ECE 415/515 –ANALOG INTEGRATED CIRCUIT DESIGN

CS AMPLIFIER FREQUENCY RESPONSE: *POLE SPLITTING*

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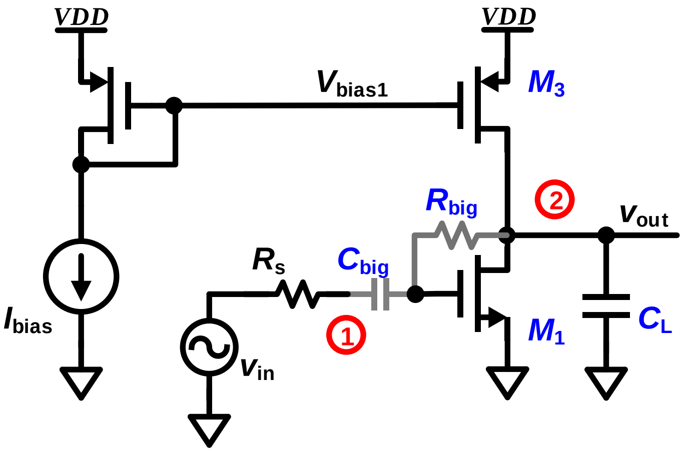
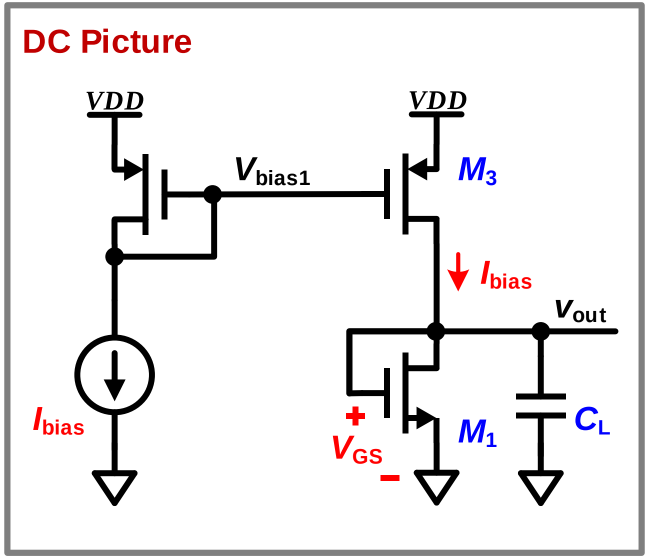
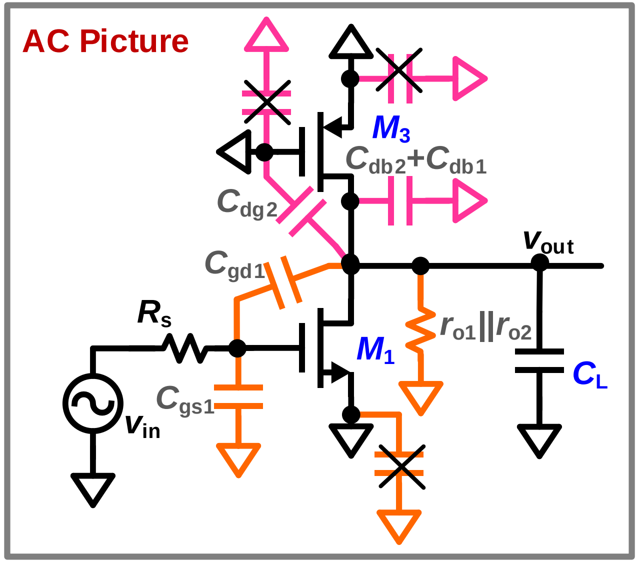
CS Amplifier With Input Source Impedance



CS Amplifier With Input Source Impedance (1)





$$A_v = -g_{m1} \cdot r_{o1}||r_{o2}$$

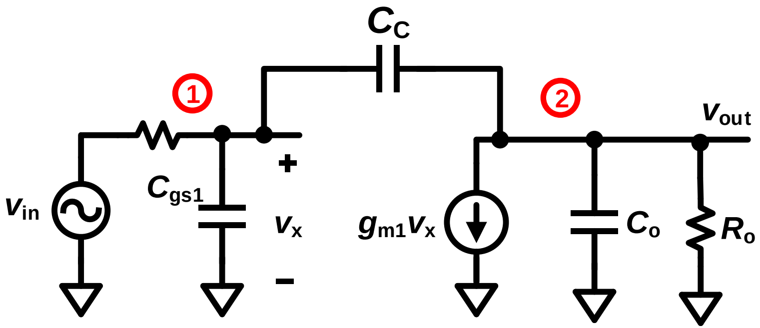
$$C_{in} = C_{gs1}$$

$$R_o = r_{o1}||r_{o2}$$

$$C_o = C_{db1} + C_{db2} + C_{dg2} + C_L$$

$$C_c = C_{gd1}$$

CS Amplifier Analysis (1)



