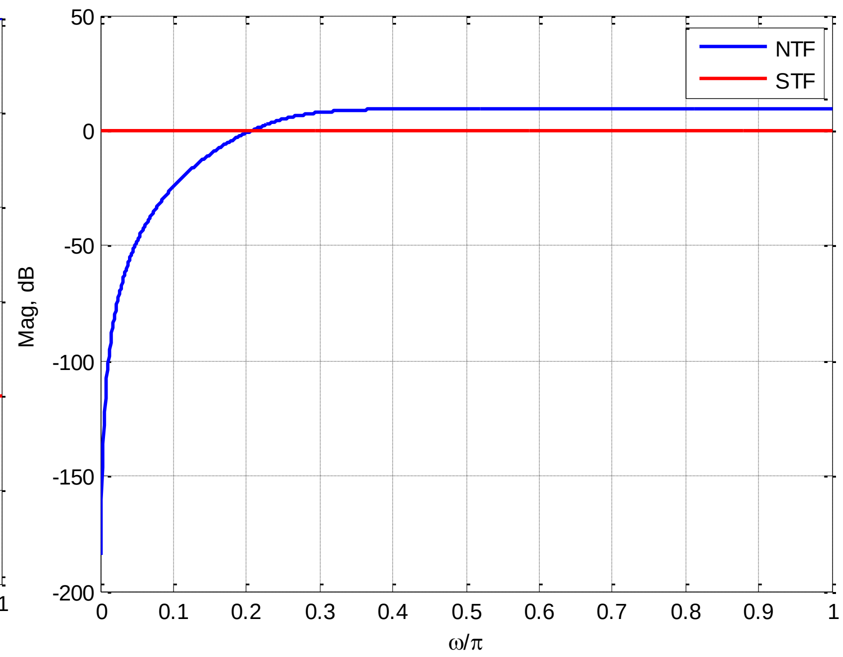
- CIFF, order = 4
- All NTF zeros at z=1, i.e. opt =0.
- OBG = 3, OSR = 16, nLev = 15.
- Low-distortion topology
  - b(1) =b(5)= 1
  - b(2:4)=0
- **a** = [2.1 1.9 0.86 0.16]
- **b** = [1 0 0 0 1]
- **c** = [1 1 1 1]
- **g** = [0 0]



