ECE 513 – Lecture 6

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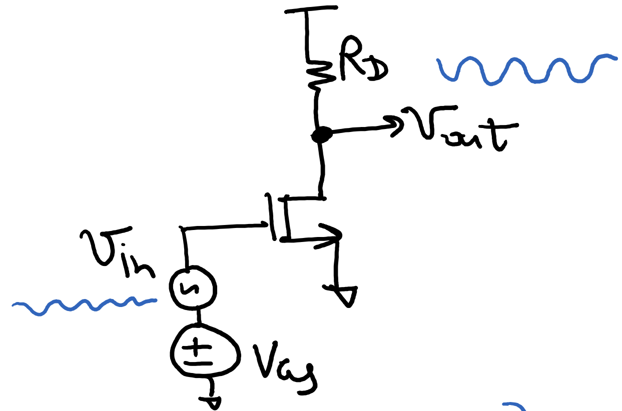
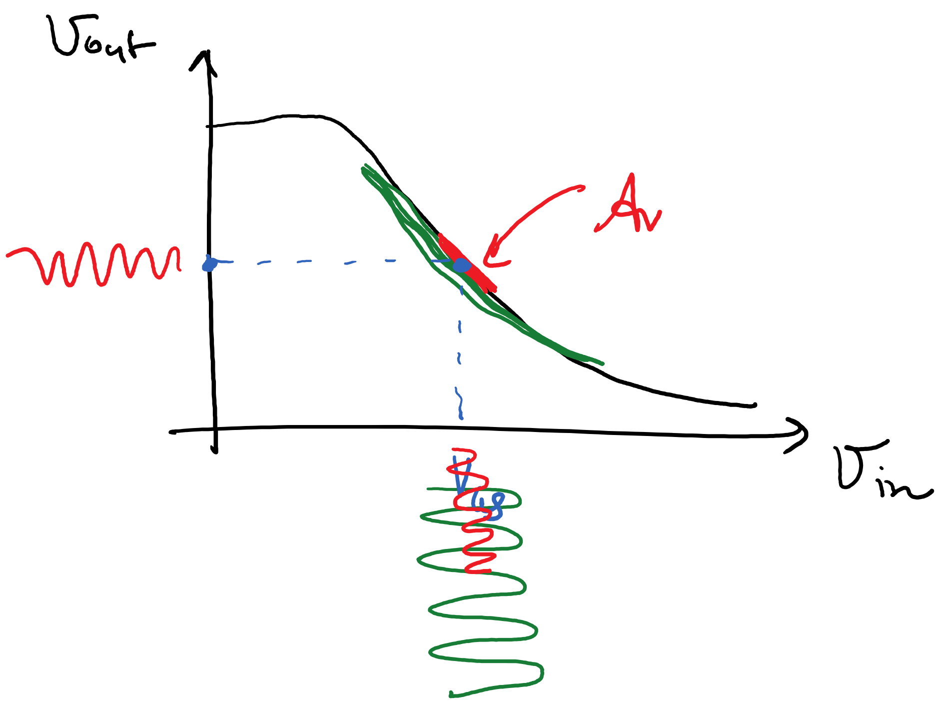
Nonlinearity:



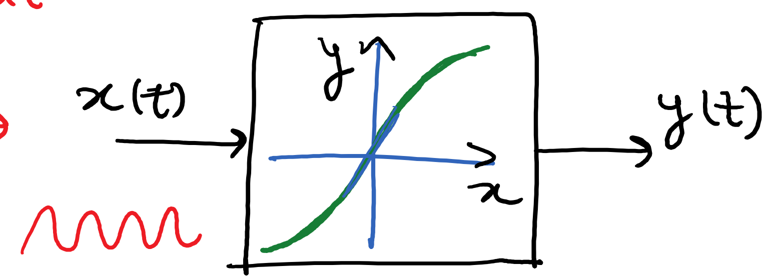
$$A_v = \frac{v_{out}}{v_{in}} = -g_m(R_D \| r_o)$$

