ECE 513- Lecture 16

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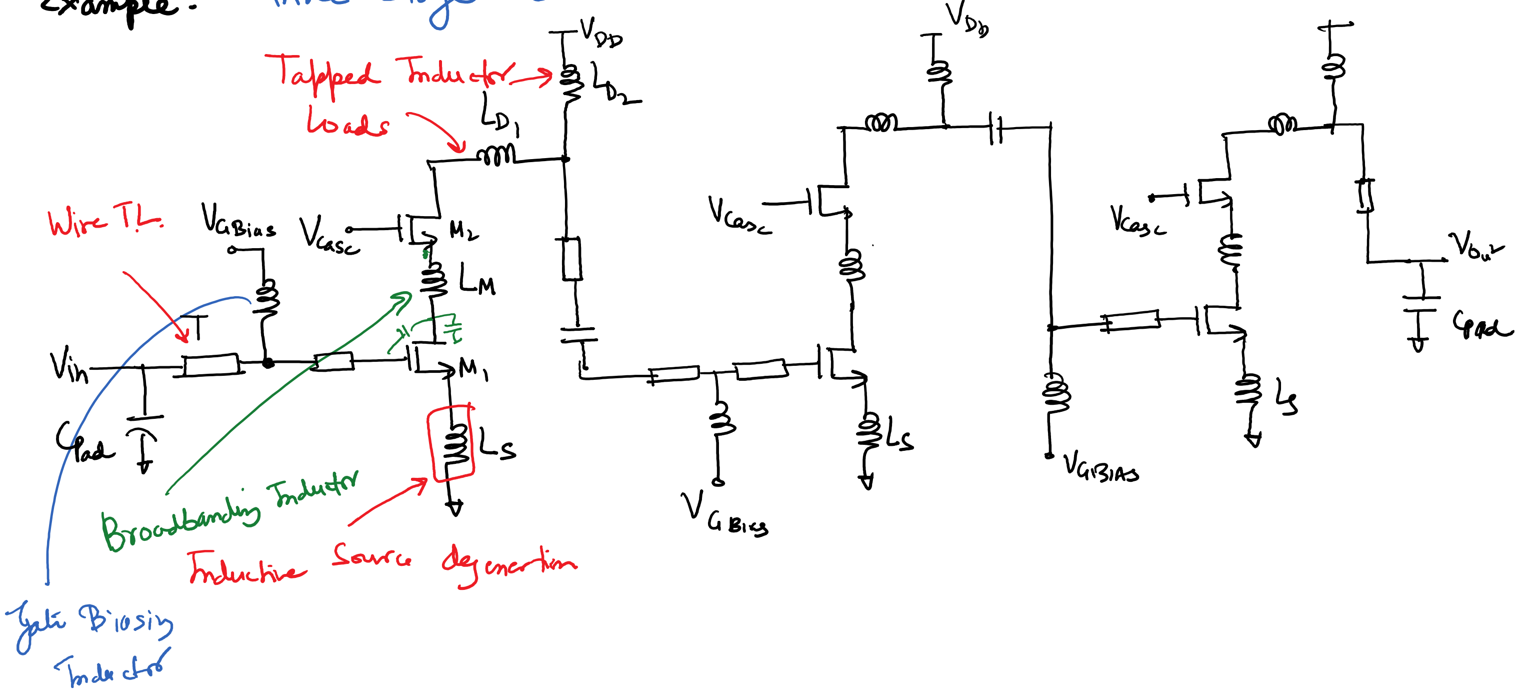
(T.H. Lee's Book)

Chapter 5:

* A higher ratio of passives to active devices (inductors, transformers, transmission lines)

Example: Three-stage LNA



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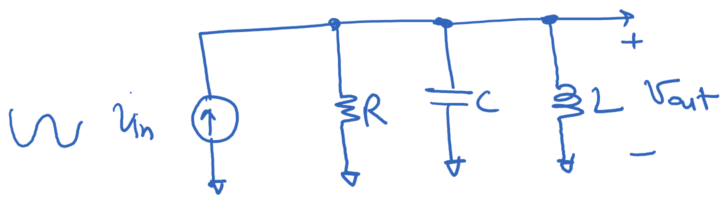
from T.H. Lee's Book

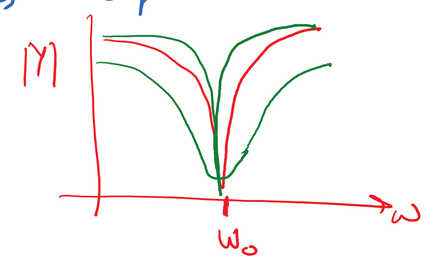
Passive RLC Networks.



