ECE 513 - Lecture 12

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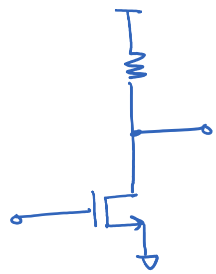
(Latex+ Beamer)

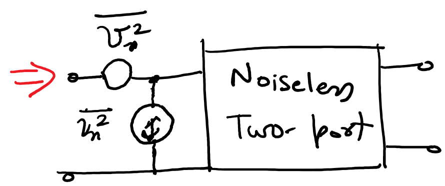
Two-port Noise:

* Maximum power transfer $(Z_{in} = Z_S^*) \Rightarrow$ power match
* Least Noise factor (Noise figure) $\Rightarrow$ noise match

LNA



$NF \simeq NF_{min}$



Linear Analysis

Two-port Noise Model

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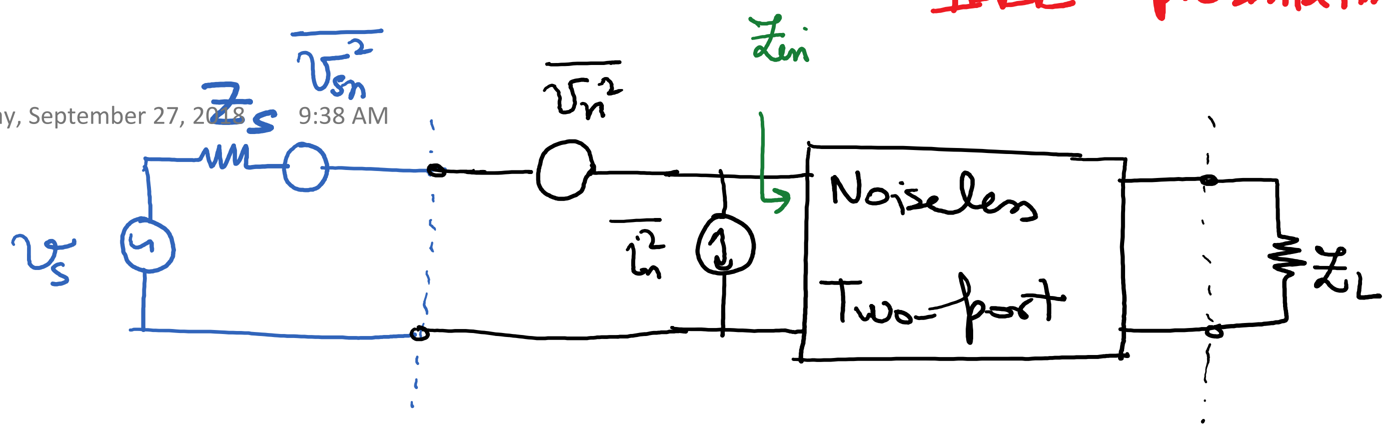
IEEE representation of the noisy two-port



* Can represent a noisy two-port with two noise sources
  - ↳ a voltage source & a current source (v_n & i_n)
    - ↳ represent all internal noise sources
      - ↳ They are usually statistically correlated

Correlation coefficient

$$c = \frac{\langle i_n, v_n^* \rangle}{\sqrt{\overline{i_n^2} \cdot \overline{v_n^2}}}$$

Correlation

$$\langle v_1, v_2^* \rangle$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_0^T v_1(t)\, v_2^*(t)\, dt$$

$= 0$ iff v_1 & v_2 are uncorrelated

* There are 'four' real numbers that completely describe the noise of the linear two-port:

$$v_n, \; i_n, \qquad c = \underbrace{\mathrm{Re}\{c\}}_{3} + j\,\underbrace{\mathrm{Im}\{c\}}_{4}$$

1 2

'c' is a complex number.

* Let's say that the two-port is driven by a signal source, v_s, with source impedance

$$Z_s = \underbrace{R_s} + jX_s$$

R_s generates noise

$$\Rightarrow \overline{v_{sn}^2} = 4kTR_s \cdot \Delta f$$

* Initially assume $c = 0 \Rightarrow v_n$ & i_n are uncorrelated

↳ we'll fix this later

To find F for the arrangement;

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① find noise power in the load due to v_{sn} alone

② find noise power in the load due to both external (v_{sn}) and internal (v_n, i_n) noise sources.

