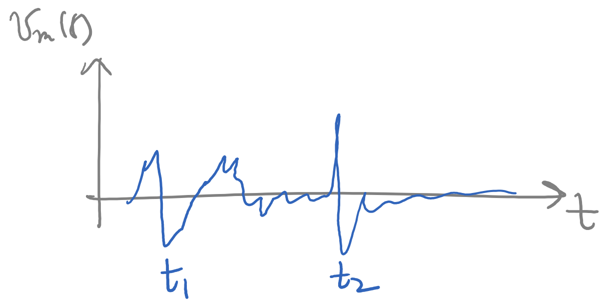
ECE 513 - Lecture 10.

Tuesday, September 18, 2018 12:12 PM

Noise:

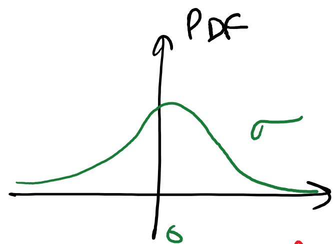
Razavi Books

noise:

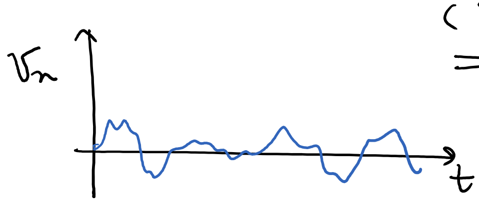


$E[v(t_1) \cdot v(t_2)] = 0$ for thermal noise
for $t_1 \neq t_2$

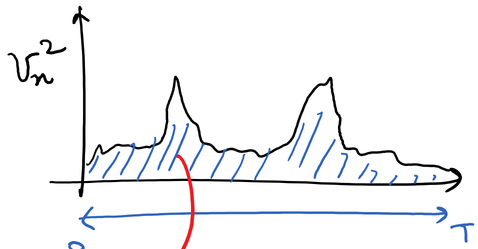
* random process

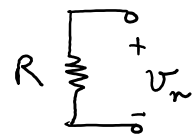


↳ noise in circuits fortunately exhibit constant average power.



$(\ )^2 \Rightarrow$





0 mean

Area = Energy of the noise

Average power

$$P_{av} = \lim_{T\to\infty} \frac{\text{Area}}{T} = \lim_{T\to\infty} \int_0^T v_n^2(t)\,dt$$

* P_{avg} is rather expressed in V^2 than Watts

root mean-square

⇒ rms voltage for noise ⇒

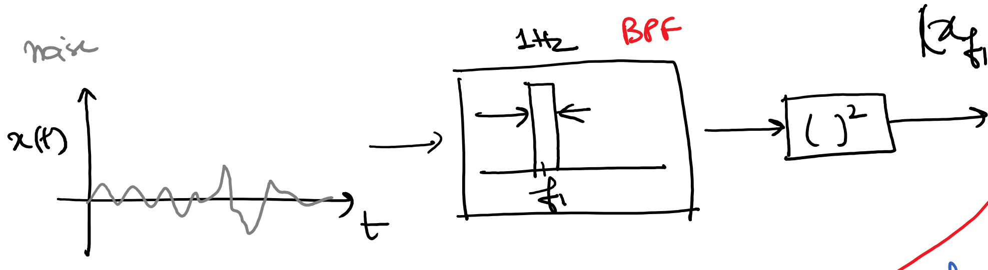
$$V_{rms} = \sqrt{P_{av}} = \sqrt{\lim_{T\to\infty} \int_0^T v_n^2(t)\,dt}$$

Thermal noise ⇒ $P_{av} = \sigma^2$ (Variance)
$V_{rms} = \sigma$ (Std. deviation)
} Gaussian

Noise Spectrum:

Thursday, September 20, 2018 9:48 AM

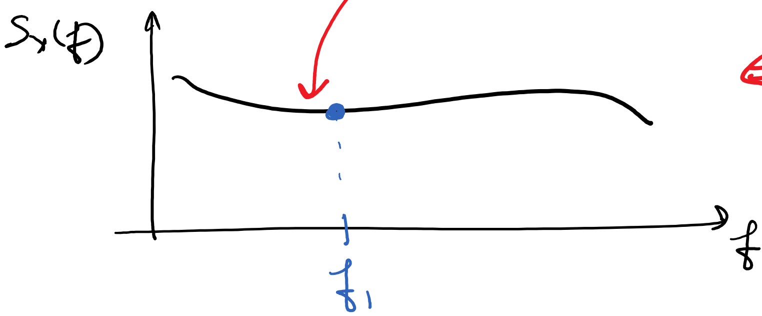
Energy $|x_{f_1}(t)|^2$ → averaged over 1Hz → power



* find Energy in 1Hz band at frequency f_1

* Sweep f_1

"Ideal Spectrum Analyzer"