$$Y = g + j\omega C + \frac{1}{j\omega L} = g + j\left(\omega C - \frac{1}{\omega L}\right)$$

Admittance goes to ∞ at DC ⇒ inductor is a short

very high frequencies ⇒ capacitor is a short



* At resonance frequency

$$\omega_0 C - \frac{1}{\omega_0 L} = 0 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

* at resonance, admittance is purely real and equal to g

↳ reactive terms cancel out

↳ introduce Q

Q: quality factor

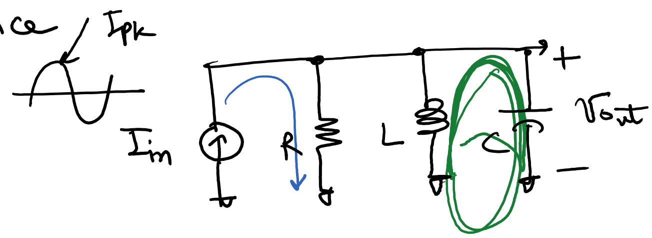
for a system is sinusoidal excitation

$$Q \triangleq \omega \cdot \frac{\text{Energy stored}}{\text{average power dissipated}} = 2\pi \cdot \frac{\text{Energy stored}}{\text{Energy dissipated in one cycle}}$$

Q → dimensionless

* If we neglect the loading of a network ⇒ unloaded-Q
including the load ⇒ loaded Q
↳ important to explicitely define the type of Q

Parallel RLC circuit at resonance



at resonance frequency, $\omega_o = \frac{1}{\sqrt{LC}}$

$V_{out} = |I_{in}R|$

* Energy in L & C sloshes back and forth with a constant sum

peak voltage across the capacitor = $I_{pk} \cdot R$

$\Rightarrow$ energy stored in the network $\Rightarrow E_{tot} = \frac{1}{2} C V_{pk}^2$
$= \frac{1}{2} C (I_{pk} R)^2$

* Average power dissipated in the resistor

$$P_{avg} = \frac{1}{2} I_{pk}^2 R$$

* Q of the network at resonance

$$Q = \omega_o \frac{E_{tot}}{P_{avg}} = \frac{1}{\sqrt{LC}} \cdot \frac{\frac{1}{2} C (\cancel{I_{pk}} R)^2}{\frac{1}{2} \cancel{I_{pk}^2} R} = \frac{R}{\sqrt{L/C}}$$

* $\sqrt{L/C}$ has the dimensions of resistance
  $\hookrightarrow$ characteristic impedance of the network

* Same as the magnitude of the capacitive and inductive reactances at resonance

$$|Z_C| = |Z_L| = \omega_o L = \frac{L}{\sqrt{LC}} = \sqrt{\frac{L}{C}}$$

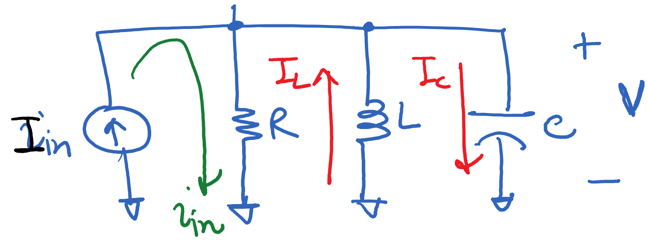
$Q = \frac{R}{\sqrt{L/C}}$ as $R \to \infty \Rightarrow Q \to \infty$

* Also,

$$Q = \frac{R}{|Z_{Lc}|} = \frac{R}{\omega_0 L} = \omega_0 R C$$

$$\boxed{Q = \frac{R}{\sqrt{L/C}} = \frac{R}{\omega_0 L} = \omega_0 R C}$$

Branch currents at resonance :



$$|I_L| = |I_C| = \frac{|V|}{Z} = \frac{|I_{in}|R}{\omega_0 L} = \frac{|I_{in}|R\sqrt{LC}}{L}$$

$$= |I_{in}|\frac{R}{\sqrt{L/C}} = Q \cdot |I_{in}|$$

Eg. $Q = 1000$ $\quad I_{in} = 1mA$

$|I_L| = |I_C| = 1A$

Bandwidth & Q

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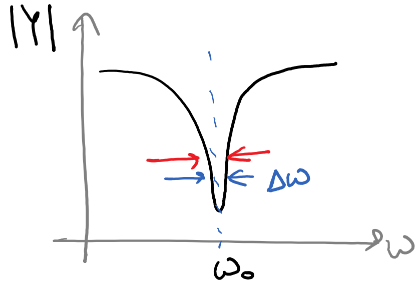


$$\omega = \omega_0 + \Delta\omega$$

$$Y = g + \frac{j}{\omega L}(\omega^2 LC - 1) = g + \frac{j}{\omega L}\left[(\omega_0 + \Delta\omega)^2 LC - 1\right]$$

