ECE 513 - Lecture 14

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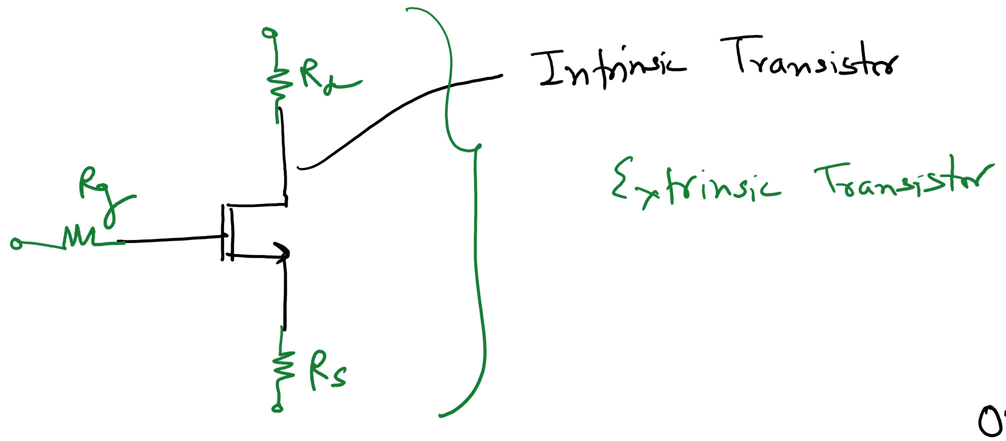