⇐ Power Spectral Density

PSD

"most of the noise sources have a predictable spectrum → PSD"

Thursday, September 20, 2018 9:55 AM

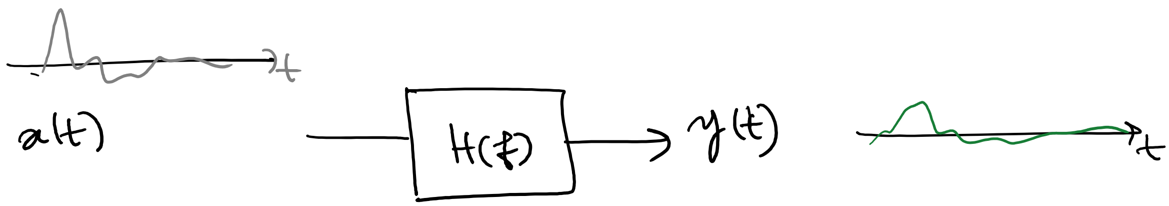
PSD: $S_x(f) \Rightarrow \frac{V^2}{Hz}$

VSD $\Rightarrow \sqrt{S_x(f)} = \frac{V}{\sqrt{Hz}}$ ← Cadence Spectre

Total noise power $= \int_{f=-\infty}^{\infty} S_x(f)\, df$

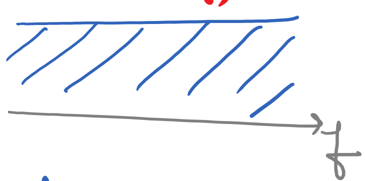
rms noise $= \sqrt{(\ )} = \sqrt{\int S_x(f)\, df}$

$= \int VSD(f)\, df$

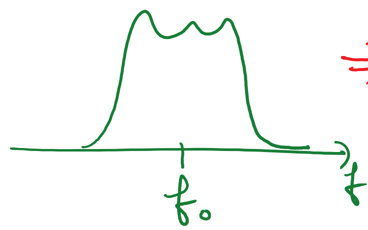


Thermal Noise

$S_x(f)$

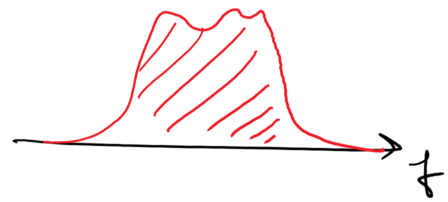


BPF



$\Rightarrow$

$S_y(f)$

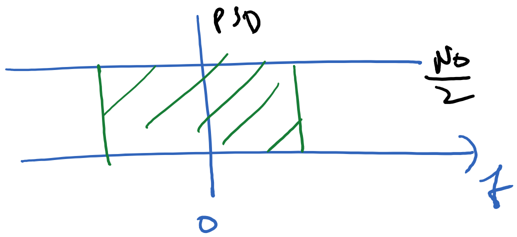


flat PSD

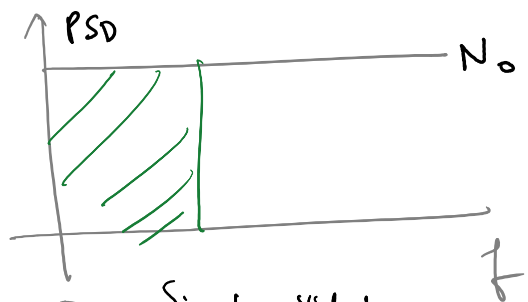
White Noise

Colored Noise

$$\boxed{S_y(f) = S_x(f) \cdot |H(f)|^2}$$



double-sided spectrum



Single-sided spectrum

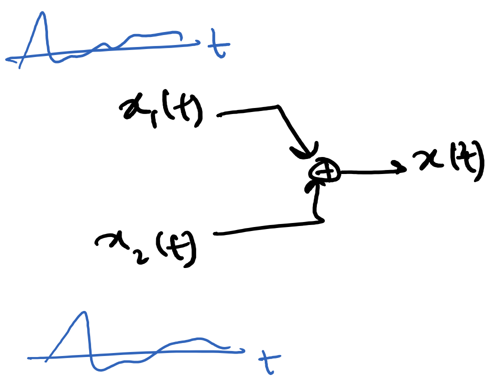
Correlated and Uncorrelated Noise Sources

Thursday, September 20, 2018 10:05 AM

* often in circuit analysis, we need to add the effect of several sources of noise.

for deterministic signals $\Rightarrow$ superposition

for random noise $\Rightarrow$ we deal with the average noise power.



$$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_0^T \left(x_1(t) + x_2(t)\right)^2 dt$$