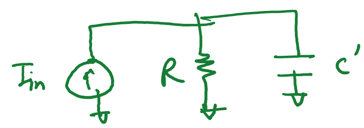
$$= g + \frac{j}{\omega L}\left[\frac{\cancel{\omega_0^2} + \Delta\omega^2 + 2\omega_0\Delta\omega}{\omega_0^2} - \cancel{1}\right]$$

$$= g + \frac{j}{\omega L}\cdot\left[\frac{\Delta\omega^2 + 2\omega_0\Delta\omega}{\omega_0^2}\right]$$

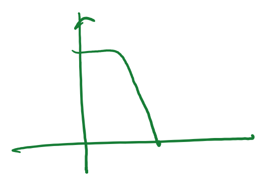
for $\omega_0 \gg \Delta\omega$

$$Y \simeq g + \frac{j}{\omega_0 L}\cdot\frac{2\omega_0\Delta\omega}{\omega_0^2}$$

$$= g + \frac{j2\Delta\omega}{\omega_0^2 \cdot L}$$

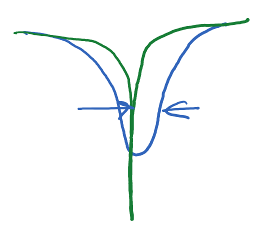


$$Y = g + j2C\Delta\omega \equiv g + jC'$$



Define $BW = \frac{1}{RC}$

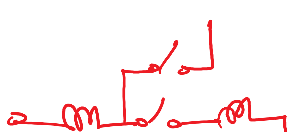
$$\Rightarrow \frac{BW}{\omega_0} = \frac{1}{RC\omega_0} = \frac{\sqrt{LC}}{RC} = \frac{\sqrt{L/C}}{R} = \frac{1}{Q}$$



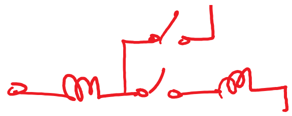
fractional bandwidth $\frac{BW}{\omega_0} = \frac{1}{Q} \Rightarrow$

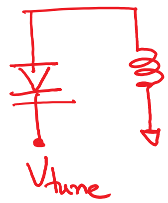
$$BW = \frac{\omega_0}{Q}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



$= \frac{1}{\quad}$





$$\omega_{LC} = \frac{1}{\sqrt{L\,(C_0+\Delta C)}}$$

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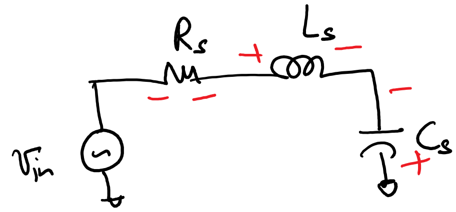
Series RLC Networks



- $\omega_0 = \dfrac{1}{\sqrt{L_s C_s}}$

- $Q = \dfrac{\sqrt{L_s / C_s}}{R_s}$ ← This is reciprocal of parallel RLC circuit.

at resonance, peak voltage across either L or C is Q-times the voltage across the resistor

Series Parallel Transformation

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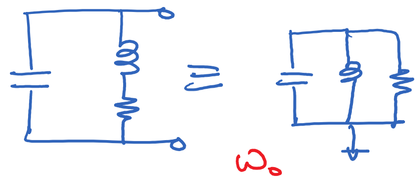
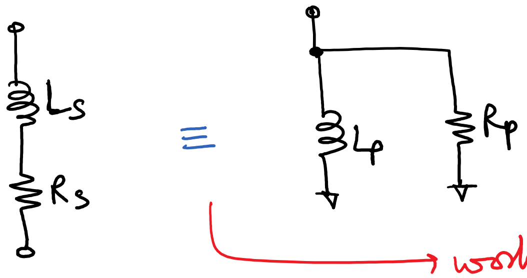




→ works only over a suitably small frequency range near resonance.