File: CIFB_4th_Order_1.m

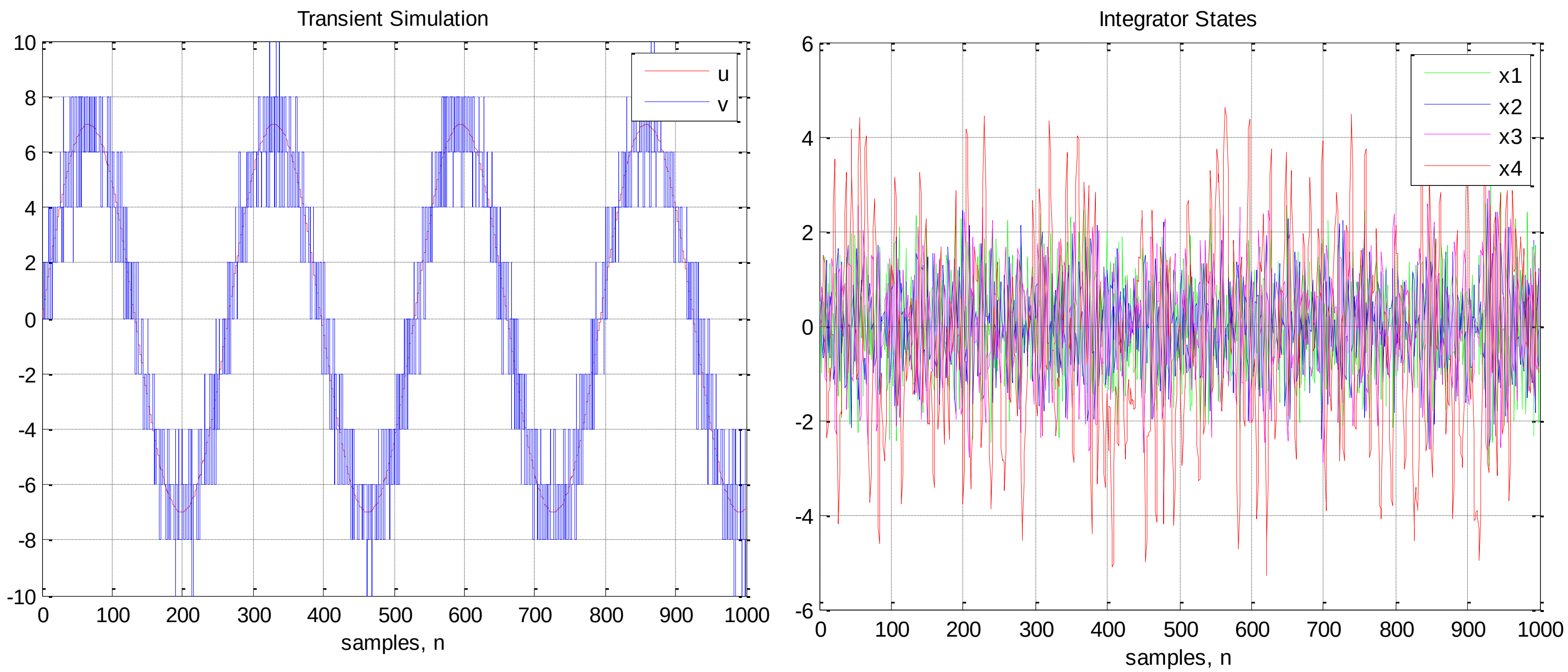
CIFF Example 1 contd. : NTF and STF





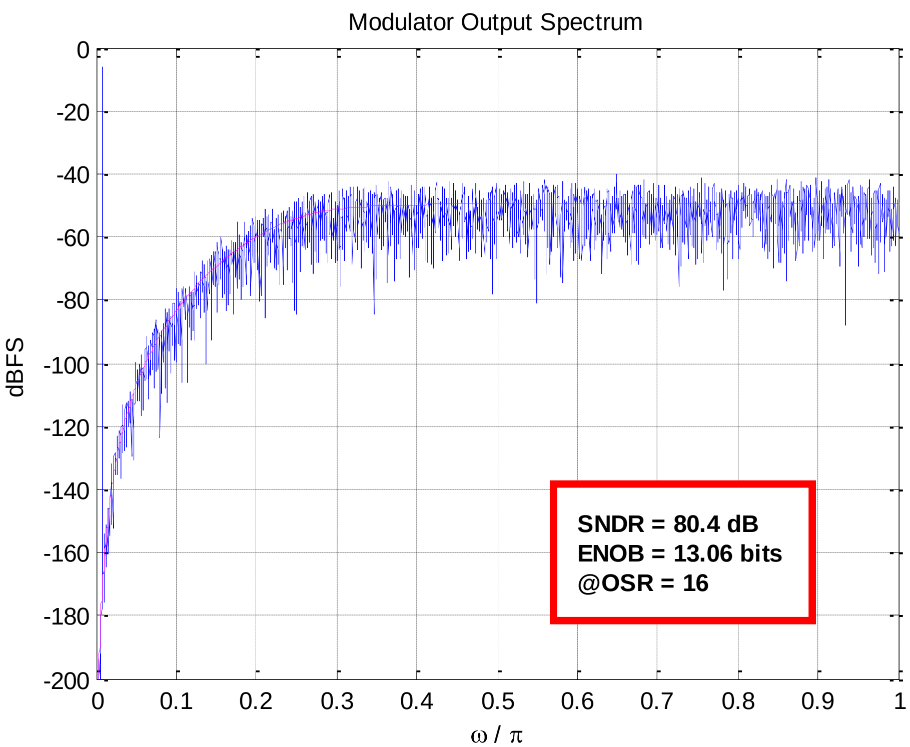
File: CIFF_4th_Order_1.m

States



File: CIFF_4th_Order_1.m

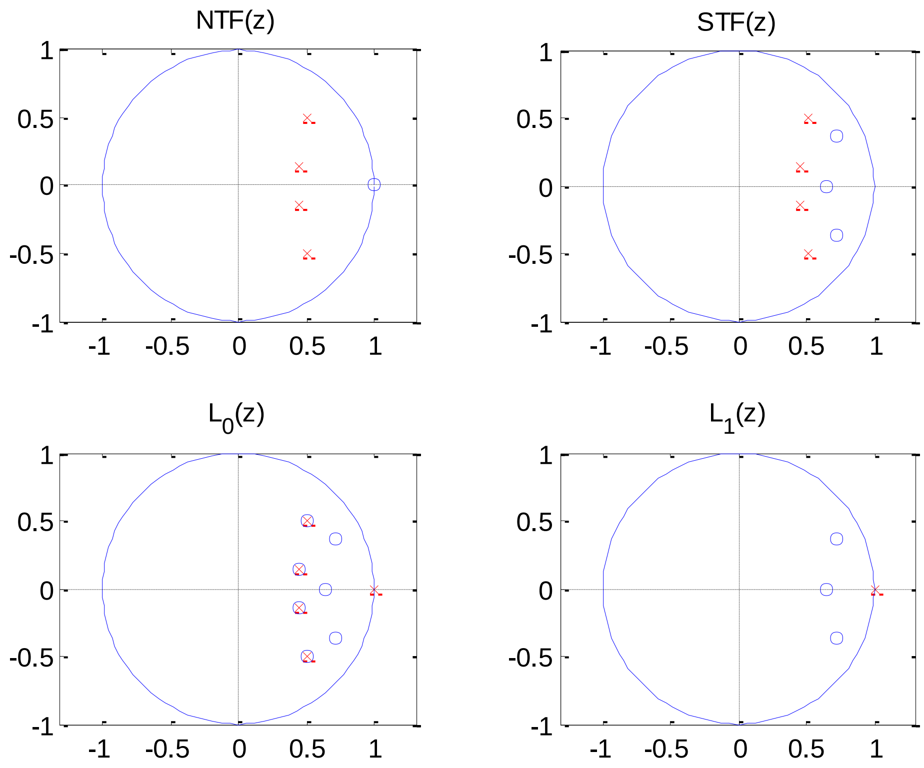
Spectrum



File: CIFF_4th_Order_1.m