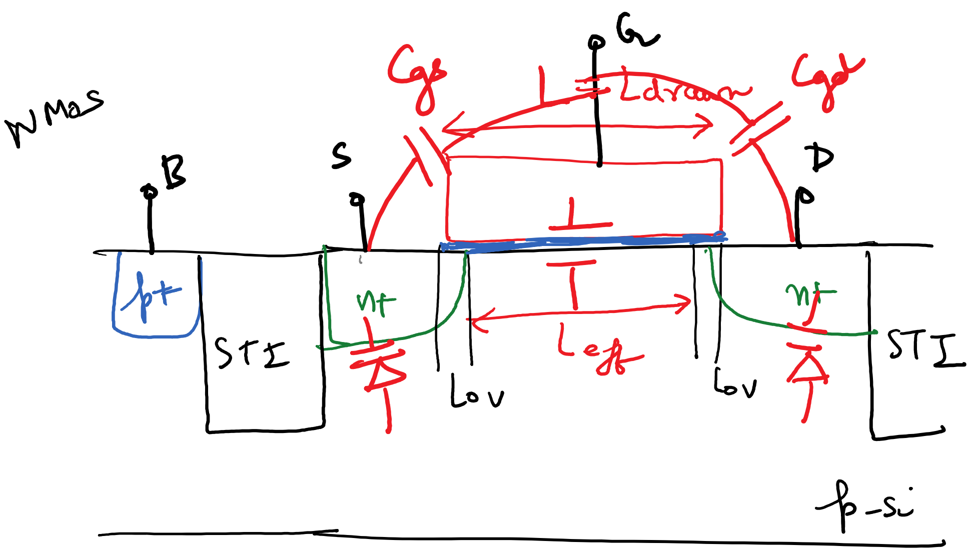
Overlap Capacitances

No channel

$$C_{gs} = C_{gd} = C'_{ox} L_{ov} \cdot W$$

$$C_{gs} = CGSO \cdot W$$

$$C_{gd} = CGDO \cdot W$$



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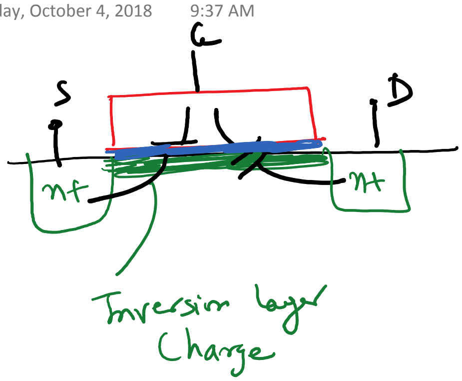
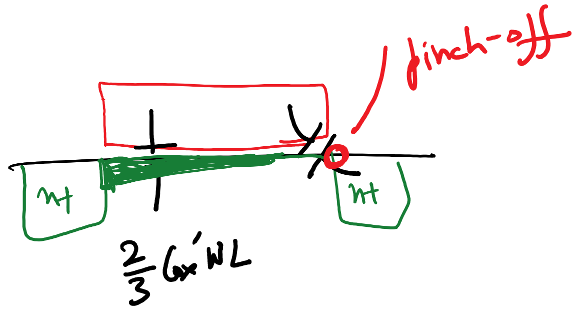


Triade:

$$C_{gs} \simeq C_{gd} = \frac{1}{2} C_{ox}' W (L - L_{ov}) + C_{ox}' W L_{ov}$$

$$\simeq \frac{1}{2} C_{ox}' W L$$

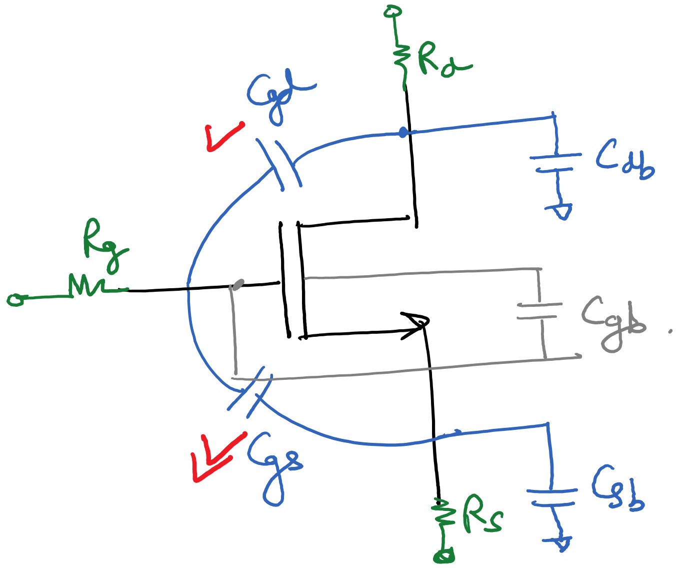
Saturation:

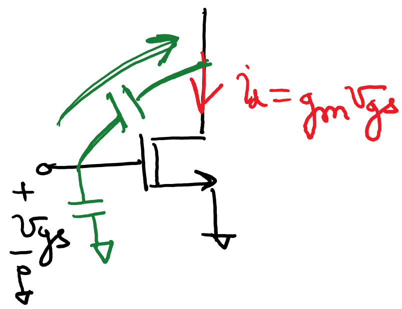


$$C_{gs} \simeq \frac{2}{3} C_{ox}' W L$$

$$C_{gd} = CGDO \cdot W$$

$$C_{gs} >> C_{gd}$$


Rd
Cgd
Cdb
Rg
Cgb
Cgs
Rs
Csb


$i_d = g_m v_{gs}$
+
v_{gs}
−

Simplified Small-Signal model

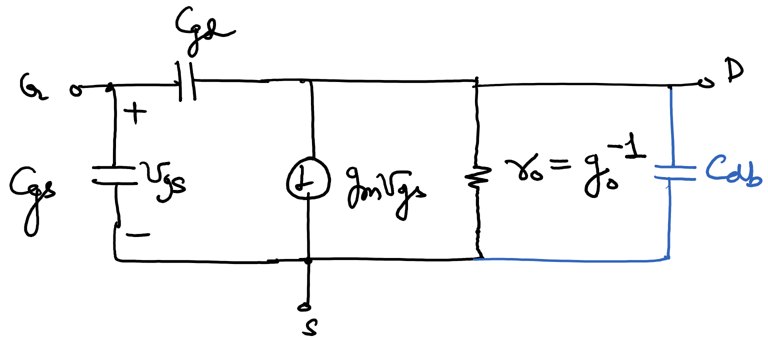
$r_o = g_o^{-1} = g_{DS}^{-1}$

Intrinsic Transistor Model

No R_s, R_d or R_g here

$\tau = 0$

$R_i = 0$

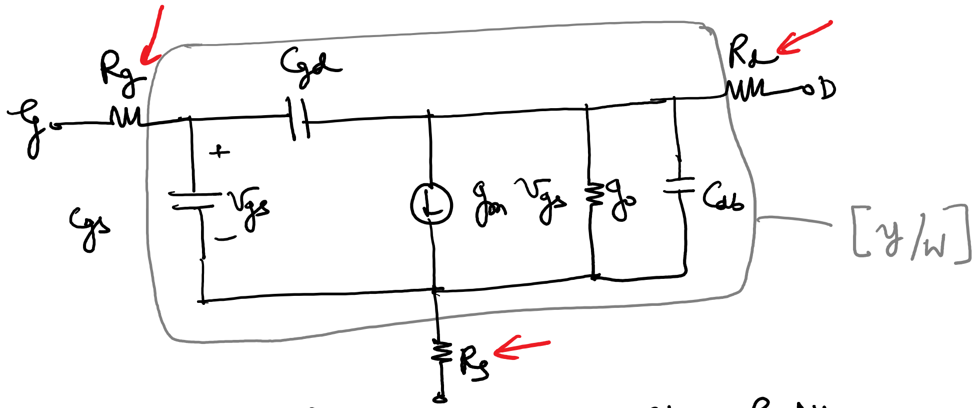


Simplified Y-parameter matrix per unit width

$$\left[\frac{y}{W}\right] = \begin{bmatrix} j\omega C_{gs}' + j\omega C_{gd}' & -j\omega C_{gd}' \\ g_m' - j\omega C_{gd}' & g_o' + j\omega (C_{db}' + C_{gd}') \end{bmatrix}$$

* g_m', g_o', C_{gs}', C_{gd}', C_{db}' are technology dependent parameters per unit width

Extrinsic Transistor Model



$$Y_{11} = \frac{y_{11} + (R_s + R_d)\Delta y}{N} \qquad Y_{12} = \frac{y_{12} - R_s \Delta y}{N}$$

$$Y_{21} = \frac{y_{21} - R_s \Delta y}{N} \qquad Y_{22} = \frac{y_{22} + (R_s + R_d)\Delta y}{N}$$

where $N = 1 + R_s(y_{11} + y_{12} + y_{21} + y_{22}) + R_d y_{22} + R_g y_{11} + \Delta y R_g^2$

& $\Delta y = y_{11} y_{22} - y_{21} y_{12}$

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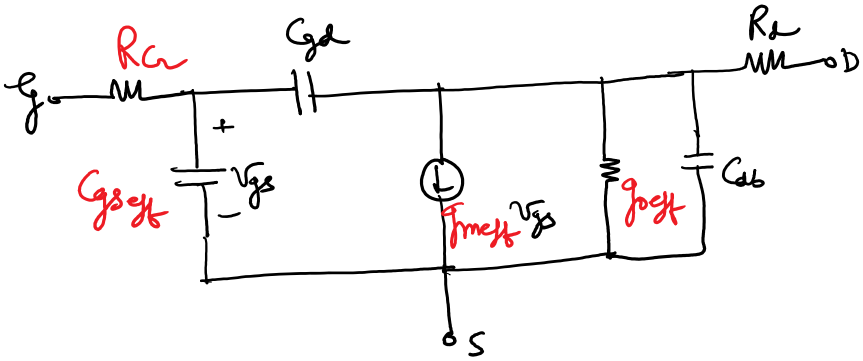
Source Degeneration



$$g_{meff} = \frac{g_m}{1 + g_m R_s}$$

$$g_{oeff} = \frac{g_o}{1 + g_m R_s}$$

$$C_{gseff} = \frac{C_{gs}}{1 + g_m R_s}$$

$$R_G = R_g + R_s$$

High-Frequency Figures of Merit

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- Cutoff frequency, f_T, (ω_T)
- Maximum oscillation frequency, f_{MAX}, (ω_{MAX})
- g_m/I_D