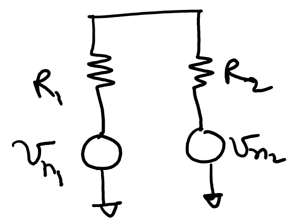
$$= \underbrace{\lim_{T\to\infty} \frac{1}{T}\int x_1^2(t)\, dt}_{} + \underbrace{\lim_{T\to\infty} \frac{1}{T}\int x_2^2(t)\, dt}_{} + \lim_{T\to\infty} \frac{1}{T}\int 2x_1(t)\cdot x_2(t)\, dt$$

$$= P_{av_1} + P_{av_2} + \underbrace{\lim_{T\to\infty} \frac{1}{T}\int 2\,x_1(t)\,x(t)\, dt}_{\text{Correlation between } x_1(t) \text{ and } x_2(t)}$$

$\rightarrow$ How similar they are

for independent devices

noise are uncorrelated



$\Rightarrow \int x_1 x_2 \, dt = 0$

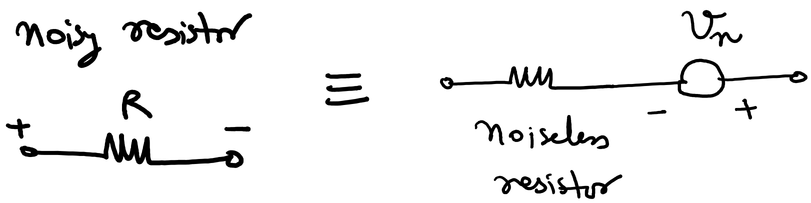
then $P_{av} = P_{av_1} + P_{av_2}$

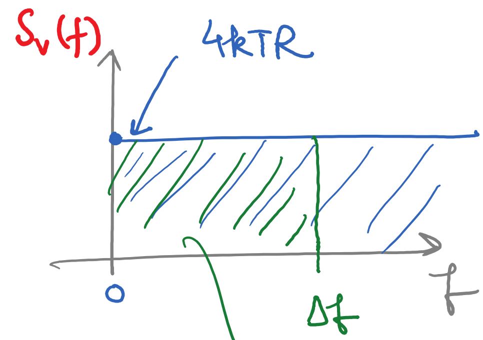
- Superposition holds for the avg. power of uncorrelated signals

↳ But not true for correlated signals.

Thursday, September 20, 2018 10:16 AM

Thermal Noise :





noise power $= 4kTR \cdot \Delta f$

PSD: $S_v(f) = 4kTR, \quad f > 0$

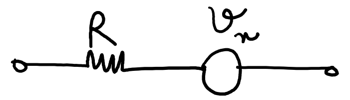
$\propto T$

$\propto R$

Boltzmann Const, $k = 1.38 \times 10^{-23}$ J/K

units $\Rightarrow \frac{V^2}{Hz}$

Thevenin

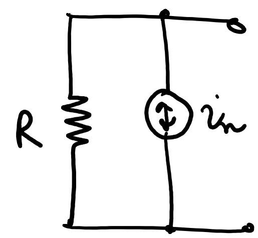


$\equiv$

$$\boxed{\overline{v_n^2} = S_v(f) = 4kTR}$$

mean square noise
per Hz

Norton

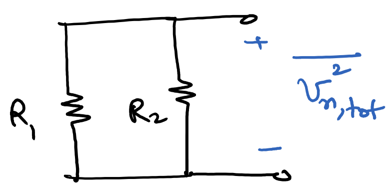


$$\boxed{\overline{i_n^2} = \frac{4kT}{R}}$$

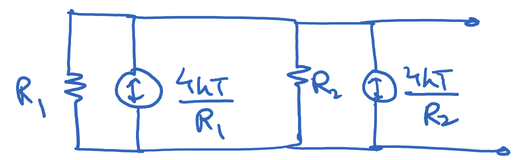
$$\overline{i_n^2} = S_I(f) = \left(\frac{v_n}{R}\right)^2 = \frac{\overline{v_n^2}}{R^2} = \frac{4kT\cancel{R}}{R^{\cancel{2}}}$$

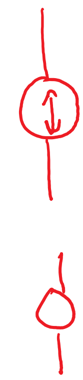
noise current spectral density $= \dfrac{4kT}{R}$

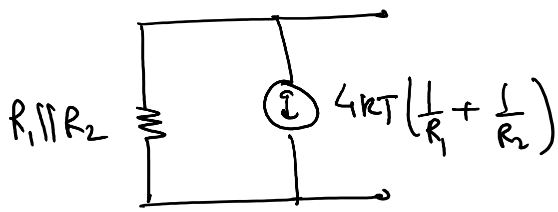
Ex.