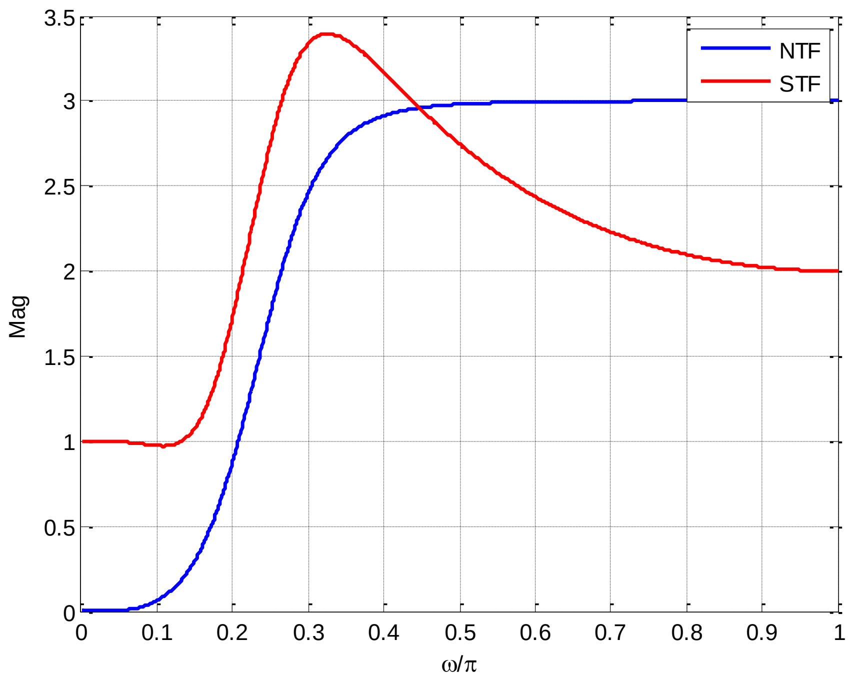
CIFF Example 2

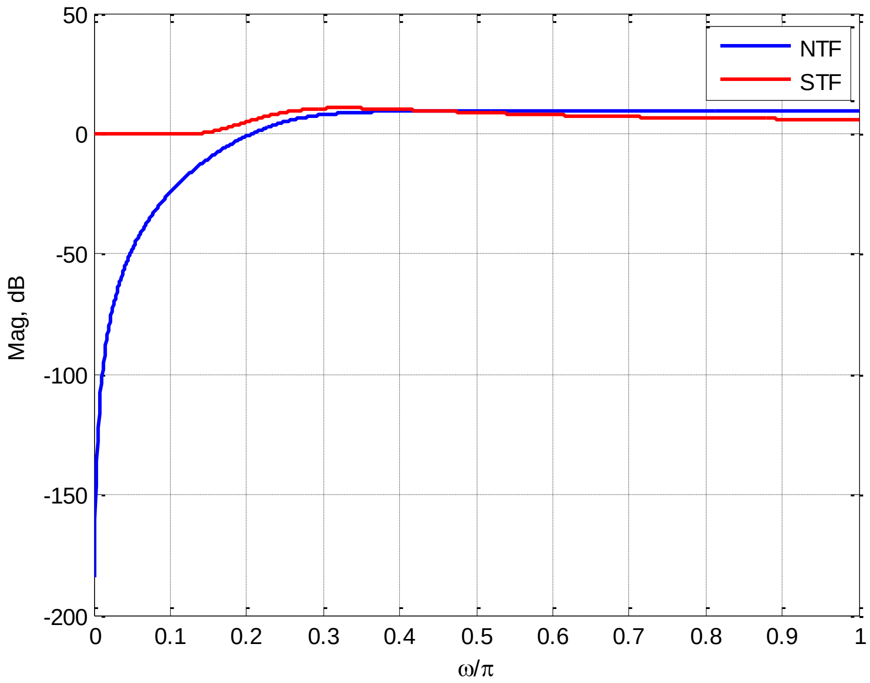
- CIFF, order = 4
- All NTF zeros at z=1, i.e. opt =0.
- OBG = 3, OSR = 16, nLev = 15.
- Only single input feed-in used
  - b(2:end)=0
- **a** = [2.1 1.9 0.86 0.16]
- **b** = [1 0 0 0 0]
- **c** = [1 1 1 1]
- **g** = [0 0]