Lets equate impedance of series and parallel LR sections

$$j\omega_0 L_s + R_s = (j\omega_0 L_p) \| R_p = \frac{(\omega_0 L_p)^2 R_p + j\omega_0 L_p R_p^2}{R_p^2 + (\omega_0 L_p)^2}$$

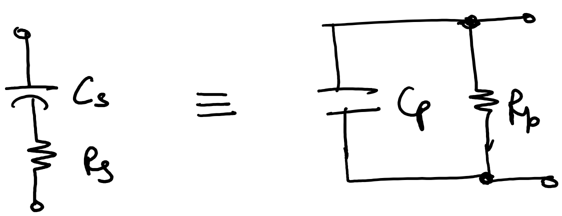
Equate real and imaginary parts

and use $Q = \dfrac{R_p}{\omega_0 L_p} = \dfrac{\omega_0 L_s}{R_s}$

$$R_p = R_s (Q^2+1)$$

$$L_p = L_s \left(\frac{Q^2+1}{Q^2}\right)$$

Similarly.



$$R_p = R_s (Q^2+1)$$

$$C_p = C_s \left(\frac{Q^2}{Q^2+1}\right)$$

Universal form

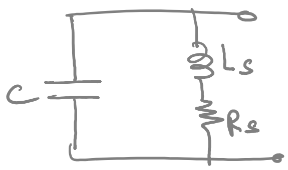
$$\boxed{\begin{aligned} R_p &= R_s\,(Q^2+1) \\ X_p &= X_s\left(\frac{Q^2+1}{Q^2}\right) \end{aligned}}$$

$X = \frac{1}{\omega_o C}$ or $\omega_o L$

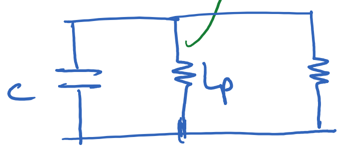
* Can convert any "impure" RLC circuit into a purely parallel one which is straightforward to analyze.

↳ But only valid for a narrow range of frequencies about ω_o.

$$Q = \frac{\omega_0 L_s}{R_s} = \frac{R_p}{\omega_0 L_p}$$

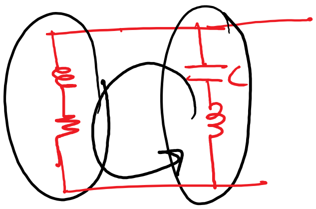


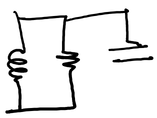
$\equiv$



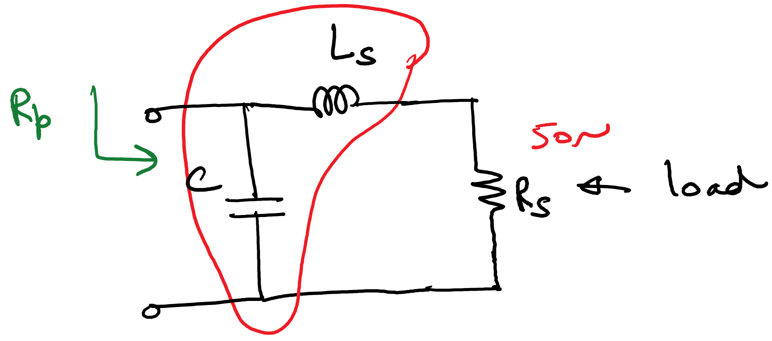
$$L_p = L_s\left(\frac{Q^2+1}{Q^2}\right)$$

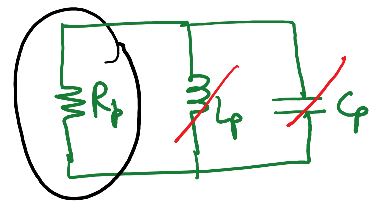
$$R_p = R_s(Q^2+1)$$



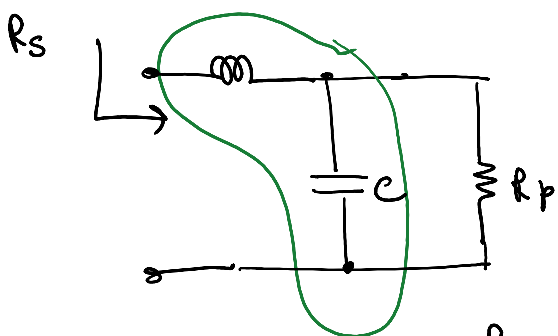


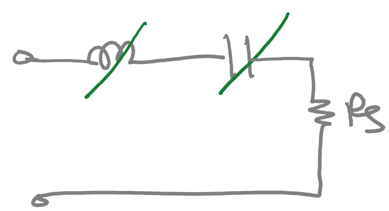
L- match





$$R_p = (Q^2+1)R_s$$





$$R_s = \frac{R_p}{Q^2+1}$$