$$\frac{v_x - v_{in}}{R_s} + v_x s C_{gs1} + (v_x - v_{out}) s C_{gd1} = 0 \qquad (1)$$

$$(v_x - v_{out}) s C_{gd1} + g_{m1} v_x + v_{out}(R_o^{-1} + sC_o) = 0 \qquad (2)$$

$$\frac{v_{out}}{v_{in}}(s) = \frac{(sC_{gd1} - g_{m1})R_o}{R_s R_o \xi s^2 + [R_s(1+g_{m1}R_o)C_{gd1} + R_s C_{gs1} + R_o(C_{gd1}+C_o)]s + 1} \qquad (3)$$

where $\xi = C_{gs_1}C_{gd1} + C_{gs1}C_o + C_{gd1}C_o$

CS Amplifier Analysis: Zero

Numerator: $$(sC_{gd1} - g_{m1})R_o = 0$$

$$\omega_z = \frac{g_{m1}}{C_{gd1}}$$ RHP zero

CS Amplifier Analysis: Poles (1)

Denominator: $$D(s) = \left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right) = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1$$

For $|\omega_{p1}| \ll |\omega_{p2}|$

$$D(s) \approx \frac{s^2}{\omega_{p1}\omega_{p2}} + \frac{s}{\omega_{p1}} + 1 \qquad (4)$$

Dominant pole by equating coefficient of 's' in Eq. (3) and (4)

$$\omega_{p1} = \frac{1}{R_s(1+g_{m1}R_o)C_{gd1} + R_sC_{gs1} + R_o(C_{gd1}+C_o)}$$

Miller input cap

Extra term due to the output node

CS Amplifier Analysis: Poles (2)

Non-dominant pole by equating coefficient of ‘s[2]’ in Eq. (3) and (4)

$$\omega_{p2} = \frac{1}{\omega_{p1}} \cdot \frac{1}{R_s R_o \xi}$$

$$= \frac{1}{\omega_{p1}} \cdot \frac{1}{R_s R_o \left[C_{gs_1} C_{gd1} + C_{gs1} C_o + C_{gd1} C_o\right]}$$

$$= \frac{R_s(1+g_{m1}R_o)C_{gd1} + R_s C_{gs1} + R_o(C_{gd1}+C_o)}{R_s R_o \left[C_{gs_1} C_{gd1} + C_{gs1} C_o + C_{gd1} C_o\right]}$$

$$\frac{v_{out}}{v_{in}}(s) = \frac{A_v\left(1 - \frac{s}{\omega_z}\right)}{\left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right)}$$

CS Amplifier Analysis: Poles (3)

If C_{gs1} is very large: $R_s C_{gs1} \gg R_s(1 + g_{m1}R_o)C_{gd1} + R_o(C_{gd1} + C_o)$

$$\omega_{p2} = \frac{R_s(1+g_{m1}R_o)C_{gd1} + R_s C_{gs1} + R_o(C_{gd1}+C_o)}{R_s R_o\left[C_{gs_1}C_{gd1} + C_{gs1}C_o + C_{gd1}C_o\right]}$$

$$= \frac{R_s C_{gs1}}{R_s R_o\left[C_{gs_1}C_{gd1} + C_{gs1}C_o + C_{gd1}C_o\right]}$$

$$\omega_{p2} \approx \frac{1}{R_o\left(C_{gd_1} + C_o\right)}$$

Extreme case only when C_{gs1} is very large

Pole Splitting: Dominant Pole

$$\omega_{p1} = \frac{1}{R_s(1+g_{m1}R_o)C_c+R_sC_{gs1}+R_o(C_c+C_o)}$$

$$\omega_{p1}|_{C_c=0} = \frac{1}{R_sC_{gs1}+R_oC_o}$$

$$\omega_{p1}|_{C_c\gg C_{gs1}} = \frac{1}{R_s(1+g_{m1}R_o)C_c+R_o(C_c+C_o)}$$

Pole Splitting: Non-Dominant Pole

$$\omega_{p2} = \frac{R_s(1+g_{m1}R_o)C_c+R_sC_{gs1}+R_o(C_c+C_o)}{R_sR_o\left[C_{gs_1}C_c+C_{gs1}C_o+C_cC_o\right]}$$

$$\omega_{p2|C_c=0} = \frac{R_sC_{gs1}+R_oC_o}{R_sR_o[C_{gs1}C_o]} \approx \frac{1}{R_sC_{gs1}} + \frac{1}{R_oC_o} \approx \boxed{\frac{1}{R_oC_o}}$$

