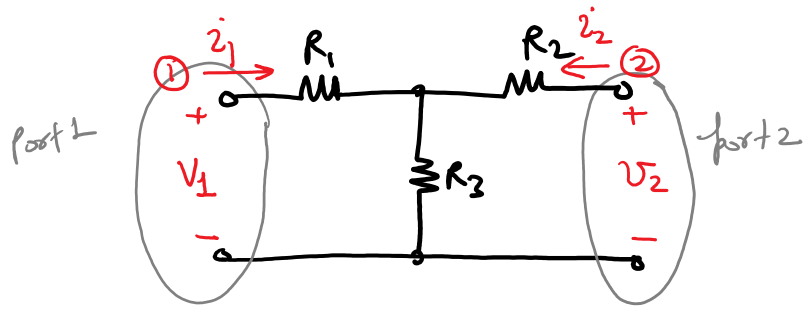
ECE 513 - Lecture 11

Tuesday, September 25, 2018 9:28 AM

Chapter 3: Two ports:

Linear Network Analysis
↳ small signal parameters.



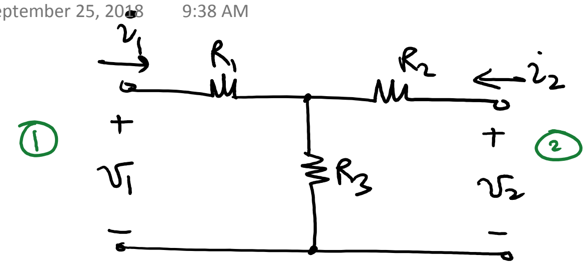
$i \Rightarrow$ independent
$v \Rightarrow$ dependent variables

$$v_1 = i_1 z_{11} + i_2 z_{12}$$
$$v_2 = i_1 z_{21} + i_2 z_{22}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$v = z\, i$$

two-port z-parameters $\Rightarrow$ can be complex

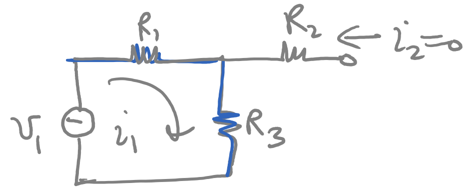


$$v_1 = Z_{11} i_1 + Z_{12} i_2$$

$$v_2 = Z_{21} i_1 + Z_{22} i_2$$

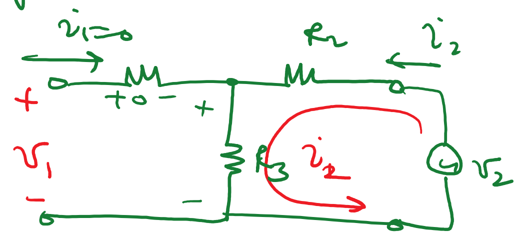
$$Z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0} \Rightarrow \text{Port ② is open circuit}$$

$$= R_1 + R_3$$



$$Z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0} \quad \text{Port ① is open circuit}$$

$$= R_3$$



$$v_1 = R_3 \cdot i_2$$

$$Z_{21} = \left.\frac{v_2}{i_1}\right|_{i_2=0} = R_3$$

$$Z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0} = R_2 + R_3$$

$$Z = \begin{bmatrix} R_1+R_3 & R_3 \\ R_3 & R_2+R_3 \end{bmatrix}$$

Tuesday, September 25, 2018 9:45 AM

Y-parameters:

$$i_1 = y_{11} v_1 + y_{12} v_2$$
$$i_2 = y_{21} v_1 + y_{22} v_2$$

(admittance parameters)

$$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

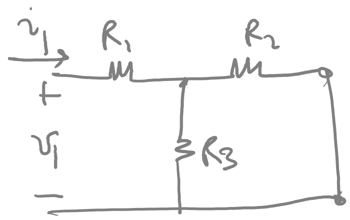
$$Y = Z^{-1}$$

$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0}$$ port 2 is short circuit

$$y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0}$$

$$y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0}$$

$$y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0}$$



$$v_1 = (R_1 + R_2 \| R_3)\, i_1$$

$$y_{11} = \frac{i_1}{v_1} = \frac{1}{R_1 + R_2 \| R_3}$$

$$y_{12} = \frac{-R_3}{(R_2 + R_1 \| R_3)(R_1 + R_3)}$$

$$y_{21} = \frac{-R_3}{(R_1 + R_2 \| R_3)(R_2 + R_3)}$$

$$y_{22} = \frac{1}{R_2 + R_1 \| R_3}$$

We can verify that

$$Y = Z^{-1}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

h-parameters.

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

↳ useful for analyzing circuits with series-shunt feedback

↳ for extraction of R_g of MOSFETs

g-params:

$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$

↳ useful in shunt-series feedback

H & g matrices are only defined for 2-ports but Z & Y can be extended to N-ports.