* We only need to calculate short-circuit noise current at the input of the noiseless two-port

↳ the circuit that follows is noiseless.

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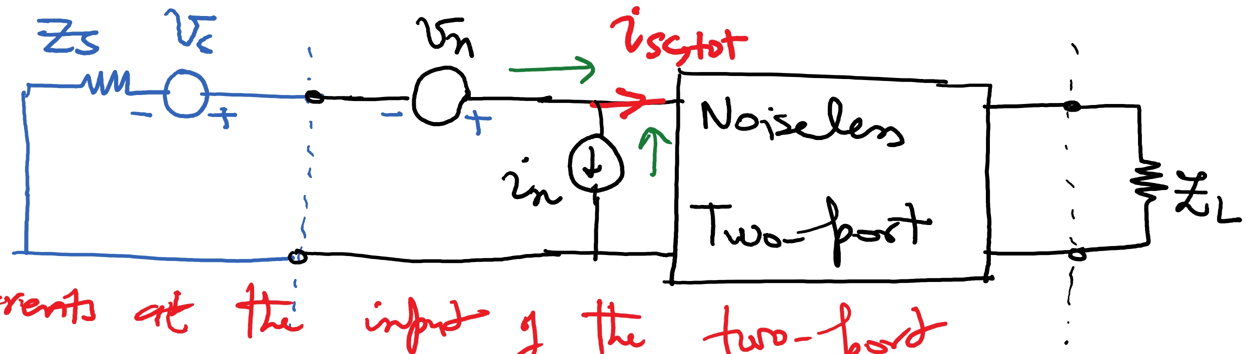

Short circuit noise currents at the input of the two-port

$$i_{sc,tot} = \frac{-v_{sn} - v_n}{Z_s} - i_n$$

$$i_{sc,s} = \frac{-v_{sn}}{Z_s}$$

$$\overline{v_n^2} = \langle v_n, v_n^* \rangle$$

$$\overline{i_{sc,tot}^2} = \left\langle \frac{v_{sn} + v_n}{Z_s} + i_n , \frac{v_{sn}^* + v_n^*}{Z_s} + i_n \right\rangle$$

$$= \frac{\overline{v_{sn}^2} + \overline{v_n^2}}{|Z_s|^2} + i_n^2$$

We used

$$\langle v_n, i_n^* \rangle = 0$$

$$\overline{i_{s,s}^2} = \frac{\overline{v_{ns}^2}}{|Z_s|^2}$$

$$F = \frac{SNR_i}{SNR_o} = \frac{P_s/N_i}{P_s/N_o} = \frac{N_o}{N_i}$$

$$\boxed{F = \frac{\overline{i^2_{sc,tot}}}{\overline{i^2_{sc,s}}}} = 1 + \frac{\overline{v_n^2}}{\overline{v_{sn}^2}} + |Z_s|^2 \cdot \frac{\overline{i_n^2}}{\overline{v_n^2}}$$

$$\Rightarrow F = 1 + \frac{\overline{v_n^2}}{4kTR_s\Delta f} + \frac{(R_s^2 + X_s^2)\,\overline{i_n^2}}{4kTR_s\Delta f} \longrightarrow (1)$$

F depends upon v_n, i_n & the R_s, X_s

* Minimum Noise Factor, F_{min}

$$\frac{\partial F(R_s, X_s)}{\partial R_s} = 0 \quad \& \quad \frac{\partial F(R_s, X_s)}{\partial X_s} = 0.$$

$$F_{min} = F\left(R_s = R_{s,opt},\ X_s = X_{s,opt}\right) = 1 + \frac{\sqrt{\overline{v_n^2}\cdot\overline{i_n^2}}}{2kT\Delta f}$$

$$\boxed{R_{s,opt} = \sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} \quad \text{and} \quad X_{s,opt} = 0} \Rightarrow Z_{s,opt} = R_{s,opt}$$

noise impedance

$$F_{min} = 1 + \frac{\sqrt{\overline{v_n^2}\cdot\overline{i_n^2}}}{2kT\Delta f}$$

* If $Z_s = R_{s,opt}$ then the maximum transfer of noise from the two-port to the input termination occurs without reflections.