small signal gain



* small signal models don't capture nonlinear phenomena

* memoryless system with input/output characteristics given by →



$$y(t) \simeq \underbrace{\alpha_1}_{\text{small-signal gain}} x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$ ← polynomial curve fit

$$\cancel{\frac{Y(s)}{X(s)}}$$

$$x(t) = A\cos\omega t$$

$$y(t) = \alpha_1 A\cos\omega t + \alpha_2 A^2\cos^2\omega t + \alpha_3 A^3 \cos^3\omega t$$

$$= \alpha_1 A\cos\omega t + \frac{\alpha_2 A^2}{2}(1+\cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3\cos\omega t + \cos 3\omega t)$$

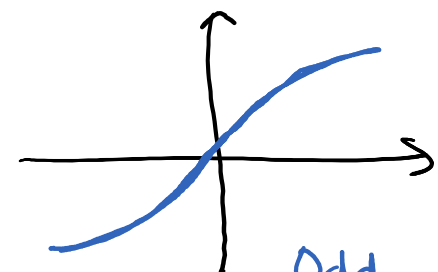
$$\left[\begin{array}{l} \cos^2\theta = \dfrac{1+\cos 2\theta}{2} \\ \cos^3\theta = \dfrac{1}{4}\left[3\cos\theta + \cos 3\theta\right] \end{array}\right.$$

ideal output $= \alpha_1 A \cos\omega t$

$$= \underbrace{\frac{\alpha_2 A^2}{2}}_{\text{DC offset}} + \underbrace{\left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right)\cos\omega t}_{\text{fundamental}} + \underbrace{\frac{\alpha_2 A^2}{2}\cos 2\omega t}_{2^{nd}\text{ harmonic}} + \underbrace{\frac{\alpha_3 A^3}{4}\cos 3\omega t}_{3^{rd}\text{ harmonic}}$$

$\Rightarrow$ Even-order harmonic results from α_j for even j

- $\hookrightarrow$ even order nonlinearity introduces DC offset
- $\hookrightarrow$ Vanish for system with odd-symmetry
  - $\hookrightarrow$ α_j, j is even $= 0$
  - $\hookrightarrow$ fully-differential circuits



$\checkmark$ n^{th} harmonic grows $\propto A^n$

HD or THD $\Rightarrow$ Total Harmonic Distortion $\frac{P_1}{P_2 + P_3 + \dots}$

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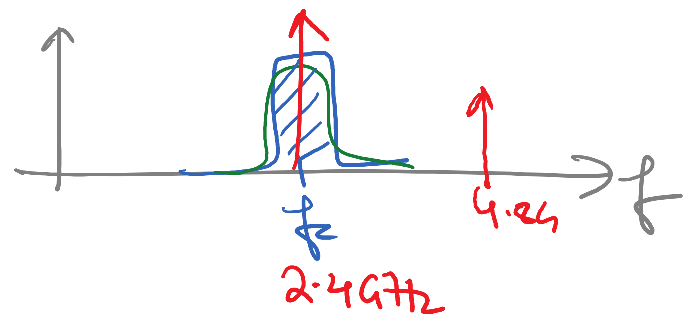

* In RF system ⇒ narrowband
harmonic distortion is irrelevant

Eg. 2.4GHz Amp produce 2^{nd} harmonic at 4.8GHz
which will be suppressed if the circuit has
narrow BW.