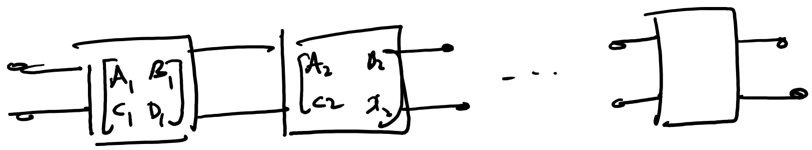
Tuesday, September 25, 2018 9:55 AM

ABCD Matrices:

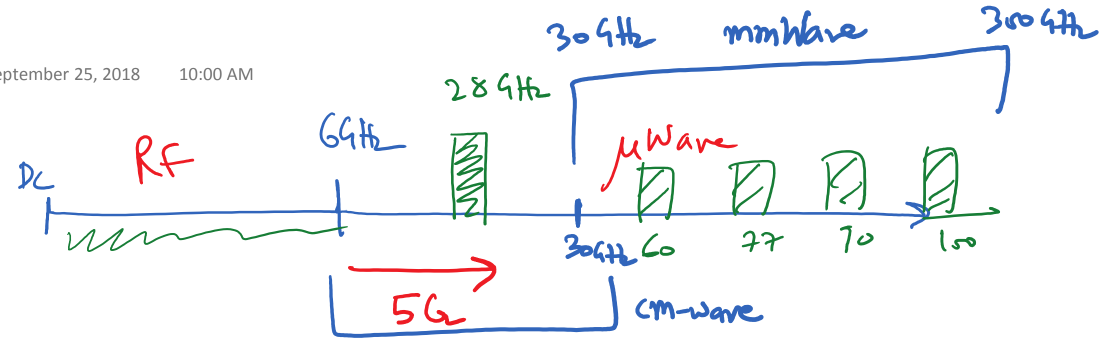
Useful for cascaded two-port analysis

$$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -i_2 \end{bmatrix}$$

$A = \frac{V_1}{V_2}\Big|_{i_2=0}$, $B = -\frac{V_1}{I_2}\Big|_{V_2=0}$, $C = \frac{I_1}{V_2}\Big|_{I_2=0}$, $D = -\frac{I_1}{I_2}\Big|_{V_2=0}$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \prod_{i=1}^{n} \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}$$



* $l \geq \frac{\lambda}{20}$ transmission line effects show up in Integrated Circuits

length or size of interconnects (wires)

@ 3 GHz

$$\lambda = \frac{3\times10^{8}}{3\times10^{9}} = 0.1\ m$$

$$= 10\ cm$$

@ 30 GHz

$$\lambda = 1\ cm$$

$$\frac{\lambda}{20} = 0.05\ cm$$

$$= 0.5\ mm$$

* at "high-frequency" need to account for possible impedance mismatches between signal source, load, and interconnect impedances.

Reflection Coefficient.

Z_0 = characteristic impedance of the TL ⇒ 50Ω

$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$= \frac{Y_0 - Y}{Y + Y_0}$$

normalized form

$$\Gamma = \frac{z - 1}{z + 1} \quad \text{where} \quad z = \frac{Z}{Z_0}$$

Γ ⇒ can be complex ⇒ typically use Smith Charts to visualize and manipulate these impedances.

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Voltage standing wave ratio

S-Parameters :

Tuesday, September 25, 2018 10:14 AM

* measurement of high frequency voltages & currents is difficult experimentally.

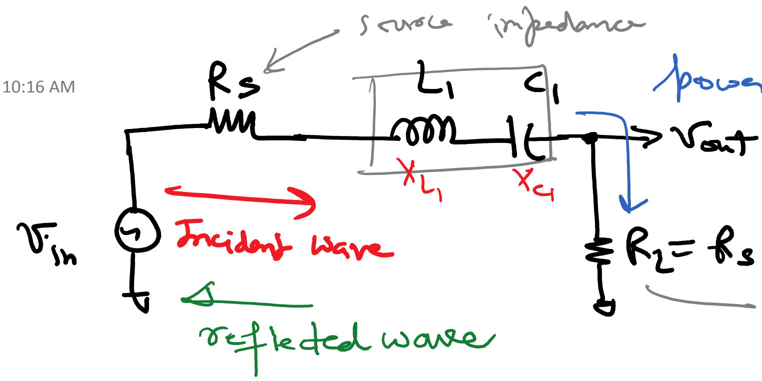
↳ opening & shorting of ports is not feasible

* transfer of power from one stage to next. instead of voltage or current.

↳ scattering parameters or "S-parameters"

Tuesday, September 25, 2018 10:16 AM

Example:

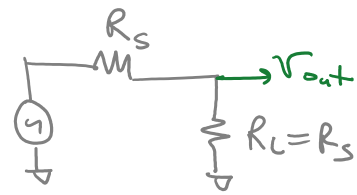


power delivered to the load

$$X_{L_1} + X_{C_1} = j\omega L_1 + \frac{1}{j\omega C_1} = j\omega L_1 - \frac{j}{\omega C_1}$$

