HW 2

ECE 4/517– Mixed Signal IC Design

Problem 1 Quantization error distribution: We discussed in the class that the quantization error ($e = v - y$) can be modeled as a random process (i.e. noise) with a uniform distribution given as $e \sim U\left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right]$ where Δ is the LSB size. In this problem, we will study the range of validity of this assumption.

1. Drive a 4-bit quantizer with a full-scale sine wave ($A = FS$), while ensuring that the input and the sampling frequency are rationally related (i.e. $f_{in} = \frac{m}{N_{FFT}} f_s$, $m, N \in I$ and m and N_{FFT} are mutually prime). Choose a large value of N_{FFT}, say $N_{FFT} = 2^{13}$. The input to the quantizer is y and the quantized output is v. The quantization error is $e = v - y$. Divide the time-domain quantizer error $e[n]$ into 100 bins and plot a histogram. Is the histogram uniform ?
2. Repeat (1) for a 6-bit, 8-bit, 10-bit and 12-bit quantizer. How does the histogram (which approximates the PDF of e) behave as the quantizer resolution if increased ? Intuitively explain your observations.
3. Repeat (1) for overloading input amplitudes when $\frac{A}{FS}$= 1, 1.2, 1.5 and 2.0. What do you observe ?

Problem 2 Quantization noise spectrum: In our simple model of the quantization noise, we assumed that the quantization noise is uncorrelated with the input and is *white* with a flat PSD in the frequency band $f \epsilon [-\frac{f_s}{2}, \frac{f_s}{2}]$ where f_s is the sampling frequency. Here, we will study this assumption.

1. Drive a 4-bit quantizer with a full-scale sine wave, while ensuring that the input and the sampling frequency are rationally related. To observe the spectral properties of the quantizer choose a large value of N_{FFT}, say $N_{FFT} = 2^{10}$ and set the input tone bin to $m = 1$. The input to the quantizer is y and the quantized output is v. The quantization error is $e = v - y$. Plot the time-domain plots for y,v and the quantization error v for one cycle of the input and observe the periodicity of $e[n]$ (see Fig. 1). Plot magnitude of the FFT of $e[n]$ given by $E[k]$. Is the quantization noise spectrum uniformly distributed in the frequency band $[-\frac{f_s}{2}, \frac{f_s}{2}]$? Can you explain the 'tonal' behavior of the quantization noise spectrum ?
2. Repeat (1) for a 6-bit, 8-bit, 10-bit and 12-bit quantizer. How does the quantization noise spectrum behave as the quantizer resolution if increased ?

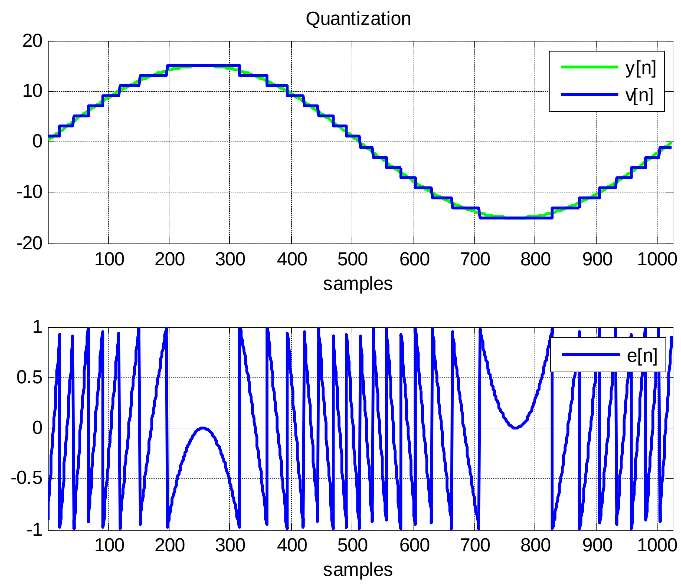

Figure 1: Quantization error waveform

Problem 3 SQNR calculation: You are given an ideal quantizer with N-bits of resolution. In your test setup, the quantizer is driven by a full-scale sine wave input with a frequency within the Nyquist bandwidth. You have captured 1024 output data points of the quantizer, and expressed the output as a Discrete Fourier Series (DFS) computed using the FFT algorithm. The magnitudes of the DFS coefficients are plotted after normalizing with the input tone magnitude (peak at 0 dBFS), on a log scale. The resulting plot is shown in Figure 2. In the plot, assume that the quantization noise floor is uniform at -63 dBFS.

1. If the sampling rate $f_s$was 100 MHz, find the frequency of the sine-wave input f_{in}.

   (a) Estimate the resolution of the quantizer N. Show your calculation steps clearly.

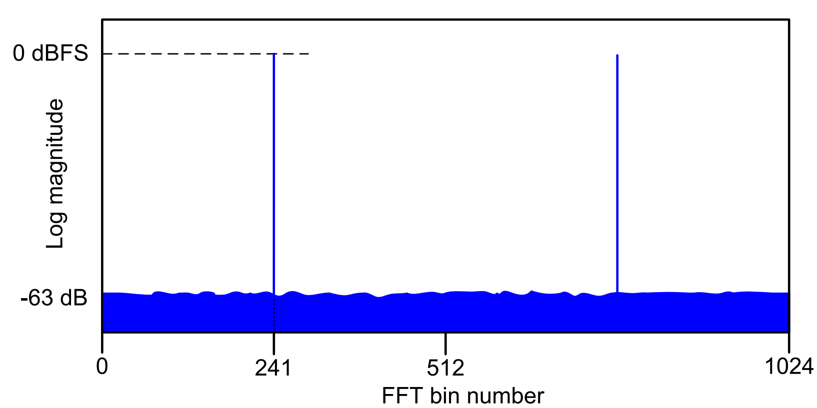

Figure 2: DFS log-magnitude plot.

Problem 4 SQNR estimation using MATLAB: Using MATLAB compute the peak Signal to Quantization Noise ratio for quantizers with resolutions of 2, 4, 6, 8, 10, 12 bits. Ensure that that the input and the sampling frequency are rationally related. Choose the FFT size of the form $N_{FFT} = 2^p$ for faster DFT computation. Make sure you add the FFT coefficient power in the signal and noise bins separately to compute the SQNR. How do the computed SQNR values compare with the relation

$$SQNR = 6.02N + 1.76 \ dB \tag{1}$$

derived in class ?

Problem 5 Quantizer SFDR: Consider an ideal N-bit quantizer with N ranging from 4 to 14, and driven by a full-scale sine wave. Find the ratio of the powers of the fundamental and the largest spur in the quantizer spectrum. This is called the Spurious Free Dynamic Range (SFDR). For consistency, use $N_{FFT} = 2^{15}$ with input at approximately $f_s/4$ (ensuring that the input and sampling frequency are rationally related).

1. Plot SFDR (in dB) versus N. Can you explain the slope of the curve ?
2. In your MATLAB code, introduce a compressive non-linearity just before a 10-bit quantizer with its normalized input-output relationship given by

$$y = A\left(\frac{x}{A} - 0.001\left(\frac{x}{A}\right)^3\right) \tag{2}$$

   where A is the full-scale amplitude. Use $N_{FFT} = 2^{15}$ with input at approximately $f_s/8$ (ensuring that the input and sampling frequency are rationally related). What is the SFDR in the presence of the non-linearity? Identify the location of the third-harmonic tone.