Gain Compression

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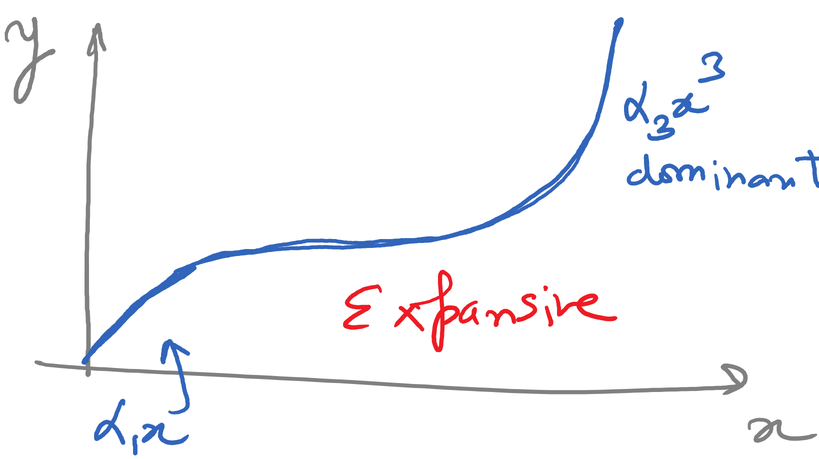
Gain experienced by $A\cos\omega t \Rightarrow \left(\alpha_1 + \frac{3\alpha_3 A^2}{4}\right)$

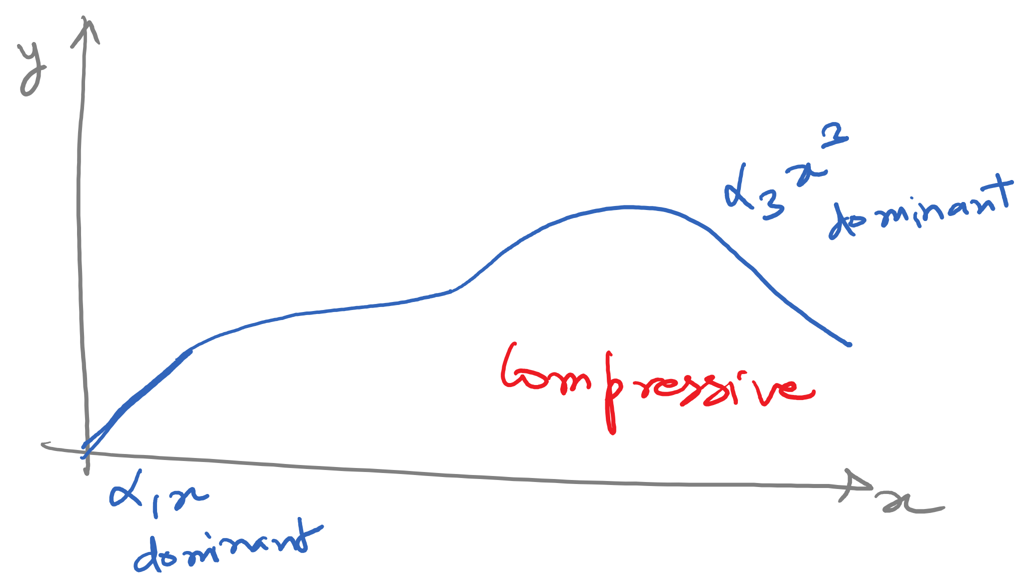
↳ varies with A

Two possibilities $\alpha_1\alpha_3$ independent of α_2

$\alpha_1\alpha_3 > 0$

$\alpha_1\alpha_3 < 0$





Most RF circuits are compressive.

$\alpha_1 \alpha_3 < 0$

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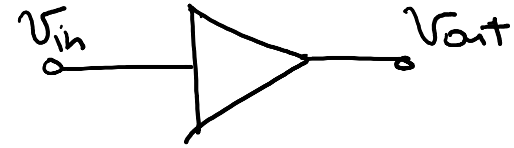