Unity current gain cutoff frequency (f_T)

→ output is short-circuited

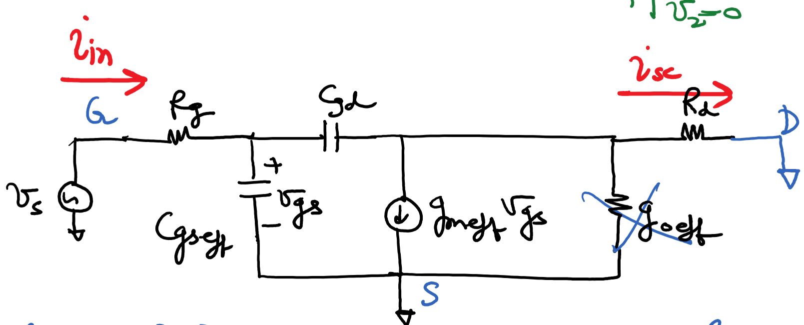
@ $f = f_T$ current gain $= 1$ or 0dB

$$|H_{21}(f = f_T)| = 1$$

$$\Rightarrow \quad \left|\frac{Y_{21}(f_T)}{Y_{11}(f_T)}\right| = 1$$

$$i_2 = h_{21} i_1 + h_{22} v_2$$

$$h_{21} = \frac{i_2}{i_1}\Big|_{v_2=0}$$



$$i_{sc} = -g_{meff} v_{gs} + (v_{gs} - i_{sc} R_d)\, sC_{gd} \longrightarrow ①$$

$$i_{in} = sC_{gs} v_{gs} + (v_{gs} - i_{sc} R_d)\, sC_{gd} \longrightarrow ②$$

$$\Rightarrow \quad v_{gs} = \frac{1 + sC_{gd}R_d}{-g_{meff} + sC_{gd}} \cdot i_{sc} \longrightarrow ③$$

$R_d \ll r_{oeff}$

$$H_{21}(s) = \frac{i_{sc}(s)}{i_{in}} = \frac{g_{meff} - sC_{gd}}{s(C_{gseff} + C_{gd} + g_{meff} R_d C_{gd}) - s^2 R_d C_{gd} C_{gseff}}$$

for $|H_{21}(f_T)| = 1$

$$\Rightarrow \quad \frac{1}{\omega_T} = \frac{1}{2\pi f_T} = \frac{C_{gseff} + C_{gd}}{g_{meff}} + C_{gd} R_d$$

$$\Rightarrow \quad \frac{1}{f_T} = \underbrace{\frac{2\pi\,(C_{gs} + C_{gd})}{g_m}}_{80-85\%} + \underbrace{2\pi C_{gd}(R_d + R_s)}_{15\%-20\%}$$

, Simplified

80 - 85 %

$$f_T = \frac{1}{2\pi}\left[\frac{g_m}{C_{gs}+C_{gd}}\right] \Bigg|\Bigg| \left[\frac{1}{(R_s+R_d)C_{gd}}\right] \simeq \frac{g_m}{2\pi C_{gs}}$$

Simplified Expression

current gain cutoff metric

* More relevant for low-frequency analog design

$C_{gs} \gg C_{gd}$

$R_s = R_d \simeq 0$

Maximum Oscillation frequency (f_{MAX})

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* In higher frequency circuits, power gain is more important than voltage or current gain

* maximum available (power) gain MAG

$$MAG(f = f_{MAX}) = 1 \quad \text{or} \quad 0dB$$

* Definition of MAG implies conjugate matched input and output impedances
  → we need to know the input and output impedance to define input and output power as well as achieve max transfer matching condition.