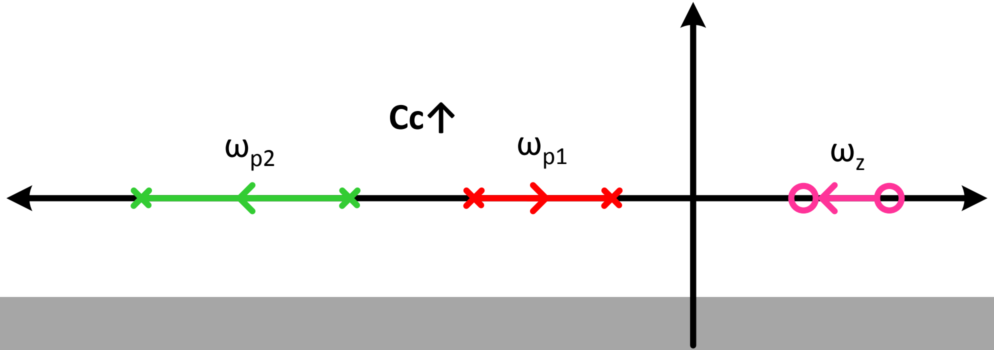
$$\omega_{p2|C_c \gg C_{gs1}} \approx \frac{R_s(g_{m1}R_o)C_c+R_o(C_c+C_o)}{R_sR_o\left[C_{gs_1}C_c+C_cC_o\right]} \approx \frac{R_s(g_{m1}R_o)}{R_sR_o\left[C_{gs_1}+C_o\right]} \approx \frac{g_{m1}}{\left[C_{gs_1}+C_o\right]}$$

$$\omega_{p2|C_c \gg C_{gs1}} \approx \boxed{\frac{g_{m1}}{C_o} = g_{m1}R_o\left(\frac{1}{R_oC_o}\right)}$$

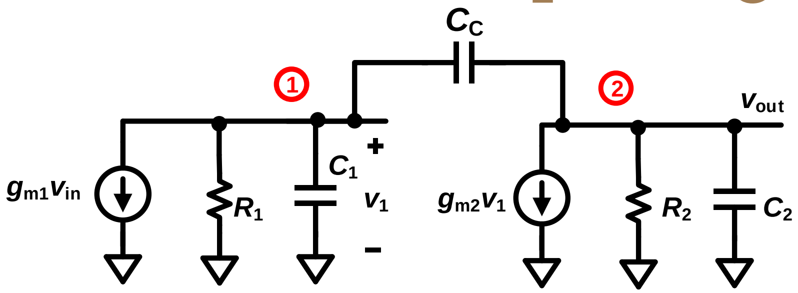
Second pole pushed to higher frequency by $g_{m1}R_o$ for a large value of C_c

Pole Splitting: Non-Dominant Pole

	Cc=0	$C_c \approx C_o >> C_{gs1}$
ω_{p1}	$\frac{1}{R_s C_{gs1} + R_o C_o}$	$\frac{1}{R_s(1+\|A_v\|)C_c + R_o(C_c + C_o)}$
ω_{p2}	$\approx \frac{1}{R_o C_o}$	$\approx \frac{g_{m1}}{C_o} = g_{m1} R_o \left(\frac{1}{R_o C_o}\right)$
ω_z	$\frac{g_{m1}}{C_{gd1}}$	$\frac{g_{m1}}{C_c}$