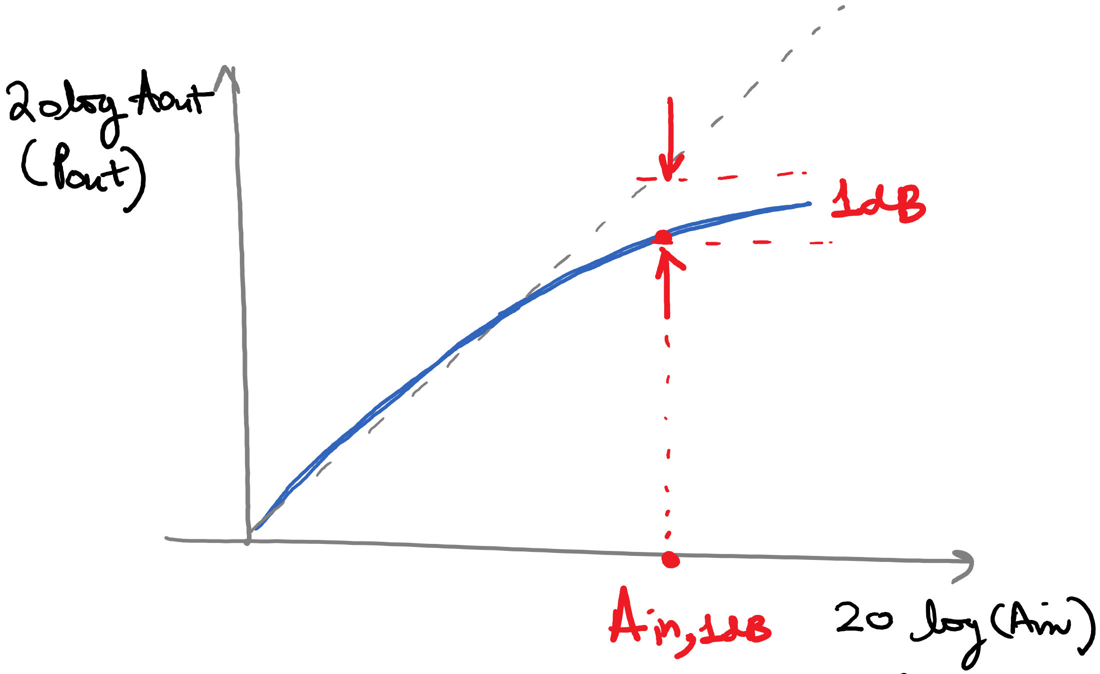

$P_{in,1dB} \rightarrow$ 1-dB Compression point

The higher $P_{in,1dB}$, the better

$$\text{Total gain}\Big|_{dB} = \text{Ideal gain}\Big|_{dB} - 1\,dB$$

$$20\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A_{in,1dB}^2\right| = 20\log|\alpha_1| - 1\,dB$$

$$\Rightarrow \quad \boxed{A_{in,1dB} = \sqrt{0.145\,|\alpha_1 \ldots}}$$

$\left|\frac{\alpha_1}{\alpha_3}\right|$ should be large for better linearity

$P_{in,1dB} = 20\log_{10} A_{in,1dB}$

$$\Rightarrow \quad \boxed{A_{in,1dB} = \sqrt{0.145 \left|\frac{\alpha_1}{\alpha_3}\right|}}$$

$$P_{in,1dB} = 20 \log_{10} A_{in,1dB}$$

1-dB compression point

$P_{1dB} \Rightarrow$ typically in the range of -20 to -25 dBm

$\Rightarrow$ 63.2 to 35.6 mVpp for 50Ω system

$\Rightarrow$ 1dB compression point represents 10% reduction in gain

Why does gain compression matter?