File: CIFB_4th_Order_2.m

CIFF Example 2 contd. : NTF and STF

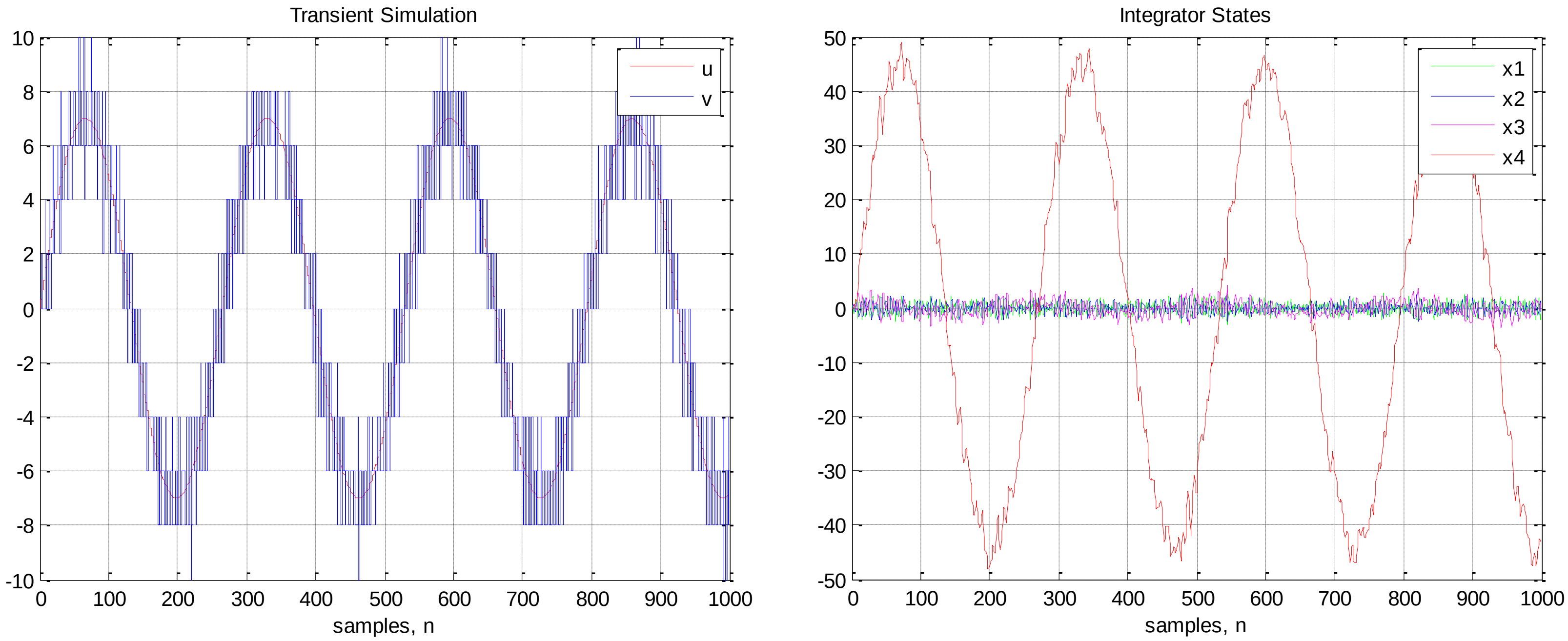




- Notice the significant STF peaking !