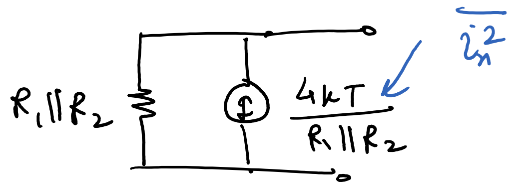
$\equiv$







$\Rightarrow$



$$\overline{v_n^2} = \overline{i_n^2}\cdot R^2$$

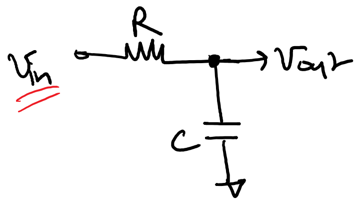
$$\overline{v_n^2} = \frac{4kT}{R_1\|R_2}\cdot(R_1\|R_2)^2$$

$$= 4kT(R_1\|R_2)$$

$\boxed{\text{ECE 614}}$

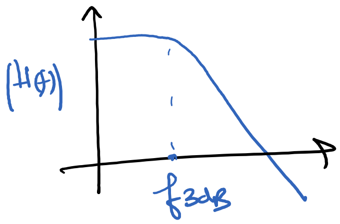
videos on noise analysis

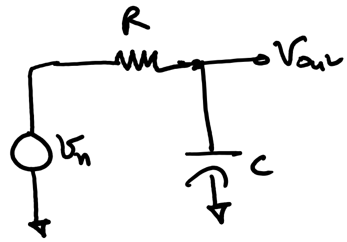
Thursday, September 20, 2018 10:32 AM

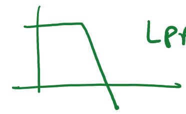


$$\frac{V_{out}}{V_{in}}(s) = \frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{1}{1+sRC} = \frac{1}{1+\left(\frac{s}{2\pi f_{3dB}}\right)}$$

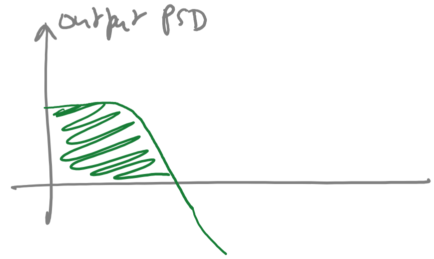
$$f_{3dB} = \frac{1}{2\pi RC}$$







LPF by the RC circuit



output PSD

$$P_{n,out} = \int_0^\infty S_{out}(f)\,df = \int_0^\infty \frac{4kTR}{\left|1+j\frac{f}{f_{3dB}}\right|^2}\cdot df$$

$$= \int_0^\infty \frac{4kTR}{1+\left(\frac{f}{f_{3dB}}\right)^2}\cdot df$$

$$x \Rightarrow \frac{f}{f_{3dB}} = 2\pi f RC$$

$$dx = df\,(2\pi RC)$$

$$= \frac{\overset{2}{\cancel{4}}kT\cancel{R}}{\cancel{2}\pi\cancel{R}C}\int_0^\infty \frac{dx}{1+x^2}$$

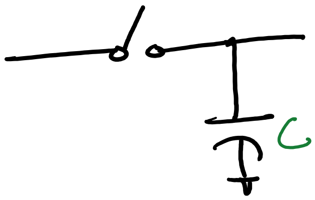
$$= \frac{2kT}{\pi C}\cdot\tan^{-1}(x)\Big|_0^\infty = \frac{\cancel{2}kT}{\cancel{\pi}C}\cdot\frac{\cancel{\pi}}{\cancel{2}} = \frac{kT}{C}$$

$$P_{n,out} = \frac{kT}{C}$$

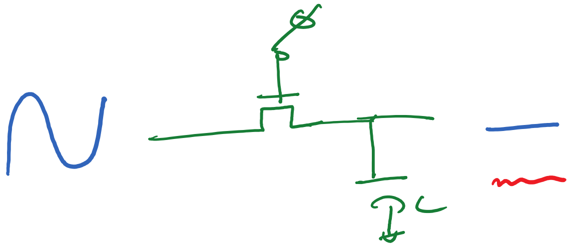
$$v_{n,out,rms} = \sqrt{\frac{kT}{C}}$$

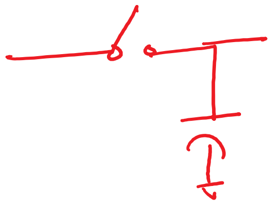
$$R\uparrow \Rightarrow \overline{v_n^2} = 4kTR\uparrow$$

$$f_{3dB} = \frac{1}{2\pi RC}\downarrow$$



$$V_{o,noise,rms} = \sqrt{\frac{kT}{C}}$$





Sample-and-hold
Circuit

v_{rms} noise $\rightarrow \sqrt{\frac{KT}{C}}$

C (pF)	$\overline{v_{n,rms}} = \sqrt{\frac{KT}{C}}$
0.1	200 μV
1	64 μV
10	20 μV
100	6.4 μV