Miller Compensation/Pole Splitting Summary



$$\frac{v_{out}}{v_{in}}(s) = \frac{A_v(1-\frac{s}{\omega_z})}{\left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)}$$

$$A_v = g_{m1}R_1g_{m2}R_2$$

$$\omega_z = +\frac{g_{m2}}{C_c}$$

$$\omega_{un} = A_v\omega_{p1} = \frac{g_{m1}}{C_c}$$

$$\omega_{p1} = \frac{1}{R_1[C_1+(1+g_{m2}R_2)C_c]+R_2(C_2+C_c)}$$

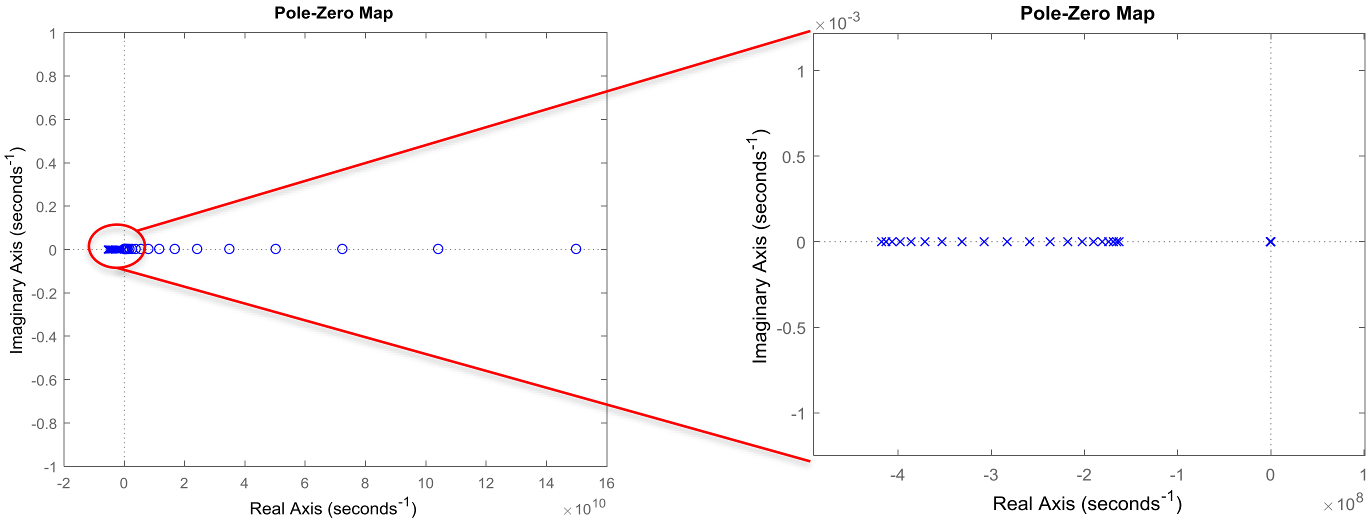
$$\approx \frac{1}{g_{m2}R_2R_1C_c}$$

$$\omega_{p2} \approx \frac{R_1(1+g_{m2}R_2)C_c+R_1C_1+R_2(C_c+C_2)}{R_1R_2[C_1C_c+C_2C_2+C_cC_2]}$$

$$\approx \frac{g_{m2}C_c}{C_1C_c+C_2C_2+C_cC_2} \propto \frac{g_{m2}}{C_2}$$

Pole Splitting Simulation (1)

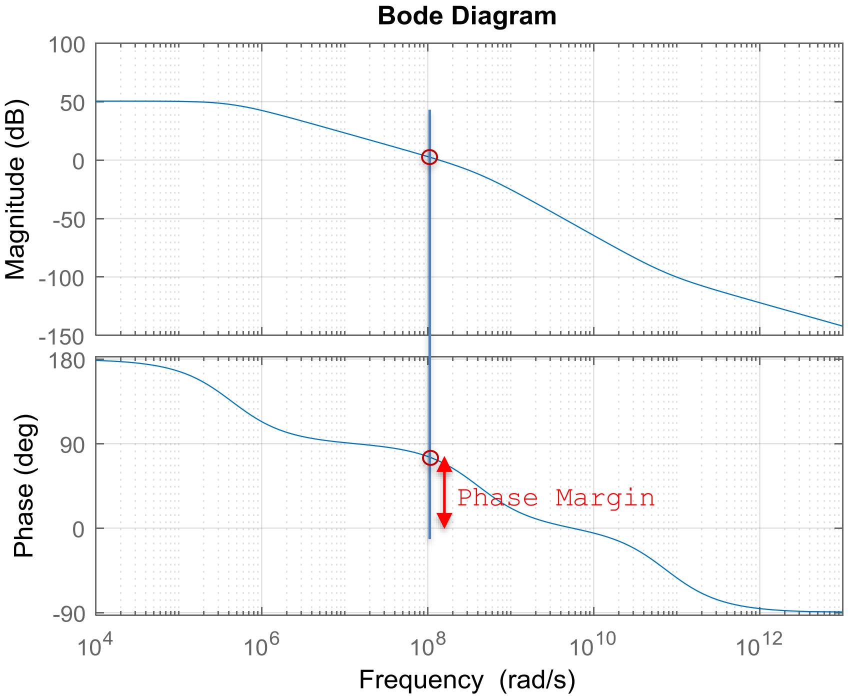
```
Rs = 100e3;
gm1=1/Rs; gm2=150e-6;
R1 = Rs; R2=2.22e6;
C1=23.3*fF; C2=1*pF;
```