File: CIFF_4th_Order_2.m

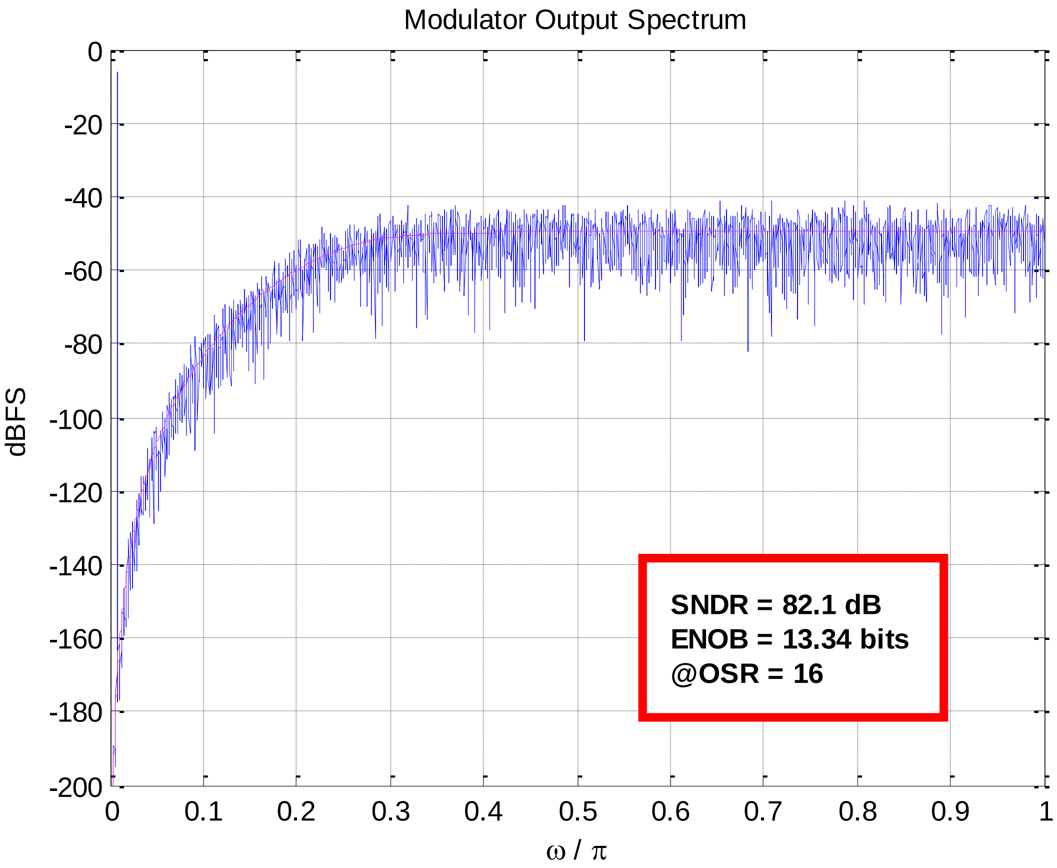
States



- ❑ Last integrator output has significant signal content
  - Use dynamic range scaling.
  - Last integrator will burn more power in this case.