Simplified model with R_g

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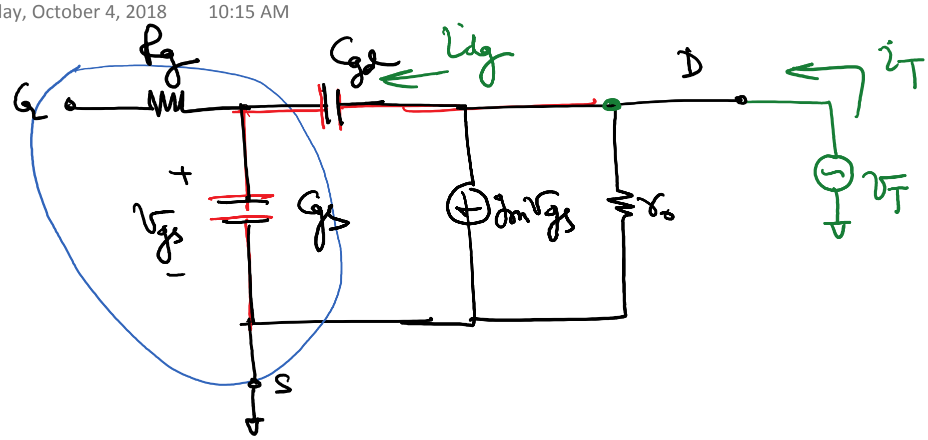


$$Z_{out} = \frac{v_T}{i_T}$$

Input impedance

$*$ $Z_{in} = R_g + \frac{1}{j\omega C_{gs}}$ $\simeq R_g$ at high frequencies as $\left|\frac{1}{j\omega C_{gs}}\right| \simeq 0$

$*$ $Z_{out} \Rightarrow$

$$i_T = \frac{v_T}{r_o} + g_m v_{gs} + i_{dg}$$

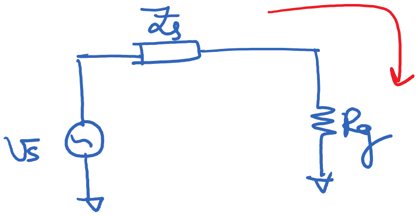
Assuming $i_{dg} \ll g_m v_{gs}$

$$\frac{v_{gs}}{v_T} = \frac{C_{gd}}{C_{gd} + C_{gs}}$$

$$Z_{out} = \frac{1}{\frac{1}{r_o} + \frac{g_m C_{gd}}{C_{gd} + C_{gs}}}$$

$$= r_o \,\|\, \frac{C_T}{g_m C_{gd}}, \quad \text{where } C_T = C_{gd} + C_{gs}$$

Conjugate Match at the input

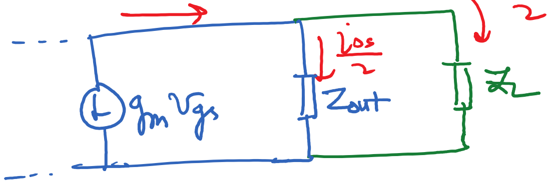


$i_{in,sc}$

$Z_s = R_g$

$\Rightarrow i_{in,sc} = \dfrac{v_s}{2R_g} \longrightarrow (1)$

Conjugate Match at the output



$\dfrac{i_{osc}}{2} = i_o$

$Z_{out} = Z_L$

$\Rightarrow R_{out} = R_L$

$i_o = \dfrac{i_{o,sc}}{2} \longrightarrow (2)$

$$MAG = \frac{\frac{1}{2} i_o^2 R_{out}}{\frac{1}{2} i_{in}^2 R_{in}} \triangleq \frac{1}{4}\left(\frac{i_{o,sc}}{i_{in,sc}}\right)^2 \cdot \frac{R_L}{R_g}$$

$P = \frac{1}{2} I^2 R_i$

we used this ratio to derive f_T

$$= \frac{1}{4}\left(\frac{f_T}{f}\right)^2 \frac{R_L}{R_g}$$

@ $f \approx f_T$

$$\frac{i_{o,sc}}{i_{in,sc}} \simeq \frac{f_T}{f}$$

$$MAG\Big|_{f=f_{MAX}} = 1$$

$$\Rightarrow \quad f_{MAX} = \frac{1}{2} f_T \sqrt{\frac{R_L}{R_g}}$$

$$R_L = R_{out} = \frac{1}{\frac{1}{r_o} + \frac{g_m C_{gd}}{C_{gs} + C_{gd}}}$$

$$= \frac{1}{r_o^{-1} + 2\pi f_T C_{gd}}$$

$$\boxed{f_{MAX} = \frac{1}{2} \frac{f_T}{\sqrt{2\pi f_T R_g C_{gd} + \frac{R_g}{r_o}}}} \longrightarrow (A)$$