↳ least amount of noise generated in the two-port is transferred to the load, resulting in the least degradation of SNR ⇒ best F.

In general v_n & i_n are correlated

Also $Z_{s,opt} \neq Z_{in}^*$ & the two-port

* Let's use $c \neq 0$

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↳ v_n & i_n are partially correlated

Let $\begin{cases} v_n = v_n \\ i_n = i_u + i_c = i_u + \underbrace{Y_{cor} v_n}_{\text{correlated}} \end{cases}$

i_u → uncorrelated

$Y_{cor} \equiv$ correlation admittance

$$\Rightarrow Y_{cor} \triangleq \frac{\overline{i_n v_n^*}}{\overline{v_n^2}} = g_{cor} + jB_{cor}$$

$Z = R + jX$

$Y = G + jB$

$1/R = G \Rightarrow$ conductance

$1/X = B \Rightarrow$ susceptance

Noise Admittance Formalism (IEEE parameters)

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Fixing R_n and $Y_{s,opt} = G_{s,opt} + jB_{opt}$

Noise resistance $R_n \triangleq \dfrac{\overline{v_n^2}}{4kT\Delta f}$

Noise conductance $g_u \triangleq \dfrac{\overline{i_u^2}}{4kT\Delta f}$

v_n

$i_n = i_u + i_c$

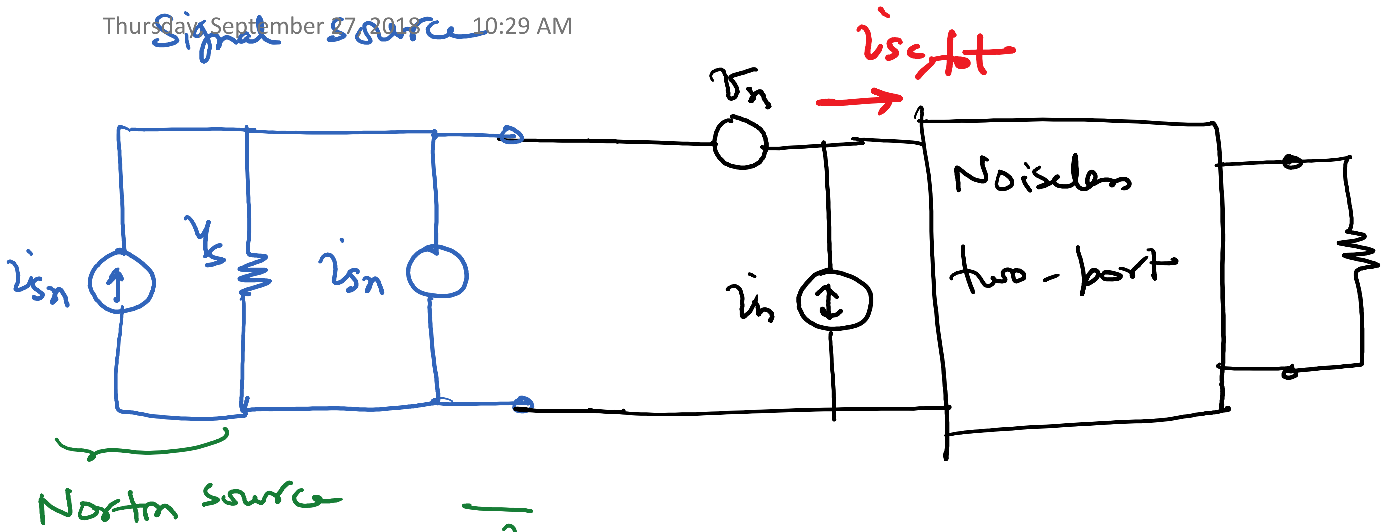
$= i_u + Y_{cor} \cdot v_n$

The short circuit currents at the input of the noiseless two-port

$$i_{sc,tot} = i_{sn} + i_n + Y_s v_n$$

$$= i_{sn} + i_u + (Y_{cor} + Y_s) v_n \longrightarrow 3$$

$$i_{sg,s} = i_{sn} \longrightarrow 4$$



$$\overline{i_{sn}^2} = 4kT\Delta f\, G_s$$

from ③ & ④

$$F = \frac{\overline{i_{sc,tot}^2}}{\overline{i_{sc,s}^2}} = 1 + \frac{\overline{i_u^2}}{\overline{i_{sn}^2}} + \frac{\overline{v_n^2}\,|Y_{cor}+Y_s|^2}{\overline{i_{sn}^2}}$$

$$= 1 + \frac{\cancel{4kT}\, G_u\, \cancel{\Delta f}}{\cancel{4kT}\, G_s\, \cancel{\Delta f}} + \frac{\cancel{4kT}\, R_n\, \cancel{\Delta f}\, |Y_{cor}+Y_s|^2}{\cancel{4kT}\, G_s\, \cancel{\Delta f}}$$