File: CIFF_4th_Order_2.m

Spectrum



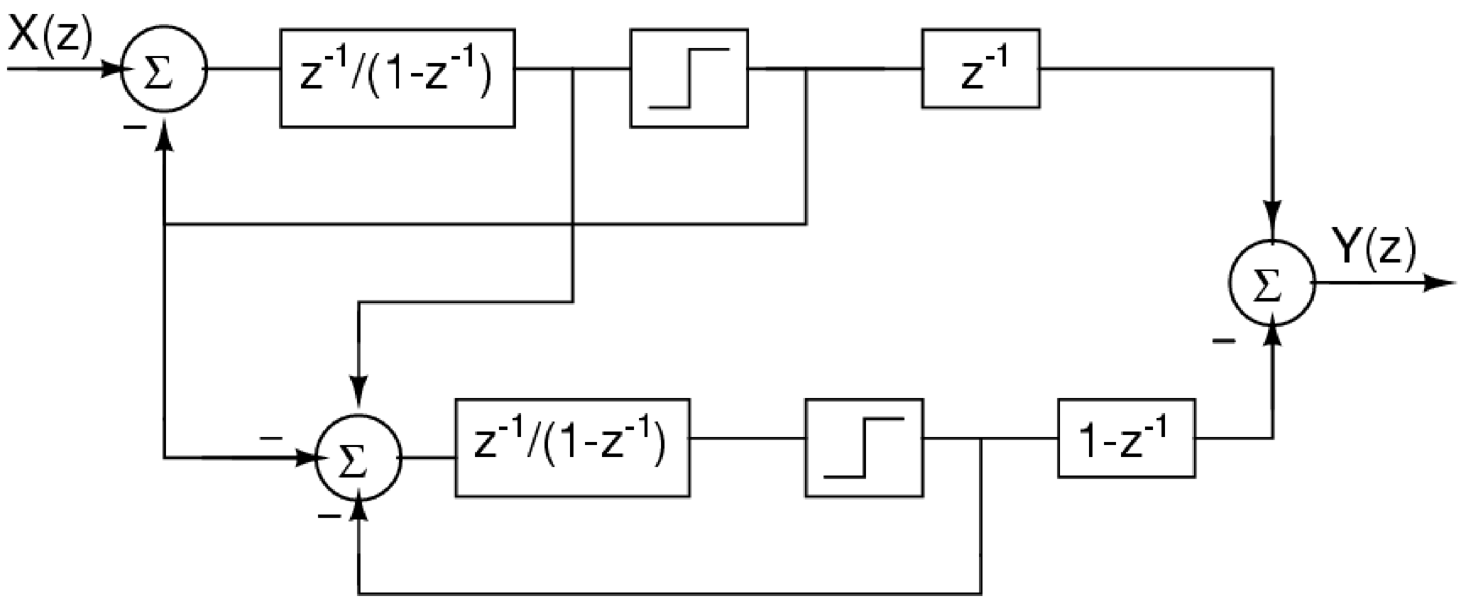
File: CIFF_4th_Order_2.m

Topologies

- Low-distortion CRFF topology
  - CRFF_4th_Order_1.m
- CRFF with single feed-in
  - CRFF_4th_Order_2.m
- Low-distortion CIFF topology with optimized NTF zeros
  - CIFF_Opt_4th_Order_1.m
- CIFF with single feed-in and optimized NTF zeros
  - CIFF_Opt_4th_Order_2.m
- STF peaking in FF topologies with single feed-in is an issue
  - CT FF DSM will have STF peaking as full-feedforward branch can't be used.
  - The feed-in coefficients b's can be strategically used to realize CIFF/CRFB topology with better out-of-band STF attenuation.

ΔΣ Modulator Architectures

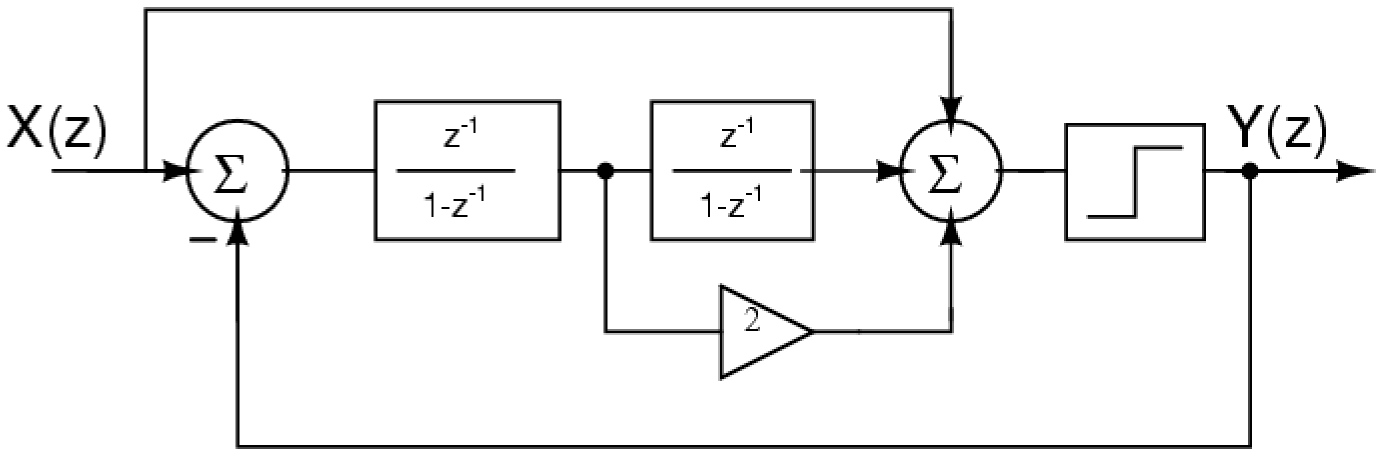
- Cascade/ MASH architecture:



•Eg. Two first order modulators are used to implement second order modulator.
•Stability concerns are relaxed but mismatch in the two forward paths should be properly monitored.