$$\sqrt{2\pi f_T R_g C_{gd} + \frac{R_g}{r_o}}$$ → A

Including R_d and R_s Book Eq 4.125

$$f_{MAX} = \frac{1}{2} \frac{f_T}{\sqrt{2\pi f_T C_{gd} (R_g + R_s) + (R_g + R_s) g_{oeff}}} \longrightarrow \text{B}$$

Simplifications

$$\frac{1}{r_o} << \frac{g_m C_{gd}}{C_{gs} + C_{gd}}$$

$$f_{MAX} \simeq \frac{1}{2} f_T \sqrt{\frac{R_d}{R_g + R_s}} = \frac{1}{2} f_T \sqrt{\frac{r_o}{R_g + R_s}}$$

$$f_{MAX} \simeq \frac{f_T}{2\sqrt{(R_s + R_g) g_{oeff}}} \longrightarrow \text{C}$$

for several transistor technologies Denominator < 1

$$\frac{(R_s + R_g)}{r_{oeff}} < 1$$

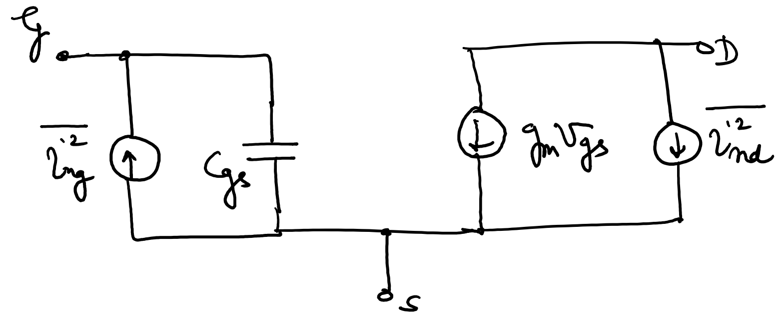
$$f_{MAX} > f_T$$

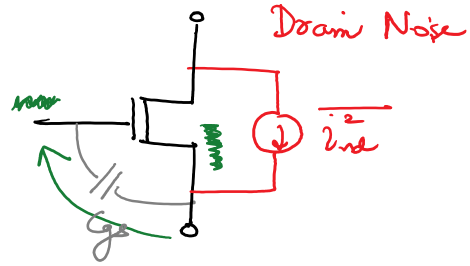
$Y_{s,opt} = g_{s,opt} + jB_{s,opt} \Rightarrow F_{min}$

Two-port Transistor Noise Model :

Van Der Ziel

Ignoring R_g





This gate noise is much higher than the thermal noise from R_g

$$\overline{i_{nd}^2} = 4kT\gamma g_{do}\Delta f$$

g_{do} → g_{DS} of transistor at $V_{DS}=0$

$g_{ds_0} = k_n' \frac{W}{L}(V_{GS}-V_{THN}) \triangleq g_m$ for long-channel

γ → Excess noise parameter technology dependent

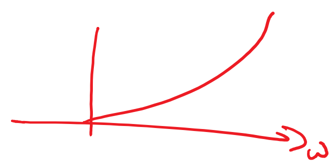
$\gamma = 2/3$ - for long channel

$\gamma \uparrow$ with $L_{min} \downarrow$

$$\overline{i_{ng}^2} = 4kT\delta g_g \Delta f$$

$$g_g = \frac{\omega^2 C_{gs}^2}{5 g_{do}}$$

blue noise



Correlated

$$C \triangleq \frac{\overline{i_{ng} \cdot i_{nd}^*}}{\sqrt{\overline{i_{ng}^2}\;\overline{i_{nd}^2}}} = -j0.395 \simeq -j0.4$$

for long-channel CMOS