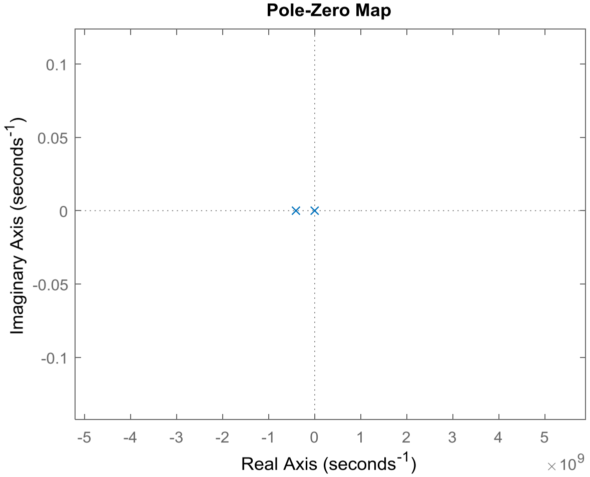


See Matlab file CommonSourcePoleSplitting.m

Pole Splitting Simulation (2)

```
Rs  =  100e3;
gm1=1/Rs;
gm2=150e-6;
R1  =  Rs;
R2=2.22e6;
C1=23.3*fF;
C2=1*pF;
Cc= 2*fF;
```

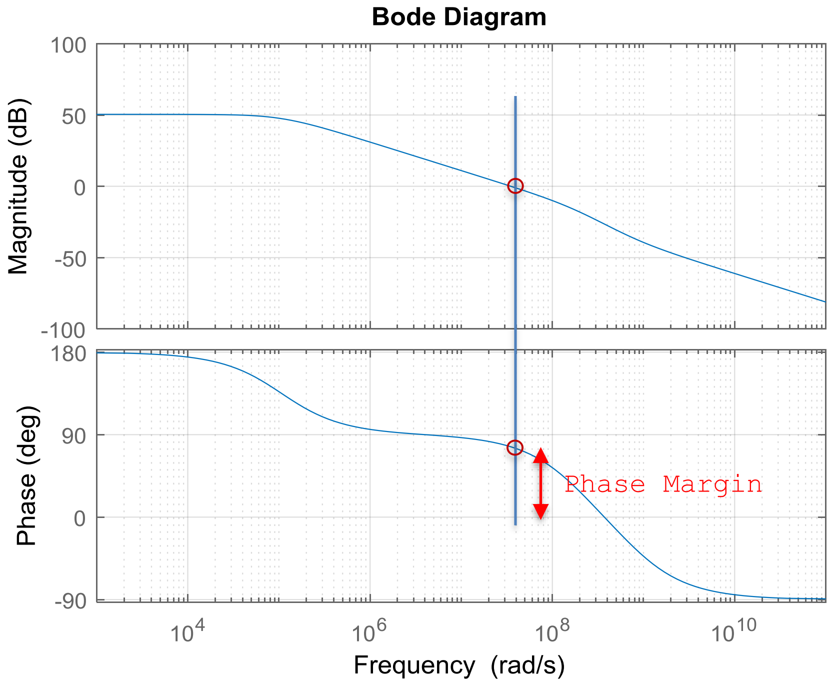


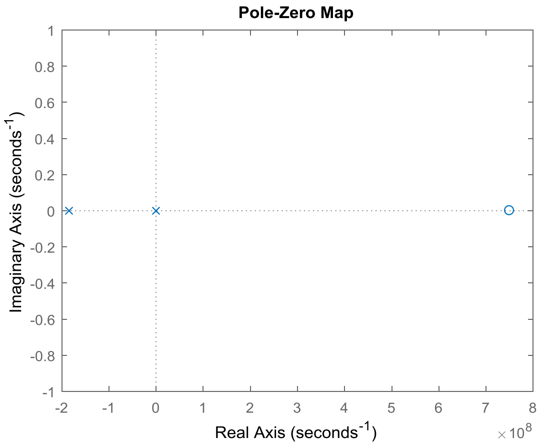


See Matlab file CommonSourceFreqResp1.m

Pole Splitting Simulation (2)