ΔΣ Modulator Architectures

- Feedforward modulators

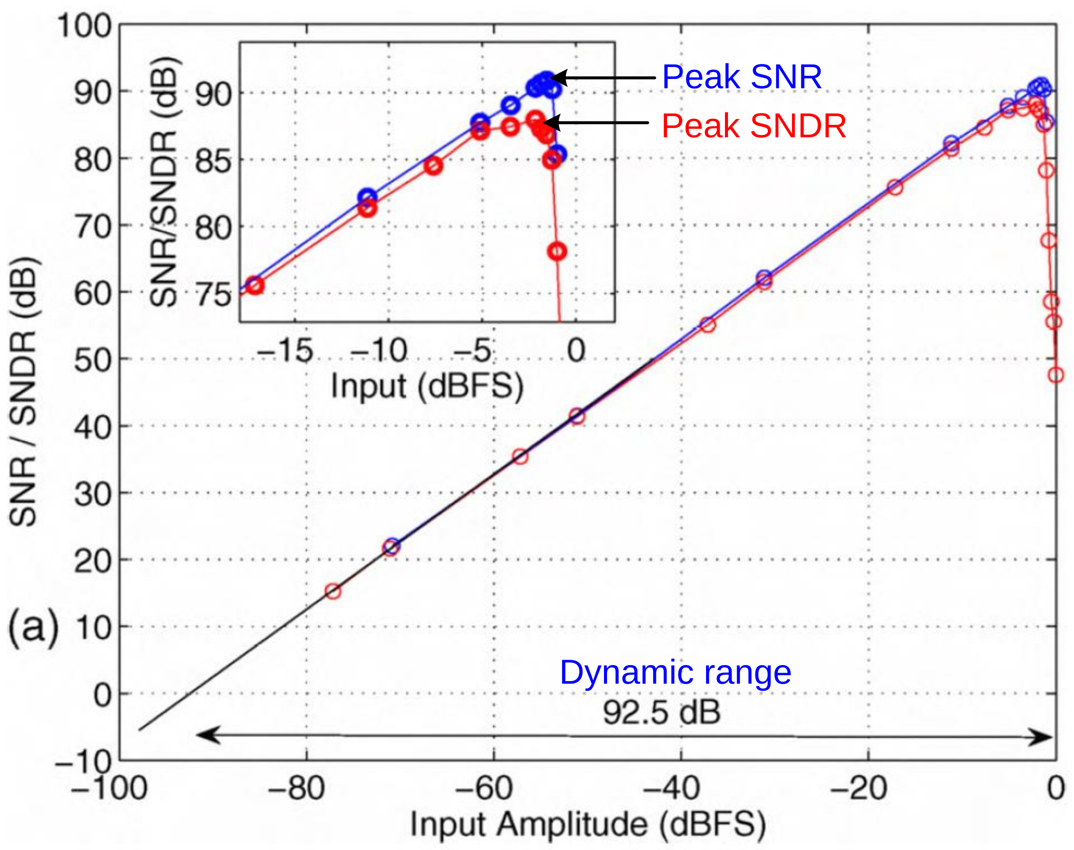


- Most popular architecture.
- Input signal is summed at Nth stage integrator output.
- Summation block may be required at higher order modulators.
- Multibit quantizer is necessary.

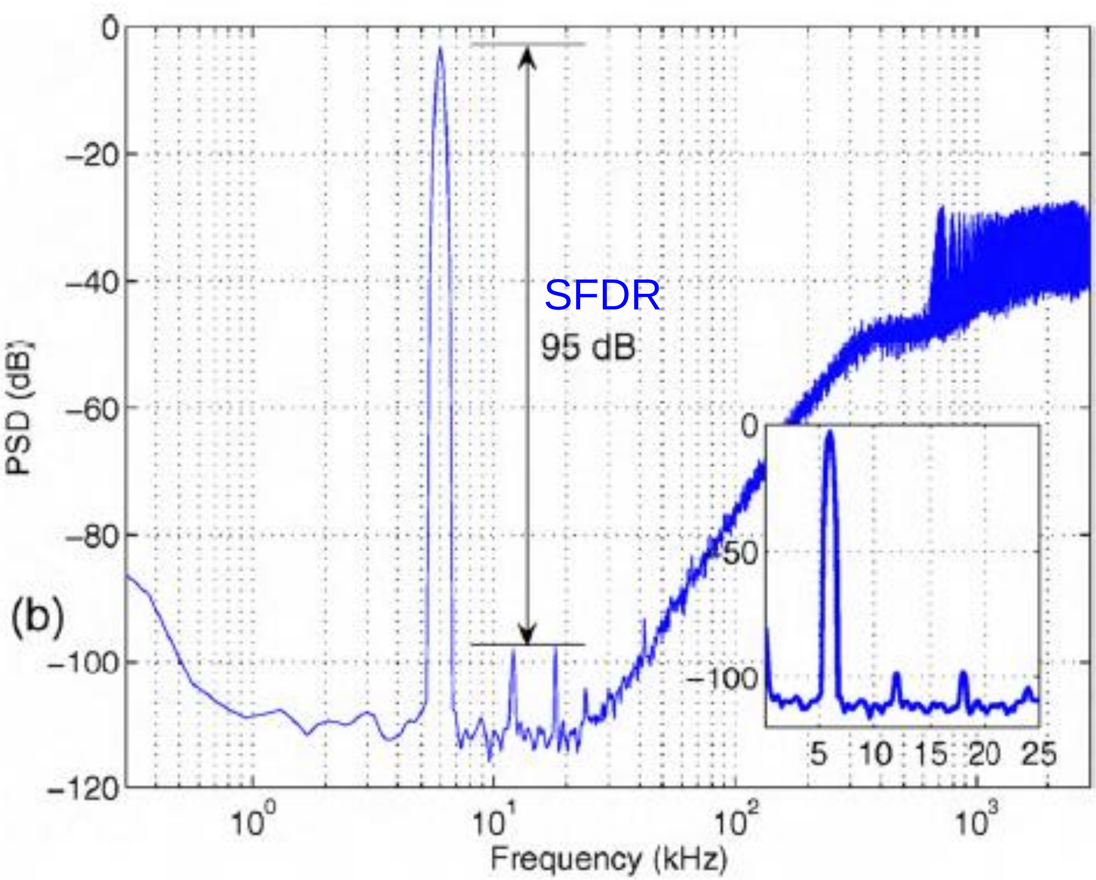
Key Terminologies :