$$F = 1 + \frac{R_n}{G_s}|Y_{cor}+Y_s|^2 + \frac{G_u}{G_s}$$

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$$Y_{cor} = G_{cor} + jB_{cor}$$

$$Y_s = G_s + jB_s$$

$$F = 1 + \frac{R_n}{G_s}\left|Y_{cor} + Y_s\right|^2 + \frac{G_u}{G_s}$$

$$= 1 + \frac{R_n}{G_s}\left[(G_{cor} + G_s)^2 + (B_{cor} + B_s)^2\right] + \frac{G_u}{G_s}$$

$$\frac{\partial F}{\partial G_s} = \frac{-R_n}{G_s^2}\left[(G_{cor} + G_s)^2 + (B_{cor} + B_s)^2\right] + \frac{2R_n}{G_s}(G_{cor} + G_s) - \frac{G_u}{G_s^2} = 0$$

$$\Rightarrow \quad G_s^2 - G_{cor}^2 - (B_{cor} + B_s)^2 = \frac{G_u}{R_n}$$

$$\Rightarrow \quad G_s^2 = G_{cor}^2 + (B_{cor} + B_s)^2 + \frac{G_u}{R_n} \longrightarrow 5$$

$$\frac{\partial F}{\partial B_s} = \frac{2R_n}{G_s}(B_{cor} + B_s) = 0$$

$$\Rightarrow B_{cor} + B_s = 0$$

$$\Rightarrow B_{s,opt} = -B_{cor} \longrightarrow (6)$$

On combining (5) & (6), we get

$$\boxed{G_{s,opt} = \sqrt{G_{cor}^2 + \frac{G_u}{R_n}} \quad \& \quad B_{s,opt} = -B_{cor}}$$

$$F_{min} = 1 + \frac{R_n}{G_{s,opt}}\left(G_{cor} + G_{s,opt}\right)^2 + \frac{G_u}{G_{s,opt}}$$

$$= 1 + \frac{1}{G_{s,opt}}\left[R_n\left(G_{cor} + G_{s,opt}\right)^2 + \left(G_{s,opt}^2 - G_{cor}^2\right)R_n\right]$$

$$= 1 + \frac{R_n}{\cancel{G_{s,opt}}}\left[\cancel{G_{cor}^2} + G_{s,opt}^{\cancel{2}} + 2G_{cor}\cancel{G_{s,opt}} + G_{s,opt}^{\cancel{2}} - \cancel{G_{cor}^2}\right]$$

$$\boxed{F_{min} = 1 + 2R_n\left(G_{cor} + G_{s,opt}\right)}$$

$$\boxed{F = F_{min} + \frac{R_n}{G_s}\left|Y_s - Y_{s,opt}\right|^2}$$

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* R_n is an indicator of the sensitivity of the NF to noise impedance mismatch

A large R_n $\Rightarrow$ NF will be very sensitive to variations in the value of Y_s (or Z_s)

$\Rightarrow$ more sensitive to process variations