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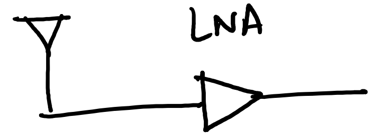
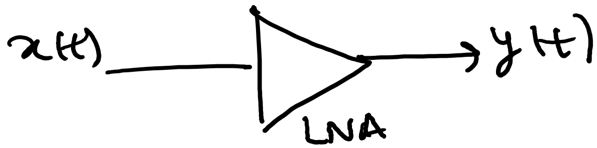
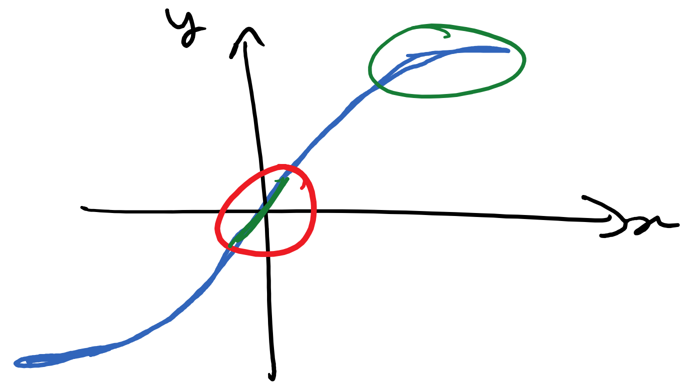
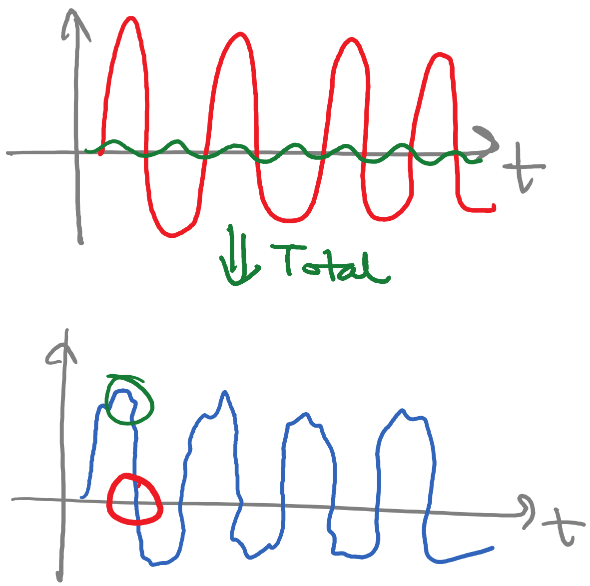
Example:

no BSF here









* Small signal is superimposed over a large sin) or

↳ LNA gain is reduced by large excursions produced by the interferer

↳ Desensitization or Blocking

↳ interferer lowers the signal to

noise ratio (SNR) for the desired
recieved signal.

Let

$$x(t) = \underbrace{A_1 \cos\omega_1 t}_{\text{Desired}} + \underbrace{A_2 \cos\omega_2 t}_{\text{Interferer / Blocker}}$$

$$y(t) = \underbrace{\left(\alpha_1 + \frac{3}{4}\alpha_3 A_1^2 + \frac{3}{2}\alpha_3 A_2^2\right)}A_1 \cos\omega_1 t + \cdots \text{ other terms}$$

$A_1 \ll A_2$

$$\simeq \left(\alpha_1 + \frac{3}{2}\alpha_3 \underline{A_2^2}\right) A_1 \cos\omega_1 t + \cdots$$

for $\alpha_1 \alpha_3 < 0 \Rightarrow$ for sufficiently large A_2

$\frac{3}{2}\alpha_3 A_2 \simeq \alpha_1$

$\Rightarrow$ gain drops to zero

$\Rightarrow$ Signal is Blocked!!

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In RF Design;

'blocking signal' or 'blocker' refers to interferers that desensitize a circuit even if they do not reduce the gain to zero.

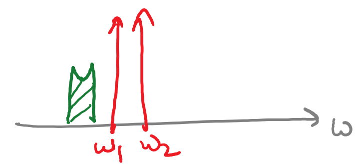
→ RF receivers must be able to withstand blockers 60 to 70 dB greater than the desired signal!

Intermodulation:

If two interferers accompany the desired signal

↳ intermodulation products are formed due to mixing.