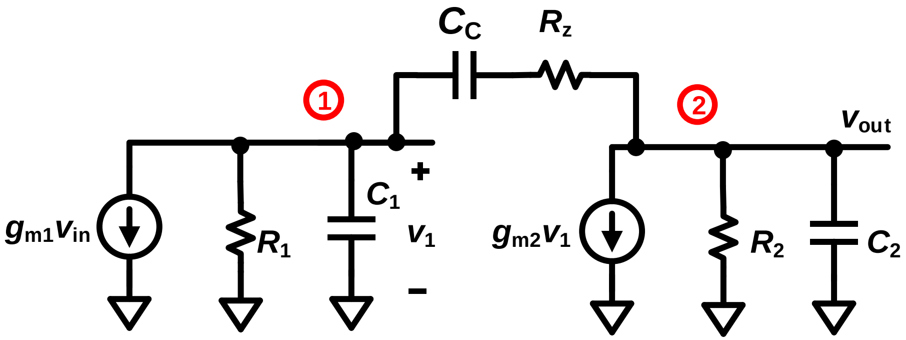
```
Rs  =  100e3;
gm1=1/Rs;
gm2=150e-6;
R1  =  Rs;
R2=2.22e6;
C1=23.3*fF;
C2=1*pF;
Cc=200*fF;
```





See Matlab file CommonSourceFreqResp1.m

Miller Compensation with Zero-Nulling Resistance



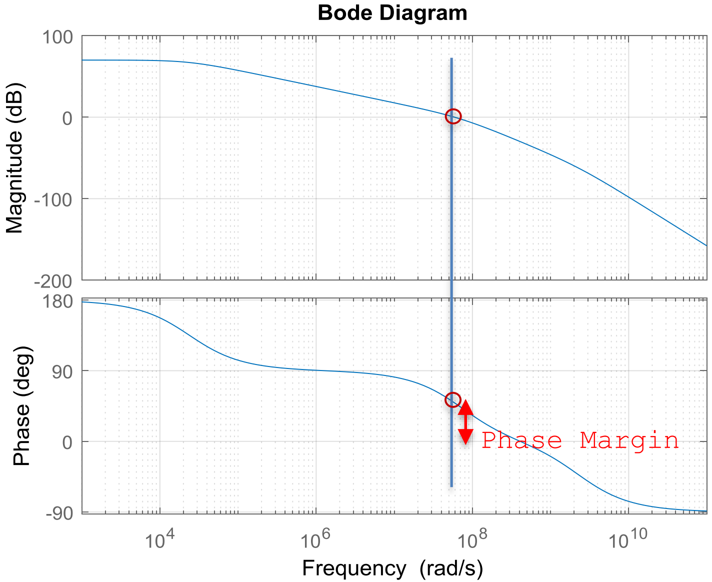
$$\omega_z = \frac{1}{C_c\left(\frac{1}{g_{m2}} - R_z\right)}$$

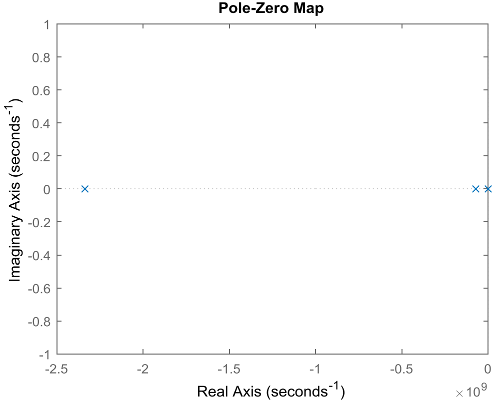
$$\omega_z \to \infty,\ R_z = \frac{1}{g_{m2}}$$

- Standard Miller compensation technique with zero nulling-R
- The RHP zero is pushed out to infinity
- Additional parasitic pole added at a higher frequency

Two-Stage Amplifier Simulation

gm1=20e-6;
gm2=80e-6;
R1 = 4e6; R2=0.5e6;
C1=40*fF; C2=1*pF;
Cc=250*fF;
Rz=12.5e3;





See Matlab file TwoStageFreqResp1.m