- SQNR – Signal to quantization noise ratio
  - Thermal/electrical noise are not included.
- SNR - Signal to noise ratio
  - Distortion is not included.
- SDNR - Signal to noise and distortion ratio
  - All noise sources are included.
- ENOB – Effective number of bits (resolution)
  - This is very important than actual number of output bits
- Dynamic Range (DR)
  - Measured with input of the modulator shorted.
- Harmonic Distortion
  - THD is usually total harmonic distortion. Or Third??
- Spur Free Dynamic Range (SFDR)
  - Very key parameter in communication systems

Frequency Domain Measurements



Spurious (tone) Free Dynamic Range (SFDR)



References

Data Conversion Fundamentals

A.1 M. Gustavsson, J. Wikner, N. Tan, *CMOS Data Converters for Communications*, Kluwer Academic Publishers, 2000.

A.2 B. Razavi, *Principles of Data Conversion System Design*, Wiley-IEEE Press, 1994.

A.3 ADC and DAC Glossary by Maxim.

A.4 B. Murmann, "ADC Performance Survey 1997-2009," [Online].

A.5 S. Pavan, N. Krishnapura, EE658: Data Conversion Circuits Course at IIT Madras [Online].

A.6 The Fundamentals of FFT-Based Signal Analysis and Measurement, T.I. App Note here.

References

Delta-Sigma Data Converters

B.1 R. Schreier, G. C. Temes, *Understanding Delta-Sigma Data Converters*, Wiley-IEEE Press, 2005 (the Green Bible of Delta-Sigma Converters).

B.2 S. R. Norsworthy, R. Schreier, G. C. Temes, *Delta-Sigma Data Converters: Theory, Design, and Simulation*, Wiley-IEEE Press, 1996 (the Yellow Bible of Delta-Sigma Converters).

B.3 S. Pavan, N. Krishnapura, "Oversampling Analog to Digital Converters Tutorial," 21st International Conference on VLSI Design, Hyderabad, Jan, 2008.

B.4 S. H. Ardalan, J. J. Paulos, "An Analysis of Nonlinear behavior in Delta-Sigma Modulators," *IEEE TCAS*, vol. 34, no. 6, June 1987.

B.5 R. Schreier, "An Empirical Study of Higher-Order Single-Bit Delta-Sigma Modulators," *IEEE TCAS-II*, vol. 40, no. 8, pp. 461-466, Aug. 1993.

B.6 J. G. Kenney and L. R. Carley, "Design of multibit noise-shaping data converters," *Analog Integrated Circuits Signal Processing Journal*, vol. 3, pp. 259-272, 1993.

B.7 L. Risbo, "Delta-Sigma Modulators: Stability Analysis and Optimization," Doctoral Dissertation, Technical University of Denmark, 1994 [Online].

B.8 R. Schreier, J. Silva, J. Steensgaard, G. C. Temes, "Design-Oriented Estimation of Thermal Noise in Switched-Capacitor Circuits," *IEEE TCAS-I*, vol. 52, no. 11, pp. 2358-2368, Nov. 2005.

References

CAD for Mixed-Signal Design

D.1 K. Kundert, “Principles of Top-Down Mixed-Signal Design,” *Designer’s Guide Community* [Online].

D.2 R. Schreier, Matlab Delta-Sigma Toolbox, 2009 [Online], [Manual], [One page summary].

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D.4 P. Malcovati, Simulink Delta-Sigma Toolbox 2, 2009. Available [Online].

Example Datasheets

E.1 A 16-bit, 2.5MHz/5 MHz/10 MHz, 30 MSPS to 160 MSPS Dual Continuous Time Sigma-Delta ADC – AD9262, Analog Devices, 2008.

E.2 A 24-bit, 192 kHz Multi-bit Audio ADC – CS5340, Cirrus Logic, 2008.