Let $x(t) = A_1 \cos\omega_1 t + A_2 \cos\omega_2 t$



$$\Rightarrow y(t) = \alpha_1 (A_1 \cos\omega_1 t + A_2 \cos\omega_2 t) + \alpha_2 (A_1 \cos\omega_1 t + A_2 \cos\omega_2 t)^2 + \alpha_3 (A_1 \cos\omega_1 t + A_2 \cos\omega_2 t)^3$$

$\omega = 2\omega_1 \pm \omega_2$: $\quad \dfrac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1 + \omega_2)t + \dfrac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1 - \omega_2)t$

$\omega = 2\omega_2 \pm \omega_1$: $\quad \dfrac{3\alpha_3 A_1 A_2^2}{4} \cos(2\omega_2 + \omega_1)t + \dfrac{3\alpha_3 A_1 A_2^2}{4} \cos(2\omega_2 - \omega_1)t$

$\omega = \omega_1, \omega_2$: $\quad \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right) \cos\omega_1 t + \left(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2\right) \cos\omega_2 t$

"intermodulation products"

$\omega = \omega_1 \pm \omega_2$:

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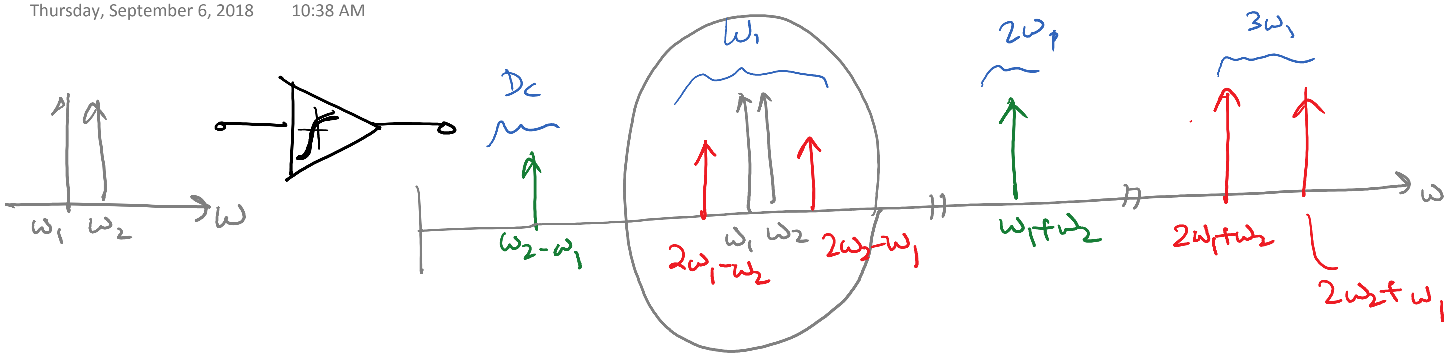
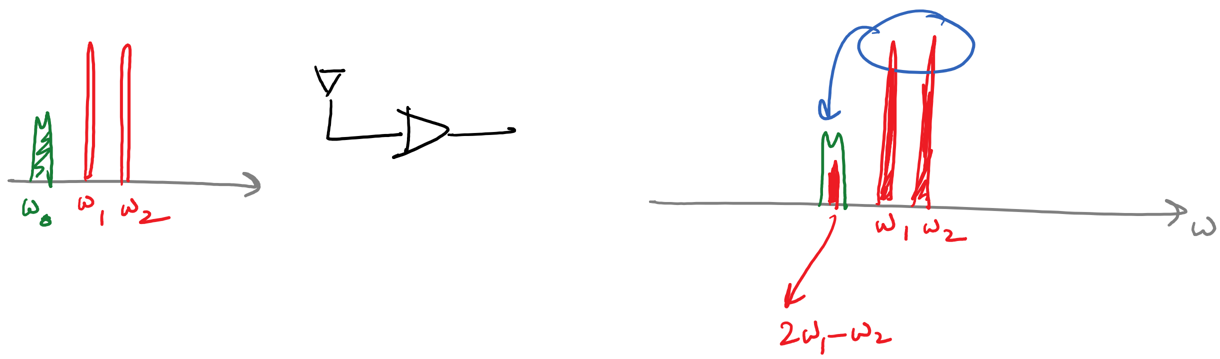
$\omega_2 > \omega_1$





Intermodulation term at $(2\omega_1-\omega_2)$ can fall into the desired channel.

Crossmodulation $\longrightarrow$ Read from the book