@ $\omega = \frac{1}{\sqrt{L_1 C_1}} = \omega_0$ resonance occurs

$$\Gamma = \frac{R_S - R_L}{R_S + R_L} = 0$$

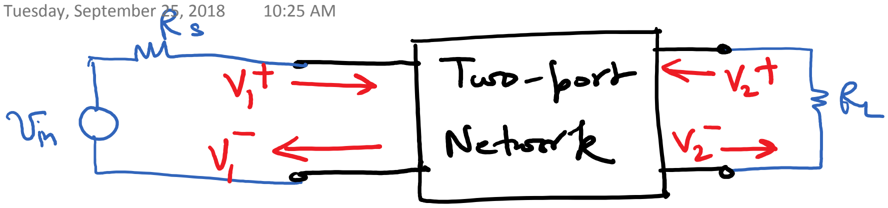


For maximum power transfer

$$Z_L = Z_S^*$$

"Conjugate Match"

* We say that the input port of the circuit generates a "reflected wave" that returns to the source



→ incident and reflected waves at the input port are V_1^+ & V_1^-

Similarly, waves at the output are V_2^+ & V_2^-

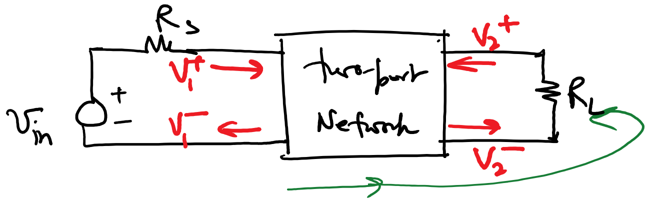
Net voltage at the input $= V_1^+ + V_1^-$

" " " " output $= V_2^+ + V_2^-$

$$\boxed{\begin{aligned} V_1^- &= S_{11} V_1^+ + S_{12} V_2^+ \\ V_2^- &= S_{21} V_1^+ + S_{22} V_2^+ \end{aligned}}$$

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \underbrace{\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}}_{[S]} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

① $$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0}$$



ratio of reflected and incident waves at the input port when the reflection from R_L (i.e. V_2^+) is zero.

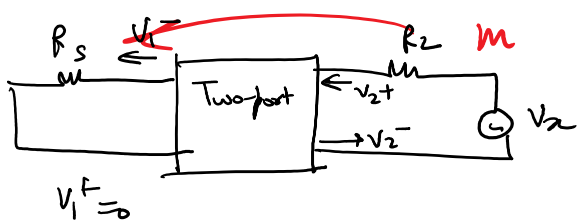
↳ represents the accuracy of input matching

② S_{12}:

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{V_1^+=0}$$

* S_{12} is the ratio of reflected wave at the input port to the incident wave into the output port when the input port is matched

↳ characterizes the reverse isolation



③ S_{21}:

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$

S_{21} is the ratio of the wave incident on the load to that going to the input, when the reflection from R_L is zero.

↳ Represents the gain of the circuit

$$\boxed{S_{21} \Rightarrow \text{Gain}}$$

④ $$S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+=0}$$

⇒ ratio of reflected and incident waves at the output when the reflection from R_s is zero ($V_1^+=0$)

↳ Represents the accuracy of the output "matching"

* S-params typically have frequency dependent complex values and often expressed in dB.

$$S_{mn}\Big|_{dB} = 20 \log |S_{mn}|$$

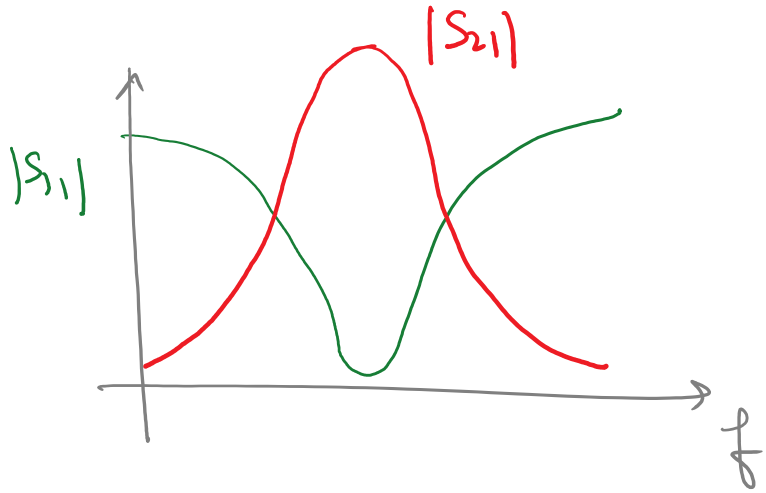
Input reflection

reverse isolation

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

output reflection

Gain



Normalized incident wave $a_n = \frac{V_n^+}{\sqrt{Z_0}}$

" reflected wave $b_n = \frac{V_n^-}{\sqrt{Z_0}}$

formalism used in the book

Avg. power for incident wave, $P_{avg,i} = \frac{V_n^+ \times I_n^{+*}}{2} = \frac{|a_i|^2}{2}$

" " " reflected wave, $P_{out,avg,i} = \frac{V_n^- \times I_n^{-*}}{2} = \frac{|b_i|^2}{2}$

power going load $\left(\frac{|a_i|^2 - |b_i|^2}{2}\right)$